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Intermediate Microeconomics W3211

Lecture 3: Preferences and Choice

Columbia University, Spring 2016

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Introduction

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The Story So Far....

- So far, we have described consumer choice as **constrained optimization:**

1. **CHOOSE a consumption bundle**
2. **IN ORDER TO MAXIMIZE preferences**
3. **SUBJECT TO the budget constraint**

- And spent some time talking about how one can represent preferences
  - Indifference curves
  - MRS
  - Utility function

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Today's Aims

1. Use our tools to describe various different types of preferences
   - 'Standard' preferences
     - Monotonic
     - Convex
   - Introduce some other handy classes of preferences
     - Perfect substitutes
     - Perfect compliments
     - Cobb Douglas
   - Varian Ch. 3 & 4, Feldman and Serrano Ch 2

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A Short Diversion: Proofs

- In your homework, you are asked to prove something

In class I claimed that two utility functions u and v represent the same preferences if and only if there is a strictly increasing function f such that u(x)=f(v(x)) for all x. I would like you to prove half of this statement: if there is such a function f, then u and v represent the same preferences

- Proving things often gets people confused.
- While they are not central to the course, I will expect you to be able to complete simple proofs
- This is not because I am mean spirited, but because this type of thinking is important
  - Both in economics and beyond
- If you are worried about this, make use of office hours
  - Talk to the TAs (or me) sooner rather than later

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A Short Diversion: Proofs

- A proof is basically just a sequence of statements which follow from each other
  - If X then Y
  - If Y then Z
- These statements don't necessarily need to be 'mathematical', but maths can be an easy way to make your argument
- There are many types of proof, but two will be particularly useful for the course
- **Direct**: If you are trying to prove that X implies Y prove a sequence of steps
  - X implies A
  - A implies B
  - ...
  - H implies Y
- **By Contradiction:** If you are trying to prove that X implies Y, show that assuming X and not Y leads to a contradiction

A Short Diversion: Proofs

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- In most cases, the proofs that you will be required to do just require manipulating the definitions.

A Short Diversion: Proofs

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- An example (from last lecture): A non transitive preference relation cannot have a utility representation.
- Start by remembering the definitions:
  - **Transitivity:** $x \succsim y$, $y \succsim z$ implies $x \succsim z$
  - **Utility representation**: there is a function u such that $x \succsim y$ if and only if $u(x) \geq u(y)$ for all x and y
- Proof by contradiction: assume that preferences are not transitive, but there IS a utility representation
  1. If preferences are non transitive, then there is some x,y and z such that $x \succsim y$, $y \succsim z$ but NOT $x \succsim z$
  2. If there is a utility representation then
     - $x \succsim y$ implies $u(x) \geq u(y)$
     - $y \succsim z$ implies $u(y) \geq u(z)$
  - This implies that $u(x) \geq u(z)$
  - But if this is a utility representation, this implies $x \succsim z$
  - Contradiction

Different Types of Preferences

Defining Standard Preferences

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Different Types of Preference

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- We now have two tools to represent preferences
  - Utility functions
  - Indifference curves (and MRS)
- We are going to use them to think about various different types of preferences
- To start with we are going to think about what we will call 'standard' preference
- This adds two assumptions to the ones that we have already made: that preferences are

1. monotonic
2. convex.

Monotonic Preferences

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- Monotonic preferences are ones in which having more is always better
- Two types of monotonicity
- **Weak** monotonicity: if bundle x has more of every good than bundle y then x is strictly preferred to y

$$x_i > y_i \text{ for every i implies } x \succ y$$

- **Strict** monotonicity: if bundle x has more of at least one good and no less of any good than bundle y then x is strictly preferred to y

$$x_i \geq y_i \text{ for every I and } x_i > y_i \text{ for some i implies } x \succ y$$

- Why might monotonicity fail?
  - Bads
  - Satiation

Monotonic Preferences

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- What do the indifference curves of monotonic preferences look like?

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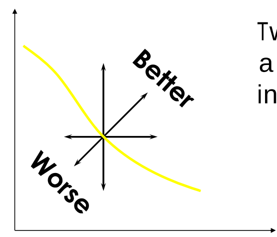
Monotonic Indifference Curves



Two goods ⇨ a **negatively sloped** indifference curve.

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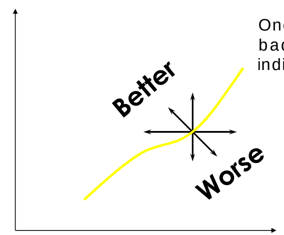
Non Monotonic Indifference Curves



One good and one bad ⇨ a **positively sloped** indifference curve.

Some amount of good 2 can compensate for Bad 1

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Satiation

- Another example of **non-monotonic** preferences
- For some goods there is a **perfect** optimal amount
- Examples
  - Salt in a dish
  - Anchovies on pizza
  - Classes of Intermediate Microeconomics
- Sometimes there is a **bundle** that is the optimal one
- Called **satiation/bliss point**

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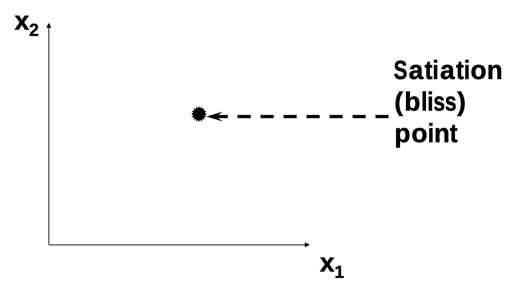
Indifference Curves Exhibiting Satiation



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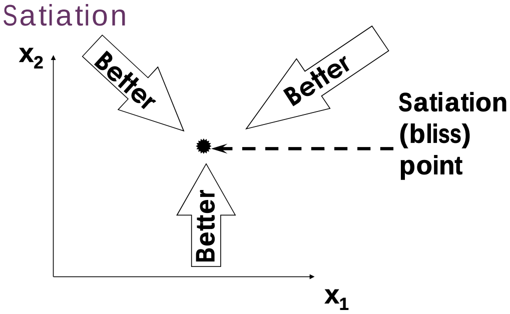
Indifference Curves Exhibiting Satiation



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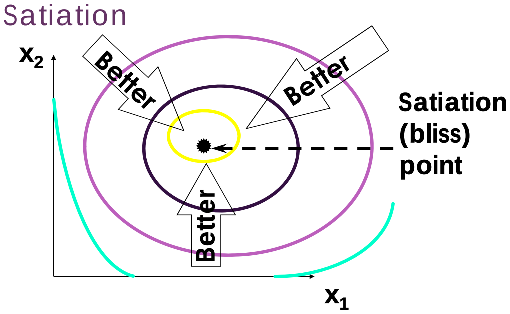
Indifference Curves Exhibiting Satiation



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Monotonic Preferences

- What do the indifference curves of monotonic preferences look like?
  - Downward sloping
  - Weakly monotonic – can be horizontal or vertical
  - Strictly monotonic – strictly downward sloping
- What does the MRS of monotonic preferences look like
  - Positive (as it is the negative of the slope of the indifference curve)
  - Weakly monotonic – can be 0 or infinity
  - Strictly monotonic – strictly positive real number
- What do monotonic utility functions look like?
  - Marginal utility is always positive:

$$MU_i = \frac{\partial u}{\partial x_i} > 0 \text{ for every i: strictly monotonic}$$

$$MU_i = \frac{\partial u}{\partial x_i} \geq 0 \text{ for every i and } \frac{\partial u}{\partial x_i} > 0 \text{ some i : weakly monotonic}$$

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Convexity

- **Convexity**: Roughly speaking - mixtures of bundles are preferred to the bundles themselves.
- E.g., say that the consumer is indifferent between x and y
- the 50-50 mixture of the bundles x and y is

$$z = (0.5)x + (0.5)y$$

Then: convexity says that z is at least as preferred as x or y

- What does a mixture mean?
- The ‘average’ of the number of each good given by the two bundles

$$\alpha x + (1-\alpha)y = \begin{pmatrix} \alpha x_1 + (1-\alpha)y_i \\ \vdots \\ \alpha x_n + (1-\alpha)y_n \end{pmatrix}$$

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Convexity

- Again, two flavors of convexity:
- **Weakly convex:** If x is indifferent to y, then any mixture of x and y is **weakly** preferred to either

$$x \sim y \text{ implies } \alpha x + (1-\alpha)y \succsim x$$

- **Strictly convex:** If x is indifferent to y, then any mixture of x and y is strictly preferred to either

$$x \sim y \text{ implies } \alpha x + (1-\alpha)y \succ x$$

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Interpretation of Convexity

- Why are we assuming convexity?
- Does it make sense?
- Well, think about constructing a meal
  - Say you were indifferent between 2lb of mash potato (and no steaks) or 2 steaks (and no mash potato)
  - It is likely that you would prefer 1lb of mash potato and 1 steak to either bundle
  - Can think of many similar examples
    - Holidays and cars
    - Visits to the dentist and visits to the theatre
    - Economics classes and novels
- What about ice cream and crab paste?
  - Arguably still yes
  - Depends on whether we think that these things are consumed at exactly the same time

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Convexity

- What do the indifference curves of convex preferences look like?

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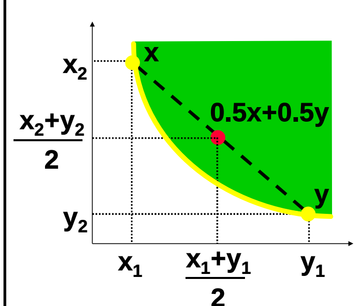
Graphically: Convexity



Convexity implies that the set of weakly preferred bundles is a **convex set**

Indifference curve is convex

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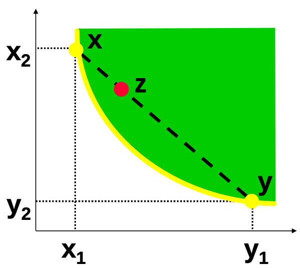
Strict Convexity



Preferences are **strictly** convex when all mixtures z are **strictly** preferred to original bundles x and y

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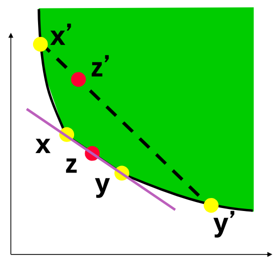
Weak Convexity.



Preferences are **weakly convex** if at least one mixture z is **equally preferred** to a component bundle

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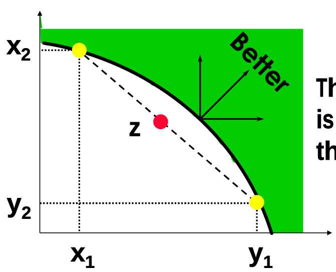
Non-Convex Preferences



The mixture z is less preferred than x or y.

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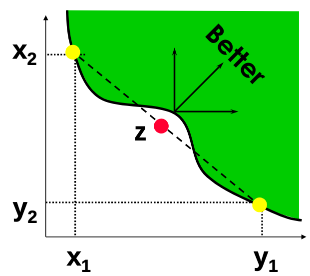
More Non-Convex Preferences



The mixture z is less preferred than x or y.

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Convexity

- What do the indifference curves of convex preferences look like?
  - They are convex
  - Weakly convex preferences – weakly convex indifference curves
  - Strictly convex preferences – strictly convex indifference curves
- What does the MRS of convex preference look like?

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MRS & Ind. Curve Properties

MRS **decreases with x_1** if and only if preferences are **strictly convex**

As x_1 becomes large MRS becomes small

Intuition? You have a lot of good 1 and you really want some good 2!

Obvious from calculus: convex function has an **INCREASING derivative**

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MRS and Convexity

- Convexity is **equivalent** to saying that MRS **decreases** with good 1
- Meaning: the amount of good 2 that the person needs to compensate them for the loss of good 1 **decreases** the more of good 1 they have
- This is very reasonable: the more you have of one good, the more you're willing to exchange for some of the other good
- From this point of view, convexity is **very natural**

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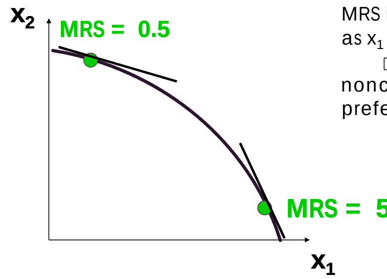
MRS & Ind. Curve Properties



MRS increases as x_1 increases ⇨ nonconvex preferences

Obvious from calculus: concave function has an **DECREASING derivative**

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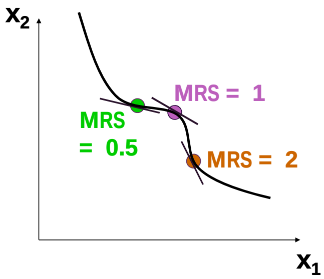
MRS & Ind. Curve Properties



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Convexity

- What do the indifference curves of convex preferences look like?
  - They are convex
  - Weakly convex preferences – weakly convex indifference curves
  - Strictly convex preferences – strictly convex indifference curves
- What does the MRS of convex preference look like?
  - Decreasing as x_1 increases
  - Weakly convex – weakly decreasing
  - Strictly convex – strictly decreasing
- What do the utility functions of convex preferences look like?
  - This is only for the math fetishists
  - Convex preferences have utility functions which are **quasi-concave**

$$u(\alpha x + (1-\alpha)y) \geq \min(u(x), u(y))$$

Different Types of Preferences

Some other beasts in the menagerie of preferences

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Other Types of Preferences

- It is now going to be useful to think about some other types of preferences
- These will be useful as we illustrate different ideas going forward
  - Perfect Substitutes
  - Perfect Compliments
  - Cobb Douglas Preferences

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Perfect Substitutes

- If a consumer always regards units of commodities 1 and 2 as equivalent, then the commodities are **perfect substitutes** and **only the total amount** of the two commodities in bundles determines their preference rank-order
- Examples
  - Suppose indifferent between 2 kinds of apples
  - Then the only thing that matters is the **total** number of apples
- Intuitively: there is a constant tradeoff between the two goods

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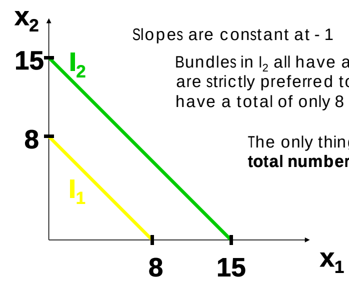
Graphically: Perfect Substitutes



Slopes are constant at - 1

Bundles in I_2 all have a total of 15 units and are strictly preferred to all bundles in I_1, which have a total of only 8 units in them.

The only thing that matters is the **total number**

What matters is not that the slope is -1 but that it is **constant**

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Perfect Substitutes

- Perfect substitutes have **linear** and **parallel** indifference curves
- The MRS is **constant**
- Utility function is also **linear**

$$u(x_1, x_2) = ax_1 + bx_2$$

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Perfect Complements

- If a consumer always consumes commodities 1 and 2 in **fixed proportion** (e.g., one-to-one), then the commodities are **perfect complements**
- Examples:
  - Right and left shoes
  - Car and tires: 1 car, 4 tires
- What do the indifference curves look like?
  - Let's think about numbers of left and right shoes which make you indifferent to having 5 left and 5 right shoes

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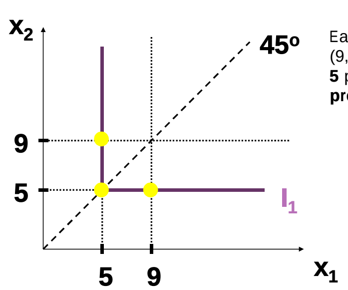
Graphically: Perfect Complements



Each of (5,5), (5,9) and (9,5) contains **5** pairs so each is **equally preferred.**

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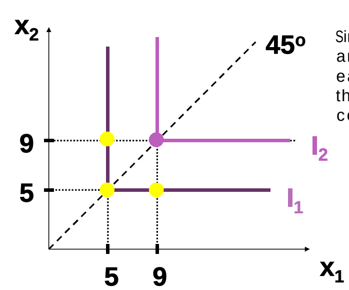
Graphically: Perfect Complements



Since each of (5,5), (5,9) and (9,5) contains 5 pairs, each is less preferred than the bundle (9,9) which contains **9 pairs**.

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Perfect Compliments

- Perfect compliments have **L-shaped** indifference curves
- The MRS is **0, infinity**, or **undefined**
- What about the utility function?
- Thinking about the shoes example, how about
  $u(x_1, x_2) = \min\{x_1, x_2\}$

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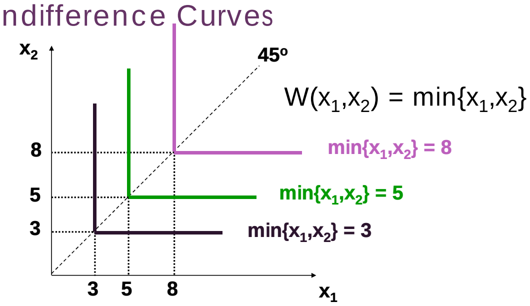
Perfect Complementarity Indifference Curves



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Cobb-Douglas

- An incredibly useful type of utility function which you will see over and over again is **cobb-douglas utility**
- Any utility function of the form
  $u(x_1, x_2) = x_1^a x_2^b$

  with a > 0 and b > 0

  E.g. $U(x_1,x_2) = x_1^{1/2} x_2^{1/2}$ (a = b = 1/2)
  $V(x_1,x_2) = x_1 x_2^3$ (a = 1, b = 3)
- The reason that you will see these preferences a lot is
  - They are standard (strictly monotonic, strictly convex)
  - They are very easy to work with

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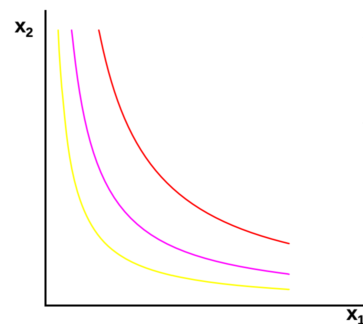
Cobb-Douglas Indifference Curves



All curves are hyperbolic, asymptoting to, but never touching any axis.

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Other Ways of Writing Cobb Douglas Preferences

- Cobb-Douglas: $U(x_1,x_2) = x_1^a x_2^b$
- We can also transform them in 2 convenient ways

1. Apply f(y)=ln(y)
   - Obtain: $V(x_1,x_2) = a \ln(x_1) + b \ln(x2)$

- Apply $f(y)=y^{1/(a+b)}$
  - Obtain $u(x_1,x_2) = x_1^{a/(a+b)} x_2^{b/(a+b)}$
  - Advantage: Exponents **sum to 1** (will be useful later)

Summary

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Summary

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- Today we classified different types of preferences
  - ‘Standard’ preferences
    - Monotonic
    - Convex
  - Introduced some other useful classes of preferences
    - Perfect substitutes
    - Perfect complements
    - Cobb Douglas