Problem Set 1 Solutions Intermediate Microeconomics

Mark Dean

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Throughout this solution set, it is assumed that all physical goods are subject to nonnegativity constraints.

- Question 1 (Budget Sets 1) Let f = number of footballs purchased, c =number of cricket balls purchased. $p_f = \$4, p_c = \$2, y = \$20$ denote the respective prices of the balls and income.
 - 1. Budget constraint: $p_f \cdot f + p_c \cdot c = y$. With numerical substitutions this becomes: 4f + 2c = 20. Letting footballs be on the horizontal axis of the commodity space, we rewrite the budget constraint in terms of c:

$$c = -2f + 10$$



2. The quantity tax $\tau_f = 0.5$ on footballs is an additional percentage means that the new budget constraint is

$$p_f(1+\tau_f)f + p_c c = y$$

. Substituting in for the values, we graph:

$$c = -3f + 10$$



3. Now we also add a quantity tax $\tau_c = 0.5$ on cricket balls. The budget constraint becomes

$$p_f(1+\tau_f)f + p_c(1+\tau_c)c = y$$

and we graph $c = -2f + \frac{20}{3}$.
c, cricket balls purchased
 $\frac{20}{3}$
 $\frac{10}{3}$
 $\frac{10}{3}$
 $\frac{5}{3}$ $\frac{10}{3}$ f, footballs purchased

4. The budget constraint with a subsidy s =\$10 is now:

$$p_f(1+\tau_f)f + p_c(1+\tau_c)c = y + s \Rightarrow 6f + 3c = 30$$

The equation for the budget constraint on the commodity space is given by: c = -2f + 10. Note that consumer behavior would be identical to the situation described in section 1, and the graph for this situation is precisely the same as the one depicted in the first section (the budget constraint is homogeneous of degree 0 in prices).

5. We can split the budget constraint in two different situations, depending on whether or not the bulk discount is "activated" or not. The budget line - becomes

$$\begin{cases} 4f + 2c = 20, & f \le 3\\ 4 \cdot 3 + 2(f - 3) + 2c = 20, & f > 3 \end{cases}$$



Question 2 (Budget Sets 2)

- 1. Since Edmund has no other source of income, if he accepts 0 sacks of garbage, he would have \$0 of income and therefore would not be able to buy any video cassettes: v = 0.
- 2. Let $u_E(v.g)$ be the utility Edmund receives from "consuming" v number of video cassettes and taking g bags of garbage. Then Edmund's constrained optimization problem is:

$$\max_{v \geq 0, g \geq 0} u(v,g) \text{ subject to } 6v \leq 2g$$

3. Putting g on the horizontal axis, we graph:

$$v = \frac{1}{3}g$$



Note that the budget set extends well beyond (in increasing g) the part depicted here.

Question 3 (Preferences and Transitivity) From the notes, recall for that two bundles of goods x and y, $x \succeq y$ denotes that either $x \sim y$ (indifference between x and y) or $x \succ y$ (x strictly preferred to y). There are several ways show the implication of these two properties. We will take the approach of clarifying the exact relationship between strict preference and indifference to weak preference. For two consumption bundles xand y, $x \succ y$ if and only if¹ $x \succeq y$ and not $y \succeq x$ (which we denote by $y \not\succeq x$). In other words,

$$x \succ y \Leftrightarrow x \succeq y \text{ and } y \not\gtrsim x.$$
 (1)

For indifference, $x \sim y$ if and only if $x \succeq y$ and $y \succeq x$:

$$x \sim y \Leftrightarrow x \succeq y \text{ and } y \succeq x.$$
 (2)

Now we use these equivalences to prove the transitivity properties. Denote (TR) to be the transitive property discussed in lecture:

$$x \succeq y \text{ and } y \succeq z \text{ implies } y \succeq z$$
 (TR)

1. Assume $x \succ y$ and $y \succ z$. We are trying to prove that $x \succ z$. By our definition \succ above, we therefore need to show *both* that $x \succeq z$ and NOT $x \succeq z$. By (1), the

¹For two logical statements A and B, A if and only if (iff) B means that A implies B and B implies A.

 $x \succ y, \ y \succ z$ imply that $x \succeq y$ and $y \nsucceq z$ and $y \succsim z$ and $z \nsucceq y$. By (TR), $x \succeq y$ and $y \succeq z$ implies $x \succeq z$.

We prove that $z \not\gtrsim x$ by contradiction. Suppose that $z \succeq x$. Then, using $y \succeq z$ from above and applying (TR), then $y \succeq x$ which contradicts the fact that $y \not\gtrsim x$ that we derived above. Therefore it must be the case that $z \not\gtrsim x$.² By (1) above, since $x \succeq z$ and $z \not\gtrsim x$, then $x \succ z$, which proves the transitivity statement above.

- 2. Suppose $x \sim y$ and $y \sim z$. By (2), then $x \succeq y, y \succeq x, y \succeq z$, and $z \succeq y$. By (TR), the first and third relations imply that $x \succeq z$ and the second and fourth relations imply that $z \succeq x$. By (2) again, we have that $x \sim z$.
- Question 4 (Utility Representations) Recall that if X is a commodity space, then a utility function $u : X \to \mathbb{R}$ represents preferences \succeq on the commodity space if the following hold true for all commodity bundles x, y:

$$x \succeq y$$
 if and only if $u(x) \ge u(y)$. (UTILDEF)

1. We prove the statement: utility functions u and v represent the same preferences if and only if there is a strictly increasing function³ f such that u(x) = f(v(x)). We suppose that utility function u represents preferences given by \succeq_1 , utility function v represents preferences⁴ given by \succeq_2 , and there is a strictly increasing function f such that u(x) = f(v(x)). By an application of (UTILDEF), for any two consumer bundles $x, y x \succeq_1 y \leftrightarrow u(x) \ge u(y)$ and that $x \succeq_2 y \leftrightarrow v(x) \ge v(y)$. By the definition of a strictly function, then $v(x) > v(b) \leftrightarrow u(x) = f(v(x)) >$ f(v(y)) = u(y). Thus

$$x \succ_1 y \leftrightarrow u(x) > u(y) \leftrightarrow v(x) > v(y) \leftrightarrow x \succ_2 y,$$

and using the fact that we can define the preference relation $\succeq_i a \succeq_i b \leftrightarrow b \neq_i a$ (for i = 1, 2, check this yourself), then the preferences represented by u and v are therefore the same $(\succeq_1 = \succeq_2)$.

The statement is no longer true if we drop the word 'strictly'. This is best shown by an example of a function that is increasing $(x > y \rightarrow f(x) \ge f(y))$ but not

²This is what we call *proving by contradiction*. Also note that completeness is still satisfied since we showed that $x \succeq z$.

³Note that this function should take the range of u and map it to the range of v, so it is NOT a function over consumer bundles.

⁴Both utility functions are defined over the same commodity space.

strictly increasing. Consider the function f(x) = c, for some real constant c.⁵ Then so long as the preferences over which v is defined (i.e. \succeq_2) gives strict preference between at leat one pair of bundles (i.e. for some $y_1, y_2, y_1 \succ_2 y_2$). Then $y_1 \succ_2 y_2$ implies that $v(y_1) > v(y_2)$, but then $u(y_1) = f(v(y_1) = c = f(v(y_2)) = u(y_2)$ using a constant f, which implies that $y_1 \sim_1 y_2$ and therefore \succeq_1 and \succeq_2 are not the same.

- 2. Let the real function f be such that u(x) = f(v(x)). We invoke the claim made in the first part and simply see if we can find a positive monotonic mapping between utilities
 - (a) Yes: $v(x_1, x_2) = f(u(x_1, x_2)) = 3u(x_1, x_2)^2 + 6.$
 - (b) No: $v(x_1, x_2) = f(u(x_1, x_2)) = -3u(x_1, x_2)^2 + 6$, but f is a decreasing function in this case.
 - (c) Yes: $f(\cdot) = \log(\cdot)$.
 - (d) No: a simple counterexample is that $u(\frac{1}{2}, \frac{1}{2}) > u(1, \frac{1}{5})$, but $v(\frac{1}{2}, \frac{1}{2}) < v(1, \frac{1}{5})$.
- 3. We use the relationship between discussed in part 1 above: if u and v represent the same preference relation over (x_1, x_2) , then we can write $u(x_1, x_2) = f(v(x_1, x_2))$ for some strictly increasing function f.⁶ We can show that the marginal utilities hold the same signs by an application of the chain rule:

$$\frac{\partial u(x_1, x_2)}{\partial x_i} = \frac{\partial f(v(x_1, x_2))}{\partial x_i} = f'(v(x_1, x_2))\frac{\partial v(x_1, x_2)}{\partial x_i}.$$

Since f is strictly increasing, then its first derivative f' is positive. Thus the sign of $\frac{\partial u(x_1,x_2)}{\partial x_i}$ must be the same $\frac{\partial v(x_1,x_2)}{\partial x_i}$ for the equality to hold and this is true for i = 1, 2.

⁵We can take c to be any constant, like c = 1, as the entire point is that all preferences would end up giving the same utility if we set u(x) = f(v(x)), regardless their relative preferences by the consumer.

⁶In other words, we can define u as the *composition* of f and v: $u = f \circ v$.