Introduction

The Story So Far....
- We now have a number of tools to analyze strategic situations
- Defined a game
- Thought about how to solve a game
- Dealt with issues of
  - Multiple equilibria (equilibrium selection)
  - Existence (mixed strategies)
- Thought about the difference between sequential and simultaneous move games

Today
- Go back to thinking about firms
- In particular, we are going to relax the assumption that firms are price takers
- Think about firms that are big enough to affect price
- Take this into account when deciding what to do
- First consider the case of Monopoly
  - Only one firm
  - Very boring game
- Then consider the case of a small number of firms
  - Oligopoly
  - More interesting game
- Varian Ch. 25/26/28
- Feldman and Serrano Ch 12/13

Monopolies
- Up until now, we have assumed that firms have been small
- In particular, they treat prices as given
- However this is clearly not always the case
- Sometimes firms are large
- Get to set their own prices
Monopolies

- How does this happen?
- Presumably monopolies make big profits
- From previous lectures, we know that excess profits should encourage new firms to enter
- Why does that not always happen?

Why Monopolies?

- a legal fiat; e.g. US Postal Service
- a patent; e.g. a new drug
- sole ownership of a resource; e.g. a toll highway
- formation of a cartel: many firms colluding to restrict output to affect prices. 2 examples:
  - OPEC: think of the '73 oil crisis
  - De Beers: controls 80% of world production of diamonds
- large economies of scale; e.g. local utility companies.

How do Monopolies Behave?

- We will assume that monopolies act to maximize profits
- Remember, that for a perfectly competitive firm, the profit maximization problem was to choose output to maximize
  \[ \pi = px - c(x) \]
- How does this change with the monopolist?
- Well, we said that a monopolist gets to choose prices
- But, there will still be a relationship between price they choose and the quantity they can sell
- Given by the demand function

The Demand Curve

- The Demand curve tells the firm how much people will buy given price p

How do Monopolies Behave?

- The latter formulation will be more convenient
  \[ \pi = p(y)y - c(y) \]
- So, what should the firm do to maximize profits?
- Let’s take first order conditions!
  \[ \text{Profit} = \text{Revenue} - \text{Cost} \]
  \[ \text{First order conditions} \]
  \[ p(y) + y \frac{dp}{dy} = \frac{dc}{dy} \]
  Marginal Revenue = Marginal Cost
- (Remember, you have to worry about corner solutions etc.)
How do Monopolies Behave?

Marginal Revenue = Marginal Cost
- This is the same condition as in the perfect competition case.
- But now marginal revenue has an additional term: \( \frac{dp}{dy} \).
- This is exactly the firm taking into account the effect that increased production has on prices.
- Side note: at the moment, we are assuming that the only thing the monopolist can do is charge a constant price per unit output.
- Later in the course we will consider some sneaky other things that a monopolist might want to do.

Marginal Revenue and Demand

For example, if the inverse demand curve is given by:

\[ p(y) = a - by \]

Then revenue is given by:

\[ py = (a - by)y \]

And so marginal revenue is given by:

\[ \frac{d(py)}{dy} = a - 2by \]

Monopolistic Output

- So what does the optimal output look like for a monopolist?

- They will choose \( y \) in order to get marginal cost to equal marginal revenue.
How do Monopolies Behave?

- How does the behavior of a monopolist compare to that of perfectly competitive firms?
- Remember, for a perfectly competitive firm:
  - Price equals marginal cost
  - The supply curve is given by the marginal cost
  - Equilibrium happens when supply equals demand

Monopolists will supply less and charge higher prices than a perfectly competitive industry.

How do Monopolies Behave?

- What happens to surplus in the monopoly case?
How do Monopolies Behave?

- What happens to surplus in the monopoly case?
- Three effects
  - Producer surplus increases
  - Consumer surplus decreases
  - Total surplus decreases
  - i.e. monopolies are inefficient

Price Mark-Up and Elasticity

- Let’s go back to the first order conditions for the monopolist
  \[ p(y) + \frac{dp}{dy} = \frac{dc}{dy} \]
- Rearranging gives
  \[ p(1 + \frac{dp}{p \, dy}) = \frac{dc}{dy} \]
  \[ p = \frac{dc}{(1 + \frac{dp}{p \, dy})} \]
  - But \( \frac{dc}{dy} \) is marginal cost, and \( \frac{dp}{p \, dy} \) is the reciprocal of the price elasticity of demand

Oligopoly

- We now want to move from one firm (monopoly) to a small number of firms (oligopoly)
- Concentrate on the case of two firms (duopoly)
- It turns out there are different ways to analyze this case, depending on how we assume firms interact
  - Sometimes called the market structure
  - We are going to think first about the case in which firms compete on quantity
  - Sometimes called Cournot competition
- In order to analyze the market, we will make use of game theoretic concepts
Assume that firms compete by choosing output levels. If firm 1 produces $y_1$ units and firm 2 produces $y_2$ units, then total quantity supplied is $y_1 + y_2$. The market price will be $p(y_1 + y_2)$. This is what makes this a game: output of firm 1 affects the price of firm 2, and vice versa.

The firms' total cost functions are $c_1(y_1)$ and $c_2(y_2)$.

Suppose firm 1 takes firm 2's output level choice $y_2$ as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

Given $y_2$, what output level $y_1$ maximizes firm 1's profit?

Then, for given $y_2$, firm 1's profit function is

$$\Pi(y_1; y_2) = (60 - y_1 - y_2)y_1 - y_1^2.$$  

So, given $y_2$, firm 1's profit-maximizing output level solves

$$\frac{\partial \Pi}{\partial y_1} = 60 - 2y_1 - y_2 - 2y_1 = 0.$$  

I.e., firm 1's best response to $y_2$ is

$$y_1 = R_1(y_2) = 15 - \frac{1}{4}y_2.$$
Quantity Competition; An Example

I.e., firm 1’s best response to \( y_2 \) is
\[
y_1 = R_1(y_2) = 15 - \frac{1}{4} y_2.
\]

Key point: Optimal output of firm 1 depends on the output of firm 2.

Quantity Competition; An Example

Similarly, given \( y_1 \), firm 2’s profit function is
\[
\Pi(y_2; y_1) = (60 - y_1 - y_2)y_2 - 15y_2 - y_2^2.
\]

So, given \( y_1 \), firm 2’s profit-maximizing output level solves
\[
\frac{\partial \Pi}{\partial y_2} = 60 - y_1 - 2y_2 - 15 - 2y_2 = 0.
\]

I.e., firm 1’s best response to \( y_2 \) is
\[
y_2 = R_2(y_1) = \frac{45 - y_1}{4}.
\]
Quantity Competition; An Example

What do we think will happen in this case?

Well, we can think of this as a game with
Two players
The action of each player is to choose a quantity
Payoff is given by the profit for each firm

How do we solve games?

Nash equilibrium!

This is an action for each player which is the best for them, given what the other player is doing.

An equilibrium is when each firm’s output level is a best response to the other firm’s output level, for then neither wants to deviate from its output level.

A pair of output levels \((y_1^*, y_2^*)\) is a Cournot-Nash equilibrium if

\[
y_1^* = R_1(y_2^*) \text{ and } y_2^* = R_2(y_1^*)
\]

Substitute for \(y_2^*\) to get

\[
y_1^* = 15 - \frac{1}{4} \left( \frac{45 - y_1^*}{4} \right)
\]

Substitute for \(y_2^*\) to get

\[
y_1^* = 15 - \frac{1}{4} \left( \frac{45 - y_1^*}{4} \right) \Rightarrow y_1^* = 13
\]

Hence

\[
y_2^* = \frac{45 - 13}{4} = 8.
\]
Quantity Competition; An Example

\[ y_1^* = R_1(y_2^*) = 15 - \frac{1}{4} y_2^* \quad \text{and} \quad y_2^* = R_2(y_1^*) = \frac{45 - y_1^*}{4}. \]

Substitute for \( y_2^* \) to get

\[ y_1^* = 15 - \frac{1}{4} \left( \frac{45 - y_1^*}{4} \right) \quad \Rightarrow \quad y_1^* = 13 \]

Hence \( y_2^* = \frac{45 - 13}{4} = 8 \).

So the Cournot-Nash equilibrium is \((y_1^*, y_2^*) = (13, 8)\).

Quantity Competition; An Example

Firm 1’s “reaction curve”

\[ y_1 = R_1(y_2) = 15 - \frac{1}{4} y_2. \]

Firm 2’s “reaction curve”

\[ y_2 = R_2(y_1) = \frac{45 - y_1}{4}. \]

So the Cournot-Nash equilibrium is \((y_1^*, y_2^*) = (13, 8)\).

Quantity Competition

Generally, given firm 2’s chosen output level \( y_2 \), firm 1’s profit function is

\[ \Pi_1(y_1; y_2) = p(y_1 + y_2)y_1 - c_1(y_1) \]

and the profit-maximizing value of \( y_1 \) solves

\[ \frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1 \frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1'(y_1) = 0. \]

The solution, \( y_1 = R_1(y_2) \), is firm 1’s Cournot-Nash reaction to \( y_2 \).

Quantity Competition

Similarly, given firm 1’s chosen output level \( y_1 \), firm 2’s profit function is

\[ \Pi_2(y_2; y_1) = p(y_1 + y_2)y_2 - c_2(y_2) \]

and the profit-maximizing value of \( y_2 \) solves

\[ \frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2 \frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2'(y_2) = 0. \]

The solution, \( y_2 = R_2(y_1) \), is firm 2’s Cournot-Nash reaction to \( y_1 \).
Collusion

- Q: Are the Cournot-Nash equilibrium profits the largest that the firms can earn in total?
- Not if they are allowed to co-operate!

Suppose two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels $y_1$ and $y_2$ that maximize

$$\Pi^M(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

This is similar to the monopolist problem!

In fact, if the firms have constant costs, it is the monopolist problem.

Typically, this will give the firm higher profit.

Sometimes called forming a cartel.

Collusion

Suppose two firms face an inverse market demand of $p(y) = 24 - y$ and have total costs of $c_1(y_1) = y^2_1$ and $c_2(y_2) = y^2_2$.

Can they make more in a cartel or if they compete?

What is each firm’s per period profit in the cartel?

$p(y) = 24 - y$, $c_1(y_1) = y^2_1$, $c_2(y_2) = y^2_2$.

If the firms collude then their joint profit function is

$$\pi^M(y_1, y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2_1 - y^2_2.$$

What values of $y_1$ and $y_2$ maximize the cartel’s profit?

Solution is $y^M_1 = y^M_2 = 4$. 

Collusion

$$\pi^M(y_1, y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2_1 - y^2_2.$$

What values of $y_1$ and $y_2$ maximize the cartel’s profit? Solve

$$\frac{\partial \pi^M}{\partial y_1} = 24 - 4y_1 - 2y_2 = 0$$
$$\frac{\partial \pi^M}{\partial y_2} = 24 - 2y_1 - 4y_2 = 0.$$

Solution is $y^M_1 = y^M_2 = 4$. 

Collusion

$\pi^M(y_1, y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y^2_1 - y^2_2.$

What values of $y_1$ and $y_2$ maximize the cartel’s profit? Solve

$$\frac{\partial \pi^M}{\partial y_1} = 24 - 4y_1 - 2y_2 = 0$$
$$\frac{\partial \pi^M}{\partial y_2} = 24 - 2y_1 - 4y_2 = 0.$$

Solution is $y^M_1 = y^M_2 = 4$. 

Collusion

Suppose the two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels $y_1$ and $y_2$ that maximize

$$\Pi^M(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

This is similar to the monopolist problem!

In fact, if the firms have constant costs, it is the monopolist problem.

Typically, this will give the firm higher profit.

Sometimes called forming a cartel.
Collusion

\[ \pi^M(y_1, y_2) = (24 - y_1 - y_2)(y_1 + y_2) - y_2. \]

\[ y^M_1 = y^M_2 = 4 \] maximizes the cartel’s profit.

The maximum profit is therefore

\[ \pi^M = $(24 - 8)(8) - $16 - $16 = $96. \]

Suppose the firms share the profit equally, getting $96/2 = $48 each per period.

Collusion

What are the firms’ profits if they do not cooperate?

\[ p(y_t) = 24 - y_t, c_1(y_t) = y_1, c_2(y_t) = y_2. \]

Given \( y_2 \), firm 1’s profit function is

\[ \pi_1(y_1; y_2) = (24 - y_1 - y_2)y_1 - y_2. \]

The value of \( y_1 \) that is firm 1’s best response to \( y_2 \) solves

\[ \frac{\partial \pi_1}{\partial y_1} = 24 - 4y_1 - y_2 = 0 \Rightarrow y_1 = R_1(y_2) = \frac{24 - y_2}{4}. \]

So both firms earn more if they form a cartel

But is such a cartel stable?

What would firm 1 want to do if firm 2 is producing at the cartel level?
Collusion

- Let’s say that firm 1 knows that firm 2 will produce at the cartel level.
- What is their best response?
- Firm 1 produces quantity $y^{CH}_1$ that maximizes firm 1’s profit given that firm 2 continues to produce $y^{M}_2 = 4$. What is the value of $y^{CH}_1$?
- $y^{CH}_1 = R_1(y^{M}_2) = (24 - y^{M}_2)/4 = (24 - 4)/4 = 5$.
- Firm 1’s profit in the period in which it cheats is therefore $\pi^{CH}_1 = (24 - 5 - 4)(5) - 5^2 = 50$.

Collusion

- So a profit-seeking cartel in which firms cooperatively set their output levels is fundamentally unstable.
- This is basically a prisoner’s dilemma
- Both firms would do better if they collude, but this is not an equilibrium
- E.g., OPEC’s broken agreements.

Collusion & Punishment Strategies

- Imagine a firm that is deciding between
  1. Staying in the cartel forever
  2. Cheating on the cartel, and then being punished forever by the other cartel member

Collusion & Punishment Strategies

- To determine if such a cartel can be stable we need to know 3 things:
  (i) What is each firm’s per period profit in the cartel?
  (ii) What is the profit a cheat earns in the first period in which it cheats?
  (iii) What is the profit the cheat earns in each period after it first cheats?
Collusion & Punishment Strategies

(i) What is the profit a cheat earns in the first period in which it cheats?
This is what firm 1 would get if they best responded to firm 2 producing the cartel output.
They would get $50

(ii) What is the profit a cheat earns in the first period in which it cheats?
This is what firm 1 would get if they best responded to firm 2 producing the cartel output.
They would get $50

(iii) What is the profit a cheat earns in the first period in which it cheats?
This depends upon the punishment inflicted upon the cheat by the other firm.

Suppose the other firm punishes by forever after not cooperating with the cheat.
Then each firm will earn the Cournot profits - $46 - in each period.
Note that this is a 'credible' threat.

To determine if such a cartel can be stable we need to know 3 things:

(i) What is each firm's per period profit in the cartel? $48.
(ii) What is the profit a cheat earns in the first period in which it cheats? $50.
(iii) What is the profit the cheat earns in each period after it first cheats? $46.

Each firm's periodic discount factor is $1/(1+r)$.
The present-value of firm 1's profits if it does not cheat is $\frac{48}{1+r} + \frac{48}{(1+r)^2} + \cdots = \frac{48 (1+r)}{r}$.
Collusion & Punishment Strategies

- Each firm’s periodic discount factor is $1/(1+r)$.
- The present-value of firm 1’s profits if it does not cheat is

$$PV_{\text{loyal}} = \frac{48}{1+r} + \frac{48}{(1+r)^2} + \cdots = \frac{(1+r)48}{r}.$$

- The present-value of firm 1’s profit if it cheats this period is

$$PV_{\text{cheat}} = \frac{50}{1+r} + \frac{46}{(1+r)^2} + \cdots = \frac{50 + \frac{46}{r}}{1+r}.$$

So the cartel will be stable if

$$\frac{(1+r)48}{r} > 50 + \frac{46}{r} \Rightarrow r < 1 \Rightarrow 1 + \frac{1}{2} > r.$$

Collusion

- Cartels can be stable if
  1. The game is played an infinite number of times
  2. The players are patient enough

Summary

- Today we have
  - Modelled monopolies
  - Modelled duopolies that compete on quantity
  - Thought about when duopolies form cartels