Intermediate Microeconomics - Spring 2016

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Income and Substitution Effect

This short note provides a practical guide to breaking down the effect of price changes into substitution and income effects along the lines of the graphs I showed in lecture 6. Note that we will be using the following definition:

Definition 1 *Consider the effect of a change in the price of good i on the demand for good i. The **substitution effect** is the effect of the change in prices keeping '‘effective' income constant - i.e. changing income to ensure that the originally chosen bundle is just affordable at the new prices. The **income effect** is then effect of the change in income at the new prices*

As discussed in class, this is slightly different to the definition of the income and substitution effects we use to derive the Slutsky equation.

So what does all this mean? Let's start with a consumer who is choosing between bundles of two goods: x_1 and x_2. Initially they face prices $\bar{p}_1$, $\bar{p}_2$ and have income $\bar{y}$. We want to know the impact on demand for good 1 of a change in the price of good 1 to p_1^* (obviously we could do the same thing for the impact on good 2 of the price of good 2).

Remember that we can figure out how much the consumer will demand of good 1 for any possible set of prices and incomes p_1, p_2 and y. We do so by solving the consumer's problem i.e. maximizing $u(x_1, x_2)$ subject to the budget constraint $p_1x_1 + p_2x_2 \leq y$. We write the resulting demand for good 1 as $x_1(p_1, p_2, y)$ See lecture 5 if you are confused.

So the demand for good 1 at the original prices and incomes is given by $x_1(\bar{p}_1, \bar{p}_2, \bar{y})$. This comes from maximizing $u(x_1, x_2)$ subject to the budget constraint $\bar{p}_1x_1 + \bar{p}_2x_2 \leq \bar{y}$

The demand for good 1 after the price change is given by $x_1(p_1^*, \bar{p}_2, \bar{y})$. This comes from maximizing $u(x_1, x_2)$ subject to the budget constraint $p_1^* x_1 + \bar{p}_2 x_2 \leq \bar{y}$

The total change in demand for good 1 is therefore given by the change in these demands - i.e. $x_1(p_1^*, \bar{p}_2, \bar{y}) - x_1(\bar{p}_1, \bar{p}_2, \bar{y})$. In order to split out the income and substitution effects, we need to solve a third decision problem. This third decision problem tells us how the consumer would have behaved if they were faced with the new prices, but had their income adjusted so that the original bundle is only just affordable. Notice that this is just a 'thought experiment' - the consumer will never actually face this third decision problem - but it will help us think about the effect of changing price $\bar{p}_1$ to p_1^*.

What does this third problem look like? We want to understand the demand of the consumer at the new prices, but with the same effective income. So we know that the consumer faces the new prices - i.e. p_1^* and p_2. But their income will be different. We want to give them the amount of income which will make the original bundle just affordable at the new prices. In other words, their income we are interested in is y^*, where

$$y^* = p_1^* x_1(\bar{p}_1, \bar{p}_2, \bar{y}) + \bar{p}_2 x_2(\bar{p}_1, \bar{p}_2, \bar{y})$$

This is just the cost of the bundle demanded at the original prices - $x_1(\bar{p}_1, \bar{p}_2, \bar{y})$, $x_2(\bar{p}_1, \bar{p}_2, \bar{y})$ calculated at the new prices p_1^*, $\bar{p}_2$.

We are interested in the demand for good 1 when the consumer is faced with the budget constraint $p_1^* x_1 + \bar{p}_2 x_2 \leq y^*$ -i.e. $x_1(p_1^*, \bar{p}_2, y^*)$. This is their demand when prices have changed, but their 'effective' income is not changed, in the sense that they can only just afford the original bundle.

Using these demand functions, we can calculate the substitution and income effects. Remember, the former is the change in demand due to the change to the new prices, but keeping 'effective' income constant. Thus it is given by

$$x_1(p_1^*, \bar{p}_2, y^*) - x_1(\bar{p}_1, \bar{p}_2, \bar{y})$$

The income effect is then the change in demand due to the change in income caused by the new prices

$$x_1(p_1^*, \bar{p}_2, \bar{y}) - x_1(p_1^*, \bar{p}_2, y^*)$$

It should be obvious that the total effect of the price changes is equal to the income plus the substitution effects.

If you look at slide 62 of lecture 6, x_1' is $x_1(\bar{p}_1, \bar{p}_2, \bar{y})$, x_1'' is $x_1(p_1^*, \bar{p}_2, y^*)$ and x_1''' is $x_1(p_1^*, \bar{p}_2, \bar{y})$.

There is a question on the practice midterm which should give you some practice on these things.