Mathematics For Economists

Mark Dean

Introductory Handout for Fall 2014 Class ECON 2010 - Brown University

1 Aims

This is the introductory course in mathematics for incoming economics PhD students at Brown in 2014. In conjunction with the Maths Camp, it has three aims

1. To provide you with the mathematical tools needed to understand your other first year courses
2. To give you a first understanding of the concepts that underlie these tools
3. To indoctrinate you into the 'theorem-proof' method of analysis

2 Administration

2.1 Contact Details

2.1.1 Mark Dean (Instructor)

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2.1.2 Victor Aguiar (TA)

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2.2 Class Times

Classes take place on Tuesdays and Thursdays 1.00-2.20.

Lab will take place on Thursdays at 2.30-3.50 in THA 101 116 E (this is on Thayer St) Labs will generally consist of Victor going through the homework from the previous week..

Mark's office hours are 3.00-5.00 on Tuesdays (though feel free to get in touch outside these times if you need help).

Victor's office hours are going to be on Mondays at 9.20-10.20 in the basement of Robinson Hall.

I am not going to be here on three Tuesdays during the semester. On these occasions Victor will have his TA sections in my usual Tuesday class slot, and I shall teach in Victor's usual Thursday Lab slot. Details can be found in the 'Timing' Section below.

2.3 Homework Schedule

Homeworks will be set on the Tuesday of each week, and will be due in the following Tuesday. These homeworks are designed to be quite challenging, and should take between 5 - 8 hours per problem set (depending on how much background you have). Feel free to work in groups if you wish though obviously there is little to be gained by simply copying the answers from someone who knows more than you.

Furthermore in previous years we have had problems with students getting hold of answers to homework questions from previous years and literally copying them out. This is dumb, lazy and surprisingly easy to spot, not least because the questions change somewhat from year to year. Please do not do this.

2.4 Grading

Grades will be assigned using an ad hoc formula that puts roughly 20% weight on homework and 40% each on the midterm and final exams

2.5 Course Materiels

The main source for the course will be my lecture notes, which I will make available on my website at some point after the relevant lecture. Hopefully, these will be pretty complete for the topics I cover but (definitionally) will not cover anything outside the course.

The 'official' textbook of the course is **Simon, C. and L. Blume, Mathematics for Economists**, (W.W. Norton, London 1994). This book is somewhat basic by the standards of modern graduate economics courses, but is well written and organized, and touches on most of the important topics. As such, it is probably worth owning as a reference book.

If you are going to get another book, I would recommend **Ok. E., Real Analysis with Economic Applications**. This is an absolutely excellent textbook, which covers a wide variety of topics in analysis and linear algebra with an economic twist. If you are going to end up as an economist that uses a lot of maths, then you will want to get hold of this book (especially if you are interested in micro theory). A useful analysis text is **Rudin, W., Principles of Mathematical Analysis** (McGraw-Hill), which is not aimed at economists, but is more complete. For those interested in macroeconomics, you are almost certainly want to get to grips with dynamic programming in a way that is beyond the scope of this course: **Lucas, R., N. Stokey with E.. Prescott, Recursive Methods in Economic Dynamics**, (Harvard University Press) is one standard reference here. Another very useful book for dynamic macroeconomics is **Galor, O. Discrete Dynamical Systems (**Springer).

There are also lots of useful lecture notes and problem sets on line, which I will try and point you in the direction of. If you want more materiel, please let me know.

3 Structure

3.1 Outline

This course comes in two parts The first part is designed to achieve aim 1 - to provide you with the basic tools you need to get you through your first year courses (and, hopefully, to be able to understand most mainstream articles in economics). This aim should largely been achieved by the maths camp taught by Michael. Victor will cover dynamics and dynamical systems in his first TA section, and fixed point theorems at some point during the semester The topics that you should understand by this point are listed in the 'prerequisites' section below. If there is something that on this list that you feel has not been adequately covered then please come and talk to me or Victor.

The items I will cover in the main syllabus are listed below. Much of this syllabus will be devoted to providing the mathematical underpinnings for the tools that you are already using. The main difference between this and what you have already done is that we will be more general, and we will prove many more of the theorems along the way. This section of the course is therefore designed to achieve aims 2 and 3 - to give you a glimpse of what 'real' maths is about, and to teach you how to use the theorem-proof method of exposition. It may therefore be the case that the immediate relevance of what you think of as economics is not obvious. However, it is still useful for two very important reasons. First, these lectures will give you a background into the tools that you are likely to spend the rest of your career using. It is therefore worth understanding where these tools come from, their limitations and some extensions. Second, some of you will go into fields in which you will need to develop a very high degree of mathematical understanding (this is true in much of theoretical macro, micro and econometrics). You people will need a lot more maths than I teach here, but hopefully this course will act as a bridge that will allow you to understand the 'proper' maths course that you need.

Some things that the course will not cover: First, I do nothing in the way of basic statistics, as this will be covered in your econometrics classes. Second, I will not be covering dynamic programming until late in the course, as you will see some of this in your Macro class.

3.2 Prerequisites

Here is a list of the topics that you should be comfortable with before we start on the 'real' syllabus. Almost all of this stuff is in Simon and Blume. For convenience I have put the relevant page numbers in brackets after each topic. Much of it is also covered in your notes from Math Camp.

Single and multivariable calculus: Definition of a derivative (24, 300), differentiable functions (29), first and second derivatives (33), partial and total derivatives (300, 307) directional derivatives (319), Jacobian and Hessian matrices (325, 329), differentiating polynomials,(27) logarithmic and exponential functions (93), expansions and approximations (34, 827), rules for differentiation (product, quotient, chain, etc) (27, 70, 326), implicit function theorem (single and multiple variables) (150, 329) and slopes of indifference curves (329), tangent plane (309) and differentiating along a parameterized curve (313), mean value theorem (824) introduction to integration (887)

Analysis: Set notation and set properties (847), open, closed and compact sets (264-272), convexity (506), cardinality of sets, sequences in $\mathbb{R}^N$ (253), summation of arithmetic and geometric sequences, limits and convergence (254), properties of functions:continuity (293), homogeneity (and Euler's theorem) (483-493) and homotheticity (500), concavity and q-concavity (505-527), one-to-one and onto functions, range, domain and image (295-299), basic fixed point theorems

Static Optimization: Unconstrained optimization:. critical points, sketching a graph (39-43), first and second order conditions (51-56, 396-407, 836), convexity/concavity (532-537), Weistrass theorem (56). Constrained Optimization: Kuhn Tucker conditions (411-445) - practical examples of using Lagrangian method (544), envelope theorem, (453, 560), meaning of the multiplier (448)

Linear Algebra: Matrix operations and properties (153), linear systems (122), solving linear systems -elimination (122), number of solutions (134) and the relationship to rank (142), determinant (188) and inverses (165), Cramer's rule (194) linear independence (237), inner products (213), quadratic forms and definiteness (375), eigenvalues, eigenvectors (579) and their properties (597), their relation to linear difference equations (585), definiteness (626) and Markov processes (615)

Basic differential and difference equations.

Definition of differential and difference equations (633), Solving linear first order equations (639), steady states (638, 684) and phase diagrams (660), systems of equations (674)

Proof Methods Direct (851), converse and contrapositive (853), Induction (855)

3.3 Syllabus

Here is a list of the topics that we will be covering in the main syllabus.

3.3.1 Definition of Real Numbers

- Ordered Fields
- Principle of Mathematical Induction
- Completeness axiom
- Extended Reals

3.3.2 Real Analysis

- Metric spaces
- Topology
- Closures, boundaries and Interior points
- Continuity
- Real Sequences, Bolzano-Weierstrass theorem
- Cauchy Sequences and Completeness
- Separability and connectedness
- Compactness
- Applications: Weierstrass theorem, Theorem of the maximum, Banach fixed point theorem, utility maximization

3.3.3 Linear Algebra

- Definition of a linear space and subspaces
- Linear dependence
- Span, basis and dimensions
- Inner products, norms and the Cauchy-Schwartz inequality
- Affinity
- Linear operators and the Fundamental theorem of calculus
- Subspaces attached to a matrix

3.3.4 Convex Analysis

- Convex sets
- Caratheodory's theorem
- Separation Theorems
- Theorems of the alternative - Farkas Lemma

3.3.5 Calculus

- Definition of derivative, differentiable function, etc
- Taylor Expansions
- Implicit function theorem

3.3.6 Static Optimization

- Unconstrained optimization - First and second order conditions
- Proof of Kuhn Tucker conditions with equality constraints
- Proof of Kuhn Tucker conditions with inequality constraints

3.3.7 Dynamic Programming

- Contraction mapping
- Blackwell's sufficient conditions
- Statement of the problem
- Finite dynamic programming
- Principle of Optimality and the Bellman equation
- Benveniste and Scheinkman
- Transversality conditions
- Hamiltonmians

3.4 Timing

This is the timing of topics that we will follow if everything goes well.There may be a bit of slippage, so you should not plan your class attendance on this basis

1. 4th Sept: Introduction/Real Numbers
2. 9th Sept: Definition of metric space, Definition of a Topology. Topology derived from a metric
3. 11th Sept: Closedness, closure points, sequential characterization of closed sets, closure, interior points, boundary. Properties of closures and interior points, Continuous functions
4. 16th Sept: Properties of continuous functions, real sequence, convergence, Blozano Weierstrass, definition of limsup, summing sequences
5. 18th Sept: Cauchy sequences and completeness, Separability and Connectedness,
6. 23rd Sept: NO CLASS (Victor will hold TA Session)
7. 25th Sept Class 1: Compactness, relation to closedness and boundedness, continuity of correspondences

8. 25th Sept Class 2: Applications: Wierstrass, Continuity and Uniform Continuity. Theorem of the Maximum. Banach fixed point theorem, utility maximization

9. 30th Sept: Definition of linear Spaces, fundamental properties of linear spaces, definition of linear subspaces, characterization of a linear subspace, Linear combinations and linear dependence.

10. 2nd Oct: Spans, Basis and Dimensions, Norms, the Euclidian Norm, relation between metric and norms.

11. 7th Oct: NO CLASS (Victor will hold TA Session)

12. 9th Oct Class 1: Minkowski's inequality Inner product, CS inequality, orthogonality

13. 9th Oct Class 2: Affinity and definition of the hyperplane, definition of a linear operator, characterization of a linear operator in $\mathbb{R}^n$

14. 14th Oct: Null Space, Fundamental Theorem of Linear Algebra, Isomorphisms, Fundamental spaces related to a matrix

15. 16th Oct: Convex sets and convex combinations, relations between the two. Convex hull, Carathedory's theorem. Properties of line segments in the closure of convex sets

16. 21st Oct: Midterm

17. 23rd Oct: Topological properties of a convex set. Defn of orthogonal projection. Existence and uniqueness of projection for closed, convex sets

18. 28th Oct: Definition and examples of separation. Separating points and sets. Separating sets that aren't closed

19. 30th Oct: Example: Second fundamental welfare theorem. Statement of Farkas Lemma

20. 4th Nov: Definition of derivative, Rolle's theorem, Taylor expansions, Implicit Function Theorem

21. 6th Nov: Unconstrained optimization, first and Second order conditions

22. 11th Nov: Constrained optimization with equality constraints – parametric curves and tangent planes, regular points, representation of the tangent plane

23. 13th Nov: First and second order necessary conditions for constrained optimization with equality constraints
24. 18th Nov: NO CLASS (Victor will hold TA Session)
25. 20th Nov Class 1: Inequality Constraints and the tangent cone
26. 20th Nov Class 2: Example of the use of KT
27. 25th Nov: Dynamic optimiziation: Hamiltonian Method
28. 2nd Dec: Dynamic optimization Bellman equations
29. 4th Dec: Dynamic optimization Examples
30. 9th Dec: Catch-up