Intermediate Microeconomics

Mark Dean

Homework 5

Due Thursday, 29th October

Question 1 Lucy has preferences between artichokes and hallibut described by

$$u(x_a, x_h) = \alpha \ln(x_a + 1) + (1 - \alpha) \ln x_h$$

and her budget constraint is given by $p_a x_a + p_h x_h = M$

1. Assuming an interior solution, show that Lucy's demand function is given by

$$x_a^* = \frac{M\alpha}{p_a} - (1 - \alpha)$$

2. This implies that for an amount x_a of artichokes, Lucy would be willing to pay

$$p_a = \frac{M\alpha}{x_a + (1 - \alpha)}$$

Use this expression to calculate the consumer surplus generated from consuming x_a anchovies. Write your answer as a function of x_a, then use the demand function to write it as a function of the parameters of the problem (prices, incomes and the parameters of the utility function)

3. Now calculate the utility that Lucy gets from being allowed to buy artichokes at price p_a (i.e. calculate the utility Lucy gets when she gets to consume artichokes at p_a compared to when she has to consume no artichokes). Is the consumer surplus a good approximation of this?

4. Why did I use the utility function $\alpha \ln(x_a + 1) + (1 - \alpha) \ln x_h$ rather than $\alpha \ln x_a + (1 - \alpha) \ln x_h$? What would the consumer surplus have been for artichokes in the second case?

Question 2 Consider a firm with a production function $f(x) = l^{\frac{1}{2}}$, who faces wages equal to 1 and price for their good equal to 5. Now look at the file Figure HW 5 Pr 2 (also in the assignments folder in Mycourses). This figure has three lines (i, ii and iii) and 5 labelled areas (A, B, C, D and E).

1. Which line is the marginal revenue line? Which is the average cost line? Which is the marginal cost line?
2. Assuming that the firm maximizes profits which area (or sum of areas - e.g. A+C+E) is equal to total revenue?
3. What areas are equal to total cost?
4. What areas are equal to profit?

Question 3 Sheffield Steel Corporation converts iron into steel. Their production function is $y = x^{\beta}$ where, y is amounts of steel and x is amounts of iron and $0 < \beta < 1$. They can buy iron at the price p_x and sell steel at the price p_y

1. Sketch the production function. Does this satisfy the three assumptions that we made in class? What happens to marginal productivity as x gets very small? What about when it gets very large?
2. Find the point of tangency between the iso profit lines and the production function as a function of the price of iron and steel. Can you always find a point of tangency for any positive price of iron and steel? What is the profit at this level of output? Will this be the profit maximizing level of output ? If not, what is?
3. Rotherham Steel Corporation is worse that Sheffield Steel Corporation. They always end up wasting the first unit of iron they buy, and get no steel out of it. Their production function is given

$$\begin{aligned} y &= 0 \text{ if } x \leq 1 \\ &= (x-1)^{\beta} \text{ for } x > 1 \end{aligned}$$

   Sketch the production function for Rotherham Steel Corporation. Find the point of tangency between the iso profit lines and the production function as a function of the price of iron and steel. Can you always find a point of tangency for any positive price of iron and steel? What is the profit at this level of output? Will this be the profit maximizing level of output ? If not, what is?

4. Sheffield Steel Corporation upgrades their production, giving a production function

$$y = x^{\beta} + \gamma x$$

Sketch the production function. What happens to marginal productivity as x gets very small or very large? Find the point of tangency between the iso profit lines and the production function as a function of the price of iron and steel. Can you always find a point of tangency for any positive price of iron and steel? What is the profit at this level of output? Will this be the profit maximizing level of output ? If not, what is?

5. Sheffield gets another upgrade, and now their production function is given by

$$y = x^{\rho}$$

where $\rho > 1$. Sketch the production function. What happens to marginal productivity as x gets very small or very large? How much will this firm want to produce at any price for iron or steel?

6. Calculate marginal cost curves for each of these producers. What goes wrong when trying to find a solution in the last case?