Reference Dependence Lecture 3

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The Story So Far

- Defined reference dependent behavior and given examples
  - Change in risk attitudes
  - Endowment effect
  - Status quo bias
- Discussed two models of reference dependent choice
  - Prospect Theory
  - Koszergi-Rabin [2006,2007]
- Latter introduced
  - Stochastic reference points
  - Personal equilibrium
Plan for Today

- Two applications
- Application: Pricing
  - Heidues and Köszegi [2008]
  - Speigler [2011]
- Contracting
  - Hart and Moore [2008]
Two Pricing Puzzles

- Prices are unresponsive to changes in costs (‘price stickiness’)
- Firms charge prices that are too similar across differentiated products with different costs (‘focal pricing’)


Two Pricing Puzzles

- Existing theories (Rational Inattention, Menu Costs) can explain the former, but have trouble with the latter

- Aim
  - Show loss aversion can explain the former
  - May also be able to explain the latter

- Intuition: Consumers form expectations over prices that will be charged

- Loss aversion means that marginal disutility of price rises is high...

- ....while marginal benefit of price falls is not so high

- Causes kink in firm’s demand function around expected price

- Makes prices ’unresponsive’
Two papers formalize this intuition

- Heidues and Köszegi [2008] (henceforth HK)
- Speigler [2011] (henceforth S)

Speigler is a ’cover version’

- But it is also simpler

Will go through this model in detail

Discuss Heidues and Koszegi afterwards
Modelling Choices - Consumer Side

- Over what are consumers loss averse?
- Does consumers expected action affect reference point?
- How to deal with stochastic reference points?
- Over what are consumers loss averse?
  - HK: Money and consumption quality
  - S: Money only
- Does consumers expected action affect reference point?
- How to deal with stochastic reference points?
Over what are consumers loss averse?

Does consumers expected action affect reference point?

- HK: Yes: personal equilibrium
- S: No: reference point is what consumers to expect firms to charge

How to deal with stochastic reference points?
• Over what are consumers loss averse?
• Does consumers expected action affect reference point?
• How to deal with stochastic reference points?
  • HK: Evaluate action at each reference point in support, calculate expectation
  • S: Consumer draws reference point from distribution and evaluates action
• HK: Salop style monopolistic competition
  • Firm’s ‘type’ located equidistantly on a ring
  • Consumer type drawn from a distribution.
  • Value of product to consumer is distance between consumer and firm type

• S: monopolist
• Both models give price stickiness
  • Result robust to these assumption
• S annot talk about focal pricing
• Monopolistic firm, single good, measure 1 of consumers
• Marginal cost of firm $c$ distributed uniformly over $m$ possible values $C$
  • $1 > c_h > c_l > 0$ respectively highest and lowest prices in $C$
• Pricing strategy $P : C \rightarrow \mathbb{IR}$
• Induces measure $\mu_P$ over prices

\[
\mu_p(x) = \frac{|c \in C | P(c) = x|}{m}
\]
Set Up - Consumer

- Consumer first draws a reference price $p^e$ from distribution $\mu_p$
- Draws valuation of good $u$ from $U[0, 1]$
- Buys product iff
  \[ p \leq u - L(p, p^e) \]
- where
  \[ L(p, p^e) = \max\{0, \lambda(p - p^e)\} \]
  \[ \lambda \geq 0 \]
- Consumer identified by
  1. Valuation $u$
  2. Reference price $p^e$
Optimization Problem - Firm

- For any given price, consumer will buy if
  \[ p + L(p, p^e) \leq u \]

- With uniform distribution, this is probability
  \[ 1 - p - L(p, p^e) \]

- For pricing strategy \( P \), there is a \( \frac{1}{m^2} \) probability that prices will be \( P(c) \) and expectations will be \( P(c^e) \) for each \( c, c^e \in C \)

- Summing across all this states gives the valuation of a pricing strategy as
  \[
  \Pi(p) = \frac{1}{m^2} \sum_{c \in C} \sum_{c^e \in C} [P(c) - c] \max\{0, 1 - P(c) - L(P(c), P(c^e))\} \]
Firm is assumed to announce strategy ex ante
Not a simultaneous move game
When we consider deviations in firm’s strategy, we assume beliefs of agents adjust as well
’Unexpected’ price hike means relative to beliefs correctly conditioning on strategy, not due to change in strategy
\[ \Pi(p) = \frac{1}{m^2} \sum_{c \in C} \sum_{c^e \in C} [P(c) - c] \max\{0, 1 - P(c) - L(P(c), P(c^e))\} \]

- Without loss aversion, expectations do not matter

\[ \Pi(p) = \frac{1}{m^2} \sum_{c \in C} [P(c) - c] \max\{0, 1 - P(c)\} \]

- For each \( c \), FOC

\[
\begin{align*}
1 + c - 2P(c) & = 0 \\
\frac{1 + c}{2} & = P^0(c)
\end{align*}
\]
Lemma

Let $P$ be an optimal pricing strategy. Then for every $c \in C$, $P(c) < 1$ and consumer demand is strictly positive.

Proof:

- Assume $P(c^*) \geq 1$ for some $c^* \in C$, then consumer demand is zero at $P(c^*)$.
- Consider $P'$ which changes $P$ only by setting $P(c^*) = 1 - \varepsilon$.
- Consumer demand is now strictly positive in this state, as

$$1 - P'(c^*) - L(P'(c^*), P'(c^*)) = 1 - P'(c^*) = \varepsilon > 0$$

- Profit also strictly positive assuming $\varepsilon$ st $\varepsilon < 1 - c$. 

Proof (cont)

- For all other $c \in C$, consumer demand is unchanged
- Consider pair $c$ $c^e$. If $c^e \neq c^*$ this is clearly true
- If $c^e = c^*$ and $P(c) = P'(c) \geq 1$ then demand is zero
- If $c^e = c^*$ and $P(c) = P'(c) < 1$ then

$$L(P(c), P(c^e)) = L(P'(c), P'(c^e)) = 0$$

for $\varepsilon$ small enough

- Deviation is therefore profitable
Lemma

For every optimal pricing strategy, $P(c)$ is increasing in $c$ and $P(c) \geq c$ for all $c$

- Proof (first statement)
  - Assume there is an optimal strategy such that $c_1 > c_2$ but $P(c_2) > P(c_1)$
  - Consider the strategy $P'$ that switches round the prices of $c_2$ and $c_1$
  - $\mu_P$ is the same as $\mu_{P'}$
  - Demand only changes with price
  - Change in profit therefore given by
    \[
    (P(c_2) - c_1)D(P(c_2)) - (P(c_1) - c_1)D(P(c_1)) + (P(c_1) - c_2)D(P(c_1)) - (P(c_2) - c_2)D(P(c_2))
    \]
    \[
    = (c_2 - c_1)(D(P(c_2)) - D(P(c_1))) > 0
    \]
  - $P(c_2) > P(c_1)$ and so $D(P(c_2) < D(P(c_1))$ (strictly by lemma 1)
Two Results

- Loss aversion reduces price volatility
- Loss aversion induces price stickiness
Result 1: Loss Aversion Reduces Price Volatility

**Theorem**

Let $P$ be the optimal pricing strategy, and $P^0$ be the optimal pricing strategy without loss aversion, then

$$P^0(c^l) \leq p(c^l) \leq P(c^h) \leq P^0(c^h)$$

- **Proof ($P^0(c^l) \leq P(c^l)$)**
  - We know that $P^0(c^l) = \frac{1}{2}(1 + c^l)$
  - Assume $p(c^l) < \frac{1}{2}(1 + c^l)$
  - Define $c^0$ to be the highest cost $c$ such that $P(c) < \frac{1}{2}(1 + c^l)$
  - Implies $P(c) < \frac{1}{2}(1 + c^l)$ for all $c \leq c^0$ (by lemma 2)
Result 1: Loss Aversion Reduces Price Volatility

Theorem

Let $P$ be the optimal pricing strategy, and $P^0$ be the optimal pricing strategy without loss aversion, then

$$P^0(c^l) \leq p(c^l) \leq P(c^h) \leq P^0(c^h)$$

- Proof ($P^0(c^l) \leq P(c^l)$): Cont
  - Consider deviation that sets $P(c) = \frac{1}{2}(1 + c^l)$ for all $c < c^0$
  - Would improve profit if no loss aversion, as moves closer to optimal price
  - Four cases to consider loss aversion $L(P'(c_1), P'(c_2))$
    - $c_1, c_2 \leq c_0 : L(P'(c_1), P'(c_2)) = 0$ as $P'(c_1) = P'(c_2)$
    - $c_2 \leq c_0 \leq c_1 : L(P'(c_1), P'(c_2)) \leq L(P(c_1), P(c_2))$ as $P(c_1) = P'(c_1)$ and $P(c_2) \leq P'(c_2)$
    - $c_0 \leq c_1, c_2 : L(P'(c_1), P'(c_2)) = L(P(c_1), P(c_2))$ as $P(c_1) = P'(c_1)$ and $P(c_2) = P'(c_2)$
    - $c_1 \leq c_0 \leq c_2 : L(P'(c_1), P'(c_2)) = 0$ as $P'(c_2) \geq P'(c_1)$
Theorem

Let $C = \{c, c + 2\varepsilon\}$. When $\varepsilon > 0$ is sufficiently small, the optimal pricing strategy is to charge constant price $\bar{p} = \frac{1}{2}(1 + c + \varepsilon)$

- Proof: First consider the case in which demand is zero when reference price is $p_l$ and actual price is $p_h$.
- Profits equal

$$\frac{1}{2}(p_l - c)(1 - p_l) + \frac{1}{2}(p_h - c - 2\varepsilon)\frac{1}{2}(1 - p_h)$$

- if $\varepsilon < \frac{3}{4}$ we can show that this is lower than the payoff of $\bar{p}$ for any $p_l, p_h$. 
Result 2: Price Stickiness

Theorem

Let $C = \{c, c + 2\varepsilon\}$. When $\varepsilon > 0$ is sufficiently small, the optimal pricing strategy is to charge constant price $\bar{p} = \frac{1}{2}(1 + c + \varepsilon)$

- Proof: (Cont); Now assume that demand is non-zero when reference price is $p_l$ and actual price is $p_h$
- Profit equals

$$
\frac{1}{2}(p_l - c)(1 - p_l) + \frac{1}{2}(p_h - c - 2\varepsilon)(1 - p_h) - \frac{\lambda}{2}(p_h - c - 2\varepsilon)\lambda(p_h - p_l)
$$

- FOC

$$
\frac{1}{2}(1 + c - 2p_l) + \frac{\lambda}{2}(p_h - c - 2\varepsilon) = 0
$$
$$
\frac{1}{2}(1 + c + 2\varepsilon - 2p_h) - \frac{\lambda}{2}(p_h - c - 2\varepsilon) - \frac{\lambda}{2}(p_h - p_l) = 0
$$
Result 2: Price Stickiness

Theorem

Let \( C = \{ c, c + 2\varepsilon \} \). When \( \varepsilon > 0 \) and \( c < \frac{1}{2} \) is sufficiently small, the optimal pricing strategy is to charge constant price

\[ \bar{p} = \frac{1}{2} (1 + c + \varepsilon) \]

- Proof: (Cont); Rearranging and using the fact that \( p_h > p_l \) gives

\[ \lambda(p_h - p_l) < 4\varepsilon - 2\lambda(p_h - c - 2\varepsilon) \]

- By theorem 1 we know \( p_h > \frac{1}{2} (1 + c) \), thus we have

\[ \lambda(p_h - p_l) < 4\varepsilon - 2\lambda(\frac{1}{2} - \frac{1}{2}c - 2\varepsilon) \]

- Assuming \( c < 1 \) means that the RHS will be negative for \( \varepsilon \) small enough - contradiction
• Embed a (similar) model in a monopolistic competition setting
• Firms product type is located on a ring (fixed)
• Consumers preferences located on a same ring (stochastic)
• Firms observe a cost and set prices
• Consumers form stochastic reference point

• Distribution over prices paid and products bought based on
  • Exogenous distribution over preferences
  • Exogenous distribution over costs
  • Pricing strategy of firm
  • Own purchasing strategy

• Prices and preferences revealed, makes purchases to maximize utility given reference point
• Strategies are an equilibrium if
  • Purchasing strategy is a personal equilibrium for consumer given price distribution generated by firms
  • Pricing strategy is optimal for firms given purchasing strategy of consumers and pricing strategies of other firms

• HK provide necessary and sufficient conditions for *Focal Equilibrium*
  • All firms charge the same price with probability 1 regardless of cost

• Intuition similar to S: If consumers expect to pay some price, a higher price is a loss

• Kink in the demand curve
• Contracts act as reference points
• Guide expectations about what parties feel they deserve
• Agents may underperform if they do not get what they expect
• Incomplete contracts may be ex-post inefficient
  • Unlike standard contracting
• Sets up trade off between flexibility and underperformance
Basic Structure

- Buyer and a Seller meet in a competitive market at date 0
  - Many buyers and sellers
  - May be uncertainty about (e.g.) preferences and costs
- Write contract at this stage
- Only some things are contractible
  - 'Perfunctory Performance' - e.g. price of trade
  - But not 'Consummate Performance' - e.g. quality
- Uncertainty resolves
- Contract refined (i.e. perfunctory performance determined)
- Level of consummate performance decided
Reference Dependent Preferences

- Agents prepared to offer 'consummate performance' only if they are well treated
- Well treated or not relative to contract they signed
- If only one contracted outcome, both parties think they have been treated fairly
- If more than one possible outcome, both judge outcome relative to the contracted outcome that was best for them
Reference Dependent Preferences

- **Utility**

\[
U_B = u_B - \sigma_s - \max[\theta(u_B^* - u_B) - \sigma_B, 0]
\]

\[
U_S = u_S - \sigma_B - \max[\theta(u_s^* - u_s) - \sigma_S, 0]
\]

Where

- \(u_i\) is utility of \(i\) from contractual outcome (assuming full consummate performance)
- \(\sigma_i\) is the ‘shading’ (i.e. reduction of consummate performance) by \(i\)
- \(u_i^*\) is maximal value of \(u_i\) over contracted outcomes
- \((u_i^* - u_i)\) ’aggrievement’ of agent \(i\)

- Assume that \(u_i\) known when \(\sigma_i\) chosen, so always the case that

\[
\theta a_i = \theta(u_i^* - u_i) = \sigma_i
\]

\[
U_i = u_i - \theta a_j
\]
• What is shading?
  • Reductions in effort that cannot be contractually punished
  • skimping on ingredients
  • low effort at work
  • bad reviews

• No direct cost to shading/non shading
  • positive cost of shading can be incorporated, negative cost less easily so
  • Why not shade all the time?
  • Seen as a model of costly punishment?
• Preferences are combination of ‘other regarding’ and ‘reference dependent’
  • Agents dislike losses relative to expectations
  • Expectations determined by contract
  • Losses can be assuaged by reducing the utility of other party

• Why are expectations the best possible outcome?
  • Self serving bias?
  • Consider alternative assumptions in the paper

• Why ‘fairness’ concerns only in period 1
  • Period 0 had choice of other people to contract with
  • Market environment
  • At period 1 ‘locked in’ to dealing with this partner
  • Market environment itself no longer salient, just contract
Example 1: No Uncertainty

- $B$ requires 1 unit of a standard good
- $B'$'s value is 100
- $S'$'s costs are zero
- Contract specifies whether or not good is traded and a price if trade does and does not take place
- What is the optimal contract?
• No price specifying contact necessary to achieve efficiency
• Assume that Nash Bargaining takes place at period 1
• Trade takes place at price 50
• Competitive equilibrium achieved at period 0 by lump some transfers
  • e.g. if there are many buyers and 1 seller then a buyer would offer 50 to seller in order to go to stage 2
• Total surplus is 100
Hart-Moore Model

- ‘No contract’ is inefficient
- Assume no price is specified at stage 0
- Assume that trade takes place at price $p$ in stage 1
- Best outcome of those possible for buyer was $p_b^* = 0$
- Best outcome of those possible for seller was $p_s^* = 100$

\[
\begin{align*}
\theta(u_b^* - u_b) &= \theta p = \sigma_b \\
\theta(u_s^* - u_s) &= \theta(100 - p) = \sigma_s
\end{align*}
\]
- Ex post utility given by

\[ U_B = (1 - \theta)(100 - p) \]
\[ U_S = (1 - \theta)p \]

- Total surplus is \((1 - \theta)100\)
- Similar effect if a mechanism is agreed upon at time 0
  - e.g. single take it or leave it offer
- However, if a contract specifying a price is agreed upon at time 0 then surplus is still 100.
• $B$ requires 1 unit of a standard good
• $B'$s value is $v$
• $S'$s costs are $c$
• $v$ and $c$ unknown at time 0 - drawn from distribution
• Assume that trade only takes place at time 1 if both parties want to
• $p_0$ no trade price
• $p_1$ trade price
• Trade occurs if

\[
\begin{align*}
\nu - p_1 & \geq -p_0 \\
p_1 - c & \geq 0
\end{align*}
\]

\[q = 1 \iff \nu \geq p_1 - p_0 \geq c\]

• Normalize $p_0$ to 0
• Compare to first best trade rule

\[q = 1 \iff \nu \geq c\]

• So a gap between $p_1$ and $p_0$ reduces trade relative to first best
Flexible contracts

- $p_0$ no trade price
- $[p^L, p^H]$ region for trade prices
- Trade occurs if

$$q = 1 \iff \exists p \in [p^L, p^H]$$

$$v \geq p - p_0 \geq c$$

- Normalize $p_0$ to 0

$$q = 1 \iff v \geq c$$

$$v \geq p^L, c \leq p^H$$
Entitlements and Shading

- Given voluntary trade, best price for seller is
  \[ p_S^* = \min(v, p^H) \]
- And best price for buyer is
  \[ p_B^* = \max(c, p^L) \]
- Aggrievement for buyer (assuming trade) is
  \[ \left( v - \max(c, p^L) \right) - (v - p) \]
- Aggrievement for seller
  \[ \left( \min(v, p^H) - c \right) - (p - c) \]
- Total aggrievement
  \[ \min(v, p^H) - \max(c, p^L) \]
Given lump sum transfers, optimal contract maximizes total surplus

\[ \int \left[ v - c - \theta \left( \min(v, p^H) - \max(c, p^L) \right) \right] dF(v, c) \]

subject to

\[ v \geq x \]
\[ v \geq p^L \]
\[ c \leq p^H \]

• Clear trade off
  • More flexible contract: more trade...
  • .... but also more shading

• Simple contracts achieves first best if \( v \) or \( c \) degenerate, or if they are separated
Example

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>9</td>
</tr>
<tr>
<td>c</td>
<td>0</td>
</tr>
</tbody>
</table>

- No simple contract will achieve first best, as no single price can guarantee trade
- Contract that specifies trading region $[9, 10]$ does achieve 1st best
- In state 1
  \[
  \min(v, p^H) - \max(c, p^L) = 9 - 9 = 0
  \]
- In state 2
  \[
  \min(v, p^H) - \max(c, p^L) = 10 - 10 = 0
  \]