Introduction

The Story So Far....
- We have solved the consumer's problem
  - Determined what we think people will do given prices and income
- We have solved for equilibrium in an endowment economy
  - Determined what we think prices and allocations will be
- This is quite impressive!
  - We have our first prediction of how a simple economy works!
- Granted it is an economy that only has consumers in it
  - We will get to firms soon enough

Today’s Aims
- We are now going to talk about the welfare properties of an equilibrium
- Arguably this is one of the most interesting, but also misunderstood lectures in the entire course
- It is where economists sometimes stop being scientists and start being policy makers
  - Positive economics: what will happen
  - Normative economics: What should happen

Today’s Aims
- Begin by defining the concept of Pareto optimality
  - Most economists would agree that if an allocation is ‘good’ it should be Pareto optimal
- Then introduce the first “fundamental theorems of welfare economics”
  - Describe the relationship between Pareto optimality and Market equilibria

Pareto Optimality
Which Allocations Do You Prefer?

<table>
<thead>
<tr>
<th>Allocation x</th>
<th>Allocation y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kendrick</td>
<td>Taylor</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Allocations describe the utility that each person gets.

If you as the government could choose between allocation X and allocation Y which would you choose?

Pareto Ranking

Economists use a very specific way of comparing allocations:

Definition: Let \((x_1, x_2), (y_1, y_2)\) be an allocation in an economy. We say it is Pareto dominated by \((z_1, z_2), (w_1, w_2)\) if

\[\begin{align*}
u^1(x_1, x_2) &\geq u^1(z_1, z_2) \\
u^2(x_1, x_2) &\geq u^2(z_1, z_2)
\end{align*}\]

And

\[\begin{align*}
u^1(y_1, y_2) &> u^1(z_1, z_2) \\
u^2(y_1, y_2) &> u^2(z_1, z_2)
\end{align*}\]

Bundle \(y\) is at least as good for everyone and better for at least one person.

What about this allocation?

No!

Makes Taylor MUCH better off

But makes Kendrick worse off

Pareto ranking is not complete: Not all bundles can be compared.
Now set up the Lagrangian:

\[ L(x_1, x_2, \omega) = u(x_1, x_2) - \rho(u(x_2w_2 + x_1w_1 + x_2) - u) \]

Taking derivatives:

\[ \frac{\partial L}{\partial x_1} = \frac{\partial u}{\partial x_1} - \rho \frac{\partial u}{\partial x_1} = 0 \]
\[ \frac{\partial L}{\partial x_2} = \frac{\partial u}{\partial x_2} - \rho \frac{\partial u}{\partial x_2} = 0 \]
\[ \frac{\partial L}{\partial \omega} = \frac{\partial u}{\partial \omega} = 0 \]

\[ \frac{\partial L}{\partial \omega} = u(x_2w_2 + x_1w_1 + x_2) - u = 0 \]

Using the first two equations gives

\[ \frac{\partial u}{\partial x_1} = \frac{\partial u}{\partial x_2} = 0 \]
Finding Pareto Optimal Points

- In other words, \( MRS_{x_1} = MRS_{x_2} \)
- Slope of the indifference curve of consumer 1 is the same as that of 2
- For a Pareto optimum, the rate at which consumer 1 trades off good a for b is the same as the rate at which consumer 2 trades off good a for b.
- This makes sense: say consumer 1 'valued' a more than consumer 2, i.e. consumer 1 was prepared to give up more b to get one unit of a than was consumer 2.
- Could this be a Pareto optimum?
  - No! Could make both consumers better off by giving 1 more of a and 2 more than b.

The Contract Curve

- Remember the Social Planner's problem is given by
  1. **CHOOSE** \((x_1^1, x_2^1) \ (x_1^2, x_2^2)\)
  2. **IN ORDER TO MAXIMIZE** \(u_1 (x_1^1, x_2^1)\)
  3. **SUBJECT TO**
     - \(u_1 (x_2^1, x_2^2) = u\)
     - Feasibility \(x_1^1 w_1 + x_2^1 w_2 = x_1^2 w_1 + x_2^2 w_2\)
- There are many such problems, with different levels of u for consumer 2.
  - i.e. different indifference curves.
  - Each of these problems has a different solution.
  - The set of solutions to all such problems is the set of Pareto optimal points.
  - This is sometimes also called the contract curve.

Equilibrium and Pareto Optimality

- So we now have two different classifications of allocations.
  - What we think will happen (the equilibrium of the economy).
  - What we think should happen (Pareto optimality).
- A natural question is: what is the relationship between these two?
  - Specifically, we may want to ask two questions
    1. Are equilibria Pareto efficient?
    2. Are Pareto efficient points equilibria?
  - To give away the punchline, the answer to both questions is a qualified yes.
  - These are two of the most fundamental theorems in economics.
  - Hence the names!
A Worked Example

First we will show that market equilibria are Pareto efficient.

Last week we calculated the equilibrium for the following economy:

1. The endowment of each agent
   - \( \mathbf{w}_1 = 3 \)
   - \( \mathbf{w}_2 = 2 \)
   - \( \mathbf{w}_3 = 1 \)
   - \( \mathbf{w}_4 = 5 \)

2. The preferences of each agent
   - \( u_1(x_1, x_2) = x_1 \)
   - \( u_2(x_1, x_2) = x_2 \)

The equilibrium allocations were:

- \( x_1^1 = \frac{3}{2} \)
- \( x_1^2 = \frac{1}{2} \)
- \( x_1^3 = \frac{1}{2} \)
- \( x_1^4 = 2 \)

And equilibrium price was \( \rho = 7 \).

Is this Pareto optimal?

Let's check:

First, fix the utility of person 2 at the level achieved in equilibrium:

- \( u_2^*(x_1) = \frac{25}{18} \)

Now figure out the maximal utility of consumer 1 given feasibility and making sure that consumer 2 gets the above utility.

From before, problem becomes:

1. CHOOSE \((x_1, x_2)\)
2. IN ORDER TO MAXIMIZE \( u_1(x_1, x_2) = x_1 \)
3. SUBJECT TO
   - \( u_2(x_1, x_2) = x_2 \)

First set up the Lagrangian:

\[
\mathcal{L}(x_1, x_2, \mu) = x_1 - \mu((4 - x_1)(7 - x_1))
\]

Taking derivatives:

- \( \frac{\partial \mathcal{L}}{\partial x_1} = x_1 + \mu(7 - x_1) = 0 \)
- \( \frac{\partial \mathcal{L}}{\partial x_2} = 4 - x_1 = 0 \)
- \( \frac{\partial \mathcal{L}}{\partial \mu} = ((4 - x_1)(7 - x_1)) = 729 \)

Using the first two equations gives:

- \( x_1 = 7/4 \)
- \( x_2 = 27/54 \)

Using the fact that \( x_1 = (4 - x_2) \) and \( x_2 = (7 - x_1) \), this implies that:

- \( x_2 = 7/4 \)

And so, plugging into the constraint on the utility of consumer 2:

\[
\frac{1}{2} (x_1)^2 = \frac{25}{18} \text{ or } x_1 = \frac{7}{4}
\]

Plugging back in to the above identities will recover the rest of the competitive equilibrium.
A Worked Example

- So this particular equilibrium is also Pareto optimal.
- Why?
- Magic!
- (It's not Magic)
- The key observation is the following:
  - For Pareto optima, the MRS of consumer 1 equals the MRS of consumer 2.
  - For a market equilibrium the MRS of consumer 1 equals the price ratio and the MRS of consumer 2 equals the price ratio.
- And therefore equal each other.

The First Fundamental Theorem of Welfare Economics

- Now you should be asking the question: was there something special about this particular example?
- No!
- The First Fundamental Theorem of Welfare Economics: if preferences are monotonic, then any competitive equilibrium is Pareto efficient.
- This result is so fundamental that we are going to prove it!

Proof of the FFTWE (by contradiction)

- Assume that $\mathbf{x}^1$, $\mathbf{x}^2$ are equilibrium allocations for some price $\mathbf{p}$.
- But they are not a Pareto optimal.
- Then there exists another feasible allocation $\mathbf{y}^1$, $\mathbf{y}^2$ such that $u^1(y^1, y^2) \geq u^1(x^1, x^2)$ and $u^2(y^1, y^2) \geq u^2(x^1, x^2)$.
- And $u^1(y^1, y^2) > u^1(x^1, x^2)$ or $u^2(y^1, y^2) > u^2(x^1, x^2)$.
- Without loss of generality, assume that $u^2(y^1, y^2) > u^2(x^1, x^2)$.

Adding these two up gives $p_0x^1_1 + y^1_1 > p_0x^2_1 + y^2_1$, which is only possible if either $y^1_2 + y^2_2 > w^1_2 + w^2_2$ or $y^1_2 + y^2_2 > w^1_2 + w^2_2$.
- Either way, $(y^1, y^2)$, $(x^1, x^2)$ are not feasible.
- Contradiction!
The First Fundamental Theorem of Welfare Economics

- This is a phenomenally powerful result
- Though see caveats in next section
- The basis of much of 'free market' economics
- To see how powerful, try to think of other ways of allocating goods to the two people in the economy that are guaranteed to be Pareto optimal
- NOT
  - Making people eat their endowment
  - Giving everyone the same amount of each good
  - Letting the government decide what each person gets
  - Fix the price, let one consumer choose how much they want to buy, then ration the other

Summary

- Today we have done the following
  1. Defined the concept of Pareto dominance and Pareto optimality as a measure of welfare
  2. Introduced the first fundamental theorems of welfare economics
     1. FFTWE: A competitive equilibrium is Pareto efficient