1

Intermediate Microeconomics W3211

Lecture 7: The Endowment Economy

Columbia University, Spring 2016

Mark Dean: mark.dean@columbia.edu

Introduction

2

3

The Story So Far….

- Remember: the course had two basic aims:
  1. Introduce you to models of **how people make choices**
     - Constrained optimization
  2. Introduce you to models of **what happens when people interact**
     - Equilibrium

- We have now done a fairly thorough job of modelling the behavior of one type of economic agent – the consumer
  - Set up the consumer's problem
  - Solved the consumer's problem
  - Derived demand functions
  - Thought about how demand changes with income and prices

4

Today's Aims

- We are now going to take our first stab in thinking about what happens when economic agents interact
- In particular, we are going to think about how **prices** and **income** are determined in an economy
  - So far we have treated these as exogenous parameters
  - But they have to come from somewhere!
  - We will assume that they come about through the interaction of economic agents buying and selling
  - They move in order to balance supply and demand
  - This is the study of **equilibrium**
- We will start with the simplest possible setting
  - Two consumers
  - Two goods
  - No firms
- This is the study of an endowment economy
  - Varian Ch. 9, 15, 16
  - Feldman and Serrano Ch 15

A Simple Endowment Economy

1: Revisiting the consumer's problem

5

6

An Endowment Economy

- We are going to think about the simplest possible example of an economy
- There are two people: 1 and 2
- There are two goods: apples and bananas
- Each person starts off with an **endowment** of apples and bananas

  (w_a^1, w_b^1): amount of apples and bananas that person 1 has

  (w_a^2, w_b^2): amount of apples and bananas that person 2 has
- People are allowed to buy and sell at market prices p_a and p_b
- Questions we want to answer:
  - What should prices be in this economy?
  - Who ends up with what stuff?

7

An Endowment Economy

- Note: Apples and bananas only exist in the endowment of the consumers
  - They are not produced
  - There are no firms
  - This is a simplification, makes it easier to see what is going on
  - Don't worry, firms will appear soon enough
- First order of business: think what the consumer's problem looks like in an endowment economy

8

Updating the Consumer's Problem

- Remember the 'standard' consumer problem

1. **CHOOSE** a consumption bundle
2. **IN ORDER TO MAXIMIZE** preferences
3. **SUBJECT TO** the budget constraint

- The budget constraint (for consumer 1) was given by

$$p_a x_a^1 + p_b x_b^1 \leq I$$

- Now, rather than getting mysterious income I, the income the consumer gets comes from their endowment

$$p_a x_a^1 + p_b x_b^1 \leq p_a w_a^1 + p_b w_b^1$$

- Or, equivalently

$$p_a(x_a^1 - w_a^1) + p_b(x_b^1 - w_b^1) \leq 0$$

- $(x_a^i - w_a^i)$ is the net demand for good I
  - Budget constraint says that the cost of net demand has to be less than zero

9

Updating the Consumer's Problem

- What happens to the budget constraint if the price of good a changes?

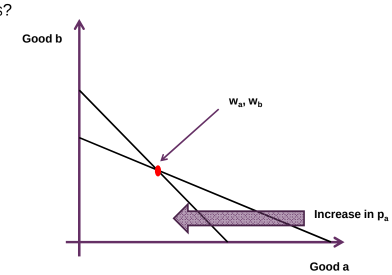
$$p_a x_a^1 + p_b x_b^1 \leq p_a w_a^1 + p_b w_b^1 \text{ implies}$$
$$x_b^1 \leq \frac{p_a}{p_b} w_a^1 + w_b^1 - \frac{p_a}{p_b} x_a^1$$

- Could buy $\frac{p_a}{p_b} w_a^1 + w_b^1$ units of good b (increasing in p_a)
- Could buy $w_a^1 + \frac{p_b}{p_a} w_b^1$ units of good a (decreasing in p_a)
- Always feasible to consume (w_a^1, w_b^1)
- Budget constraint pivots round the endowment

10

Updating the Consumer's Problem

- What happens to the budget constraint if the price of good a changes?



11

Updating the Consumer's Problem

- Another question: what happens when the prices of both goods change in proportion?
- A quick example
  - $w_a^1 = 2$
  - $w_b^1 = 3$
  - $p_a = 2$
  - $p_b = 1$
- Budget constraint is

$$x_b^1 \leq \frac{p_a}{p_b} w_a^1 + w_b^1 - \frac{p_a}{p_b} x_a^1$$
$$x_b^1 \leq 2 * 2 + 3 - 2x_a^1$$
$$x_b^1 \leq 7 - 2x_a^1$$

12

Updating the Consumer's Problem

- Now proportionally increase prices of both a and b
- A quick example
  - $w_a^1 = 2$
  - $w_b^1 = 3$
  - $p_a = 4$
  - $p_b = 2$
- Budget constraint is

$$x_b^1 \leq \frac{p_a}{p_b} w_a^1 + w_b^1 - \frac{p_a}{p_b} x_a^1$$
$$x_b^1 \leq 2 * 2 + 3 - 2x_a^1$$
$$x_b^1 \leq 7 - 2x_a^1$$

Updating the Consumer's Problem

- The only thing that matters for the consumers problem is the ratio of the price of good a to that of good b
- This is obvious from the budget constraint
$$x_b^1 \leq \frac{p_a}{p_b} w_a^1 + w_b^1 - \frac{p_a}{p_b} x_a^1$$
- This means that we can **normalize** the price of good b to 1
  - i.e. assume that the price of good b is 1
  - Think only about changes in the price of good a
  - What we **really** mean by the price of good a is $\frac{p_a}{p_b}$
- This will make our life a lot easier
- But is quite a subtle point. Make sure you understand it!

Updating the Consumer's Problem

- We have now changed the parameters of the consumer's problem
- Rather than the parameters being p_a, p_b, I they are now p_a, w_a^1, w_b^1
- We can similarly think of the demand function in terms of these parameters:
  - $x_a^1(p_a, w_a^1, w_b^1)$ is the demand of person 1 for good a, given prices p_a and endowment (w_a^1, w_b^1)
  - $z_a^1(p_a, w_a^1, w_b^1)$ =$x_a^1(p_a, w_a^1, w_b^1)$ -w_a^1 is the **net demand** of person 1 for good a

A Worked Example

- A worked example:

Choose x_a^1, x_b^1 to Maximize $u^1(x_a^1, x_b^1)$=$x_a^1 x_b^1$

Subject to $p_a x_a^1 + x_b^1 \leq p_a w_a^1 + w_b^1$

- First set up the Lagrange Function
$$L(x_a^1, x_b^1, \mu) = x_a^1 x_b^1 - \mu(p_a x_a^1 + x_b^1 - p_a w_a^1 - w_b^1)$$
- Then take derivatives

A Worked Example

$$L(x_a^1, x_b^1, \mu) = x_a^1 x_b^1 - \mu(p_a x_a^1 + x_b^1 - p_a w_a^1 - w_b^1)$$

- Then take derivatives
$$\frac{\partial L}{\partial x_a^1} = x_b^1 - \mu p_a = 0$$
$$\frac{\partial L}{\partial x_b^1} = x_a^1 - \mu = 0$$
$$\frac{\partial L}{\partial \mu} = p_a x_a^1 + x_b^1 - p_a w_a^1 - w_b^1 = 0$$
- First two equations gives
$$\frac{x_b^1}{x_a^1} = p_a$$
- Last equation gives the
$$p_a x_a^1 + x_b^1 = p_a w_a^1 + w_b^1$$

A Worked Example

- Substituting in gives

$p_a x_a^1 + p_a x_a^1 = p_a w_a^1 + w_b^1$ which implies
$$x_a^1(p_a, w_a^1, w_b^1) = \frac{1}{2}(w_a^1 + \frac{w_b^1}{p_a})$$
$$x_b^1(p_a, w_a^1, w_b^1) = \frac{1}{2}(p_a w_a^1 + w_b^1)$$
- And so
$$z_a^1(p_a, w_a^1, w_b^1) = \frac{1}{2}(\frac{w_b^1}{p_a} - w_a^1)$$
$$z_b^1(p_a, w_a^1, w_b^1) = \frac{1}{2}(p_a w_a^1 - w_b^1)$$

A Simple Endowment Economy

2: The Edgeworth Box

19

The Edgeworth Box

- For the consumer's problem we found it handy to draw graphs which allowed us to see what is going on
- It will be equally handy for us to do so for our simple 2 person, 2 good economy
- However, the graph we need to draw is a bit more complicated
- The Edgeworth Box!

20

The Edgeworth Box

- First step: how can we represent all the stuff that there is in the economy?
- With a box!
- Width of the box is **total** amount of good a
- Height of the box is the **total** amount of good b

21

Fig 2: An Edgeworth Box

22

The Edgeworth Box

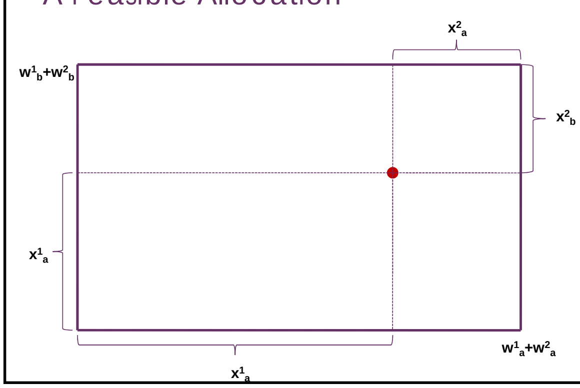
- Second step, how do we represent a **feasible allocation** in the Edgeworth Box?
  - A feasible allocation is an amount of each good for person 1 and 2 that uses up the total allocation
  - $x_a^1, x_b^1, x_a^2, x_b^2$ such that

$$x_a^1+x_a^2=w_a^1+w_a^2$$
$$x_b^1+x_b^2=w_b^1+w_b^2$$

- Any point in the box is a feasible allocation

23

A Feasible Allocation



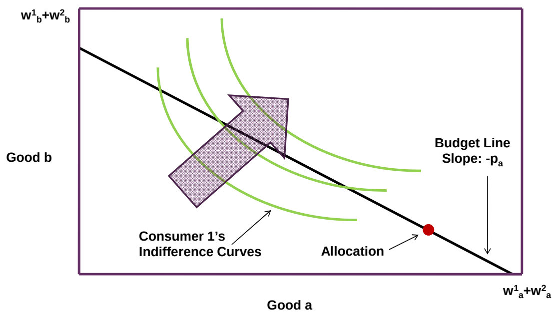
24

The Edgeworth Box

- Third step, we can represent the consumer's problem for person 1

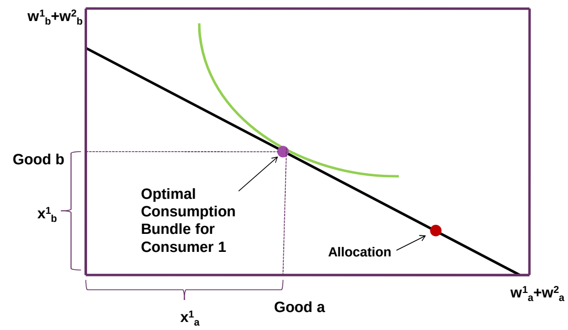
25

Consumer 1 in the Edgeworth Box



26

Fig 5: Consumer 1's Optimal Bundle



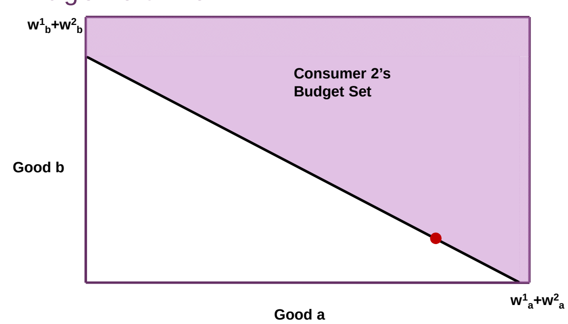
27

The Edgeworth Box

- Fourth step (and this is the clever bit), we can represent the consumer's problem for person 2

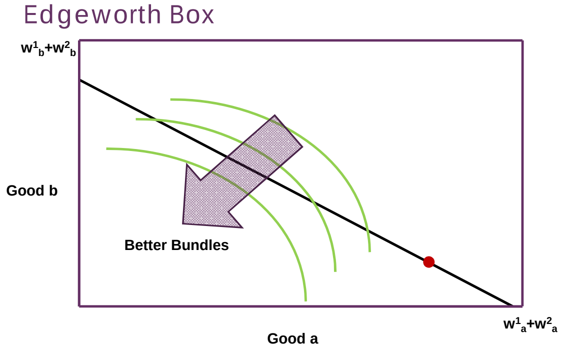
28

Consumer 2' Budget Set in the Edgeworth Box



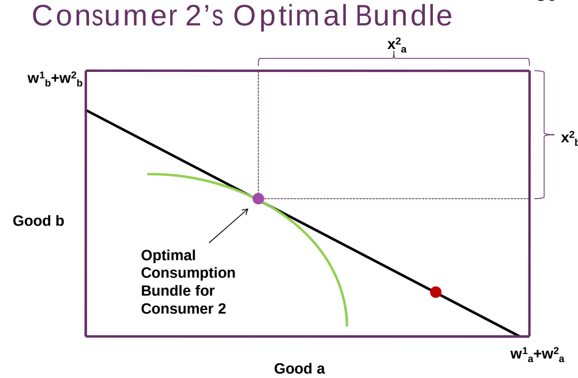
29

Consumer 2's Preferences in the Edgeworth Box



30

Consumer 2's Optimal Bundle



A Simple Endowment Economy

3: Equilibrium

31

Equilibrium

32

- We are now ready to calculate the **equilibrium** for our nice, simple, two person, two good exchange economy
- An important first step: we better **define** what we mean by an equilibrium

Equilibrium

33

- An equilibrium is a description of what we think will happen in an economy
- We can think of it as a **prediction** of where we might expect the economy to end up
- Consists of two types of thing
  - Prices of the various goods
  - Quantities of each good that each person receives.
- A handy tip: If I ask you, in an exam or homework, to calculate an equilibrium, you **have** to tell me what the prices and quantities are
  - If you haven't, you haven't answered the question!

Equilibrium

34

- So in our simple, two person two good economy, an equilibrium consists of
  - A price of good a: p_a (remember we don't need a price for good b)
  - The amount of stuff that person 1 gets: x_a^1 and x_b^1
  - The amount of stuff that person 2 gets: x_a^2 and x_b^2

Equilibrium

35

- What properties do you think an equilibrium should have?
- i.e., if I told you that p_a, x_a^1, x_b^1, x_a^2 and x_b^2 were the equilibrium for the economy, what would you expect to be true?
- It may be useful to think of the equilibrium as the **resting point** of the economy

The Definition of an Equilibrium

36

- We define an equilibrium in the following way: p_a, x_a^1, x_b^1, x_a^2 and x_b^2 form an equilibrium if two things are true

1. Optimality: The amount that each person gets is what they want, given the prices and their endowments
   - $x_a^1 = x_a^1(p_a, w_a^1, w_b^1)$
   - $x_b^1 = x_b^1(p_a, w_a^1, w_b^1)$
   - $x_a^2 = x_a^2(p_a, w_a^2, w_b^2)$
   - $x_b^2 = x_b^2(p_a, w_a^2, w_b^2)$
2. Market Clearing: Supply equals demand for each good
   - $x_a^1 + x_a^2 = w_a^1 + w_a^2$
   - $x_b^1 + x_b^2 = w_b^1 + w_b^2$

37

Finding an Equilibrium

- So now we know what an equilibrium is
- How do we find one?
- Luckily we have another recipe!

1. Calculate the demand for each consumer and each good as a function of prices
   - i.e. calculate $x_a^1(p_a, w_a^1, w_b^1)$, $x_b^1(p_a, w_a^1, w_b^1)$, $x_a^2(p_a, w_a^2, w_b^2)$, $x_b^2(p_a, w_a^2, w_b^2)$
2. Find the price at which markets clear
   - i.e. find the price p_a^* such that

$$x_a^1(p_a^*, w_a^1, w_b^1)+x_a^2(p_a^*, w_a^2, w_b^2)= w_a^1+w_a^2$$
$$x_b^1(p_a^*, w_a^1, w_b^1)+x_b^2(p_a^*, w_a^2, w_b^2)= w_b^1+w_b^2$$

3. Use this price to calculate what each person gets

38

A Worked Example

- Let's work through an example
- First, we need to describe the economy

1. The **endowment** of each agent
   - w_a^1=3
   - w_b^1=2
   - w_a^2=1
   - w_b^2=5
2. The **preferences** of each agent
   - $u^1(x_a^1, x_b^1)$=$x_a^1 x_b^1$
   - $u^2(x_a^2, x_b^2)$=$x_a^2 x_b^2$

39

A Worked Example

- Stage 1: Calculate the demand for each consumer and each good as a function of prices
- Luckily we have already done this for these preferences (earlier in the lecture)

$$x_a^1\left(p_a, w_a^1, w_b^1\right) = \frac{1}{2}\left(w_a^1 + \frac{w_b^1}{p_a}\right) = \frac{3}{2} + \frac{1}{p_a}$$
$$x_b^1\left(p_a, w_a^1, w_b^1\right) = \frac{1}{2}\left(p_a w_a^1 + w_b^1\right) = \frac{3p_a}{2} + 1$$
$$x_a^2\left(p_a, w_a^2, w_b^2\right) = \frac{1}{2}\left(w_a^2 + \frac{w_b^2}{p_a}\right) = \frac{1}{2} + \frac{5}{2p_a}$$
$$x_b^2\left(p_a, w_a^2, w_b^2\right) = \frac{1}{2}\left(p_a w_a^2 + w_b^2\right) = \frac{p_a}{2} + \frac{5}{2}$$

40

A Worked Example

- Stage 2: Find the price at which markets clear
- Let's first think about the market for apples

$$x_a^1(p_a^*, w_a^1, w_b^1)+x_a^2(p_a^*, w_a^2, w_b^2) = w_a^1+w_a^2$$
$$\frac{3}{2} + \frac{1}{p_a^*} + \frac{1}{2} + \frac{5}{2p_a^*} = 3 + 1$$
$$4 + \frac{7}{p_a^*} = 8$$
$$p_a^* = \frac{7}{4}$$

41

A Worked Example

- Great! We have the price that clears the market for apples!
- But there is also the market for bananas!
- And we only have one price
- This sounds like trouble
- Let's see what happens in the market for bananas

$$x_b^1(p_a^*, w_a^1, w_b^1)+x_b^2(p_a^*, w_a^2, w_b^2) = w_b^1+w_b^2$$
$$\frac{3p_a}{2} + 1 + \frac{p_a}{2} + \frac{5}{2} = 2 + 5$$
$$7 + 4p_a^* = 14$$
$$p_a^* = \frac{7}{4}$$

42

A Worked Example

- The same price clears the market for apples and bananas!
- A miracle!
- (it's not a miracle. I will explain why in a minute)
- First, is it the case that $p_a^* = \frac{7}{4}$ is an equilibrium?
- **NO!**
- An equilibrium consists of p_a^*, x_a^1, x_b^1, x_a^2 and x_b^2

43

A Worked Example

- Stage 3: Use this price to calculate what each person gets

$$x_a^1(p_a^*, w_a^1, w_b^1) = \frac{3}{2} + \frac{4}{7} = \frac{29}{14}$$
$$x_b^1(p_a^*, w_a^1, w_b^1) = \frac{21}{8} + 1 = \frac{29}{8}$$
$$x_a^2(p_a^*, w_a^2, w_b^2) = \frac{1}{2} + \frac{20}{14} = \frac{27}{14}$$
$$x_b^2(p_a^*, w_a^2, w_b^2) = \frac{7}{8} + \frac{5}{2} = \frac{27}{8}$$

- The equilibrium of this economy is these allocations, plus the price $p_a^* = \frac{7}{4}$

44

Walras' Law

- So let's go back to the miracle
- We had two markets (apples and bananas) and only one price (p_a^*)
- How did the one price clear the two markets?
- It turns out this is always true
- **Walras law:** The price that clears the market for good a will also clear the market for good b
- Why (strap in for some algebra)

45

Walras' Law

- Both agents obey their budget constraint

$$p_a x_a^1 + x_b^1 = p_a w_a^1 + w_b^1$$
$$p_a x_a^2 + x_b^2 = p_a w_a^2 + w_b^2$$

- So

$$p_a x_a^1 + x_b^1 + p_a x_a^2 + x_b^2 = p_a w_a^1 + w_b^1 + p_a w_a^2 + w_b^2$$
$$p_a(x_a^1 + x_a^2 - w_a^1 - w_a^2) + (x_b^1 + x_b^2 - w_b^1 - w_b^2) = 0$$

- If the market for good a clears then $(x_a^1 + x_a^2 - w_a^1 - w_a^2) = 0$
- Implies $(x_b^1 + x_b^2 - w_b^1 - w_b^2) = 0$
- Market for good b clears

A Simple Endowment Economy

4: Equilibrium in Pictures

46

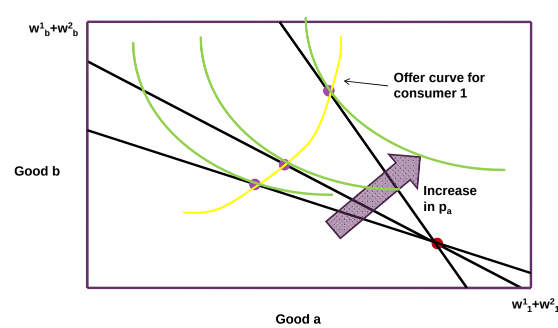
47

Equilibrium in the Edgeworth Box

- Having introduced the Edgeworth box, it might be useful to think about what equilibrium looks like in this diagram.
- First lets think what happens to the demand of consumer 1 as we change the price p_a^*

48

Equilibrium in the Edgeworth Box



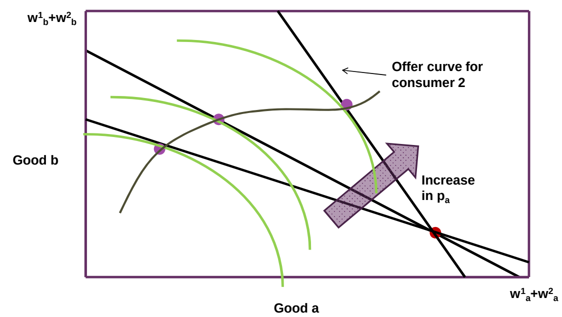
49

Equilibrium in the Edgeworth Box

- Having introduced the Edgeworth box, it might be useful to think about what equilibrium looks like in this diagram.
- First lets think what happens to the demand of consumer 1 as we change the price p_a
- Offer curve of consumer a traces out their optimal bundle as p_a changes
- Similarly, we can trace out the offer curve of consumer 2

50

Equilibrium in the Edgeworth Box



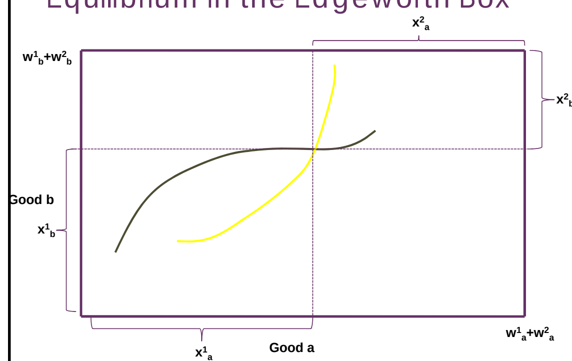
51

Equilibrium in the Edgeworth Box

- Having introduced the Edgeworth box, it might be useful to think about what equilibrium looks like in this diagram.
- First lets think what happens to the demand of consumer 1 as we change the price p_a
- Offer curve of consumer a traces out their optimal bundle as p_a changes
- Similarly, we can trace out the offer curve of consumer 2
- Where is equilibrium going to occur?

52

Equilibrium in the Edgeworth Box



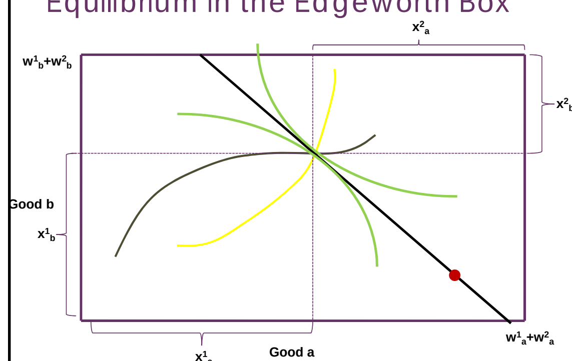
53

+ Equilibrium in the Edgeworth Box

- In the Edgeworth box, equilibrium occurs where the offer curves intersect

54

Equilibrium in the Edgeworth Box



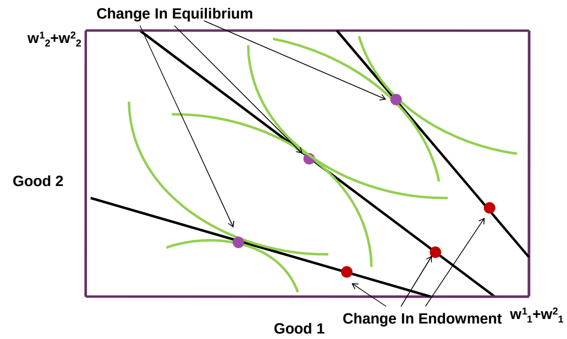
55

Equilibrium in the Edgeworth Box

- In the Edgeworth box, equilibrium occurs where the offer curves intersect
- This is also the point at which three things are tangent
  - The price line
  - The indifference curve of person 1
  - The indifference curve of person 2
- This will be important for the next lecture!
- Notice: different endowments will give rise to different equilibria

56

Equilibrium



57

Equilibrium, Supply and Demand

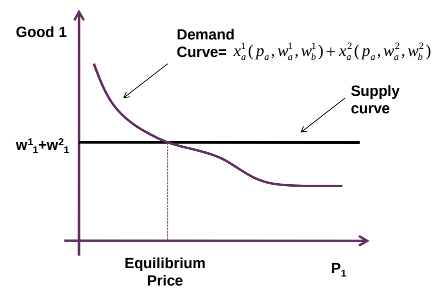
- Here is a final way of thinking about equilibrium
- Remember the market clearing condition:

$$x_a^1(p_a^*, w_a^1, w_b^1)+x_a^2(p_a^*, w_a^2, w_b^2)= w_a^1+w_a^2$$

- Left hand side is the total (or **market**) demand for good a
  - Typically (though not always) downward sloping
- Right hand side is the total supply of good a
- Equilibrium occurs when prices make supply equal to demand

58

Equilibrium, Supply and Demand



Summary

59

60

Summary

- Today we have done the following

1. Defined a simple endowment economy
   - Rather than income, consumers have endowments of each good
   - We have defined the consumer's problem in this case
   - Also defined the demand function $x_a^i(p_a, w_a^i, w_b^i)$
2. Introduced the Edgeworth Box as a way of representing a simple two person, two good endowment economy
   - Shown how it represents feasible allocations
   - Shown how it can represent the consumer's problem for both consumer 1 and 2

- Defined the concept of an equilibrium
- Shown how to solve for the equilibrium of a simple endowment economy