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Intermediate Microeconomics W3211

Lecture 8: Equilibrium and Efficiency 1

Columbia University, Spring 2016

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Introduction

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The Story So Far….

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- We have solved the consumer's problem
  - Determined what we think people will do given prices and income
- We have solved for equilibrium in an endowment economy
  - Determined what we think prices and allocations will be
- This is quite impressive!
  - We have our first prediction of how a simple economy works!
- Granted it is an economy that only has consumers in it
  - We will get to firms soon enough

Today's Aims

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- We are now going to talk about the welfare properties of an equilibrium
- Arguably this is one of the most interesting, but also misunderstood lectures in the entire course
- It is where economists sometimes stop being scientists and start being policy makers
  - Positive economics: what will happen
  - Normative economics: What should happen

Today's Aims

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- Begin by defining the concept of Pareto optimality
  - Most economists would agree that if an allocation is 'good' it should be Pareto optimal
- Then introduce the first 'fundamental theorems of welfare economics'
  - Describes the relationship between Pareto optimality and Market equilibria

Pareto Optimality

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Which Allocations Do You Prefer?

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Allocation x		Allocation y	
Kendrick	Taylor	Kendrick	Taylor
3	5	5	3
10	10	1	1
100	10	10	10
10	10	9.9	10000

- Allocations describe the utility that each person gets
- If you as the government could choose between allocation X and allocation Y which would you choose?

Pareto Ranking

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- Economists use a very specific way of comparing allocations:

Definition: Let (x_a^1, x_b^1), (x_a^2, x_b^2) be an allocation in an economy. We say it is **Pareto dominated** by (y_a^1, y_b^1), (y_a^2, y_b^2) if

$$u^1(y_a^1, y_b^1) \geq u^1(x_a^1, x_b^1) \text{ and}$$
$$u^2(y_a^2, y_b^2) \geq u^2(x_a^2, x_b^2)$$

And

$$u^1(y_a^1, y_b^1) > u^1(x_a^1, x_b^1) \text{ or}$$
$$u^2(y_a^2, y_b^2) > u^2(x_a^2, x_b^2)$$

- Bundle y is at least as good for everyone and better for at least one person

Which Allocations Do You Prefer?

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Which Allocations Do You Prefer?

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Which Allocations Do You Prefer?

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Kendrick	Taylor	Kendrick	Taylor
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- What about this allocation?
- No!
- Makes Taylor MUCH better off
- But makes Kendrick worse off
- Pareto ranking is not **complete:** Not all bundles can be compared

Which Allocations Do You Prefer?

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Allocation x		Allocation y	
Kendrick	Taylor	Kendrick	Taylor
10	10	9.9	10000

- Why are we not prepared to say that y is better that x
  - Kendrick only loses 0.1 units of utility, but Taylor gains 9990
  - Surely this is a good deal?

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Which Allocations Do You Prefer?

Allocation x		Allocation y	
Kendrick	Taylor	Kendrick	Taylor
10	10	9.9	10000

- Not necessarily, for two reasons
  1. Remember, utility numbers don't mean anything beyond bigger or smaller
     - We could find another utility representation which would mean that Taylor only gains 0.0001 units
  2. Even if we equated utility with happiness, we would be making interpersonal comparisons
     - Would you be prepared for one person to lose 1000 utility units if 1000 people gained 1 utility unity each?
     - Some people might say yes, some no
     - Everyone should agree that Pareto optimality is a good thing
- You (as a person) may prefer y to x. You (as an economist) do not

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Pareto Optimality

- Along with the definition of Pareto dominance comes the definition of **Pareto optimality**

Definition: An allocation is **Pareto optimal** if it is not Pareto dominated by any other **feasible** bundle

- i.e., in our simple 2-person, 2-good economy, a bundle is Pareto optimal if there is no other way of dividing up the endowments to make at least one person better off without making the other person worse off
- Pareto optimality is also sometimes referred to as **Pareto Efficiency**

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Finding Pareto Optimal Points

- We can find Pareto Optimal points by solving a constrained optimization problem:

1. **CHOOSE** (x_a^1, x_b^1), (x_a^2, x_b^2)
2. **IN ORDER TO MAXIMIZE** $u^1\,(x_a^1, x_b^1)$
3. **SUBJECT TO**
   - $u^2(x_a^2, x_b^2) = \text{u}$
   - Feasibility $x_a^1 + x_a^2 = w_a^1 + w_a^2$ and $x_b^1 + x_b^2 = w_b^1 + w_b^2$

- This is sometimes called the **social planner's problem**
- Maximize the utility of consumer 1 while fixing consumer 2's utility at u
- The solution must be Pareto optimal
  - No way to improve the utility of consumer 1 without reducing the utility of consumer 2

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Finding Pareto Optimal Points

1. **CHOOSE** (x_a^1, x_b^1), (x_a^2, x_b^2)
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   - $u^2(x_a^2, x_b^2) = \text{u}$
   - Feasibility $x_a^1 + x_a^2 = w_a^1 + w_a^2$ and $x_b^1 + x_b^2 = w_b^1 + w_b^2$

- What do solutions to the social planner's problem look like?
- We can solve the problem in two stages
- First, use feasibility to get rid of x_a^2 and x_b^2
  - $x_a^2 = w_a^1 + w_a^2 - x_a^1$
  - $x_b^2 = w_b^1 + w_b^2 - x_b^1$

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Finding Pareto Optimal Points

- Problem becomes

1. **CHOOSE** (x_a^1, x_b^1)
2. **IN ORDER TO MAXIMIZE** $u^1(x_a^1, x_b^1)$
3. **SUBJECT TO**
   - $u^2(w_a^1 + w_a^2 - x_a^1, w_b^1 + w_b^2 - x_b^1) = \text{u}$

- Now set up the Lagrangian:

$$L(x_a^1, x_b^1, \mu) = u(x_a^1, x_b^1) - \mu(u(w_a^1 + w_a^2 - x_a^1, w_b^1 + w_b^2 - x_b^1) - \text{u}\,)$$

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Finding Pareto Optimal Points

$$L(x_a^1, x_b^1, \mu) = u^1(x_a^1, x_b^1) - \mu(u^2(w_a^1 + w_a^2 - x_a^1, w_b^1 + w_b^2 - x_b^1) - \text{u})$$

- Taking derivatives:

$$\frac{\partial L}{\partial x_a^1} = \frac{\partial u^1}{\partial x_a^1} - \mu\frac{\partial u^2}{\partial x_a^2} = 0$$

$$\frac{\partial L}{\partial x_b^1} = \frac{\partial u^1}{\partial x_b^1} - \mu\frac{\partial u^2}{\partial x_b^2} = 0$$

$$\frac{\partial L}{\partial \mu} = u^2(w_a^1 + w_a^2 - x_a^1, w_b^1 + w_b^2 - x_b^1) - \text{u} = 0$$

- Using the first two equations gives

$$\frac{\partial u^1}{\partial x_a^1} \Big/ \frac{\partial u^1}{\partial x_b^1} = \frac{\partial u^2}{\partial x_a^2} \Big/ \frac{\partial u^2}{\partial x_b^2}$$

Finding Pareto Optimal Points

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- In other words
  $$MRS^1_{a,b} = MRS^2_{a,b}$$
- Slope of the indifference curve of consumer 1 is the same as that of 2
- For a Pareto optimum, the rate at which consumer 1 trades off good a for b is the same as the rate at which consumer 2 trades off good a for b
- This makes sense: say consumer 1 'valued' a more than consumer 2
  - i.e. consumer 1 was prepared to give up more b to get one unit of a than was consumer 2
- Could this be a Pareto optimum?
- No! Could make both consumers better off by giving 1 more of a and 2 more than b

Pareto Efficiency in the Edgeworth Box

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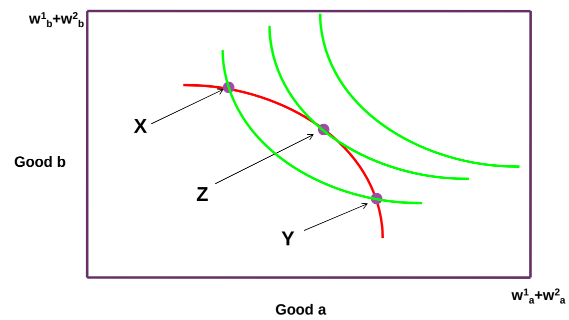

The Contract Curve

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- Remember the Social Planner's problem is given by

1. **CHOOSE** (x_a^1, x_b^1), (x_a^2, x_b^2)
2. **IN ORDER TO MAXIMIZE** $u^1\ (x_a^1, x_b^1)$
3. **SUBJECT TO**
   - $u^2(x_a^2, x_b^2) = \mathrm{u}$
   - Feasibility $x_a^1 + x_a^2 = w_a^1 + w_a^2$ and $x_b^1 + x_b^2 = w_b^1 + w_b^2$

- There are many such problems, with different levels of u for consumer 2
  - I.e. different indifference curves
- Each of these problems has a different solution
- The set of solutions to all such problems is the set of Pareto optimal points
- This is sometimes also called the **contract curve**

The Contract Curve

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Equilibrium and Pareto Optimality

The First Fundamental Theorem of Welfare Economics

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Equilibrium and Pareto Optimality

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- So we now have two different classifications of allocations
  - What we think will happen (the equilibrium of the economy)
  - What we think should happen (Pareto optimality)
- A natural question is: what is the relationship between these two?
- Specifically, we may want to ask two questions
  1. Are equilibria Pareto efficient?
  2. Are Pareto efficient points equilibria?
- To give away the punchline, the answer to both questions is a qualified yes
- These are two of the most fundamental theorems in economics
- Hence the names!

A Worked Example

- First we will show that market equilibria are Pareto efficient
- Last week we calculated the equilibrium for the following economy

1. The **endowment** of each agent
   - w_a^1=3
   - w_b^1=2
   - w_a^2=1
   - w_b^2=5
2. The **preferences** of each agent
   - $u^1(x_a^1, x_b^1)$=$x_a^1 x_b^1$
   - $u^2(x_a^2, x_b^2)$=$x_a^2 x_b^2$

A Worked Example

- The equilibrium allocations were

$$x_a^1(p_a^*, w_a^1, w_b^1) = \frac{3}{2} + \frac{4}{7} = \frac{29}{14}$$
$$x_b^1(p_a^*, w_a^1, w_b^1) = \frac{21}{8} + 1 = \frac{29}{8}$$
$$x_a^2(p_a^*, w_a^2, w_b^2) = \frac{1}{2} + \frac{20}{14} = \frac{27}{14}$$
$$x_b^2(p_a^*, w_a^2, w_b^2) = \frac{7}{8} + \frac{5}{2} = \frac{27}{8}$$

- And equilibrium price was $p_a^* = \frac{7}{4}$
- Is this Pareto optimal?

A Worked Example

- Let's check
- First, fix the utility of person 2 at the level achieved in equilibrium:

$$u^2(x_a^2, x_b^2) = \frac{27}{14}\frac{27}{8} = \frac{729}{112}$$

- Now figure out the maximal utility of consumer 1 given feasibility and making sure that consumer 2 gets the above utility

A Worked Example

- From before, problem becomes

1. **CHOOSE** (x_a^1, x_b^1)
2. **IN ORDER TO MAXIMIZE** $u^1(x_a^1, x_b^1)$=$x_a^1 x_b^1$
3. **SUBJECT TO**
   - $u^2(w_a^1+w_a^2 - x_a^1, w_b^1+w_b^2 - x_b^1) = (4 - x_a^1)(7 - x_b^1) = \frac{729}{112}$

- First set up the Lagrangian:

$$L(x_a^1, x_b^1, \mu) = x_a^1 x_b^1 - \mu((4 - x_a^1)(7 - x_b^1) - \frac{729}{112})$$

A Worked Example

$$L(x_a^1, x_b^1, \mu) = x_a^1 x_b^1 - \mu((4 - x_a^1)(7 - x_b^1) - \frac{729}{112})$$

- Taking derivatives:

$$\frac{\partial L}{\partial x_a^1} = x_b^1 + \mu(7 - x_b^1) = 0$$
$$\frac{\partial L}{\partial x_b^1} = x_a^1 + \mu(4 - x_a^1) = 0$$
$$\frac{\partial L}{\partial \mu} = ((4 - x_a^1)(7 - x_b^1) - \frac{729}{112})$$

- Using the first two equations gives

$${x_b^1}/{x_a^1} = {(7 - x_b^1)}/{(4 - x_a^1)}, \text{ or}$$
$$x_b^1 = \frac{7x_a^1}{4}$$

A Worked Example

$$x_b^1 = \frac{7x_a^1}{4}$$

- Using the fact that $x_a^2 = (4 - x_a^1)$ and $x_b^2 = (7 - x_b^1)$, this implies that

$$x_b^2 = \frac{7x_a^2}{4}$$

- And so, plugging into the constraint on the utility of consumer 2

$$\frac{7}{4}(x_a^2)^2 = \frac{729}{112} \text{ or}$$
$$x_a^2 = \frac{27}{14}$$

- Plugging back in to the above identities will recover the rest of the competitive equilibrium

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A Worked Example

- So this particular equilibrium is also Pareto optimal
- Why?
- Magic!
- (It's not Magic)
- The key observation is the following:
  - For Pareto optima, the MRS of consumer 1 equals the MRS of consumer 2
  - For a market equilibrium the MRS of consumer 1 equals the price ratio and the MRS of consumer 2 equals the price ratio
    - And therefore equal each other

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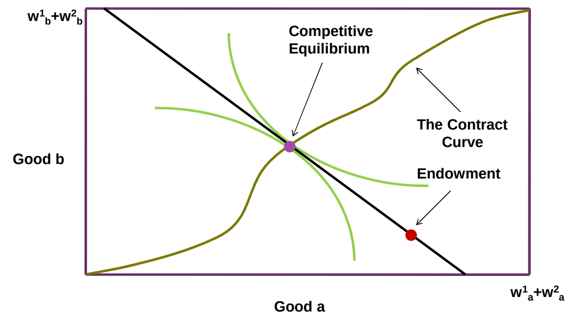
Competitive Equilibria Lie on the Contract Curve



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The First Fundamental Theorem of Welfare Economics

- Now you should be asking the question: was there something special about this particular example?
- No!

The First Fundamental Theorem of Welfare Economics: If preferences are monotonic, then any competitive equilibrium is Pareto efficient

- This result is so fundamental that we are going to prove it!

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The First Fundamental Theorem of Welfare Economics

- Proof of the FFTWE (by contradiction)
- Assume that (x_a^1, x_b^1), (x_a^2, x_b^2) are equilibrium allocations for some price p_a
- But they are not a Pareto optimal
- Then there exists another feasible allocation (y_a^1, y_b^1), (y_a^2, y_b^2) such that

$$u^1(y_a^1, y_b^1) \geq u^1(x_a^1, x_b^1) \text{ and}$$
$$u^2(y_a^2, y_b^2) \geq u^2(x_a^2, x_b^2)$$

And

$$u^1(y_a^1, y_b^1) > u^1(x_a^1, x_b^1) \text{ or}$$
$$u^2(y_a^2, y_b^2) > u^2(x_a^2, x_b^2)$$

- Without loss of generality, assume that $u^2(y_a^2, y_b^2) > u^2(x_a^2, x_b^2)$

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The First Fundamental Theorem of Welfare Economics

- Now, as (x_a^1, x_b^1), (x_a^2, x_b^2) is part of an equilibrium, and $u^2(y_a^2, y_b^2) > u^2(x_a^2, x_b^2)$, it must be the case that that consumer 2 could not afford y_a^2, y_b^2 given prices p_a
  - This follows from optimality, so
  $$p_a y_a^2 + y_b^2 > p_a w_a^2 + w_b^2$$
- Similarly, as for consumer 1 $u^1(y_a^1, y_b^1) \geq u^1(x_a^1, x_b^1)$, it must be the case that
$$p_a y_a^1 + y_b^1 \geq p_a w_a^1 + w_b^1$$
- (Note that this is where we are using monotonicity)

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The First Fundamental Theorem of Welfare Economics

- Adding these two up gives
$$p_a y_a^2 + y_b^2 + p_a y_a^1 + y_b^1 > p_a w_a^2 + w_b^2 + p_a w_a^1 + w_b^1$$
- Or, rearranging
$$p_a(y_a^2 + y_a^1 - w_a^2 - w_a^1) + (y_b^2 + y_b^1 - w_b^2 - w_b^1) > 0$$
- Which is only possible if either
$$y_a^2 + y_a^1 > w_a^2 + w_a^1 \text{ or}$$
$$y_b^2 + y_b^1 > w_b^2 + w_b^1$$
- Either way, (y_a^1, y_b^1), (y_a^2, y_b^2) are not feasible
- Contradiction!

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The First Fundamental Theorem of Welfare Economics

- This is a phenomenally powerful result
  - Though see caveats in next section
- The basis of much of 'free market' economics
- To see how powerful, try to think of other ways of allocating goods to the two people in the economy that are guaranteed to be Pareto optimal
- NOT
  - Making people eat their endowment
  - Giving everyone the same amount of each good
  - Letting the government decide what each person gets
  - Fix the price, let one consumer choose how much they want to buy, then ration the other

Summary

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Summary

- Today we have done the following

1. Defined the concept of Pareto dominance and Pareto optimality as a measure of welfare
2. Introduced the first fundamental theorems of welfare economics
   1. FFTWE: A competitive equilibrium is Pareto efficient