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Intermediate Microeconomics W3211

Lecture 12:
Perfect Competition 2: Cost Minimization

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Introduction

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The Story So Far….

- We have now introduced the idea of a firm
- An economic agent who
  - Buys inputs, converts them to output, which they then sell
  - Does so to maximize profit
- We defined the firms' problem

1. **CHOOSE** **inputs and output** - $y, x_1, x_2, \ldots$
2. **IN ORDER TO MAXIMIZE** **profit** - $p_y y - p_1 x_1 - p_2 x_2 - p_3 x_3 - \cdots$
3. **SUBJECT TO** **technological constraints** - $y \leq f(x_1, x_2, x_3, \ldots.)$

- Introduced two ways to solve a simple version of the firm's problem with only one input
  - Pictures
  - Substituting output out of the optimization problem

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Today

- Use a different method for solving the firm's problem
- Substitute to remove the **inputs** from the problem
  - Figure out the cost of producing any level of output
  - Figure out the level of output which maximizes profit
- Initially we will do this in the case where there is only one input
  - A bit redundant, as we already have two methods for solving this problem
  - But will help fix ideas
- We will then move on to the case of multiple inputs
  - The 'cost based' approach will come in very handy here

The Firm's Problem with One Input

Solving by Calculating the Cost Function

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The Case of One Input

- Remember: we are working with the simple version of the firm's problem where there is only one input

1. **CHOOSE** y, x_1
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 x_1$
3. **SUBJECT TO** $y \leq f(x_1)$

- As we have seen, one way to solve it is to use the constraint to get rid of output as a choice variable
- Another way is to remove x_1 as a choice variable
- This is going to seem a bit redundant (and long winded) in the case of only one input
- But it will be very useful when the problem gets more complicated

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Approach 3: Remove Output

- Assuming that firms don't throw away output we have
$$y = f(x_1)$$
- This implies
$$f^{-1}(y) = x_1$$
- Where f^{-1} is the **inverse** of the function f

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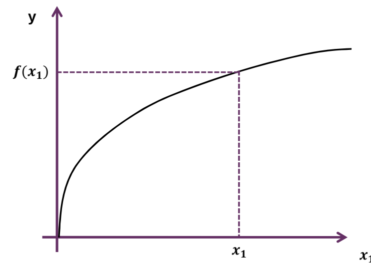
Inverse functions



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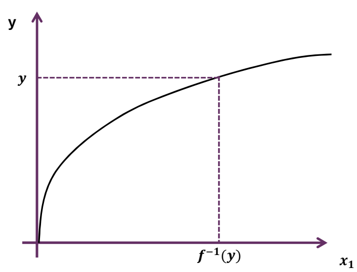
Inverse functions



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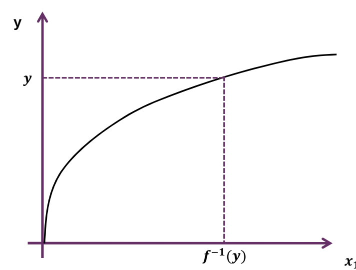
Inverse functions



- Do functions always have well behaved inverses?

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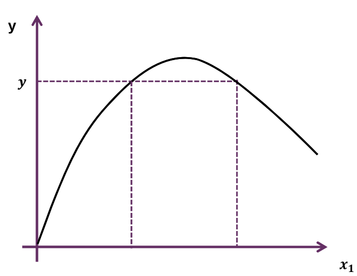
Inverse functions



- No!
- But they will do if the function is **strictly increasing** (which we have assumed)

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Approach 3: Remove Output

- We can therefore rewrite the firm's problem as

1. **CHOOSE** y
2. **IN ORDER TO MAXIMIZE** $p_y y - p_1 f^{-1}(y)$

- We can rewrite $p_1 f^{-1}(y)$ as $c(p_1, y)$: the cost of producing output y
- The profit function becomes
$$p_y y - c(p_1, y)$$

Revenue of producing y minus cost of producing y

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Approach 3: Remove Output

- The first order conditions become

$$p_y = \frac{\partial c(p_1, y)}{\partial y}$$

- Or

Marginal revenue = Marginal cost

MR(y)=MC(y)

- This make sense
- Keep producing output until the point at which the additional revenue from one extra unit is no more than the additional cost

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Marginal With Respect to What?

- Note that the terms ‘marginal revenue’ and ‘marginal cost’ are slightly confusing
- We have already used these terms last lecture to mean something different
- Marginal with respect to what?
- In lecture 1 it was with respect to an additional unit of **input**
- Here it is with respect to an additional unit of **output**
- Should always be precise, but if people aren’t this is usually what they mean

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Relationship between Marginal Productivity and Marginal Cost

- Remember

$$c(p_1, y) := p_1 f^{-1}(y)$$

- So

$$\frac{\partial c(p_1, y)}{\partial y} = p_1 \frac{\partial f^{-1}(y)}{\partial y}$$

- What does this look like?
- Luckily, we can use the inverse function theorem to write

$$\frac{\partial f^{-1}(y)}{\partial y} = \frac{1}{\partial f(x_1)/\partial x_1}$$

- Where $f(x_1) = y$

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Marginal Cost

- So

$$MC(y) = \frac{\partial c(p_1, y)}{\partial y} = \frac{p_1}{\partial f(x_1)/\partial x_1}$$

- So our assumptions about f tell us something about marginal cost

1. $\frac{df}{dx_1} > 0$: $MC(y)$ is **positive**
2. $\frac{d^2 f}{dx_1^2} < 0$: $MC(y)$ is **increasing**

- (2) is the property of **increasing marginal cost**
- Means that graphically we have

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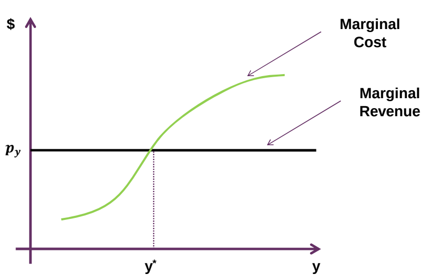
Marginal Revenue and Marginal Cost



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Average Cost

- It is also going to be handy for us to think about **Average Cost**

$$AC(y) = \frac{c(p_1, y)}{y}$$

- Why? Well, one reason is that profits are only positive if price is above average cost

$$\pi(y) = p_y y - c(p_1, y) > 0$$

- If and only if

$$p_y > \frac{c(p_1, y)}{y} = AC(y)$$

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Marginal Cost and Average Cost

- What is the relationship between Average Cost and Marginal Cost?
- Average cost
  - Rises if Marginal Cost is above Average Costs
  - Falls if Marginal Cost is below Average Cost
- To see this, take derivatives of average cost

$$AC(y)=\frac{c(p_1,y)}{y}$$

$$\frac{\delta AC(y)}{\delta y}=\frac{\frac{\delta c(p_1,y)}{\delta y}}{y}-\frac{c(p_1,y)}{y^2}$$

$$=\frac{1}{y}\left[\frac{\delta c(p_1,y)}{\delta y}-\frac{c(p_1,y)}{y}\right]$$

The Firm's Problem with Multiple Inputs

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The Case of Multiple Inputs

- Up to now, the firm's problem has been very simple
- They only used one input in their production
  - E.g. only had to decide how many workers to hire
  - Either because this was the only input, or because we were thinking about the 'short run' where other inputs are fixed
- More generally, firms may have to make choices about multiple inputs
  - E.g. how many workers to hire **and** how many machines to buy
- This clearly makes the firm's problem more complex
- Have to choose both the level of output **and** the level of the different inputs

1. **CHOOSE inputs and output -** $y, x_1, x_2, \ldots$
2. **IN ORDER TO MAXIMIZE profit -** $p_y y - p_1 x_1 - p_2 x_2 - p_3 x_3 - \cdots$
3. **SUBJECT TO technological constraints -** $y \leq f(x_1, x_2, x_3, \ldots.)$

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The Case of Multiple Inputs

- How can we solve it?
- Easiest way is to **split the problem in two**

1. For each level of output, calculate the cheapest way to produce that output
   - This means you can calculate the cost function
   - i.e. the cost of producing any given level of output
2. Use the resulting cost function to choose the profit maximizing level of output
   - Just as we did in the one-input case

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The Case of Multiple Inputs

- Remember, in the case of one input, we said that we could rewrite the firm's problem as

1. **CHOOSE** y
2. **IN ORDER TO MAXIMIZE** $p_y y - c(p_1, y)$

- Where $c(p_1, y)$ was the cost of producing output y (when the price of the input was p_1)
- When there was only one input, we could calculate the cost function using the inverse of the production function

$$c(p_1, y) = p_1 f^{-1}(y)$$

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The Case of Multiple Inputs

- In the case where there is more than one input, we can still write the second part of the firm's problem as

1. **CHOOSE** y
2. **IN ORDER TO MAXIMIZE** $p_y y - c(p_1, p_2, \ldots, y)$

- But now where does the cost function come from?

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The Case of Multiple Inputs

- If there are multiple inputs, there are many different ways of producing the same output
- For example, maybe we could produce 1 car using either 10 people and 1 machine, or 5 people and 2 machines
- If people cost $2 and machines $3, then the first approach would cost $23, and the second approach $16
- What is the 'cost' of producing one car?
- The **cheapest** possible way of combining people and machines in order to produce one car!

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The Case of Multiple Inputs

- This means that before solving for the profit maximizing output we need to solve another problem
- The **cost minimization** problem

1. **CHOOSE inputs** $x_1, x_2, \ldots$
2. **IN ORDER TO MINIMIZE costs** $p_1x_1 + p_2x_2 + \cdots$
3. **SUBJECT TO achieving output** $y = f(x_1, x_2, x_3, \ldots.)$

- Once we solve this we can figure out the best collection of inputs to produce every y
  - $x_1^*(y), x_2^*(y), \ldots.$
- Which will allow us to figure out the (lowest) cost of producing y

$$c(p_1, p_2, \ldots y) = p_1x_1^*(y) + p_2x_2^*(y) + \cdots$$

- Which can then be fed back into the firms profit maximization problem

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The Cost Minimization Problem

- So how do we solve the cost minimization problem?
- Let's draw some pictures!
- First let's draw some iso-cost lines
- We will make life easy by assuming only two inputs
- Costs are given by

$$c = p_1x_1 + p_2x_2$$

- So an iso cost line is

$$x_2 = \frac{c}{p_2} - \frac{p_1}{p_2}x_1$$

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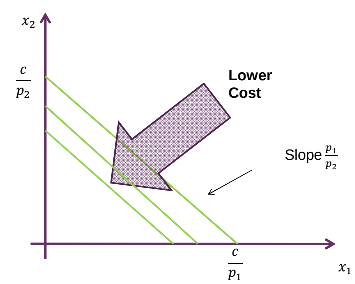
Iso-Cost Line



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The Cost Minimization Problem

- Iso cost lines are the thing we want to minimize
- Want to get on the **left most** cost line that we can
- What does the constraint look like?
- Need to achieve output y, so must pick x_1, x_2 such that y = $f(x_1, x_2)$
- Depends on what the production function f looks like!

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The Cost Minimization Problem

- To make some progress we need to make some assumptions
- We say a production function is
  - Weakly monotonic: $x_1 > \hat{x}_1$ and $x_2 > \hat{x}_2$ implies that

    $$f(x_1, x_2) > f(\hat{x}_1, \hat{x}_2)$$
  - Strictly monotonic: $x_1 \geq \hat{x}_1$ and $x_2 \geq \hat{x}_2$ with one inequality strict implies that

    $$f(x_1, x_2) > f(\hat{x}_1, \hat{x}_2)$$
  - Weakly convex: $f(x_1, x_2) = f(\hat{x}_1, \hat{x}_2)$ implies that

    $$f(\alpha x_1 + (1-\alpha)\hat{x}_1, \alpha x_2 + (1-\alpha)\hat{x}_2) \geq f(x_1, x_2)$$
  - Strictly convex: $f(x_1, x_2) = f(\hat{x}_1, \hat{x}_2)$ implies that

    $$f(\alpha x_1 + (1-\alpha)\hat{x}_1, \alpha x_2 + (1-\alpha)\hat{x}_2) > f(x_1, x_2)$$

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The Cost Minimization Problem

- Generally we will assume that the production function is
  - Weakly or strictly monotone
  - Weakly or strictly convex
- These properties should look familiar!
- We made exactly the same assumptions about preferences!
- What do they mean in this case?
- **Monotone:** if you use more inputs, you get more outputs
- **Convex:** if you can produce one car with either 10 machines and 0 workers, or 0 machines and 10 workers, you will produce more that one car with 5 machines and 5 workers
  - Produce more with a 'mix' of workers and machines

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The Cost Minimization Problem

- As with preferences, we can draw the production function on our graph using **iso-output** lines
  - Find all the x_1, and x_2 such that $f(x_1, x_2) = y$ for some y
  - These are basically the same as indifference curves
- What do iso-output lines look like?
- Well, if we assume that technology is monotone and convex, we know that they must
  - Be downward sloping
  - Not cross
  - Move to higher levels of output in a North Easterly direction
  - Be convex

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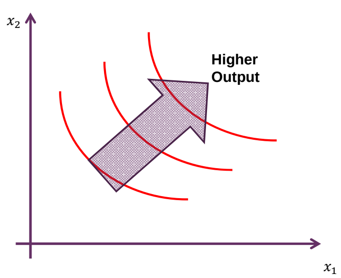
Iso-Output Lines



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The Cost Minimization Problem

- So now we can represent the firm's cost minimization problem in pictures
- A given output level y fixes an iso-output line
- We want to get on the left most (lowest) iso-cost line while staying on this iso-output line

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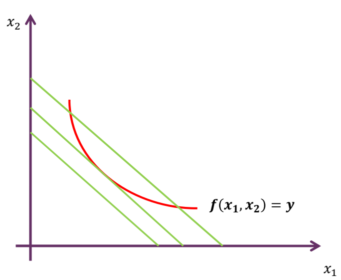
Iso-Output Lines



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The Cost Minimization Problem

- This should look strikingly familiar
- It is the consumer's problem!
- (or, more accurately, it is the consumer's dual problem)

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The Cost Minimization Problem

- In order to make the problem look **even more** like the consumer's problem, we tend to focus on production functions which look very much like the utility functions we have worked with
- **Cobb Douglas:**

$$f(x_1, x_2) = x_1^a x_2^b$$

- **Perfect Complements (often called Liontief):**

$$f(x_1, x_2) = \min(ax_1,\ bx_2)$$

- **Perfect Substitutes**

$$f(x_1, x_2) = ax_1 + bx_2$$

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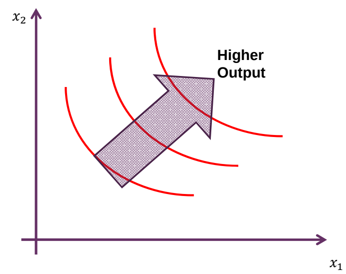
Iso-Output Lines – Cobb Douglas



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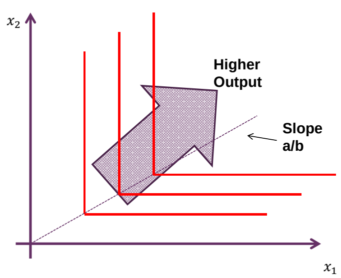
Iso-Output Lines – Liontief



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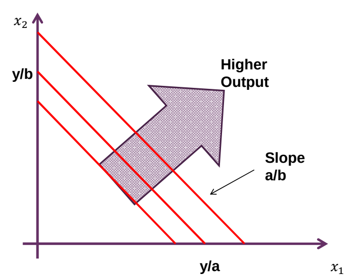
Iso-Output Lines – Perfect Substitutes



Summary

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Summary

- Today we have studied a new way to solve the firm's problem
  - Find the cost of each level of output
  - Choose output level that maximizes profit
- Applied it to the case when the firm uses only one input
  - Invert the production function to get the cost function
- Applied it to the case where the firm uses multiple inputs
  - Solve the cost minimization problem to get the cost function