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Intermediate Microeconomics W3211

Lecture 18: Equilibrium with Firms 2

Columbia University, Spring 2016
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Introduction

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The Story So Far....

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- Last lecture we talked about equilibrium with firms and consumers
- Distinguished between partial equilibrium and general equilibrium
- Showed how to find a partial equilibrium
- Showed how supply and demand graphs could be used for policy analysis

Today

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- Do a partial equilibrium analysis of tax policy
- Show how we can solve for a (simple) general equilibrium with firms

Partial Equilibrium

The Effect of Quantity Taxes

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Quantity Taxes

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- One particularly important type of policy we might want to examine is the effect of taxes
- A quantity tax levied at a rate of $t is a tax of $t paid on each unit traded.
- If the tax is levied on sellers then it is an **excise** tax.
- If the tax is levied on buyers then it is a **sales** tax.

Quantity Taxes

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- Questions we might want to ask
  - What is the effect of a quantity tax on a market's equilibrium?
  - How are prices affected?
  - How is the quantity traded affected?
  - Who pays the tax?
  - How are gains-to-trade altered?

Quantity Taxes

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- A tax rate t makes the price paid by buyers, p_b, higher by t from the price received by sellers, p_s.
- If there is a excise tax

$$p_s = p_b - t$$

- If there is a sales tax

$$p_b = p_s + t$$

- In both cases

$$p_b - p_s = t$$

Quantity Taxes

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- Even with a tax the market must clear.
- I.e. quantity demanded by buyers at price p_b must equal quantity supplied by sellers at price p_s.

$$D(p_b) = S(p_s)$$

Quantity Taxes

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$p_b - p_s = t$ **and** $D(p_b) = S(p_s)$

describe the market's equilibrium.
Notice that these two conditions apply no matter if the tax is levied on sellers or on buyers.

Quantity Taxes

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This is an Important Point!

Quantity Taxes

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describe the market's equilibrium.
Notice that these two conditions apply no matter if the tax is levied on sellers or on buyers.

This is an Important Point!

The equilibrium conditions are not affected by who faces the tax

Quantity Taxes & Market Equilibrium

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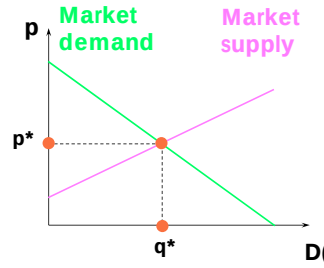

No tax

D(p), S(p)

Quantity Taxes & Market Equilibrium

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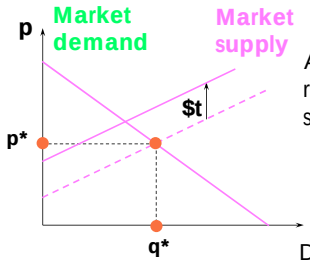

An excise tax raises the market supply curve by $t

D(p), S(p)

Quantity Taxes & Market Equilibrium

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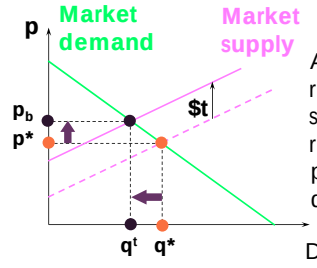

An excise tax raises the market supply curve by $t, raises the buyers' price and lowers the quantity traded.

D(p), S(p)

Quantity Taxes & Market Equilibrium

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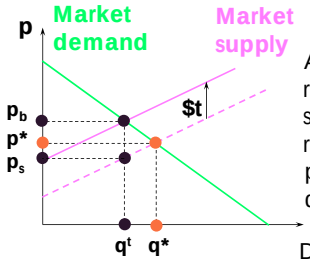

An excise tax raises the market supply curve by $t, raises the buyers' price and lowers the quantity traded.

D(p), S(p)

And sellers receive only $p_s = p_b - t$.

Quantity Taxes & Market Equilibrium

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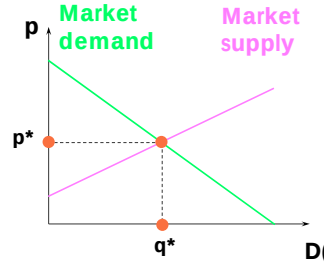

No tax

D(p), S(p)

Quantity Taxes & Market Equilibrium

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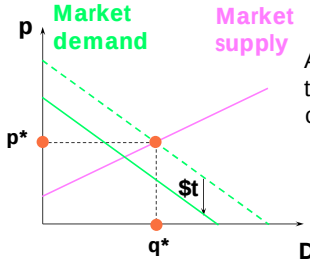

An sales tax lowers the market demand curve by $t

D(p), S(p)

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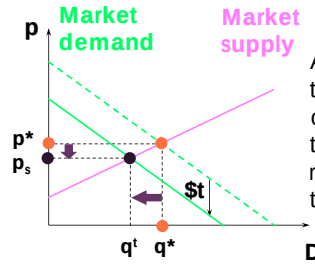
Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by $t, lowers the sellers' price and reduces the quantity traded.

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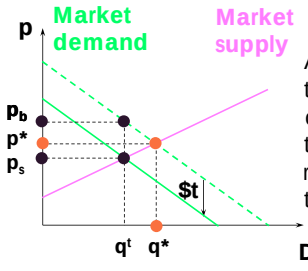
Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by $t, lowers the sellers' price and reduces the quantity traded.

And buyers pay $p_b = p_s + t$.

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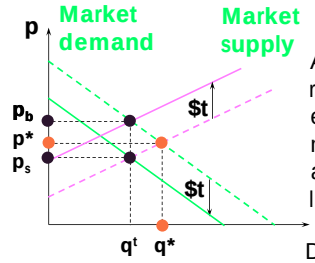
Quantity Taxes & Market Equilibrium



A sales tax levied at rate $t has the same effects on the market's equilibrium as does an excise tax levied at rate $t.

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Quantity Taxes & Market Equilibrium

- Who pays the tax of $t per unit traded?
  - The important thing is not who physically hands over the tax, but who is affected by the tax
  - i.e. for the buyer, the difference between the buyers price and the equilibrium price with no tax
  - for the seller, the difference between the sellers price and the equilibrium price with no tax
- The division of the $t between buyers and sellers is the incidence of the tax.

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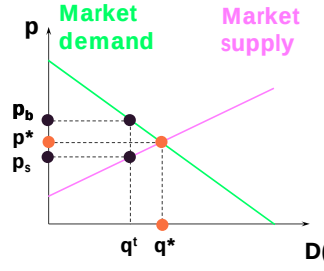
Quantity Taxes & Market Equilibrium



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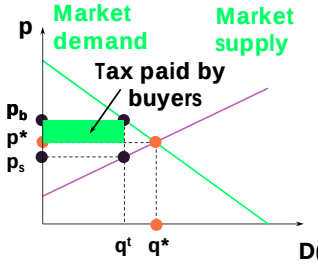
Quantity Taxes & Market Equilibrium



Quantity Taxes & Market Equilibrium

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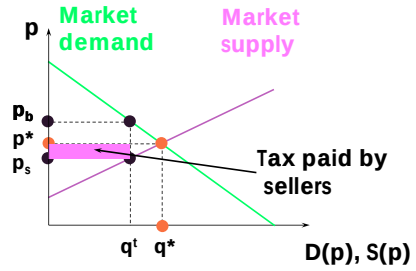

Quantity Taxes & Market Equilibrium

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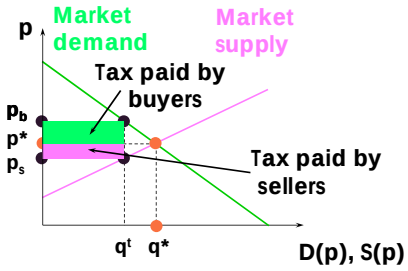

Tax Incidence and Own-Price Elasticities

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- The incidence of a quantity tax depends upon the own-price elasticities of demand and supply.
- Remember, the demand elasticity of a good is **proportional** change in quantity over **proportional** change in price

$$\varepsilon_D = \frac{dq/q}{dp_b/p_b}$$

- We can also define the supply elasticity in the same way

$$\varepsilon_S = \frac{dq/q}{dp_s/p_s}$$

Tax Incidence and Own-Price Elasticities

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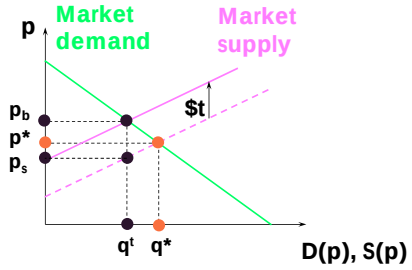

Tax Incidence and Own-Price Elasticities

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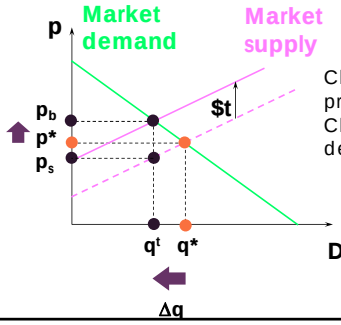

Change to buyers' price is $p_b - p^*$. Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

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Around $p = p^*$ the own-price elasticity of demand is approximately

$$\varepsilon_D \approx \frac{\frac{\Delta q}{q^*}}{\frac{p_b - p^*}{p^*}}$$

Tax Incidence and Own-Price Elasticities

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Around p = p* the own-price elasticity of demand is approximately

$$\varepsilon_D \approx \frac{\frac{\Delta q}{q^*}}{\frac{p_b - p^*}{p^*}} \Rightarrow p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}.$$

Tax Incidence and Own-Price Elasticities

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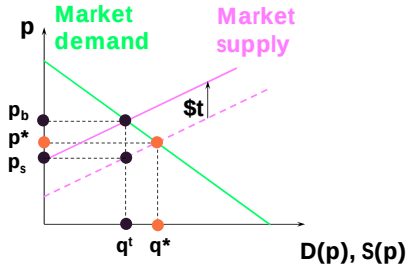

Tax Incidence and Own-Price Elasticities

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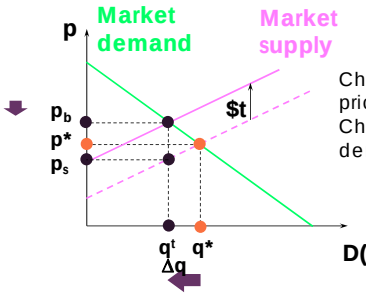

Change to sellers' price is p_s - p*. Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

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Around p = p* the own-price elasticity of supply is approximately

$$\varepsilon_S \approx \frac{\frac{\Delta q}{q^*}}{\frac{p_S - p^*}{p^*}}$$

Tax Incidence and Own-Price Elasticities

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Around p = p* the own-price elasticity of supply is approximately

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Tax Incidence and Own-Price Elasticities

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- We will define the **tax incidence** as the ratio of the price change for the buyer to the price change for the seller

$$\frac{p_b - p^*}{p^* - p_s}$$

  - Note, because prices rise for buyers and fall for sellers, this way of doing things will ensure that the number is positive
- A high number means that the incidence falls on the buyer
- A low number means the incidence falls on the seller

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Tax Incidence and Own-Price Elasticities

Tax incidence = $\dfrac{p_b - p^*}{p^* - p_s}$.

$p_b - p^* \approx \dfrac{\Delta q \times p^*}{\varepsilon_D \times q^*}$. $\quad p_s - p^* \approx \dfrac{\Delta q \times p^*}{\varepsilon_S \times q^*}$.

So $\dfrac{p_b - p^*}{p^* - p_s} \approx -\dfrac{\varepsilon_S}{\varepsilon_D}$.

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Tax Incidence and Own-Price Elasticities

Tax incidence is $\dfrac{p_b - p^*}{p^* - p_s} \approx -\dfrac{\varepsilon_S}{\varepsilon_D}$.

The fraction of a $t quantity tax paid by buyers rises as supply becomes more own-price elastic or as demand becomes less own-price elastic.

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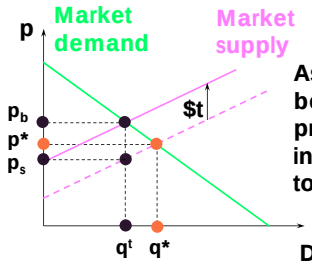
Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

D(p), S(p)

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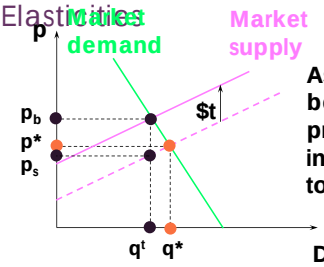
Tax Incidence and Own-Price Elasticities



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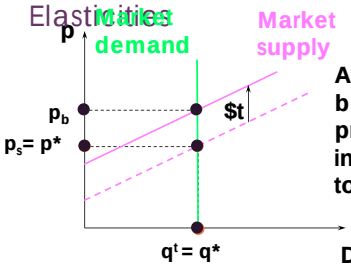
Tax Incidence and Own-Price Elasticities



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D(p), S(p)

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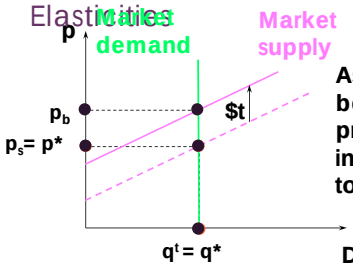
Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

D(p), S(p)

When $\varepsilon_D = 0$, buyers pay the entire tax, even if it is levied on the sellers.

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Deadweight Loss and Own-Price Elasticities

- This makes sense
- Think of a good that is very price inelastic
  - E.g. cigarettes
  - People who are addicted will buy cigarettes whatever the price
- If there is a tax on cigarettes what happens?
- People buy the same amount of cigarettes and just pay more
- This is one of the reasons that tax of cigarettes is so high

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Tax Incidence and Own-Price Elasticities

Tax incidence is

$$\frac{p_b - p^*}{p^* - p_S} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

Similarly, the fraction of a $t quantity tax paid by sellers rises as supply becomes less own-price elastic or as demand becomes more own-price elastic.

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Deadweight Loss and Own-Price Elasticities

- A quantity tax imposed on a competitive market reduces the quantity traded and so reduces gains-to-trade (*i.e.* the sum of Consumers' and Producers' Surpluses).
- The lost total surplus is the tax's deadweight loss, or excess burden.

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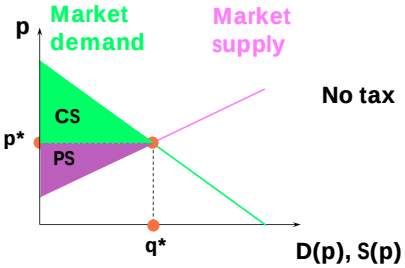
Deadweight Loss and Elasticities



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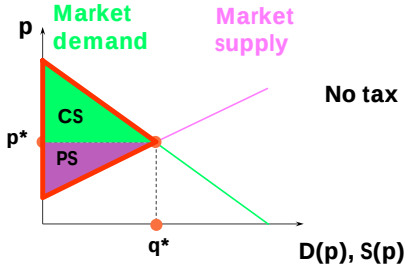
Deadweight Loss and Elasticities



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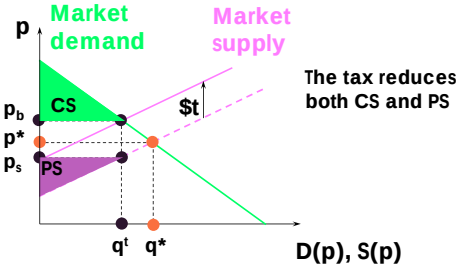
Deadweight Loss and



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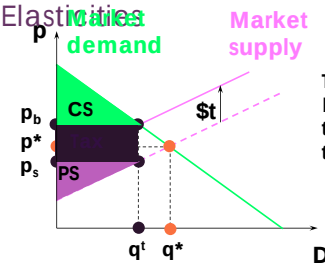
Deadweight Loss and Own-Price Elasticities



The tax reduces both CS and PS, transfers surplus to government

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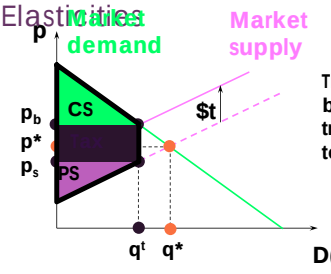
Deadweight Loss and Own-Price Elasticities



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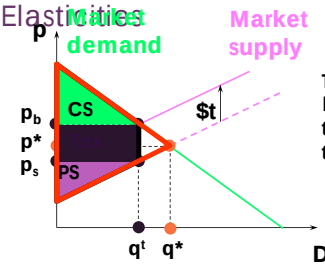
Deadweight Loss and Own-Price Elasticities



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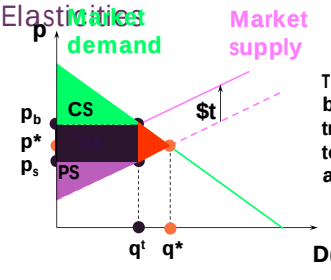
Deadweight Loss and Own-Price Elasticities



The tax reduces both CS and PS, transfers surplus to government, and lowers total surplus.

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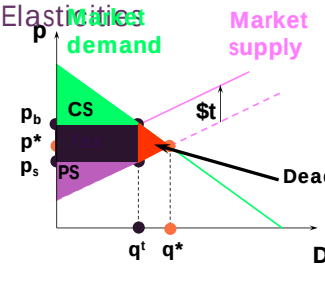
Deadweight Loss and Own-Price Elasticities



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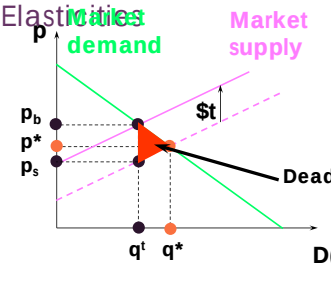
Deadweight Loss and Own-Price Elasticities



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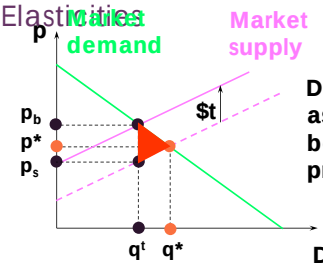
Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

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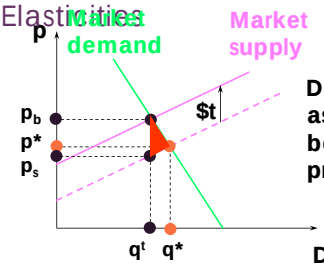
Deadweight Loss and Own-Price Elasticities



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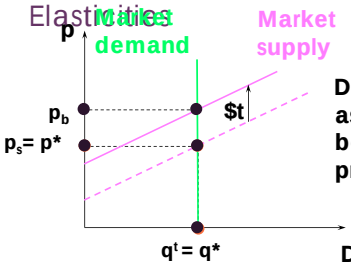
Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

When $\varepsilon_D = 0$, the tax causes no deadweight loss.

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Deadweight Loss and Own-Price Elasticities

- Deadweight loss due to a quantity tax rises as either market demand or market supply becomes more own-price elastic.
- If either $\varepsilon_D = 0$ or $\varepsilon_S = 0$ then the deadweight loss is zero.
- This is another reason why there is such a high tax on cigarettes!

General Equilibrium with Firms

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General Equilibrium with Firms

- As we discussed last lecture, partial equilibrium is part of the story
- When we look at markets one at a time, we miss out on the fact that different markets effect each other
- In order to rectify this error, we need to switch to **general equilibrium**
- Here we solve for all the markets in an economy simultaneously

General Equilibrium with Firms

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- Why might we be interested in general equilibrium?
- Well, when there were only consumers, we proved the **first and second fundamental theorem of welfare economics**
  - (Under certain conditions) Any equilibrium is pareto efficient and any pareto efficient allocation could be supported as an equilibrium
- Does this result survive if we have firms?
- The answer is generally yes
  - With the same caveats we had with the consumer re externalities, price taking, equity etc

The Robinson Crusoe Economy

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- In order to demonstrate this I am going to tell you a story
- The story of Robinson Crusoe
- As you probably know, Robinson Crusoe was stranded on a desert island
- Had to subsist only on coconuts until he was rescued

The Robinson Crusoe Economy

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- As is less well known (except to economists), he went a bit bonkers while on the desert island
- In particular he developed a particularly handy form of multiple personality disorder
- Specifically, he split himself into two economic units
  - RC, the consumer
  - RC Inc., the firm

The Robinson Crusoe Economy

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- RC Inc acts as the firm in the economy
  - Hires labor from RC at the wage rate w
  - Uses the labor to produce coconuts, which are sold at the market price (normalized to 1)
  - Does so to maximize profits
- RC acts as the consumer in the market
  - Buys coconuts and sells labor to maximize utility
    - Decides how much to work at wage w
    - Uses that money to buy coconuts at the market price 1
    - Also owns the firm, so collects any profits the firm makes

Consumer and Firm Problems

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- RC's Inc's problem

1. **CHOOSE coconuts c and labor l**
2. **IN ORDER TO MAXIMIZE profits $\pi(c, l) = c - wl$**
3. **SUBJECT TO technology constraint $c \leq f(l)$**

- RC's consumer problem

1. **CHOOSE coconuts c and labor l**
2. **IN ORDER TO MAXIMIZE utility $u(c, 24 - l)$**
3. **SUBJECT TO the budget constraint $c \leq wl + \pi$**

Why Are We Doing This?

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- This (admittedly a little artificial) set up allows us to study **general equilibrium** with firms and consumers
- How many markets are there in this economy?
- Two
  - Market for coconuts
  - Market for labor
- We can figure out equilibrium in both markets at the same time
- Key question: Is equilibrium pareto efficient?

Pareto Efficiency

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- What is pareto efficiency here?
- Well, there is only one person (Robinson)
- So we want to make him as well off as possible
- To do this we can solve the **planner's problem**

1. **CHOOSE coconuts c and labor l**
2. **IN ORDER TO MAXIMIZE utility $u(c, 24 - l)$**
3. **SUBJECT TO technology constraint $c \leq f(l)$**

- Notice this is what we would get if we 'merged' RC and RC Inc

Pareto Efficiency

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- How can we solve the planner's problem?
- Draw some pictures!

Robinson Crusoe Inc's Production Function

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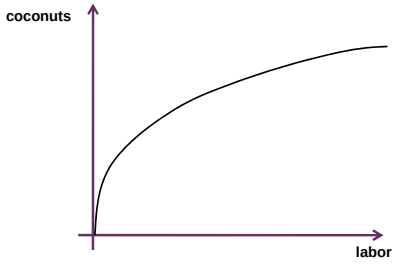
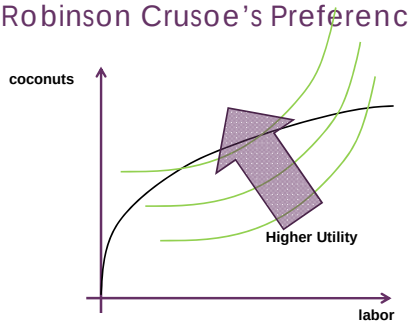


Robinson Crusoe's Preferences

70



Pareto Optimality for Robinson

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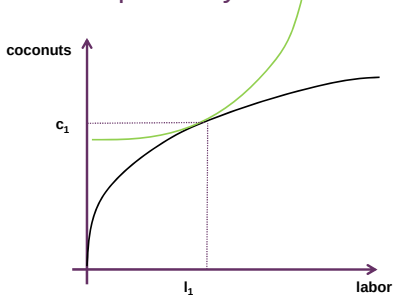

Pareto Efficiency

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- Pareto efficiency occurs at the point at which the slope of the **production function** is equal to the slope of the **indifference curve**
- What does this mean?
- Slope of production function
  - Marginal Product of Labor
  - Rate at which one additional unit of labor creates one additional coconut
- Slope of the indifference cure
  - Marginal Rate of Substitution
  - The number of coconuts that the consumer has to be given to make them indifferent about supplying one more unit of labor

Pareto Efficiency

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- This makes sense!
- If the MRS is lower that the marginal product of labor, can make RC better off by working harder and having more coconuts
- If MRS is higher than the marginal product of labor, can make RC better off by working less hard and having less coconuts
- At the optimum the two have to be equal

Equilibrium

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- We want to know whether equilibrium in this economy is pareto efficient
- In order to figure this out, we need to know what an equilibrium is!

Equilibrium

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- An **Equilibrium with consumer and firms** is
  - A set of prices w
  - A consumption decision for RC (i.e. an c_c and l_c)
  - A production decision for RC Inc (i.e. an c_f and l_f)
- Such that
  - The allocation is feasible
    $$c_c = c_f \text{ and } l_c = l_f$$
  - c_c and l_c are optimal for the consumer given prices w
  - c_f and l_f are optimal for the firm given prices w

Equilibrium

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- Let's draw some more pictures
- First, let's think about the Firm's optimization problem
- We have already solved this type of problem once
- We draw iso-profit lines
  $$\pi = c - wl$$
- So
  $$c = \pi + wl$$
- Highest possible iso profit line will occur at the point of tangency with the production function

Competitive Behavior of RCI

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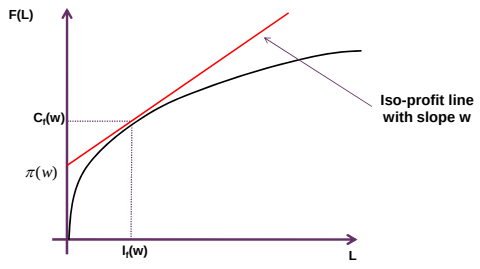

Equilibrium

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- Next, let's think about the consumer's optimization problem
- They will try to get on the highest possible indifference curve, subject to the budget constraint
- What is the budget constraint?
  $$\boldsymbol{c \leq wl + \pi}$$

Competitive Behavior of RC

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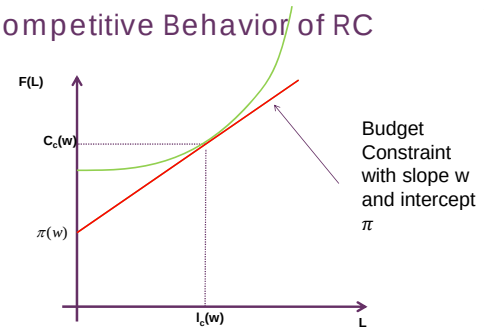

Equilibrium

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- So now, for a given w, we can figure out the optimal behavior of the firm, and their resulting profits
- And the optimal behavior of the consumer, given that they receive such profits
- What does an equilibrium look like?

Not an Equilibrium

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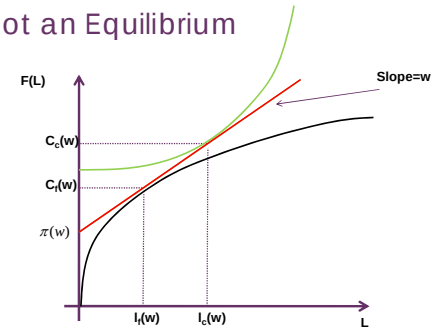

An Equilibrium

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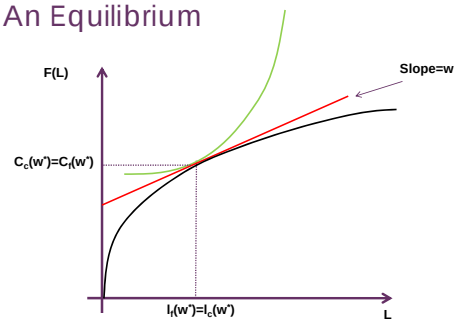

Equilibrium

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- At the equilibrium
  - The marginal product of the firm will equal the wage rate
  - The marginal rate of substitution of the consumer will equal the wage rate
- This means that marginal product of the firm equals the marginal rate of substitution for the consumer
- i.e. this is the solution to the planner's problem!
- The competitive equilibrium is Pareto Efficient!
- The FFWE carries over to this setting!

Equilibrium

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- What is going on?
- Well, it is the same as in the Edgeworth Box cases
- For optimality we need the MRS of the individual to equal the MP of the firm
- In a competitive equilibrium the consumer will set the MRS equal to the price ratio, and the firm will set the MP equal to the price ratio
- Because consumers and firms face the same prices, this will equalize MRS and MP
- Magic!
- But again subject to the usual caveats.

Summary

Summary

- Today we have dealt with

1. Partial Equilibrium with taxes
2. General equilibrium with firms