Intermediate Microeconomics

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Homework 7

Due Tuesday, 1st December

Question 1 Consider a monopolist who is selling quince. Their cost function is given by $c(q) = cq$. The demand function for quince is given by $q = Ap^{-\varepsilon}$, where A, c and ε are positive numbers

1. Sketch the marginal cost, demand, and marginal revenue curves for the firm. What is the price elasticity of demand implied by the demand curve?

2. Calculate the profit maximizing price and quantity for the firm. Does the firm's mark up increase or decrease as ε increases?

3. Graphically explain the results from part 2. What is happening to demand and marginal revenue as ε increases?

Question 2 Quiche inc. is a monopolistic producer of quiche. They produce quiche with a cost function $c(q) = cq$. They sell quiche to Canadians, who have a demand function $p = a - bq_c$ and Finns, who have a demand function $p = a - dq_f$ Assume that both for Canadians and Finns, consumer surplus is equal to the monetary value they get from being allowed to buy quiche, relative to not being allowed to buy any quiche. Also assume that there are the same umber of Canadians and Finns.

1. Imagine that Quiche inc. is restricted to linear pricing, and cannot distinguish between Canadians and Finns (in other words, they have to set some price p and sell to anyone who wants to buy at that price). What would the optimal price and quantity be? How much profit would they make?

2. Someone points out the obvious ways to differentiate between Canadians and Finns, so Quiche Inc. can charge different prices to the two different groups. However, quiche consumer groups pass a law outlawing two part tariffs. What are the optimal prices that Quiche Inc. will charge the two different groups? Will there be any difference in the prices charged to the two groups, or the amount sold? Expalin these differences, or lack thereof? How much profit will Quiche Inc. make?

3. Now assume that a quiche producer's advocacy group gets the law repealed, so Quiche Inc can charge a separate two part tarrif for Finns and Canadians. What two part tarrif will they charge each of the two groups? How much profit will they make?

Question 3 Quark Industries sells quarks to two groups - scientists, who have a demand curve $p = d - bq_s$, and artists, who have demand $p = e - bq_a$, where $d > e$. As usual, their costs are given by $c(q) = cq$. As before assume consumer surplus is equal to the monetary value that each group gets from being allowed to buy quarks, relative to not being allowed to buy any quarks.

1. For the moment, assume that the scientists are busy fixing the LHC, and so only artists are buying quarks. If Quark Industries makes a 'take it or leave it' offer to the artists by which they can either buy an amount of quarks q_a^* for a total cost f_a^*, or buy no quarks, what is profit maximizing offer that Quark Industries can make?

2. Now the scientists have fixed the LHC, and also want to buy quarks. Quark Industries cannot distinguish between artists and scientists. If they are already offering the artists the profit maximizing contract q_a^*, f_a^*, what is the profit maximizing offer q_s^*, f_s^* such that the scientists will choose this offer rather than q_a^*, f_a^*. What is the total profit that quiche inc. will make.

3. Say that, for some reason, Quark industries decides to sell an amount $q_a^* - \alpha$ to the artists. What is the maximum price they could charge? What now is the profit maximizing offer they can offer the scientists? What is the total profit of the firm.

4. What is the profit maximizing level of α?

Question 4 Some questions about choice under uncertainty

1. When we talked about choice under *certainty,* we said that if some utility function u was to represent preferences then any function v that is a strictly positive transformation

of u would also represent preferences. Is this still true for the VNM utility function in choice under uncertainty? In other words, if

$$p \ \succ \ q \text{ if and only if}$$

$$p_a u(a) + p_b u(b) + \dots \ > \ q_a u(a) + q_b u(b) + \dots$$

and v is a strictly positive transformation of u, is it the case that

$$p \ \succ \ q \text{ if and only if}$$

$$p_a v(a) + p_b v(b) + \dots \ > \ q_a v(a) + q_b v(b) + \dots$$

Does it work for $v(x) = \alpha + \beta u(x)$?

2. A function u is concave if $u(\lambda x + (1-\lambda)y) > \lambda u(x) + (1-\lambda)u(y)$. Show that, if a person has a concave utility function, they will be risk averse

3. (Allais paradox) Consider the following 4 gambles :

    A   100% chance of $1 million

    B   89% chance of $1 million, 1% chance of nothing, 10% chance of $5 million

    C   89% chance of nothing, 11% chance of $1 million

    D   90% chance of nothing 10% of $5 Million

Show that

$$A \ = \ 89\%(\$1 \text{ million for sure}) + 11\%(\$1 \text{ million for sure})$$

$$C \ = \ 89\%(\text{zero for sure}) + 11\%(\$1 \text{ million for sure})$$

$$B \ = \ 89\%(\$1 \text{ million for sure}) +$$

$$11\%(1/11 \text{ chance of getting nothing and } 10/11 \text{ chance of getting } \$5 \text{ million})$$

$$D \ = \ 89\%(\text{zero for sure}) +$$

$$11\%(1/11 \text{ chance of getting nothing and } 10/11 \text{ chance of getting } \$5 \text{ million})$$

and so, if someone satisfies the independence axiom, then if they prefer A to B they have to prefer C to D