# Intermediate Microeconomics

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Homework 6

**Due** Wednesday, 30th March

**Question 1** If a production function exhibits diminishing marginal product with respect to each input, does it have to exhibit decreasing returns to scale? Hint - think about the Cobb Douglas case.

**Question 2** Sheffield Steel Corporation converts iron into steel. Their production function is $y = x_1^{\beta}$ where, $y$ is amounts of steel and $x_1$ is amounts of iron and $0 < \beta < 1$. They can buy iron at the price $p_1$ and sell steel at the price $p_y$

1. Sketch the production function. Does this satisfy the three assumptions that we made in class? What happens to marginal productivity as $x$ gets very small? What about when it gets very large?

2. Find the point of tangency between the iso profit lines and the production function as a function of the price of iron and steel. Can you always find a point of tangency for any positive price of iron and steel (for $x_1 > 0$)? If a point of tangency exists with $x_1 > 0$, what is the profit at this level of output? Will this be the profit maximizing level of output ? If not, what is?

3. Rotherham Steel Corporation is worse that Sheffield Steel Corporation. They always end up wasting the first unit of iron they buy, and get no steel out of it. Their production function is given

$$
\begin{aligned}
y &= 0 \text{ if } x_1 \leq 1 \\
&= (x_1 - 1)^{\beta} \text{ for } x_1 > 1
\end{aligned}
$$

Sketch the production function for Rotherham Steel Corporation. Find the point of tangency between the iso profit lines and the production function as a function of the price of iron and steel. Can you always find a point of tangency for any positive price of iron and steel? If a point of tangency exists with $x_1 > 0$, what is the profit at this level of output? Will this be the profit maximizing level of output ? If not, what is?

4. Sheffield Steel Corporation upgrades their production, giving a production function

$$y = x_1^\beta + \gamma x_1$$

with $\gamma > 0$. Sketch the production function. What happens to marginal productivity as $x_1$ gets very small or very large? Find the point of tangency between the iso profit lines and the production function as a function of the price of iron and steel. Can you always find a point of tangency for any positive price of iron and steel? If a point of tangency exists with $x_1 > 0$, what is the profit at this level of output? Will this be the profit maximizing level of output ? If not, what is?

5. Sheffield gets another upgrade, and now their production function is given by

$$y = x_1^\rho$$

where $\rho > 1$. Sketch the production function. What happens to marginal productivity as $x_1$ gets very small or very large? How much will this firm want to produce at any price for iron and steel?

6. Calculate marginal cost curves for the production function $y = x_1^\beta$ and $y = x_1^\rho$. What goes wrong when trying to find a solution in the last case?

**Question 3** Bongo Inc. make books ($b$), which they sell at $p_b$ and Casgwent ltd. make capacitors ($c$) which they sell at $p_c$ Both companies need land ($l$) and energy ($e$) to make their products. Bongo's production function is given by $b = l_b^{\frac{1}{2}} e_b^{\frac{1}{4}}$, where $l_b$ and $e_b$ are the land and energy used by Bongo, while Casgwent's production function is given by $c = l_c^{\frac{1}{4}} e_c^{\frac{1}{2}}$, where $l_c$ and $e_c$ are the land and energy used by Casquent

1. Do these firms exhibit increasing, decreasing, or constant returns to scale?
2. Calculate the marginal rate of technical substitution for each firm as a function of labor and energy used. If both firms used the same amount of labor and energy which would

have the higher rate of technical substitution (i.e. the ratio of the marginal product of land to the marginal product of energy)?

3. Say that land costs $p_l$ per unit and energy costs $p_e$ per unit. Calculate each firm's demand for land and energy as a function of these prices and the firm's output. At any set of input prices, which firm will use a higher ratio of land to energy.
4. Calculate the cost functions for both firms.
5. Disco LLT make discoballs ($d$), also using land and energy with the production function $d = \min(3l_d, 2e_d)$. Does this firm exhibit increasing, decreasing or constant returns to scale? Calculate their demand for land and energy as a function of $d$, $p_l$ and $p_e$. Also calculate their cost function .