Reference Dependence Lecture 3

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The Story So Far

- Defined reference dependent behavior and given examples
    - Change in risk attitudes
    - Endowment effect
    - Status quo bias
- Discussed two models of reference dependent choice
    - Prospect Theory
    - Koszergi-Rabin [2006,2007]
- Latter introduced
    - Stochastic reference points
    - Personal equilibrium

Plan for Today

- Two applications
- Application: Pricing
  - Heidues and Köszegi [2008]
  - Speigler [2011]
- Contracting
  - Hart and Moore [2008]

Two Pricing Puzzles

- Prices are unresponsive to changes in costs (‘price stickiness’)
- Firms charge prices that are too similar across differentiated products with different costs ('focal pricing’)

Two Pricing Puzzles

- Existing theories (Rational Inattention, Menu Costs) can explain the former, but have trouble with the latter
- Aim
    - Show loss aversion can explain the former
    - May also be able to explain the latter
- Intuition: Consumers form expectations over prices that will be charged
- Loss aversion means that marginal disutility of price rises is high...
- ....while marginal benefit of price falls is not so high
- Causes kink in firm's demand function around expected price
- Makes prices 'unresponsive'

- Two papers formalize this intuition
  - Heidues and Köszegi [2008] (henceforth HK)
  - Speigler [2011] (henceforth S)
- Speigler is a 'cover version'
  - But it is also simpler
- Will go through this model in detail
- Discuss Heidues and Koszegi afterwards

Modelling Choices - Consumer Side

- Over what are consumers loss averse?
- Does consumers expected action affect reference point?
- How to deal with stochastic reference points?

Modelling Choices - Consumer Side

- Over what are consumers loss averse?
  - HK: Money and consumption quality
  - S: Money only
- Does consumers expected action affect reference point?
- How to deal with stochastic reference points?

Modelling Choices - Consumer Side

- Over what are consumers loss averse?
- Does consumers expected action affect reference point?
  - HK: Yes: personal equilibrium
  - S: No: reference point is what consumers to expect firms to charge
- How to deal with stochastic reference points?

Modelling Choices - Consumer Side

- Over what are consumers loss averse?
- Does consumers expected action affect reference point?
- How to deal with stochastic reference points?
  - HK: Evaluate action at each reference point in support, calculate expectation
  - S: Consumer draws reference point from distribution and evaluates action

Modelling Choices - Supply Side

- HK: Salop style monopolistic competition
  - Firm's 'type' located equidistantly on a ring
  - Consumer type drawn from a distribution.
  - Value of product to consumer is distance between consumer and firm type
- S: monopolist
- Both models give price stickiness
  - Result robust to these assumption
- S annot talk about focal pricing

Set Up - Firm

- Monopolistic firm, single good, measure 1 of consumers
- Marginal cost of firm c distributed uniformly over m possible values C
  - $1 > c_h > c_l > 0$ respectively highest and lowest prices in C
- Pricing strategy $P : C \to \mathbb{R}$
- Induces measure μ_p over prices

$$\mu_p(x) = \frac{|c \in C | P(c) = x|}{m}$$

Set Up - Consumer

- Consumer first draws a reference price p^e from distribution μ_p
- Draws valuation of good u from $U[0,1]$
- Buys product iff

$$p \leq u - L(p, p^e)$$

- where

$$\begin{aligned} L(p, p^e) &= \max\{0, \lambda(p - p^e)\} \\ \lambda &\geq 0 \end{aligned}$$

- Consumer identified by
  1. Valuation u
  2. Reference price p^e

Optimization Problem - Firm

- For any given price, consumer will buy if

$$p + L(p, p^e) \leq u$$

- With uniform distribution, this is probability

$$1 - p - L(p, p^e)$$

- For pricing strategy P, there is a $\frac{1}{m^2}$ probability that prices will be $P(c)$ and expectations will be $P(c^e)$ for each $c, c^e \in C$
- Summing across all this states gives the valuation of a pricing strategy as

$$\Pi(p) = \frac{1}{m^2} \sum_{c \in C} \sum_{c^e \in C} [P(c) - c] \max\{0, 1 - P(c) - L(P(c), P(c^e))$$

A Note on Timing

- Firm is assumed to announce strategy ex ante
- Not a simultaneous move game
- When we consider deviations in firm's strategy, we assume beliefs of agents adjust as well
- 'Unexpected' price hike means relative to beliefs correctly conditioning on strategy, not due to change in strategy

Benchmark - No Loss Aversion

$$\Pi(p) = \frac{1}{m^2} \sum_{c \in C} \sum_{c^e \in C} [P(c) - c] \max\{0, 1 - P(c) - L(P(c), P(c^e))$$

- Without loss aversion, expectations do not matter

$$\Pi(p) = \frac{1}{m^2} \sum_{c \in C} [P(c) - c] \max\{0, 1 - P(c)\}$$

- For each c, FOC

$$\begin{aligned} 1 + c - 2P(c) &= 0 \\ \frac{1+c}{2} &= P^0(c) \end{aligned}$$

Lemma

Let P be an optimal pricing strategy. Then for every $c \in C, P(c) < 1$ and consumer demand is strictly positive

- Proof:
    - Assume $P(c^*) \geq 1$ for some $c^* \in C$, then consumer demand is zero at $P(c^*)$
    - Consider P' which changes P only by setting $P(c^*) = 1 - \varepsilon$
    - Consumer demand is now strictly positive in this state, as

$$
\begin{aligned}
& 1 - P'(c^*) - L(P'(c^*), P'(c^*)) \\
= \;& 1 - P'(c^*) = \varepsilon > 0
\end{aligned}
$$

    - Profit also strictly positive assuming ε st $\varepsilon < 1 - c$

Characterizing Optimal Strategy

- Proof (cont)
  - For all other $c \in C$, consumer demand is unchanged
  - Consider pair c c^e. If $c^e \neq c^*$ this is clearly true
  - If $c^e = c^*$ and $P(c) = P'(c) \geq 1$ then demand is zero
  - If $c^e = c^*$ and $P(c) = P'(c) < 1$ then

$$L(P(c), P(c^e)) = L(P'(c), P'(c^e) = 0$$

    for ε small enough

- Deviation is therefore profitable

Lemma

For every optimal pricing strategy, $P(c)$ is increasing in c and $P(c) \geq c$ for all c

- Proof (first statement)
  - Assume there is an optimal strategy such that $c_1 > c_2$ but $P(c_2) > P(c_1)$
  - Consider the strategy P' that switches round the prices of c_2 and c_1
  - μ_P is the same as $\mu_{P'}$
  - Demand only changes with price
  - Change in profit therefore given by

$$\begin{aligned} & (P(c_2) - c_1)\, D(P(c_2)) - (P(c_1) - c_1)\, D(P(c_1)) \\ & + (P(c_1) - c_2)\, D(P(c_1)) - (P(c_2) - c_2)\, D(P(c_2)) \\ = \quad & (c_2 - c_1)(D(P(c_2)) - D(P(c_1))) > 0 \end{aligned}$$

  - $P(c_2) > P(c_1)$ and so $D(P(c_2) < D(P(c_1))$ (strictly by lemma 1)

Two Results

- Loss aversion reduces price volatility
- Loss aversion induces price stickiness

Result 1: Loss Aversion Reduces Price Volatility

Theorem

Let P be the optimal pricing strategy, and P^0 be the optimal pricing strategy without loss aversion, then

$$P^0(c^l) \leq p(c^l) \leq P(c^h) \leq P^0(c^h)$$

- Proof ($P^0(c^l) \leq P(c^l)$)
  - We know that $P^0(c^l) = \frac{1}{2}(1 + c^l)$
  - Assume $p(c^l) < \frac{1}{2}(1 + c^l)$
  - Define c^0 to be the highest cost c such that $P(c) < \frac{1}{2}(1 + c^l)$
  - Implies $P(c) < \frac{1}{2}(1 + c^l)$ for all $c \leq c^0$ (by lemma 2)

Result 1: Loss Aversion Reduces Price Volatility

Theorem

Let P be the optimal pricing strategy, and P^0 be the optimal pricing strategy without loss aversion, then

$$P^0(c^l) \leq p(c^l) \leq P(c^h) \leq P^0(c^h)$$

- Proof ($P^0(c^l) \leq P(c^l)$): Cont
  - Consider deviation that sets $P(c) = \frac{1}{2}(1 + c^l)$ for all $c < c^0$
  - Would improve profit if no loss aversion, as moves closer to optimal price
  - Four cases to consider loss aversion $L(P'(c_1), P'(c_2))$
  - $c_1, c_2 \leq c_0 : L(P'(c_1), P'(c_2)) = 0$ as $P'(c_1) = P'(c_2)$
  - $c_2 \leq c_0 \leq c_1 : L(P'(c_1), P'(c_2)) \leq L(P(c_1), P(c_2))$ as $P(c_1) = P'(c_1)$ and $P(c_2) \leq P'(c_2)$
  - $c_0 \leq c_1, c_2 : L(P'(c_1), P'(c_2)) = L(P(c_1), P(c_2))$ as $P(c_1) = P'(c_1)$ and $P(c_2) = P'(c_2)$
  - $c_1 \leq c_0 \leq c_2 : L(P'(c_1), P'(c_2)) = 0$ as $P'(c_2) \geq P'(c_1)$

Result 2: Price Stickiness

Theorem

Let $C = \{c, c + 2\varepsilon\}$. When $\varepsilon > 0$ is sufficiently small, the optimal pricing strategy is to charge constant price $\bar{p} = \frac{1}{2}(1 + c + e)$

- Proof: First consider the case in which demand is zero when reference price is p_l and actual price is p_h.
- Profits equal

$$\frac{1}{2}(p_l - c)(1 - p_l) + \frac{1}{2}(p_h - c - 2\varepsilon)\frac{1}{2}(1 - p_h)$$

- if $\varepsilon < \frac{3}{4}$ we can show that this is lower than the payoff of $\bar{p}$ for any p_l ,p_h

Theorem

Let $C = \{c, c + 2\varepsilon\}$. When $\varepsilon > 0$ is sufficiently small, the optimal pricing strategy is to charge constant price $\bar{p} = \frac{1}{2}(1 + c + e)$

- Proof: (Cont); Now assume that demand is non-zero when reference price is p_l and actual price is p_h
- Profit equals

$$\frac{1}{2}(p_l - c)(1 - p_l) + \frac{1}{2}(p_h - c - 2\varepsilon)(1 - p_h)$$
$$-\frac{\lambda}{2}(p_h - c - 2\varepsilon)\lambda(p_h - p_l)$$

- FOC

$$\begin{aligned} \frac{1}{2}(1 + c - 2p_l) + \frac{\lambda}{2}(p_h - c - 2\varepsilon) &= 0 \\ \frac{1}{2}(1 + c + 2\varepsilon - 2p_h) - \frac{\lambda}{2}(p_h - c - 2\varepsilon) - \frac{\lambda}{2}(p_h - p_l) &= 0 \end{aligned}$$

Theorem

Let $C = \{c, c + 2\varepsilon\}$. When $\varepsilon > 0$ and $c < \frac{1}{2}$ is sufficiently small, the optimal pricing strategy is to charge constant price $\bar{p} = \frac{1}{2}(1 + c + \varepsilon)$

- Proof: (Cont); Rearranging and using the fact that $p_h > p_l$ gives
$$\lambda(p_h - p_l) < 4\varepsilon - 2\lambda(p_h - c - 2\varepsilon)$$
- By theorem 1 we know $p_h > \frac{1}{2}(1 + c)$, thus we have
$$\lambda(p_h - p_l) < 4\varepsilon - 2\lambda(\frac{1}{2} - \frac{1}{2}c - 2\varepsilon)$$
- Assuming $c < 1$ means that the RHS will be negative for ε small enough - contradiction

Heidues and Köszegi [2008]

- Embed a (similar) model in a monopolistic competition setting
- Firms product type is located on a ring (fixed)
- Consumers preferences located on a same ring (stochastic)
- Firms observe a cost and set prices

- Consumers form stochastic reference point
- Distribution over prices paid and products bought based on
  - Exogenous distribution over preferences
  - Exogenous distribution over costs
  - Pricing strategy of firm
  - Own purchasing strategy
- Prices and preferences revealed, makes purchases to maximize utility given reference point

Heidues and Köszegi [2008]

- Strategies are an equilibrium if
  - Purchasing strategy is a personal equilibrium for consumer given price distribution generated by firms
  - Pricing strategy is optimal for firms given purchasing strategy of consumers and pricing strategies of other firms
- HK provide necessary and sufficient conditions for *Focal Equilibrium*
  - All firms charge the same price with probability 1 regardless of cost
- Intuition similar to S: If consumers expect to pay some price, a higher price is a loss
- Kink in the demand curve

Hart and Moore [2008]

- Contracts act as reference points
- Guide expectations about what parties feel they deserve
- Agents may underperform if they do not get what they expect
- Incomplete contracts may be ex-post inefficient
  - Unlike standard contracting
- Sets up trade off between flexibility and underperformance

Basic Structure

- Buyer and a Seller meet in a competitive market at date 0
  - Many buyers and sellers
  - May be uncertainty about (e.g.) preferences and costs
- Write contract at this stage
- Only some things are contractible
  - 'Perfunctary Performance' - e.g. price of trade
  - But not 'Consumate Performance' - e.g. quality
- Uncertainty resolves
- Contract refined (i.e. perfunctory performance determined)
- Level of consummate performance decided

Reference Dependent Preferences

- Agents prepared to offer 'consummate performance' only if they are well treated
- Well treated or not relative to contract they signed
- If only one contracted outcome, both parties think they have been treated fairly
- If more than one possible outcome, both judge outcome relative to the contracted outcome that was best for them

- Utility

$$\begin{aligned} U_B &= u_B - \sigma_s - \max[\theta(u_B^* - u_B) - \sigma_B, 0] \\ U_S &= u_S - \sigma_B - \max[\theta(u_s^* - u_s) - \sigma_S, 0] \end{aligned}$$

  Where

  - u_i is utility of i from contractual outcome (assuming full consummate performance)
  - σ_i is the 'shading' (i.e. reduction of consummate performance) by i
  - u_i^* is maximal value of u_i over contracted outcomes
  - $(u_i^* - u_i)$ 'aggrievement' of agent i

- Assume that u_i known when σ_i chosen, so always the case that

$$\begin{aligned} \theta a_i &= \theta(u_i^* - u_i) = \sigma_i \\ U_i &= u_i - \theta a_j \end{aligned}$$

- What is shading?
  - Reductions in effort that cannot be contractually punished
  - skimping on ingredients
  - low effort at work
  - bad reviews
- No direct cost to shading/non shading
  - positive cost of shading can be incorporated, negative cost less easily so
  - Why not shade all the time?
  - Seen as a model of costly punishment?

Comments

- Preferences are combination of 'other regarding' and 'reference dependent'
  - Agents dislike losses relative to expectations
  - Expectations determined by contract
  - Losses can be assuaged by reducing the utility of other party
- Why are expectations the best possible outcome?
  - Self serving bias?
  - Consider alternative assumptions in the paper
- Why ‘fairness’ concerns only in period 1
  - Period 0 had choice of other people to contract with
  - Market environment
  - At period 1 'locked in' to dealing with this partner
  - Market environment itslef no longer salient, just contract

Example 1: No Uncertainty

- B requires 1 unit of a standard good
- $B's$ value is 100
- $S's$ costs are zero
- Contract specifies whether or not good is traded and a price if trade does and does not take place
- What is the optimal contract?

- No price specifying contact necessary to achieve efficiency
- Assume that Nash Bargaining takes place at period 1
- Trade takes place at price 50
- Competitive equilibrium achieved at period 0 by lump some transfers
  - e.g. if there are many buyers and 1 seller then a buyer would offer 50 to seller in order to go to stage 2
- Total surplus is 100

- 'No contract' is inefficient
- Assume no price is specified at stage 0
- Assume that trade takes place at price p in stage 1
- Best outcome of those possible for buyer was $p_b^* = 0$
- Best outcome of those possible for seller was $p_s^* = 100$

$$\begin{aligned} \theta(u_b^* - u_b) &= \theta p = \sigma_b \\ \theta(u_s^* - u_s) &= \theta(100 - p) = \sigma_s \end{aligned}$$

- Ex post utility given by

$$\begin{aligned} U_B &= (1-\theta)(100-p) \\ U_S &= (1-\theta)p \end{aligned}$$

- Total surplus is $(1-\theta)100$
- Similar effect if a mechanism is agreed upon at time 0
  - e.g. single take it or leave it offer
- However, if a contract specifying a price is agreed upon at time 0 then surplus is still 100.

Example 2; Ex Ante Uncertainty

- B requires 1 unit of a standard good
- $B's$ value is v
- $S's$ costs are c
- v and c unknown at time 0 - drawn from distribution
- Assume that trade only takes place at time 1 if both parties want to

- p_0 no trade price
- p_1 trade price
- Trade occurs if

$$\begin{aligned} v - p_1 &\geq -p_0 \\ p_1 - c &\geq 0 \\ q &= 1 \Leftrightarrow v \geq p_1 - p_0 \geq c \end{aligned}$$

- Normalize p_0 to 0
- Compare to first best trade rule

$$q = 1 \Leftrightarrow v \geq c$$

- So a gap between p_1 and p_0 reduces trade relative to first best

- p_0 no trade price
- $[p^L, p^H]$ region for trade prices
- Trade occurs if

$$
\begin{aligned}
q &= 1 \Leftrightarrow \exists\ p \in [p^L, p^H] \\
v &\geq p - p_0 \geq c
\end{aligned}
$$

- Normalize p_0 to 0

$$
\begin{aligned}
q &= 1 \Leftrightarrow v \geq c \\
v &\geq p^L,\ c \leq p^H
\end{aligned}
$$

- Given voluntary trade, best price for seller is

$$p_S^* = \min(v, p^H)$$

- And best price for buyer is

$$p_B^* = \max(c, p^L)$$

- Aggreivement for buyer (assuming trade) is

$$\Big(v - \max(c, p^L)\Big) - (v - p)$$

- Aggrievement for seller

$$\Big(\min(v, p^H) - c\Big) - (p - c)$$

- Total aggrievement

$$\min(v, p^H) - \max(c, p^L)$$

Optimal Contract

- Given lump sum transfers, optimal contract maximizes total surplus

$$\int \left[v - c - \theta\left(\min(v, p^H) - \max(c, p^L)\right)\right] dF(v, c)$$

subject to

$$\begin{aligned} v &\geq x \\ v &\geq p^L \\ c &\leq p^H \end{aligned}$$

- Clear trade off
  - More flexible contract: more trade...
  - .... but also more shading
- Simple contracts achieves first best if v or c degenerate, or if they are separated

Example

	State 1	State 2
v	9	20
c	0	10

- No simple contract will achieve first best, as no single price can guarantee trade
- Contract that specifies trading region $[9, 10]$ does achieve 1st best
- In state 1

$$\begin{aligned} & \min(v, p^H) - \max(c, p^L) \\ = \ & 9 - 9 = 0 \end{aligned}$$

- In state 2

$$\begin{aligned} & \min(v, p^H) - \max(c, p^L) \\ = \ & 10 - 10 = 0 \end{aligned}$$