1

Intermediate Microeconomics W3211

Lecture 11: Perfect Competition 1: The Firm's Problem

Columbia University, Spring 2016
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Introduction

2

The Story So Far….

3

- We have now thought very carefully about how to analyze an economy with only one type of economic agent
- The consumer!
- We solved the consumer's problem
  - Thought about how they would behave in isolation, given prices and income
- Thought about what an economy consisting of many consumers would look like
  - Equilibrium

The Next Stage

4

- One obvious omission from our analysis so far
- The world consists of different types of economic agents!
- We are now going to partly rectify this problem
- Introduce a second type of economic agent: The firm
- Our analysis will take the same path as it did with the consumer
  - Set up and solve the firm's problem
  - Think about an economy in which firms and consumers interact together
- However, we will have some choices to make along the way
  - About what it is that firms get to choose

Today

5

- Think about what a firm is
- Think about how to set up the firm's problem
  - Describe various versions of the problem
  - Pick the version we are going to start off working with
    - Perfect competition
- Solve the simplest version of the firm's problem
- Varian Ch. 19-23, Feldman and Serrano Ch. 8

The Firm's Problem

Also: what is a firm?

6

7

The Firm

- Just as with the consumer, we want a model which will allow us to make predictions about how a firm will behave
- Just as with the consumer, we are going to assume that firms are **smart**
  - They make the best possible decisions given their goals and constraints
  - Allow us to model firms using constrained optimization
  - Arguably a better assumption for the firm than it was for the consumer
- In order to do so, we want to think a little bit about what a firm is
  - What does it do?
  - What is its mission?
  - How does it differ from a consumer?
- Over to you….

8

The Firm

1. A firm converts things from one type to another
2. It does so to maximize profits

9

The Firm

1. A firm **converts** things from one type to another

- Think about a mining company
  - Things go in to the company
    - Effort of workers
    - Machinery
    - Electricity
    - Land
  - Things come out of the company
    - Coal
- We call these
  - **Inputs**: $x_1, x_2, x_3, \ldots$.
  - **Output**: y
- The rate at which it does this is called the firm's **technology**
  - $f(x_1, x_2, x_3, \ldots)$ is the maximum amount of output the firms can produce if it uses inputs $x_1, x_2, x_3, \ldots$.
  - This is called the **production function**

10

The Firm

- Arguably all firms can be thought of this way, even if this is not obvious at first glance
- Shop?
  - Inputs: goods at the warehouse, labor, downtown boutique
  - Output: goods on the high street

11

The Firm

2. A firm **maximizes profit**

- Say that the output is sold at a unit price p_y
- Input i can be bought at price p_i
  - e.g. if input 1 is labor then p_1 is the wage rate
- The profit of selling y units and using inputs $x_1, x_2, x_3, \ldots$ is
$$p_y y - p_1 x_1 - p_2 x_2 - p_3 x_3 \ldots.$$

12

The Firm

- Why might firms maximize profits?
  - Firms are owned by **individuals**
  - Privately owned, partnerships, corporations
  - More profit for the firm means more income for the individual
  - Allows them to move to more preferred consumption bundles
- Is it always the case that firms maximize profits?
  - No! Social concerns, empire building, satisfaction
  - But it will be a useful simplification to start off with!

13

The Firm's Problem

- We want to model the firm as solving a constrained optimization problem

1. **CHOOSE** <some alternative>
2. **IN ORDER TO MAXIMIZE** <some objective>
3. **SUBJECT TO** <some constraints>

14

What Do Firms Choose?

- First what do they choose?
- This is not necessarily an easy question
- First, do firms get to set prices?
- On the one hand, we didn't let consumer's set prices
  - They could only buy and sell as much as they wanted at the market price
- On the other, it seems that firms get to choose what prices they charge for stuff
  - Apple gets to set the price of an i-phone
  - Vivian Westwood gets to set the price of her men's navy porcelain rose t-shirt

15

What Do Firms Choose?

16

What Do Firms Choose?

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17

What Do Firms Choose?

- So what should we assume?
- There is no right answer to this question
- Different assumptions may be appropriate for different markets
- We will begin by studying one **extreme** case
- Perfect competition:
  - Firms can sell as much of their product as they like at the market price
  - Treat the market price as **exogenous** (i.e. something they do not get to choose)
- Appropriate when there are many **small** producers selling the **same** thing
  - E.g. wheat farmers in the US
  - Coffee producers in Nicaragua
- The amount that each firm produces is too small to affect the market price

18

What Do Firms Choose?

- Later in the course we will consider another extreme case
- Monopoly:
  - One **large** producer
  - Firm gets to set whatever price they want
  - The amount that they can sell is determined by the demand function of the consumer
- Even later, we will think about some middle cases
  - Duopoly
  - Oligopoly
- We will need some new tools to be able to think about this
  - This is a **strategic** setting
  - Firm A needs to think about the price that firm B will set, and visa versa
  - Requires the tools of **game theory**

19

The Firm's Problem – Perfect Competition

- We want to model the firm as solving a constrained optimization problem
1. **CHOOSE <some alternative>**
2. **IN ORDER TO MAXIMIZE <some objective>**
3. **SUBJECT TO <some constraints>**
- So firms do not get to choose prices
- What do they choose?
  - Inputs and output!

20

The Firm's Problem – Perfect Competition

- We want to model the firm as solving a constrained optimization problem
1. **CHOOSE** $y, x_1, x_2 \ldots$ each greater than equal to zero
2. **IN ORDER TO MAXIMIZE <some objective>**
3. **SUBJECT TO <some constraints>**
- So firms do not get to choose prices
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21

The Firm's Problem – Perfect Competition

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  - Inputs and output!
- In order to maximize….

22

The Firm's Problem – Perfect Competition

- We want to model the firm as solving a constrained optimization problem
1. **CHOOSE** $y, x_1, x_2 \ldots$ each greater than equal to zero
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 x_1 - p_2 x_2 - p_3 x_3$
3. **SUBJECT TO <some constraints>**
- So firms do not get to choose prices
- What do they choose?
  - Inputs and output!
- In order to maximize….
  - Profit

23

The Firm's Problem – Perfect Competition

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  - Inputs and output!
- In order to maximize….
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- Subject to….

24

The Firm's Problem – Perfect Competition

- We want to model the firm as solving a constrained optimization problem
1. **CHOOSE** $y, x_1, x_2 \ldots$ each greater than equal to zero
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 x_1 - p_2 x_2 - p_3 x_3$
3. **SUBJECT TO** $y \leq f(x_1, x_2, x_3, \ldots)$
- So firms do not get to choose prices
- What do they choose?
  - Inputs and output!
- In order to maximize….
  - Profit
- Subject to….
  - Technology (i.e. the production function)

The Firm's Problem with One Input

25

The Case of One Input

26

- We are going to begin by solving the simplest version of the consumer problem
- Where there is only one input

1. **CHOOSE** $y \geq 0, x_1 \geq 0$
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 x_1$
3. **SUBJECT TO** $y \leq f(x_1)$

The Case of One Input

27

- There are two possible interpretations

1. The firm only requires one input
   - E.g. comedian: effort in, jokes out
2. We are thinking of the **short run** problem of the firm
   - E.g. a factory employs people and machines
   - Can change the number of workers at any time
   - Machines take longer to change
   - In the short run the firm chooses only one input: labor
   - Treats number of machines as fixed

The Case of One Input

28

1. **CHOOSE** $y \geq 0, x_1 \geq 0$
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 x_1$
3. **SUBJECT TO** $y \leq f(x_1)$

- So how can we solve this problem?
- We are going to try three different approaches, each of which will (hopefully) give some insight

1. Pictures
2. Substituting out to remove output
3. Substituting out to remove labor (next lecture)

The Firm's Problem with One Input

Solving with Pictures

29

Approach 1: Pictures

30

- In the case of the consumer problem, we found it useful to draw pictures to get some intuition
- We drew one line that represented the constraint
  - Budget constraint in the consumer problem
  - Technology constraint here
- And a bunch of other lines which represented the objective
  - Indifference curves for the consumer
  - Iso profit lines for the firm

Approach 1: Pictures

31

- Iso-profit lines

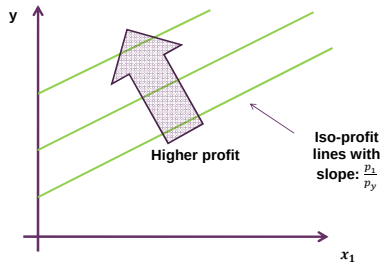
$$\pi = p_y y - p_1 x_1$$

- So

$$y = \frac{\pi}{p_y} + \frac{p_1}{p_y} x_1$$

Iso Profit Lines

32



Approach 1: Pictures

33

- What about the constraint $y \leq f(x_1)$?
- Depends on what we assume about the technology
- We are going to make three assumptions to make our life easier

1. $f(x_1) > 0$: always produce a positive amount
2. $\frac{df}{dx_1} > 0$: The more of the input, the more is produced
3. $\frac{d^2 f}{dx_1^2} < 0$: Decreasing marginal productivity

- Why (3)?
  - Hire better workers first?
  - More workers per machine makes them less productive?
- Also called **decreasing returns to scale**
  - Doubling input does not double output

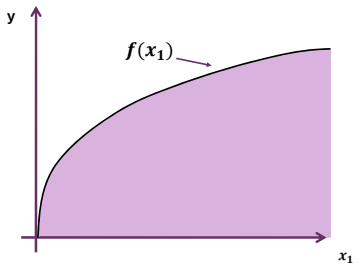
Approach 1: Pictures

34

- Assumption will make it easy to solve problem
  - Though it is easy to think of cases when it is not true
- These assumptions mean that the technology constraint looks like this

The Technology Constraint

35



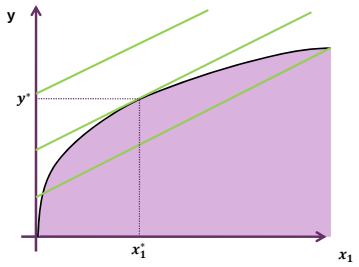
Approach 1: Pictures

36

- So now we have a picture of
  - The constraint
  - What we want to maximize
- How do we solve?
- Same way as we did in the consumer's problem!

Solving the Firm's Problem

37



Approach 1: Pictures

38

- As with the consumer's problem, we play the game of trying to get onto the highest possible iso-profit line while staying in the technological constraint
- As with the consumer's problem, there are three types of solution
  - Tangency points
  - Kinks
  - Corner solutions
- We will focus on tangency points
  - Don't forget the other types of solution!

Approach 1: Pictures

39

- What does a tangency point look like in this case?
- Iso profit lines are

$$y = \frac{\pi}{p_y} + \frac{p_1}{p_y}x_1$$

- And so have slope $\frac{p_1}{p_y}$
- The technology constraint is given by $f(x_1)$
- And so has slope $\frac{df}{dx_1} = f'(x_1)$
- So tangency condition is

$$f'(x_1)=\frac{p_1}{p_y}$$

A Worked Example

40

- Lets assume that technology is given by

$$f(x_1)=(x_1)^{\frac{1}{2}}$$

- Then the tangency condition says

$$f'(x_1)=\frac{p_1}{p_y}$$

$$\frac{1}{2(x_1)^{\frac{1}{2}}} = \frac{p_1}{p_y}$$

$$x_1^* = \left(\frac{p_y}{2p_1}\right)^2 \text{ and } y^* = \frac{p_y}{2p_1}$$

A Worked Example

41

$$x_1^* = \left(\frac{p_y}{2p_1}\right)^2 \text{ and } y^* = \frac{p_y}{2p_1}$$

- $y^*(p_y, p_1)$ is the firm's **supply function**
  - Reports how much the firm is willing to supply given prices
  - Like the demand curve of the consumer – says how supply changes with parameters of the problem
- $x_1^*(p_y, p_1)$ is the firm's **demand function** (for input x_1^*)

The Firm's Problem with One Input

Substituting to Remove Output

42

Approach 2: Get rid of output

43

- Remember, the consumer's problem is

1. **CHOOSE** y, x_1
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 x_1$
3. **SUBJECT TO** $y \leq f(x_1)$

- Assuming that prices are positive, the firm will never throw away output
- It will always be optimal to set $y = f(x_1)$
- We can rewrite as an unconstrained problem

1. **CHOOSE** x_1
2. **IN ORDER TO MAXIMIZE** $p_y f(x_1) - p_1 x_1$

Approach 2: Get rid of output

44

- So we want to maximize

$$p_y f(x_1) - p_1 x_1$$

- First order conditions give

$$p_y f'(x_1) - p_1 = 0 \text{ or}$$

$$f'(x_1) = \frac{p_1}{p_y}$$

- i.e. the tangency condition!
- Notice also that the second derivative of the objective function is $p_y \frac{d^2 f}{dx_1^2}$
- We want this to be **negative** for an optimum
- Which it will be if $\frac{d^2 f}{dx_1^2} < 0$ i.e. diminishing marginal productivity

An Alternative Interpretation

45

- Remember that the profit function is given by

$$p_y f(x_1) - p_1 x_1$$

Revenue of using x_1 minus cost of using x_1

- First order conditions give

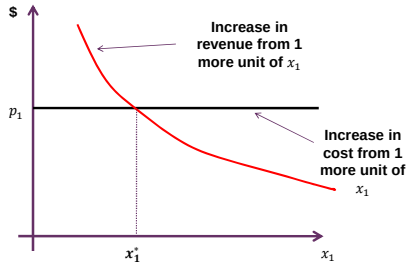
$$p_y f'(x_1) = p_1$$

Marginal Revenue at x_1 equals marginal cost

- This makes sense: keep employing workers until the additional cost of an additional worker outweighs the additional revenue
- Notice, diminishing marginal productivity implies marginal revenue decreases as x_1 increases

Marginal Revenue and Marginal Cost

46



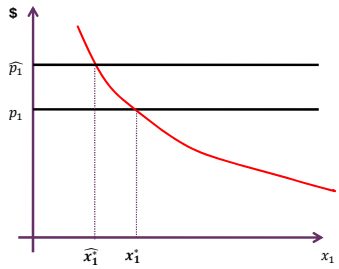
Comparative Statics

47

- We can use these graphs to easily see what happens to output as prices change
- First, what happens when p_1 increases to $\widehat{p_1}$?

Increase in Input Prices

48



Comparative Statics

49

- We can use these graphs to easily see what happens to output as prices change
- First, what happens when p_1 increases to $\widehat{p_1}$
- Input and output fall
- Why?
  - Marginal cost rises
  - So marginal revenue also needs to rise
  - Given diminishing marginal productivity, means use of input has to fall

Comparative Statics

50

- We can also see this by differentiating the tangency condition with respect to p_1

$$f'(x_1^*(p_y, p_1)) = \frac{p_1}{p_y}$$

- implies

$$\frac{d^2f}{dx_1^2}\frac{dx_1^*}{dp_1} = \frac{1}{p_y}$$

- Right hand side is positive
- $\frac{d^2f}{dx_1^2}$ is negative
- Implies $\frac{dx_1^*(p_y, p_1)}{dp_1}$ must be negative

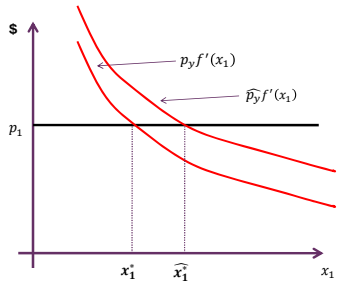
Comparative Statics

51

- Second, what happens when p_y increases to $\widehat{p_y}$
- Remember, marginal revenue given by $p_y f'(x_1)$

Marginal Revenue and Marginal Cost

52



Comparative Statics

53

- Second, what happens when p_y increases to $\widehat{p_y}$
- Inputs and outputs rise
- Why?
  - Marginal cost unchanged
  - Marginal revenue increases at every level of x_1
  - To equate marginal cost and marginal revenue, output must rise, so productivity falls

Comparative Statics

54

- Again, we can also see this by differentiating the tangency condition with respect to p_y

$$f'(x_1^*(p_y, p_1)) = \frac{p_1}{p_y}$$

- implies

$$\frac{d^2f}{dx_1^2}\frac{dx_1^*}{dp_y} = -\frac{p_1}{(p_y)^2}$$

- Right hand side is negative
- $\frac{d^2f}{dx_1^2}$ is negative
- Implies $\frac{dx_1^*(p_y, p_1)}{dp_y}$ must be positive

Summary

55

Summary

56

- Today we discussed what a firm is
- Set up the firm's problem for the case of **perfect competition**
- Solved the firm's problem for the simple case of a single input
- Derived the resulting supply function, and considered comparative statics