Credit Constraints and the Measurement of Time Preferences

Mark Dean†  Anja Sautmann‡

January 24, 2019

Abstract

Incentivized experiments are often used to identify the time preferences of households in developing countries. We argue theoretically and empirically that experimental measures may not identify preferences, but are a useful tool for understanding financial shocks and constraints. Using data from an experiment in Mali we find that subject responses vary with savings and financial shocks, meaning they provide information about credit constraints and can be used to test models of risk sharing. We use our model and data to determine that changes in consumption are driven by substantial unsmoothed ‘preference’ shocks, which are quantitatively important relative to income shocks.

JEL: D90, D03, D14, C90, C81, B41

*We would like to thank in particular Andrew Foster and Demian Pouzo for their helpful suggestions and comments, as well as David Atkin, Daniel Björkegren, Yoram Halevy, Kyle Hyndman, Greg Kaplan, Supreet Kaur, Aprajit Mahajan, Dilip Mokherjee, Sriniketh Nagavarapu, Suresh Naidu, Andriy Norets, Nancy Qian, Marta Serra Garcia, Jesse Shapiro, Duncan Thomas, Martin Uribe, David Weil and seminar participants at UC Berkeley, Boston University, Columbia University, and Yale University, and at the 27th BREAD conference, the NBER Summer Institute 2014, and SITE 2014. We also thank Seunghoon Na for excellent research assistance in providing the numerical solution to the model used in appendix C. All errors are exclusively ours. We recognize support from the Aga Khan Foundation, the Economic and Social Research Council (Development Frontiers Grant ES/K01207X/1), the Brown University Seed Fund award, and the Population Studies and Training Center at Brown University in the completion of this project. Dean gratefully acknowledges the support of NSF grant 1156090. Special thanks go to Mali Health for their help in the design and implementation of the project.

†Department of Economics, Columbia University. Email mark.dean@columbia.edu.

‡Corresponding author: Brown University and Massachusetts Institute of Technology, 400 Main Street, Cambridge, MA 02141, USA. Tel +1-617-258-0667, email: sautmann@mit.edu.
Experimental methods have become an important part of the applied economist’s toolkit. They are regularly used to identify individual-specific preferences, in particular those that govern intertemporal choices. Typically, such time preference experiments measure the subject’s relative valuation for money received in two different periods. In order to identify underlying personal discount factors directly from experimental choices it must be assumed that these choices are divorced from outside conditions. As many have pointed out (for example Frederick et al. [2002]), without this ‘narrow bracketing’ assumption, experimental trade-offs may be affected by prevailing credit market conditions.

In this paper we use a panel data set from poor households in Mali to demonstrate that narrow bracketing does not hold in (at least some) developing country contexts. This implies that experimental choices do not directly identify time preference parameters. Instead, they measure the Marginal Rate of Intertemporal Substitution (MRS) for consumption. This makes them a tool to understand many other questions of interest to economists, such as the relative importance of different types of financial shocks affecting households, and their ability to cope with these shocks through insurance or intertemporal consumption smoothing (Townsend [1994], Ligon [1998], Karlan et al. [2014] are examples of influential papers which have addressed these issues).

In order to understand what can be learned from time preference experiments, we propose a model that integrates experimental decisions with the subject’s broader intertemporal optimization problem. We assume a decision maker with quasi-hyperbolic preferences who suffers income shocks as well as preference shocks that affect their marginal utility from consumption expenditure (due, for example, to sickness of a household member). The effective interest rate at which households can borrow and save depends negatively on their current savings stock. This reduced-form model of ‘soft’ or ‘partial’ credit constraints is easily tractable, and can accommodate many existing models of credit rationing.

The model predicts that, if subjects take into account their broader economic circumstance when making experimental choices, measured MRS equals the individual interest rate at the optimal level of savings in each period, which in turn equals the ratio between marginal utility of consumption today and expected discounted marginal utility of wealth tomorrow. Importantly, this conclusion does not require that subjects arbitrage the experimental payments, but only that they adjust outside consumption optimally to non-experimental shocks and take the resulting change in their “real-world MRS” into account when making choices in the experiment.

This finding suggests that experimentally measured MRS provides a tool to learn about

---

1See for example Ashraf et al. [2006], Tanaka et al. [2010], Mahajan and Tarozzi [2011], Schaner [2015].
2See Frederick et al. [2002] for a comprehensive overview.
credit constraints and financial shocks, and to test models of consumption smoothing. For example, the covariance of MRS with other financial variables helps to identify the credit regime under which a household is operating. Our partial credit constraints model predicts that positive income shocks decrease measured MRS, that preference shocks increase it, and that a higher stock of savings is directly linked with a lower interest rate and therefore MRS. The same relationships do not hold in the extreme cases of a household without credit constraints (the ‘no constraints’ model) or one that is completely unable to borrow and save (the ‘complete constraints’ model). We also show how MRS data can be used to learn about the relative importance of income and preference shocks in driving changes in measured consumption (i.e. spending). If spending is primarily driven by income shocks, then our model predicts that high spending coincides with relatively low marginal utility of consumption and low measured MRS. However, if spending is driven by preference shocks that increase the marginal utility of consumption, then high spending correlates with high marginal utility and relative impatience (high MRS).

Our approach suggests many other applications for experimental measures of MRS both on its own, and in conjunction with other financial variables. The covariance of MRS with individual and collective shocks can serve to test models of informal risk sharing (e.g Townsend [1994], Ligon [1998], Kinnan [2014]). Because the MRS directly traces out the household’s effective interest rate at different savings rates, reductions in its variance can be used to measure the success of interventions designed to remove credit constraints and facilitate intertemporal consumption smoothing (e.g. Karlan et al. [2014]). Existing approaches to answering questions of this type typically require the parameterized estimation of marginal utility over time, as in the consumption Euler equation, a strategy well-known to be complicated by approximation bias and measurement error (see for example Ludvigson and Paxson [2006], Carroll [2001], Alan et al. [2009]). By contrast, experimentally elicited MRS is easy to collect, does not require a parameterized utility function, and its measurement can be carried out independently from, or in addition to, survey measures of household consumption.

These applications imply a valuable role for experimental MRS measures in the applied economist’s toolkit. The degree to which households can smooth consumption through credit and insurance is of great importance in development and household economics. Incomplete financial markets lead individuals to resort to second-best risk management mechanisms and hamper investment in both human and physical capital. This can lead to allocative distortions and may create poverty traps (see e.g. Rosenzweig and Wolpin [1993] for a well-known example).

We apply our model in a unique panel data set from 1013 household in Mali which
measures intertemporal trade-offs and financial data in three consecutive weeks. We find that measured MRS responds as predicted to exogenous preference and income shocks. This rules out the narrow bracketing and ‘no constraints’ model. Moreover, we find a negative correlation of MRS with savings, ruling out the ‘complete constraints’ model in favor of partial credit constraints. To our knowledge, our paper is the first to document the simultaneous correlation of MRS with income shocks, preference shocks, and savings, allowing us to identify the credit regime that best describes our sample.

Next, we use the data to show that preference shocks are quantitatively important, demonstrated by the fact that households are *more* impatient in periods in which they are spending more. This positive correlation is driven by expenses on adverse events and on food and necessities, identifying them as important sources of uninsured risk. We can use our model to bound the size of the preference shocks and find that in our data they explain at least 46% of the total expenditure variance. An equivalent nonparametric test that uses only financial data might build on the correlation of spending and (stock) savings, but would be complicated by correlated measurement error due to the direct relationship between changes in savings and expenditure.\(^3\) The existence of quantitatively important preference shocks not only has potential policy implications, but is also important from a methodological standpoint. For example, such shocks make expenditure a poor proxy for household income or (marginal) consumption utility, as is needed in parameterized Euler equation estimates (see Deaton and Zaidi [2002]).\(^4\)

Our results have relevance for the large literature which uses experiments to measure underlying preference parameters in both experimental and applied settings (e.g. Ashraf et al. [2006], Mahajan and Tarozzi [2011], Augenblick et al. [2015]). Our model implies that MRS measurements cannot be used to directly identify time preference parameters. Individual preference reversals are not generally indicative of time inconsistency, but can be the result of negative financial shocks. In the presence of incomplete credit constraints, choices which exhibit present bias on average in a population or in a single household over time can result from present-biased preferences ($\beta < 1$) only under some conditions, and their absence does not indicate time consistency.\(^5\) The reason is that the ability of even imperfect arbitrage across periods limits the present self’s ability to dictate its future

\(^3\)While measurement error in expenditure is likely to also appear in measured flow savings as the residual of income minus expenditure, this is not true of experimentally measured MRS, making it plausibly less susceptible to the problem of correlated measurement error.

\(^4\)The fact that income and spending have correlations with MRS of opposite sign also provides further evidence against full credit constraints: if households could not borrow or save, then the two would be identical.

\(^5\)In the case of complete credit constraints average choices do reveal underlying preference parameters.
An emerging empirical literature has considered the possibility that experimentally measured time preferences depend on subjects’ current financial situation. What sets our data apart is its level of detail, which allow us to use the relationships between MRS and financial variables to differentiate between models of credit constraints and learn what types of financial shock hit our sample population. We further discuss our relationship with the empirical literature in section 4.

1 Integrated Choices in Time Preference Experiments

Consider the sequences of decisions shown in table 1. In Set A, the subject is asked to make a series of choices between receiving money today ($a_0$) and receiving money in one week’s time ($a_1$), here denominated in CFA, or West African Francs (CFA 300 equal approximately USD 0.60 at market exchange rates and USD 1.60 in PPP terms). In Set B, the subject makes choices between money in one week’s time ($b_1$) and money in two weeks’ time ($b_2$). ‘Multiple price list’ (MPL) experiments of this kind have been used in many experimental investigations into time preferences; for example Coller and Williams [1999], Andersen et al. [2008], and Benhabib et al. [2010]. From the top to the bottom of each list, the earlier payout becomes more attractive. The parameter of interest is the point at which the subject switches to choosing the early over the late payment.

---

Table 1: A Multiple Price List Experiment.

<table>
<thead>
<tr>
<th>Set A</th>
<th>Set B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Today ($a_0$)</td>
<td>in 1 week ($a_1$)</td>
</tr>
<tr>
<td>CFA 50</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 100</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 150</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 200</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 250</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 300</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 350</td>
<td>CFA 300</td>
</tr>
<tr>
<td>CFA 400</td>
<td>CFA 300</td>
</tr>
</tbody>
</table>

---

6 McClure et al. [2007] and Augenblick et al. [2015] have proposed to use choices over primary rewards or real effort tasks to estimate discount parameters (and Augenblick et al. [2015] have also found a significant difference between the measured MRS for money and effort). Our results support this proposal, as well as using the demand for commitment to identify sophisticated time-inconsistent decision makers, as in Mahajan and Tarozzi [2011]. See section 1.6.1.

7 See for example Krupka and Stephens [2013], Meier and Sprenger [2015], Ambrus et al. [2015], Carvalho et al. [2016], Giné et al. [2018].

8 The questions are typically either hypothetical or one question is selected at random for payment to avoid wealth/portfolio effects.
Typically, behavior in MPL experiments has been understood in the context of ‘narrow bracketing’ models, in which decisions in the laboratory are treated in isolation from the outside world (see for example Ashraf et al. [2006], Andersen et al. [2008], Benhabib et al. [2010]). It is assumed that subjects ignore both their current outside consumption and their cost of saving and borrowing. However, if subjects experience financial shocks, narrow bracketing may break down and outside conditions could intrude on experimental decisions about receiving money at different points in time. This may be especially true in a developing-country context, where households are poor and markets are incomplete, meaning that financial shocks are salient because they substantially affect the household’s utility from consumption. We therefore propose an integrated model of experimental choices when there is quasi-hyperbolic discounting (in the manner of Laibson [1997]), the decision maker experiences income and preference shocks, and the financial market imposes ‘soft’ credit constraints.

While some of the ideas incorporated here have been previously discussed, as far as we are aware, ours is the first model that combines all these elements, making it possible to answer a rich set of theoretical questions. The tractability of the model, which stems in part from the assumption of ‘smooth’ credit constraints, allows us to make clear predictions about what can be learned about time preferences from MRS measurements, how measured MRS should covary with other financial variables, and what this tells us about the financial constraints and shocks that affect the household (we note, however, that the key results in section 1.5 do not depend on the assumption that the interest rate function is differentiable). The assumption of soft credit constraints also means that MRS experiments can convey more interesting information than in the case of no credit constraints (in which they simply report the market interest rate) or the complete credit constraint case (in which the household makes no dynamic allocation decisions). Allowing for quasi-hyperbolic discounting enables us to discuss the identification of present bias from the data, and ensures that our results are robust to such preferences. We further discuss the relationship of our model with the theoretical literature in section 4.

---

9 See Cubitt and Read [2007] for a discussion. See also Schechter [2007] on the importance of related assumptions for the measurement of risk preferences.

10 For example in Pender [1996], Frederick et al. [2002], Cubitt and Read [2007], Meier and Sprenger [2015].
1.1 Set-Up

We model a decision maker \( i \) whose preferences are described by

\[
u_i(c_{i0}, \rho_{i0}) + \beta_i E_0 \sum_{t=1}^{\infty} \delta_t^t u_i(c_{it}, \rho_{it}).\]

Utility is time-separable and given by the instantaneous utility function \( u_i(c, \rho) \), where \( c \) is period consumption, and \( \rho \) is a stochastic (individual-specific) preference parameter that is drawn independently from a distribution \( F_\rho \) in each period. We use \( \rho \) to model the effect of shocks to the marginal utility of consumption and assume that the derivative of \( u_i \) with respect to \( c \) is everywhere increasing in \( \rho \) (see below). \( \delta_t \) is the discount factor, and the parameter \( 0 < \beta_i \leq 1 \) indexes the degree of present bias. These preferences are quasi-hyperbolic and time-inconsistent if \( \beta_i < 1 \).

The resource constraint is given by

\[
c_{it} = w_{it} - s_{it} \\
w_{it} = y_{it} + (1 + r_i(s_{i,t-1}))s_{i,t-1} \\
= y_{it} + R_i(s_{i,t-1}) \\
w_{i0} \text{ given.}
\]

The variable \( s_{it} \) represents the stock of savings at the end of period \( t \), which can be either positive or negative. \( y_{it} \) is \( i \)'s current income, assumed to be drawn independently from a distribution \( F_y \) in each period, and \( w_{it} \) is cash-on-hand in \( t \). \( R_i(s_{it}) = (1 + r_i(s_{it}))s_{it} \) describes the gross returns to saving and thus the intertemporal budget constraint. From here on out we will suppress the person index \( i \) to ease notation.

We assume that the returns to saving fall as savings \( s_t \) increase, or equivalently that the cost of borrowing rises with the amount of credit. We further assume that \( R \) is continuously differentiable\(^{12}\) and obeys

\[
0 < R'(s_t) = 1 + r(s_t) + r'(s_t)s_t, \text{ and} \\
0 > R''(s_t) = 2r'(s_t) + r''(s_t)s_t.
\]

These imply that the resources available in period \( t + 1 \) are increasing in \( s_t \), but that the marginal rate of return to savings is decreasing in \( s_t \).

\(^{11}\)The assumption that the consumer is infinitely lived is not fundamental to the results.

\(^{12}\)As noted above, the differentiability assumption makes the analysis easier but is not essential for our central predictions.
The return function $R$ is a reduced-form way of modeling the (potentially individual-specific) credit and savings constraints that households in developing countries face. We argue in section 1.3 that many models of incomplete financial markets naturally lead to a concave $R$. In section 1.7.3 we discuss the possibility of $R$ varying over time.

1.2 The Euler Equation and Marginal Rate of Intertemporal Substitution

We use the results of Harris and Laibson [2001] to identify an Euler equation for the quasi-hyperbolic consumer of our model. These authors characterize the set of perfect equilibria in stationary Markov strategies of the game between the different ‘selves’ of the consumer in different periods. It is assumed that consumers are sophisticated about the behavior of their future selves.\textsuperscript{13} Because shocks are independent over time, the only state variables at time $t$ are cash on hand $w_t$ and the realization of the preference shock $\rho_t$.

Harris and Laibson [2001] provide a set of conditions under which the equilibrium of such a game can be described by what they call the Strong Hyperbolic Euler Equation (SHEE). We assume the corresponding conditions hold here.\textsuperscript{14} Essentially, they are regularity conditions on the utility function $u$ and the distributions $F_\rho$ and $F_y$, plus the assumption that $\beta$ is not too far from 1 so that the equilibrium consumption function is Lipschitz continuous (Harris and Laibson show that this condition is satisfied for a wide range of parameters).

**Definition:** A consumption function $c : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}_+$ satisfies the Strong Hyperbolic Euler Equation with credit and savings constraints if the following holds for every $w_t, \rho_t \in \mathbb{R}$:

\[
\begin{align*}
    u'(c(w_t, \rho_t), \rho_t) &= R'(s_t) \beta \delta E_t \left[ V'(w_{t+1}) \right] \\
    &= R'(s_t) E_t \left[ \left( \beta \delta \frac{\partial c_{t+1}}{\partial w_{t+1}} + \delta \left( 1 - \frac{\partial c_{t+1}}{\partial w_{t+1}} \right) \right) u'(c(w_{t+1}, \rho_{t+1}), \rho_{t+1}) \right]
\end{align*}
\]

This version of the SHEE has the familiar Euler equation interpretation. $u'(c(w_t, \rho_t), \rho_t)$ is the marginal utility of consumption at $t$, while $R'(s_t)$ is the rate at which money today can be converted into money tomorrow. The expectation term on the right hand side of

\textsuperscript{13}We conjecture that the key predictions of the model are robust to the assumption that the decision maker is naive.

\textsuperscript{14}These are: (1) The utility function $u$ is strictly increasing and twice continuously differentiable on $[0, \infty)$; (2) Relative risk aversion is bounded away from zero and below infinity, i.e. $0 \leq \frac{-u''(c, \rho)}{u'(c, \rho)} \leq \bar{\pi} < \infty$, on $[0, \infty)$ for all $\rho$ in the support of $F_\rho$; (3) The distribution function $f_y$ is twice continuously differentiable and has a support that is bounded away from zero and below infinity; (4) The distribution $f_\rho$ is twice continuously differentiable; (5) $\max \{\delta, \delta R(s)^{1-\bar{\pi}}\} \leq 1$ for $s > 0$; and (6) The hyperbolic discounting factor satisfies $\beta \in [0, 1]$ and the model is parameterized such that the equilibrium consumption function is Lipschitz continuous ($\beta$ is close to 1).
equation (1) is the expected discounted marginal value of cash on hand in period \( t + 1 \) from the point of view of the agent at time \( t \) (denoted \( V(w_{t+1}) \)). The optimal allocation equalizes the marginal value of consuming funds right away and handing them to one’s next-period self.

The SHEE differs from the standard Euler equation in two ways. First, the marginal utility of income tomorrow from the perspective of time \( t \) is discounted by

\[
d_{t+1} \equiv \beta \delta \frac{\partial c(w_{t+1}, \rho_{t+1})}{\partial w_{t+1}} + \delta \left( 1 - \frac{\partial c(w_{t+1}, \rho_{t+1})}{\partial w_{t+1}} \right)
\]

rather than a time-invariant discount factor (Harris and Laibson [2001]). This effective discount factor is a weighted average of the short-run discount factor \( \beta \delta \) and the long-run discount factor \( \delta \), where the (time-variant) weight is given by the future propensity to consume. An intuitive interpretation is that the decision-maker applies \( \beta \delta \) to the share of wealth that is used for consumption in the next period, but \( \delta \) to the funds that her future self will save. Second, the standard market interest rate term \( 1 + r \) is replaced by the savings-dependent rate of return \( R'(s_t) \).

Rearranging, we can express the MRS of the consumer as\(^{15}\)

\[
MRS_t \equiv \frac{u'(c(w_t, \rho_t), \rho_t)}{E_t\left[ d_{t+1} u'(c(w_{t+1}, \rho_{t+1}), \rho_{t+1}) \right]} = R'(s_t).
\]

1.3 Credit and Savings Constraints and the Shape of \( R \)

Our assumptions above imply that the marginal returns to saving \( R'(s_t) \) are decreasing in the (possibly negative) stock of savings \( s_t \). What motivates a concave \( R \)? To begin with, note that we assume that “credit and savings” in an incomplete market may encompass a range of technologies used to shift consumption between periods: formal and informal borrowing, physical storage of money or assets like gold, or investment in an enterprise or capital good that yields returns later. The function \( R \) describes the effective return earned by the portfolio of instruments that the household uses at each level of saving. \( R \) is linear only if the household borrows and saves all funds at the same constant rate of return.

There are several underlying possible reasons for a concave \( R \). First, various kinds of market imperfections – from lenders with market power to information asymmetries – can mean that individual demand for financial instruments affects the interest rate (price) paid. Second, if there are large frictions in the capital market, households may

\(^{15}\)Note that this is equal to the expectation of the stochastic discount factor.
save by investing in a productive asset or enterprise (e.g. Rosenzweig and Wolpin, 1993). Diminishing returns to capital in the production function then imply decreasing returns to savings. Indeed, De Mel et al. [2008] have shown empirically that small businesses in Sri Lanka that receive a cash or in-kind grant exhibit lower returns if their owners report higher household wealth to begin with.\footnote{Investments used for consumption smoothing over short periods will likely be in flexible capital; the urban households in our sample might invest in stock for resale, fuel, or wood, fabric, or other supplies for making goods for their clients, for example. By contrast, investments in a durable asset or into starting a business may require a minimum amount \( I \), and combined with a borrowing constraint, this would lead to a convexity in the return function \( R \); the household can only save at a low rate of return up to \( I \), but realize higher returns for greater savings (but cannot borrow funds to cover the gap between their savings and \( I \); see e.g. Banerjee and Newman, 1993, Aghion and Bolton, 1997). We are not modeling this possibility here, assuming that it mostly applies to rarely occurring large investments which effectively shift \( R \) permanently. Note that we do allow the (short-term) return function \( R \) to vary between households.}

Last, concavity of \( R \) also arises if the household has access to a range of different borrowing and savings tools with different rates of return and with limits to the amounts that can be borrowed or saved through each.\footnote{Empirically, limits on the amounts that can be borrowed through one instrument are common, especially in informal markets and when borrowers cannot provide collateral. Poor households are known to use complex and varied portfolios of financial instruments (see Collins et al. [2009] for examples). There is also evidence for substantial savings constraints and an unfilled demand for savings instruments, to the point that some savers are willing to accept negative interest rates (Ashraf et al. [2006], Dupas and Robinson [2013a]). This indicates that they cannot even store the proverbial “cash under the mattress”, possibly due to demands for transfers from others, the risk of theft or loss, and inflation (Dupas and Robinson [2013b]).} We assume that households will use these different instruments in order of attractiveness. For example, when reducing its debt, the household will first pay off the loan with the highest interest rates, or when short of money, it will first exhaust its store credit before taking a loan with interest. This means that the household uses more expensive forms of credit the more it borrows, and earns lower interest rates the more it saves. As long as the available instruments individually exhibit constant or decreasing returns, the resulting returns function is continuous and concave, and can be approximated arbitrarily well by a differentiable \( R \). An extreme version of such limits is a hard credit constraint, as in classical liquidity constraint models, where the agent is free to borrow at the market interest rate down to a hard limit \( B \) (e.g. buffer-stock models of wealth and precautionary savings, Zeldes [1989], Deaton [1991]). Adverse selection, moral hazard, and contract enforcement problems can all lead to this form of credit rationing, because restricting loan sizes helps lenders mitigate the risks of intentional and involuntary default (see e.g. Ghosh et al. [2000] for an overview). This model can be recast as a linear budget constraint with a “kink” at \( B \), which is another limit case of our model. That said, it can be shown that the key predictions from section...
1.5 are robust to a concave and piecewise linear (non-differentiable) return function, as well as the possibility of infinite slope implied by a hard constraint.

In summary, the assumption of a concave \( R \) provides a tractable framework that can capture a range of possible frictions to the cost of borrowing and lending. The curvature of \( R \) indexes the degree to which the consumer is credit constrained: the more concave the function, the more the rate of return varies with the amount saved or borrowed. At one extreme, \( R \) is globally linear, and equation 2 implies that the MRS of the consumer is constant and equal to \( 1 + r \). We call this the no-constraints case. At the other extreme, as the second derivative \( R''(0) \to -\infty \), the cost of borrowing goes to infinity, while the rate of return on savings goes to zero.\(^{18}\) In the limit, no borrowing or savings are possible, so that the propensity to consume equals one, \( d_{t+1} = \beta \delta \), and the identity \( w_t = y_t = c_t \) holds in every period. The MRS is therefore

\[
MRS_t = \frac{u'(y_t, \rho_t)}{\beta \delta E_tu'(y_{t+1}, \rho_{t+1})}.
\]

We call this the complete-constraints case.

Given how we justify the shape of \( R_i \), we need to account for the possibility that these constraints on the intertemporal allocation of funds change over time, for example because the returns to investment vary over time. We explore this in section 1.7.3.

1.4 Choice in MPL Experiments

Assume that the subject has optimized her consumption plan given her current period’s income \( y_t \) and realized shock \( \rho_t \). Her current level of consumption is \( c_t^* \) and her savings are \( s_t^* \). Now she is offered the experimental choice of a payoff of \( a_1 \) in one period vs. \( a_0 \) immediately. We will now show that her experimental choices reveal her MRS, as long as these payoffs are small. This is regardless of whether experimental payouts in the current period must be consumed immediately, or current period consumption can be adjusted.

**Proposition 1.** Consider the decision maker’s preferences between receiving \( a_0 > 0 \) immediately and \( a_1 > 0 \) in the next period, and the set of all such payments with the ratio \( a_1/a_0 = \hat{R} \). There exists a non-zero bound on \( a_0 \) below which the decision maker will strictly prefer the earlier payment if

\[
R'(s_t^*) = \frac{u'(c_t^*, \rho_t)}{E_t[c(w_{t+1}, \rho_{t+1}), \rho_{t+1}]} > \hat{R},
\]

regardless of whether \( a_0 \) must be consumed immediately or if the decision maker can

\(^{18}\)This can be seen by taking the Taylor series expansion \( R'(s) \approx R'(0) + R''(0)s \). As \( R''(0) \to -\infty \), \( R'(s) \to \infty \) for \( s < 0 \). For \( s > 0 \), \( R'(s) \) is bounded below at zero by assumption.
adjust her consumption and saving decision in period \( t \). The later payment will be strictly preferred if the inequality is reversed.

**Proof.** See appendix A.

The proposition shows that the pairwise choices in the MPL experiment provide an interval estimate of the MRS under either protocol. At the point of indifference between earlier and later payments the relative marginal value of money in the two periods is (approximately) equal, both in terms of its consumption value, and its investment value. The subject’s experimental choices approximate the slope of the budget constraint \( R'(s^*_t) \) when they can arbitrage the payoffs, and the slope of the indifference curve at \( s^*_t \) without arbitrage, and at the optimum these are equal.

A similar argument can be used to determine the subject’s choice between future payments at \( t = 1 \) and \( t = 2 \), evaluated at period \( t = 0 \).

**Proposition 2.** Consider the decision maker’s preferences between receiving \( b_1 > 0 \) at \( t + 1 \) and \( b_2 > 0 \) at \( t + 2 \), and the set of all such payments with the ratio \( \frac{b_2}{b_1} = \hat{R} \). There exists a non-zero bound on \( b_1 \) below which the decision maker will strictly prefer the earlier payment if

\[
\frac{E_t[d_{t+1}u'(c_{t+1}, \rho_{t+1})]}{E_t[d_{t+1}d_{t+2}u'(c_{t+2}, \rho_{t+2}) + O_b]} > \hat{R}
\]

(4)

where \( O_b \) is a term that equals zero if either the decision maker is time-consistent, or the interest rate is fixed (the no-constraints case). The later payment will be strictly preferred if the inequality is reversed.

**Proof.** See appendix A.

Note that \( d_{t+2} \) is the discount rate that the \( t+1 \) self applies in trade-offs between periods \( t + 1 \) and \( t + 2 \). We will discuss this result and its implications, and especially the role of \( O_b \), for measuring time preferences in section 1.6.1.

### 1.5 Predictions of the Partial Credit Constraints Model

Next, we use the model to make predictions about the relationships between measured MRS and savings, income shocks, and preference shocks, and show how these can differentiate between credit regimes. We consider here a model with a fixed returns function \( R \) in which all income and preference variation is exogenous and independent over time. We discuss the implications of endogenously chosen labor supply, serially correlated shocks, and time-variant returns to savings \( R \) in section 1.7. All the predictions in this section hold for the cases of exponential as well as quasi-hyperbolic discounting (\( \beta = 1 \) and \( \beta < 1 \)).
First, we consider the impact of exogenous variation in income on the MRS in the same period. It is straightforward to show that, all else equal, higher income is associated with higher savings, and therefore lower measured MRS.

**Prediction (Income shocks and MRS):** Consider a decision maker who holds savings from the previous period $s_{t-1}$ and has preference parameter $\rho_t$. For any two possible income realizations $y_t, y'_t$ and associated $MRS_t, MRS'_t$, $y_t > y'_t$ implies $MRS_t < MRS'_t$.

**Proof.** See appendix B.

Next, we consider the preference parameter $\rho$. The notion that the derivative of $u$ may vary randomly for a given level of $c$ is motivated by the observation that measured consumption spending and true “value of consumption” do not always perfectly line up. For example, if we think of the $c$ in the utility function as consumption expenditure, we have to account for variation in spending that does not translate into immediate utility gains. An example would be spending in response to events like the theft of a productive asset, illness of a family member, or damage to one’s house; the expenditure to “undo” such an adverse event does not actually increase the decision maker’s utility in the same way as, say, buying a meal would. A household that is subject to such an adverse event has a higher marginal utility to additional consumption spending than a household with the same level of $c$ but without this event. Such a preference shock will lead to an increase in measured consumption and a reduction in savings, which will in turn increase MRS.

**Prediction (Preference shocks and MRS):** Consider a decision maker with cash on hand $w_t$. For any two realizations of the preference shock $\rho_t, \rho'_t$ and the associated $MRS_t, MRS'_t$, $\rho_t < \rho'_t$ (and therefore $\frac{\partial u(c, \rho_t)}{\partial c} < \frac{\partial u(c, \rho'_t)}{\partial c}$ for all $c$) implies $MRS_t < MRS'_t$.

**Proof.** See appendix B.

Note that these predictions refer to exogenous changes in income and preferences (shocks). Endogenous (chosen) changes in income, for example from increased labor supply, are generally positively related with MRS. For example, if a household raises additional funds in response to a preference shock, then this income component will be positively related with MRS (see section 1.7.1 for an extension of the model allowing for endogenous changes in income). We exploit the prediction that income sources with greater exogenous variation should be more strongly negatively related to MRS in section 3.1.

Our third prediction uses that an increase in $s_t$ is directly associated with a fall in $MRS_t$ through the shape of the returns function $R$. Note that, even though the level of savings $s_t$ is endogenously chosen by the household and may depend on the shape of $R$, $MRS_t = R'(s_t)$ holds with equality in each period.
Table 2: Predicted Relationships between MRS and Financial Variables

<table>
<thead>
<tr>
<th>Expected Relationship with MRS</th>
<th>Savings $s_t$</th>
<th>Inc. shocks $y_t$</th>
<th>Pref. shocks $\rho_t$</th>
<th>Spending $c_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrow Bracketing ($R$ irrelevant)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No Credit Constraints ($R' = 1 + r$)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Full Credit Constraints ($R' = 0/-\infty$)</td>
<td>0</td>
<td>$-$</td>
<td>$+$</td>
<td>same as income</td>
</tr>
<tr>
<td>Partial Credit Constraints ($R'' &lt; 0$)</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>indeterminate</td>
</tr>
</tbody>
</table>

**Prediction (Savings and MRS):** For any two possible savings levels $s_t$, $s'_t$ and associated $MRS_t$, $MRS'_t$, $s_t > s'_t$ implies $MRS_t < MRS'_t$.

*Proof.* This follows from $MRS_t = R'(s_t)$ (equation (2)) and the fact that $R'' < 0$.

Also of interest is the relationship between household spending and MRS. As discussed in section 1.6.3, spending and MRS may be positively or negatively related in our model, depending on the relative role of income and preference shocks. In the full-constraints model, however, consumption and income are the same (save for reporting and measurement error), and we would therefore expect that spending is negatively related with $MRS$. In the no-constraints or narrow bracketing models, there is no relationship.

These predictions allow us to distinguish our partial credit constraints model from narrow bracketing as well as from the no-constraints and complete-constraints limit cases, as summarized in table 2. The predictions regarding preference and income shocks differentiate our model with credit constraints from narrow bracketing and the no-constraints version of the model. Both predict no relationship between shocks and MRS, in the first case by assumption, and in the second because such shocks do not affect the effective interest rate faced by the household. The relationships of MRS with savings and spending serve to differentiate the partial- and complete-constraints version of our model. Under complete constraints, savings are zero, so that any difference between income and spending is due to measurement error. In this case we would therefore not expect a relationship between savings and MRS (see section 3.2), while spending and income must have the same relationship with MRS.

The above results pertain to the sign of the relationship between MRS and alternative realizations of other variables (income, spending, and preference shocks) in the same period. We show in appendix B that the covariance between MRS and each of the variables calculated from a $T$-length sample will, under mild conditions, have the same sign in expectation. Thus, the covariance between MRS and preference shocks will be
positive, while that between MRS and income will be negative.\textsuperscript{19} In section 1.7.2 we use simulations to show that these correlations hold for a range of parameters in a parametric version of our model when income or preference shocks are serially correlated.

1.6 Implications

As we show below, the predictions of our proposed model for the relationship of MRS with shocks and savings are supported by the data. We can therefore use the model to ask what can be learned from experiments on intertemporal trade-offs. We first discuss what conclusions can be drawn about a subject’s time preference parameters. Then we show that the responsiveness of measured MRS to real-world variables means that such experiments can be exploited for empirical research on the effect of financial shocks on household finances and the availability of intertemporal consumption smoothing and (self-) insurance. We propose the use of experimentally obtained MRS as a direct, independent measurement of the intertemporal marginal rate of substitution, an object that is otherwise unobserved and must be estimated using parametric assumptions about the utility function. We illustrate the approach with a novel example, showing that repeated MRS measures in conjunction with consumption data can be used to understand the importance of shocks to spending relative to disposable income.

1.6.1 Implications for the Measurement of Time Preference Parameters

Section 1.4 shows that, if our model is correct, measured MRS from decision A will typically not directly reflect the discount factor, as is often assumed in time preference experiments. Moreover, decision A and B together do not generally give us information about the (level of) present bias or the value of $\beta$.

To see this, substitute the Euler equation into our expression for $\frac{b_2}{b_1}$ to get

$$\frac{b_2}{b_1} \approx \frac{E_t \left[ d_{t+1} \cdot E_{t+1} \left[ d_{t+2} u'(c_{t+2}; \rho_{t+2}) \right] \cdot R'(s_{t+1}) \right]}{E_t \left[ d_{t+1} d_{t+2} u'(c_{t+2}; \rho_{t+2}) + O_b \right] + \text{Cov} \left( d_{t+1} \cdot E_{t+1} \left[ d_{t+2} u'(c_{t+2}; \rho_{t+2}) \right], R'(s_{t+1}) \right)} \times \frac{E_t \left[ d_{t+1} d_{t+2} u'(c_{t+2}; \rho_{t+2}) \right]}{E_t \left[ d_{t+1} d_{t+2} u'(c_{t+2}; \rho_{t+2}) + O_b \right]}$$

(5)

The first term in this expression is the expected future interest rate, which, in a stationary economy, is (approximately) equal to the expectation of $\frac{d_1}{a_0} \approx R'(s_t)$. This is the rate at which money can be transferred between periods outside the experiment in period $t + 1$.\textsuperscript{19}
This term appears here because the subject at time $t$ can only choose when payments are received, not when they are consumed. This means to a first approximation that the best they can do is maximize expected discounted income.

The covariance term arises from the fact that self $t$ must predict both future consumption utility and the future interest rate. Consider for example the exponential discounting case, where $d_t = \delta$, and assume there are no preference shocks. In this case, the covariance term is positive and the term in brackets is greater than $\frac{2\delta}{\delta_0}$ on average, since both $u'$ and $R'$ vary negatively with $s_{t+1}$. This argument continues to hold with quasi-hyperbolic discounting if the marginal propensity to consume does not respond too strongly to financial shocks. The covariance term disappears only if either there are no credit constraints ($R'$ is constant) or if credit constraints are very high and savings vary little with income ($c_{t+2}$ becomes independent of $s_{t+1}$).

The last term is a multiplier that equals one if $O_b$ equals zero. From Proposition 2, this is the case if either there are no credit constraints, or the decision maker is not present biased. We show in appendix A that the term $O_b$ will be positive if the decision maker is present biased (as long as $\beta$ is not too far from one) and the interest rate varies with savings, as in our model.\(^{20}\)

One approach to identifying time inconsistency in the literature has been to use preference reversals between decisions A and B to conclude that there is present bias, without necessarily identifying $\beta$ exactly. Note that, without narrow bracketing, any individual preference reversal may be due to financial shocks. Moreover, the covariance term tends to bias any estimate of $\frac{b_2}{b_1}$ upwards, making decision B on average less patient than decision A. However, if $O_b$ is positive, which can only be the case if $\beta < 1$, the term in brackets in the expression above is multiplied by a number less than one. Thus, assuming that the economy is stationary, and our model of partial credit constraints is correct, we would observe decision B to be on average more patient than decision A only if there is present bias, either on the individual level (when observing many decisions for one person) or on the population level. Due to the covariance term, however, the converse is not true, that is, the presence of present bias does not imply that decision B must be more patient than A on average.

Can there be any further progress on identifying $\beta$ and $\delta$ from decisions A and B? The answer is yes, but only in some special cases. Specifically, equation (2) shows that the MRS

\(^{20}\)The bracketed term in the above equation is the indifference point one would get from replacing the transfer $b_2$ in period $t+2$ with the appropriately discounted amount in period $t+1$. The $O_b$ term comes about because the time $t$ decision maker is not indifferent between these two options if there is both present bias and partial constraints: By changing the timing of the payment, they can manipulate the interest rate faced by the $t+1$ decision maker, and so indirectly modify future consumption choices.
identifies the (effective) discount factor if 
\[ u'(c(w_t, \rho_t), \rho_t) = E_t[u'(c(w_{t+1}, \rho_{t+1}), \rho_{t+1})]. \]
This holds for example if the household has globally linear utility or if consumption is constant in all periods because there are no shocks.\(^{21}\) The expression also holds in expectation if \( u' \) is stable over time and the decision maker is subject to complete credit constraints, so that marginal utility in each period is determined only by realized income and preference shocks. Moreover, we have
\[
E \left( \frac{a_1}{a_0} \right) = E \left( \frac{u'(y_t, \rho_t)}{\beta \delta E_t u'(y_{t+1}, \rho_{t+1})} \right) = \frac{1}{\beta \delta}, \quad \text{and}
\]
\[
E \left( \frac{b_2}{b_1} \right) = E \left( \frac{E_t \left[ u'(y_{t+1}, \rho_{t+1}) \right]}{\delta E_t \left[ u'(y_{t+2}, \rho_{t+2}) \right]} \right) = \frac{1}{\delta},
\]
where \( E(\cdot) \) denotes the unconditional expected value. As a result, the average difference between decision A and B can be used to identify time inconsistency on average.  

Our framework also points to possible alternative approaches to identifying time preference parameters. One is to make use of a long-run equilibrium argument, building on the idea that the steady-state asset allocation and interest rate depend on the discount factor.\(^{22}\) Krusell and Smith [2003] show that in a quasi-hyperbolic model without uncertainty, the set of equilibria and therefore equilibrium realizations of the rate of return on assets depends in monotonic ways on \( \beta \) and \( \delta \). Thus, observing long-run average MRS allows some inference on time preference parameters. One may conjecture that a parallel result holds under uncertainty, so that different time preference types exhibit distinct (sets of) stationary equilibrium ergodic distributions and different average \( R'(s_t) \) in the long run, potentially allowing a ranking of individuals by their effective discount factor. Characterizing this connection may be a promising direction for future research.  

Another strategy would be to combine financial and experimental data to estimate \( \beta \) and \( \delta \) from a structural model, using expression (3) for decision A and (4) for decision B. This requires measurement of wealth, consumption, and preference shocks, as well as the utility function curvature (for example by measuring risk aversion as suggested by Andersen et al. [2008]). An advantage is that this method works even in the no-constraints case; intuitively, for a given MRS, a more patient decision maker will have a lower level of consumption today relative to tomorrow. The main disadvantage lies in the very strong

\(^{21}\) In the case of linear utility, the decision maker would adjust savings until reaching the point at which \( R'(s_t) = \frac{1}{E_t(d_{t+1})} \).

\(^{22}\) As an illustration, in the simplest case of no uncertainty and exponential discounting, a steady state can only occur at \( R'(s) = \frac{1}{\delta} \), meaning that measured MRS reveals the inverse discount rate. Note, however, that in general no such steady state exists in the no-constraints model where \( R'(s) = 1 + r \) for all \( s \), and therefore \( \frac{1}{\delta} = 1 + r \) may not hold.
Some authors measure discount factors for primary rewards (see e.g. McClure et al. [2007]) or effort (Augenblick et al. [2015]), which may be harder to arbitrage between different time periods and less affected by preference shocks (although a subject who has to carry out an experimental task or consumes a reward may still choose to reschedule other work or consumption). Another possibility is to use hypothetical questions, which may be more amenable to narrow bracketing. The difference between decisions A and B can therefore be used to detect time-inconsistent decision makers. However, it is worth noting that Krupka and Stephens [2013] find that changes in inflation rates, which alter effective interest rates, have effects even on hypothetical discount rates. Yet another approach, suggested by Ambrus et al. [2015], is to condition experimental payments on the stability of future income.

Our results also support using demand for commitment to identify time inconsistent preferences, as in Mahajan and Tarozzi [2011]. These authors use experimental preference reversals jointly with observed take-up of a commitment contract for malaria net retreatment to stochastically identify population shares and preference parameters of time-consistent as well as naïve and sophisticated present-biased agents. Consistent with our argument, they find that experimental preference reversals have little predictive power for the estimated time preference parameters of their sample.

1.6.2 Implications for the Measurement of Consumption Smoothing and Insurance

While our results are somewhat pessimistic about identifying time preference parameters from experimental measures of MRS, they suggest that these measures can instead help us understand the financial shocks and constraints that affect a household. Repeated MPL experiments can be used to measure the variance of individual MRS over time and between

\[ \frac{\partial}{\partial \alpha} \]
subjects, and the covariance between MRS and other financial variables. Measuring MRS in this way is significantly easier than (for example) inferring changes in marginal utility from the variance of consumption, and is unaffected by the problem of preference shocks (which we show to be important in section 3).

This methodology has many applications. As we have shown in section 1.5, the relationship between MRS and other financial variables can help to determine the credit regime faced by a household. Furthermore, the better a household’s ability to smooth financial shocks, the lower should be the overall variance of its MRS, as well as its MRS response to exogenous shocks (e.g. promised future payments). This could be used to test the impact of programs designed to improve household consumption smoothing, for example of the type evaluated in Karlan et al. [2014].

MRS measurements can also be used to test predictions about the first-order conditions for intertemporal consumption allocation over time. Starting with Hall [1978], a large literature has examined systematic deviations of observed consumption choices from the path prescribed by (a linear approximation of) the Euler equation due to factors such as credit constraints (see e.g. Zeldes [1989], Runkle [1991] for early examples). Other models make predictions for the effects of incentive constraints in problems of risk sharing on (inverse) MRS, and test them using implications for consumption allocations over time (e.g. Rogerson [1985], Green and Oh [1991], Ligon [1998], Golosov et al. [2003], Kocherlakota and Pistaferri [2009], Attanasio and Pavoni [2011], Karaivanov and Townsend [2014], Kinnan [2014]). Yet it has long been recognized that the estimation of (log-linearized) Euler equations is hampered by approximation bias (e.g. Ludvigson and Paxson [2006], Carroll [2001]), whereas nonlinear GMM approaches lead to inconsistent estimates when there is measurement error (see Alan et al. [2009] for a discussion). Correlated measurement error can bias the results when studying consumption and income over time (Runkle [1991]), which is a concern given that these variables are often difficult and costly to measure (Grosh and Glewwe [2000]).

Experimental measures of MRS may be able to address some of these issues, assuming that the measurement error in them is independent from concurrent measures of financial variables. As a simple, illustrative example, consider the classical test of full insurance (Townsend [1994], Deaton [1997], Mace [1991]). Without a savings technology, the Pareto optimal choice by a social planner in period 0 will allocate consumption in any period t and any state of nature s such that weighted marginal utility is equalized across individuals. This means that the full-insurance model in its purest form predicts that MRS is the same for all individuals in any given period, and this prediction can be tested with experimental MPL data. Weaker predictions, such as whether or not MRS is related to individual-
specific shocks (in addition to group-level shocks) can also be tested.\textsuperscript{25}

Townsend [1994] and others conduct equivalent tests with consumption and income data only, by using a specific utility function (typically CARA or CRRA) to predict the co-movement of individual and group consumption or to test the residual effect of individual income on consumption. Early applications of these tests show problems with this approach; for example, Mace [1991] carries out the test for both a power and an exponential utility function and rejects full insurance in one case but not the other. As Kinnan [2014] and many others have pointed out, measurement error in right-hand side variables or correlated measurement error in individual consumption and income\textsuperscript{26} may also lead to a spurious effect of individual income on MRS. By contrast, the use of experimental data does not require estimating MRS from consumption, and measurement error in the experimental data is less likely to be correlated with measurement error in the data on financial shocks. This is particularly important in the presence of preference shocks, which drive a wedge between consumption expenditure and utility. As we discuss below, our data suggests such shocks are quantitatively important.

1.6.3 Application: The Relative Importance of Income and Preference Shocks

A novel application of MRS measurements in this spirit is to assess the relative importance of income and preference shocks by studying the relationship between total consumption and measured MRS. In our model, this relationship is indeterminate: on the one hand, a positive income shock (such as an unexpected payment) will lead to an increase in consumption expenditure,\textsuperscript{27} accompanied by a fall in MRS. On the other hand, a preference shock which increases the marginal utility of consumption (such as the illness of a child) will lead to a rise in consumption spending as well as a rise in measured MRS. Thus, the two different types of shock can lead to either a positive or negative relationship between consumption spending and measured MRS. The average relationship can therefore tell us something about the relative importance of each type of shock.

We formalize this claim using a particular type of preference shock in which the household must make some expenditure which does not generate utility in the way that regular consumption does. As a simple example, consider a household that usually spends all its...

\textsuperscript{25}In an earlier version of this paper we discuss the predictions of the Townsend mutual insurance model for measured MRS more formally (Dean and Sautmann [2014], section 5.2) and argue that the correlations of individual shocks with MRS are a rejection of the full-insurance hypothesis. One could perform further analysis by studying the co-movement of the MRS measures of the risk-sharing group, or relate MRS changes to aggregate shocks.

\textsuperscript{26}Introduced for example by errors in pricing household production and consumption, see Deaton [1997].

\textsuperscript{27}Harris and Laibson [2001] show that the consumption function is strictly increasing in wealth provided that $\beta$ is sufficiently close to 1 and $f_y$ and $u$ are three times continuously differentiable.
income on food. In one period, a relative becomes ill, and they assist them by spending $10 on medicine. Their total expenditure on food and medicine of $x$ then leads to the same marginal utility of consumption that would be associated with the expenditure level $x - $10 absent the need to buy medicine. Such shocks therefore cause variation in the marginal utility of consumption at given observed consumption levels. We can write $u(c_t, \rho_t) = u(n_t)$ where $n_t = c_t - \rho_t$: utility depends on “net consumption” $n_t$. The individual maximizes

$$E \sum_{t=0}^{\infty} u(n_t)$$

with wealth at $t = 0$ given and under the constraint $s_t + n_t = w_t - \rho_t$. The decision maker chooses $n_t$ conditional on wealth $w_t$ and preference shock $\rho_t$. Note that preference shocks act essentially as negative income shocks; they reduce the funds available for consumption and savings $s_t + n_t$. We assume that the researcher observes consumption expenditure $c_t$ but not $n_t$ or $\rho_t$.

In order to examine the relative importance of shocks to $w_t$ and $\rho_t$, we examine the relationship between spending $c_t$ and $MRS_t$. Since $R'' < 0$, this is determined by the negative of the covariance of savings and spending. In the canonical model without preference shocks, we have $c_t = n_t$, and any increase in spending is caused by an increase in income. An increase in income also leads to higher savings and so to a lower MRS. By contrast, a preference shock – i.e. an increase in $\rho_t$ – increases $c_t$, but reduces savings, leading to a higher MRS.

Taking a Taylor series expansion of $s$ around the average levels of the wealth and preference shocks $\bar{w}$ and $\bar{\rho}$, we get

$$s_t(w_t, \rho_t) \approx \bar{w} - \bar{\rho} - n(\bar{w}, \bar{\rho}) + \left(1 - \frac{\partial n_t}{\partial w_t}\right)(w_t - \bar{w}) - \left(1 + \frac{\partial n_t}{\partial \rho_t}\right)(\rho_t - \bar{\rho})$$

where $\frac{\partial n_t}{\partial w_t}$ is the marginal propensity to consume out of wealth and $\frac{\partial n_t}{\partial \rho_t}$ the marginal effect of preference shocks on consumption. Note that $(w_t - \bar{w})$ incorporates both income shocks and differences in the current stock of savings relative to $\bar{w}$. We can similarly approximate spending as

$$c_t(w_t, \rho_t) \approx c(\bar{w}, \bar{\rho}) + \frac{\partial n_t}{\partial w_t}(w_t - \bar{w}) + \left(1 + \frac{\partial n_t}{\partial \rho_t}\right)(\rho_t - \bar{\rho}).$$
Assuming that preference shocks are distributed independently of wealth, this implies

\[ \text{Cov}(s_t, c_t) \approx \left(1 - \frac{\partial n_t}{\partial w_t}\right) \frac{\partial n_t}{\partial w_t} \text{Var}(w_t) - \left(1 + \frac{\partial n_t}{\partial \rho_t}\right)^2 \text{Var}(\rho_t). \]

For the purpose of consumption choice, changes to \( w_t \) are equivalent to negative changes to \( \rho_t \), so we have \(-\frac{\partial n_t}{\partial w_t} = \frac{\partial n_t}{\partial \rho_t}\), and therefore

\[ \text{Cov}(s_t, c_t) \approx \left(1 - \frac{\partial n_t}{\partial w_t}\right) \left[ \frac{\partial n_t}{\partial w_t} \text{Var}(w_t) - \left(1 - \frac{\partial n_t}{\partial w_t}\right) \text{Var}(\rho_t) \right]. \] (6)

Equation (6) says that the covariance of savings and spending is the difference between the variance of income and preference shocks, weighted by \( \frac{\partial n_t}{\partial w_t} \), and scaled by the marginal propensity to save. If the marginal propensity to consume is high, net consumption and therefore total spending closely follows income, but total spending is relatively unaffected by preference shocks, since those are almost entirely compensated by (net) consumption changes \( n_t \). If \( \frac{\partial n_t}{\partial w_t} \) is low, income shocks have little effect on net consumption, but preference shocks translate almost entirely into spending changes. Taken together, this means that MRS and consumption will be positively related if

\[ \frac{\partial n_t}{\partial w_t} \text{Var}(w_t) < \left(1 - \frac{\partial n_t}{\partial w_t}\right) \text{Var}(\rho_t) \] (7)

and negative otherwise.\(^29\) Thus, with an estimate of \( \frac{\partial n_t}{\partial w_t} \) we can bound the relative variance of the two types of shock. In section 3.3 we apply this approach to our data and deduce from the positive relationship between spending and MRS that the sample households are subject to preference shocks that explain at least 46% of the variance of their total expenditure.

Observe that one could in principle examine the covariance between consumption spending and the stock of savings directly. However, the stock of savings is difficult to measure, especially in populations that do not use formal banking, and typical measures involve rough approximations based on asset ownership. More importantly, the fluctuation in the stock of savings is directly determined by current consumption expenditure. Any measurement error in this variable would therefore lead to spurious correlations. Lastly, while the marginal propensity to save scales the relationship between shocks and savings

\(^{28}\)Here we can think of expectations being taken with regard to the distribution of \( \rho_t \) and \( w_t \) conditional on \( w_{t-1} \), though the following approximation holds for any distribution in which wealth and preference shocks are independent.

\(^{29}\)Following the logic of the proof in appendix B, under mild conditions this ensures that the expectation of the sample covariance calculated from a \( T \)-length sample will have the same sign.
(eq. (6)), it does not scale the relationship between MRS and shocks. For example, a household that faces high credit constraints may have a low propensity to save, but a very strong relationship between saving and MRS – in fact the latter is the cause of the former.

1.7 Extensions of the Basic Model and Implications for Model Predictions

Our baseline model makes a number of simplifying assumptions. Here we discuss the robustness of the results of section 1.5 to three generalizations: endogenous labor supply, inter-temporally correlated shocks, and temporary shocks to the individual return function $R_t$. Detailed results for all three cases are included in appendix C.

1.7.1 Endogenous Labor Supply

While some income variation is exogenous and beyond the control of the household, the household can likely also endogenously change their income in response to shocks, most obviously by adjusting the time or effort spent on working. In appendix C we analyze a model which allows for this possibility. Under mild conditions, the results of 1.5 go through: MRS is negatively related to exogenous income shocks, positively related to preference shocks and negatively related to savings. We can also use the model to show that endogenous increases in income will be positively related to MRS.\(^{30}\) For example, a preference shock that increases the marginal utility from consumption will lead to an increase in endogenous (labor) income as well as a decrease in savings and higher measured MRS. Similarly, households will partly compensate an exogenous drop in income by increasing their labor supply.

This implies that sources of income which are more under the household’s control will be less negatively or even positively related to MRS. Moreover, the endogenous income response has an attenuating effect on the relationship between $MRS_t$ and total income, but also between $MRS_t$ and preference shocks. We will discuss these issues more in section 3.

Last, note that some types of spending may respond to shocks in similar ways as labor supply. Specifically, consider spending that does not directly enter the utility function, but is a response to a preference shock, say, the repair of a motorbike. The amount spent can be adjusted to some degree at the cost of a reduction in utility (e.g. spending time searching for used motorbike part and doing the repair oneself). An argument similar to the above applies; instead of increasing income by increasing effort, the household can reduce spending by increasing effort, and this endogenous response may attenuate...\(^{30}\) Assuming that changes are driven by shocks to income and preferences, and not shocks to the wage rate.
the effect of the relationship between MRS and spending on shocks as well as MRS and income shocks. We address this below by instrumenting for spending on shocks with the occurrence of the shock (see section 3).

1.7.2 Serially Correlated Income and Preference Shocks

Our baseline model assumes that income and preference shocks are independent over time. There are, however, circumstances in which this may be unrealistic, especially for income – for example, poor business conditions in one week may be predictive of bad conditions in the following week, lowering income today as well as expected income tomorrow.

Do our results hold up in the case of correlated shocks? For the relationship between savings and MRS the answer is yes, as the two are linked by an identity. For the relationship between shocks and MRS, the answer depends on the degree of correlation. Consider first the question of whether income shocks are negatively related to MRS. In the case of the partial constraints model, this boils down to the question of whether the marginal propensity to consume from the shock is less than one: if so, then a positive income shock will lead to increased savings, and so a fall in MRS. If not, then the shock will lead to a decrease in savings and so an increase in MRS, the opposite of our prediction.

The question of when correlated income shocks lead to an increase in savings has been studied in macroeconomics (see for example Uribe and Schmitt-Grohé [2017]). Broadly speaking, the answer is that as long as the shock raises current income more than permanent income, savings will increase, because the decision maker wishes to move resources into the future. Thus, if an AR(1) income process has a parameter of less than one, a positive shock will increase savings, as current income increases more than lifetime income. In the case of quadratic utility, exponential discounting and constant interest rates, this result can been readily established analytically (see Uribe and Schmitt-Grohé [2017] chapter 2). Maliar and Maliar [2004] show that the same result holds in the special case of quasi-hyperbolic discounting and exponential utility with no credit constraints. Outside these simple cases, and in particular in the presence of partial credit constraints, analytical results can no longer be obtained. We therefore numerically simulate our more complex model to show that the same intuition holds for a wide range of parameterizations (see appendix C).

The same reasoning holds true when considering correlated preference shocks: as long as shocks are not ‘too’ correlated, our results go through. This can be seen directly in the special case of section 1.6.3, where preference shocks work as negative income shocks.
1.7.3 Shocks to the Returns Function $R_i$

So far we have assumed that the interest rate faced by the household is determined only by the level of savings through $R$. However, interest rates may vary for other reasons, for example if investment opportunities available to the household change over time. In appendix C we consider a model which allows for stochastic changes in $R$ which are known at the time of investment (in the same way that income is stochastic, but $y_t$ is known when $c_t$ is chosen). We show that, if the household is in debt, changes in $R$ will lead to a positive relation between savings and measured MRS everything else equal. If savings are positive, then the relation may be positive or negative, depending on whether the income or substitution effect of the interest rate change dominates.

These results mean that shocks to the interest rate would introduce noise in the estimations we perform in section 3. If savings are negative or the substitution effect dominates the interest rate effect on savings, then the relationship of MRS with savings studied in section 3.2 constitutes a test of the relative importance of exogenous shocks to $R$, and endogenous changes to the interest rate due to changes in $s$. The fact that we find a negative relationship implies that the latter are more important. If instead the household is a net saver and the income effect of the interest change dominates, then the negative relationship between MRS and savings could be due to either endogenous or exogenous changes in interest rates. However, our finding that MRS is affected by income and preference shocks is independent evidence for the endogenous channel (curvature of $R$), as we argue further in section 3.31

2 Data

We now apply the insights from the model to data from MPL experiments that were carried out as part of a larger panel survey in Fall 2012 in Bamako, Mali. The survey was the baseline of a randomized control trial for a health care program for children. We collected demographic information at the start of the survey, and household members answered detailed questions on income and spending every week. The head of the household participated in multiple price list time preference experiments in four consecutive visits.

Table 3 shows summary statistics for the population of subjects. There are in total 1013 households for which we have some information on demographics and time preference measurements. Our participants are overwhelmingly male and in prime working age, although only a small proportion hold a salaried job. About a third had more than four years of schooling and half are literate. There are on average more than six people in

31Note that shocks to the interest rate will additionally introduce a relationship of MRS with spending, as well as a positive relationship with (endogenous) income.
Table 3: Characteristics of experimental subjects.

<table>
<thead>
<tr>
<th></th>
<th>Observations</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Respondent:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Holds salaried employment</td>
<td>1008</td>
<td>12.20%</td>
<td>32.70%</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Male</td>
<td>1009</td>
<td>87.20%</td>
<td>33.40%</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Under 25 years</td>
<td>1009</td>
<td>4.96%</td>
<td>21.70%</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>45 years and over</td>
<td>1009</td>
<td>26.26%</td>
<td>44.00%</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>At least four years of school</td>
<td>1009</td>
<td>34.49%</td>
<td>47.56%</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Can read and write</td>
<td>1009</td>
<td>50.05%</td>
<td>50.00%</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Household:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of members</td>
<td>1013</td>
<td>6.29</td>
<td>3.15</td>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>Children under 15</td>
<td>1013</td>
<td>3.31</td>
<td>2.03</td>
<td>0</td>
<td>13</td>
</tr>
</tbody>
</table>

Sample: 1017 respondents with at least one completed MPL. Four subjects without demographic information.

a household, a majority of them are children under 15 years old. The sample is fairly characteristic for the area, but there is selection to the degree that survey participants were chosen according to the criteria of the NGO providing the health care program. All households have children under five and had to pass a proxy-means test for income. If a household member had a savings account or was holding a salaried job at the time of the proxy test, the household was not eligible for the program and did not participate in the survey. Labor market and income structures of our sample are predominantly urban. Domestic production is relatively unimportant, and the household heads typically report income earning activities like sale and resale, motorcycle or car repair, taxi driving, or tailoring.

It is clear that our sample differs from typical undergraduate student populations who form the subject pool for many time preference experiments. Plausibly, poor households in urban Mali are more credit constrained and experience more frequent financial shocks than this group (though see Halevy [2015]). However, under the assumptions of our model, it is equally problematic to use MRS experiments to extract time preference information from unconstrained, low-shock individuals as it is from constrained, high-shock subjects. Credit constraints and financial shocks only make the issues easier to see.

The time preference experiment consists of a set of multiple price list choices over payoffs at different points in time as shown in table 1. These MPLs measure trade-offs between money in the current week and the next (A), and next week and one week after (B). Each decision in the MPL is a choice between a payment of CFA 300 (about US$ 0.60) at the later point in time, and a payment varying from CFA 50 to CFA 400 (US$ 0.10-0.80) at the earlier point in time. The experimental design follows the standard MPL procedure used in the literature, with the exception that we allow for negative interest rates by offering trade-offs between a higher payoff earlier and a lower payoff later. This

---

32In one week, an additional MPL experiment was carried out which is not used here, concerning choices between payouts two and three weeks away.
is motivated by the idea that a severely savings-constrained household would actually prefer to exchange high amounts today for lower amounts tomorrow, and indeed we see a number of households choose this option (see below).

One decision from all MPL choices during the current visit was selected for payout, using a random draw at the end of the experiment. Subjects then either received their immediate monetary payout, or a written receipt that stated the date and amount of any future payout the subject was owed. In the following weeks, the surveyors used their own notes and the subjects’ receipts to make payouts due from past decisions. As the surveyors visited the household every week, transaction costs were the same for current and future payments. In order to establish subjects’ trust, the first time-preference experiment consisted only of choices over payouts in the future to make salient that the surveyors actually return and make payments owed in later weeks. These choices are not used here.

Table 4 shows a summary of the remaining three weeks of MPL choices. The top row shows the number of observations. As is fairly typical in these types of experiments, 10-14% of subjects were recorded as making inconsistent decisions within a price list, with repeated switches between earlier and later payoffs. Education and literacy are associated with consistency; for instance, illiterate subjects make on average 15.4% inconsistent choices, but literate subjects only 8.7% (different at the 1% significance level). The other demographic variables have no effect on consistency. In the remainder of the table, only consistent choices are reported (we use the inconsistent choices in our conditional logit estimates, see below).

The row labeled “Avg. switch to earlier payment” reports the lowest earlier payoff that was chosen on average. For example, if the subject chooses CFA 300 in one week over CFA 150 today, but CFA 200 today over CFA 300 next week, the number used is CFA 200. The lowest possible value is therefore CFA 50, the highest value was set to 450 (for individuals who chose the later payment always). Due to the discrete experimental choices, we cannot report exact “indifference points” between earlier and later payments. We will discuss this issue in more detail below.

The next rows in the table show the week-to-week correlations of decisions in A and in B, and the proportions of subjects who made the same, more patient, or less patient decisions in A compared to B. Subjects’ decisions in the different MPL experiments are clearly related: a sizable proportion choose the same switch point in both decision A and decision B, and across weeks. However, the correlation is far from perfect; up to 30% of subjects choose different switch points in A and B in the same week, and the correlation of choices between weeks is only around 0.67-0.72. In 10-15% of cases subjects make a
more “patient” choice in decision A than decision B. The table also shows that at least 7% of subjects are willing to pay a weakly negative interest rate, that is, they choose CFA 300 in one week over CFA 350 right now. None of these patterns can be explained by the quasi-hyperbolic model in the standard “narrow bracketing” framework, but they are possible in the presence of financial shocks.

Table 4: Experimental choices.

<table>
<thead>
<tr>
<th>All observations</th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision</td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>Consistent</td>
<td>830 (85.3%)</td>
<td>836 (85.9%)</td>
<td>871 (89.9%)</td>
</tr>
<tr>
<td>Avg. switch to earlier payment (CFA)</td>
<td>157.2</td>
<td>155.8</td>
<td>153.6</td>
</tr>
<tr>
<td>Implied average MRS</td>
<td>4.78</td>
<td>4.7</td>
<td>4.73</td>
</tr>
<tr>
<td>Paying negative interest rate (MRS&lt;1)</td>
<td>9.64%</td>
<td>8.25%</td>
<td>7.35%</td>
</tr>
<tr>
<td>Week to week correlation for A</td>
<td>btw. weeks 1 and 2: 0.67</td>
<td>btw. weeks 2 and 3: 0.71</td>
<td>btw. weeks 1 and 2: 0.69</td>
</tr>
<tr>
<td>Week to week correlation for B</td>
<td>69.85%</td>
<td>70.37%</td>
<td>76.31%</td>
</tr>
<tr>
<td>Equal choice in A and B</td>
<td>15.14%</td>
<td>14.02%</td>
<td>10.09%</td>
</tr>
<tr>
<td>More patient (lower MRS) in A</td>
<td>15.01%</td>
<td>15.61%</td>
<td>15.01%</td>
</tr>
<tr>
<td>More patient (lower MRS) in B</td>
<td>15.01%</td>
<td>15.61%</td>
<td>15.01%</td>
</tr>
</tbody>
</table>

Table 11 in appendix D shows the distribution of switch points in decision A for consistent subjects by week. There is bunching at the most patient and most impatient choice, with a large proportion of subjects choosing the earlier payment always. 14% of subjects each week choose always the higher of the two payments in each of the eight choices, implying an interest rate between 1 and 1.167.Aside from these three most frequently observed choices, there is significant and varying dispersion in choices across the three weeks.

The lack of (average) present bias in our data, in the sense of more impatient choices in A over B, may seem surprising, but is consistent with other studies that take care to minimize differences in transaction costs and risk between present and future payments. Andreoni and Sprenger [2012] and Augenblick et al. [2015] estimate present-bias parameters $\beta$ for money that are never significantly less than 1. Halevy [2015], who repeatedly visited subjects in class in order to make payments, also found little present bias. Repeated visits – in class, or at home as in our study – may not only eliminate transaction costs for the subjects but also reduce self-selection into experiment participation based, for example, on current financial need. However, based on the analysis of section 1.6.1, the lack of present bias in our data does not mean that we can conclude that households are not time inconsistent. For the remainder of the paper we focus on the data from decision A.

In addition to time preference measures and demographics, our data contains weekly

---

33 See Dean and Sautmann [2014] for analysis of the relationship between decision B and financial variables.
Table 5: Weekly income by source, consumption, adverse events, and the resulting change in savings (income minus spending).

<table>
<thead>
<tr>
<th></th>
<th>Min</th>
<th>Median</th>
<th>Mean</th>
<th>Max</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>0</td>
<td>31</td>
<td>59.46</td>
<td>1309</td>
<td>92.96</td>
</tr>
<tr>
<td>Labor income</td>
<td>0</td>
<td>28</td>
<td>51.33</td>
<td>952</td>
<td>80.06</td>
</tr>
<tr>
<td>Nonlabor income</td>
<td>0</td>
<td>0</td>
<td>7.95</td>
<td>1001</td>
<td>41.54</td>
</tr>
<tr>
<td>&quot;exogenous&quot; (1)</td>
<td>0</td>
<td>0</td>
<td>4.99</td>
<td>1001</td>
<td>34.99</td>
</tr>
<tr>
<td>&quot;endogenous&quot; (2)</td>
<td>0</td>
<td>0</td>
<td>3.02</td>
<td>500</td>
<td>21.68</td>
</tr>
<tr>
<td>Spending</td>
<td>0</td>
<td>66.37</td>
<td>98.45</td>
<td>1210</td>
<td>106.42</td>
</tr>
<tr>
<td>Spending on food and hh. necessities</td>
<td>0</td>
<td>21.4</td>
<td>27.16</td>
<td>1040</td>
<td>32.3</td>
</tr>
<tr>
<td>Adverse event spending</td>
<td>0</td>
<td>0</td>
<td>5.80</td>
<td>600</td>
<td>23.57</td>
</tr>
<tr>
<td>Adverse event occurred?</td>
<td>0</td>
<td>0</td>
<td>33.20%</td>
<td>1</td>
<td>47.10%</td>
</tr>
<tr>
<td>Savings increase (income - spending)</td>
<td>-1176</td>
<td>-27</td>
<td>-38.59</td>
<td>1147</td>
<td>82.07</td>
</tr>
</tbody>
</table>

(1) rent received, loan repayment, and formal transfers, (2) tontine payouts, informal transfers, sales. All amounts converted to US$.

income and spending data. The household head and one mother\textsuperscript{34} in the household were interviewed. Income data was collected by source, and can be broadly categorized into labor income (income from self employment, sales, contracting work, or salaried work) and non-labor income (rent collected, government and military pensions and other transfers, loan repayment, etc.). Spending was split into different categories and includes any monetary outlays of the household, purchases of food and household goods, spending on fuel, rent, electricity, and heat, personal expenses of the household head, transfers to other households, business expenses including labor cost, and payments into a savings club or to pay off a debt.

Table 5 reports summary statistics of the financial variables of interest in US$. Of particular importance for our analysis is the expenditure category of “adverse events”. Subjects were asked whether they had incurred any large, unexpected expenditure since the surveyor’s last visit due to “damage to an item your household owns; damage to a building; loss, theft, or destruction of a good; loss or theft of animals; or illness to a household member”. If they answered yes, they were asked how much money was spent on repairs, replacement or (in the case of illness) treatment. We use such events to proxy for preference shocks of the type discussed in section 1.5.

We also break out labor income and different types of non-labor income according to the degree by which the household can affect the size and timing of payments, in order to separate income shocks from endogenous variation in earnings (see section 3.1). As these numbers show, a typical experimental payment is about 2% of the weekly median

\textsuperscript{34}Since households were selected for the presence of small children, the woman interviewed is typically the (first) wife of the household head who makes spending decisions.
household income.

Some notes on data quality and the match with the model variables are in order. First, savings as reported here are a flow variable. The stock of savings $s_t$ is unobserved, because our survey did not collect information on cash and other liquid assets held from week to week.\footnote{Information on household wealth, while available, is noisy and only includes relatively illiquid assets. Subjects are generally reluctant to give information on cash and other savings in the house. Within the time constraints of the health survey in which this data was collected, we expect that, even if liquid asset information had been gathered, it would likely not be precise enough to reflect week-to-week variation in the relevant $s_t$ accurately.} We discuss this issue when testing the relationship of MRS and savings. Second, spending does not directly correspond to consumption, but rather represents the outflow of cash, whereas ‘true’ consumption is unobserved. The model addresses this by allowing for preference shocks which do not directly contribute to consumption utility. Empirically, we test the effects of preference shocks by using reports on adverse events. Third, we may be concerned that households selectively participate in the survey depending on their financial outcomes in a given week, or that individuals who make inconsistent choices differ from those who do not. Comparing households that have some weeks of missing or inconsistent data with households that do not (411 out of 2559 observations), we find that they have on average lower spending, income, and savings. The largest difference is in income (significant at the 14% level); households with missing MRS data report on average $54 compared to $61 weekly income. Occurrence of and spending on adverse events is nearly identical for both types of households, so they seem to be subject to similar shocks. Lastly, using our information on consumption and income, we calculate flow savings to be negative on average. While it is possible that our sample of households as a whole is dissaving, this discrepancy is not atypical for household surveys and commonly interpreted as a sign of under-reported income (see e.g. Deaton [1997]). The income distribution is also more skewed than the spending distribution, suggesting that households have rare high income realizations that were not observed in our short panel. In general, it is likely that our financial data exhibits measurement error. We will address this again when discussing individual empirical tests.

3 Analysis

In this section we test the predictions from section 1.5, before using our model to understand the financial constraints of our sample households. We emphasize that the aim of this analysis is to estimate moments of the data which allow us to differentiate between different models of financial constraints. While using structural methods to estimate the parameters of the underlying model might be possible in principle in practice the data
requirements are extreme, as we discuss in section 1.6.1.

The empirical model we use to test the effects of income and preference shocks is

\[ MRS_{it} = \alpha_i + \lambda X_{it} + \gamma_t + \epsilon_{it}, \]

where \( X_{it} \) is the financial variable of interest converted to US$ 100.\(^{36}\) \( MRS_{it} \) represents the marginal rate of substitution measured by the MPL experiment.

The individual fixed effect \( \alpha_i \) implies that we are looking at deviations of \( MRS_{it} \) and \( X_{it} \) from their individual-specific averages. This accounts for ex-ante differences between households in income and spending levels (which determine for example what constitutes a “positive” or “negative” shock), the savings stock, and the returns function \( R \). It is likely that \( R \) is stable for a given household over the relatively short span of the experiment, but there may be variation in the interest rate function between households. For example, if household 1 faces a lower interest rate than household 2 at all savings levels, it may induce household 2 to save more, leading to a positive inter-household correlation between savings and MRS (see also section 1.7.3). Similarly, the savings stock is endogenous to past shocks and the shape of the returns function; for example, households that face strict borrowing limits may accumulate a higher buffer stock. The savings stock may also differ for individuals with same \( R \) but different \( \beta \) or \( \delta \). For all these reasons we focus on the within-subject response of the MRS to shocks.

In some specifications we also include period fixed effects \( \gamma_t \) to control for potential period-specific preference changes and time trends, for example due to festivals, holidays, weather changes, changing financial market conditions, or other sample-wide events. The error term \( \epsilon_{it} \) captures the measurement and approximation error in the experimentally measured MRS, as well as any variance in intertemporal trade-offs not explained by the financial variables.

This specification enables us to test several model implications. First, under the narrow bracketing model, any variation in MRS is due to measurement error unrelated to any financial variable. Similarly, under the no-constraints version of the model, the interest rate is fixed. Thus, both these models are rejected if we reject the null hypothesis \( \lambda = 0.\)\(^{37}\)

\(^{36}\)Or a vector of financial variables, see below.

\(^{37}\)Could the no-constraints model be correct, yet financial shocks are correlated with the interest rate due to factors other than the curvature of \( R \)? Note first that the results we document below are robust to the inclusion of time dummies, which control for aggregate changes in the interest rate, e.g. due to macroeconomic conditions. Note further that shocks to the interest rate as discussed in section 1.7.3 would lead to a relationship of MRS with endogenous (labor) income and consumption spending, but not with financial shocks. Thus, some third event would have to cause both a change in in the individual-specific interest rate and individual-specific financial shocks, at a high enough frequency to be picked up in our three-week panel. Given the nature of the shocks we examine (for example the sickness of a family
As discussed in section 1.5, the complete and partial constraints versions of the model imply that the expected covariance calculated from a \( T \) length sample is weakly negative for exogenous income shocks and MRS, and weakly positive for preference shocks and MRS. The parameter \( \lambda \) is proportional to this expected covariance. Under standard conditions, the OLS estimator will converge in probability to \( \lambda \), and thus can be used to examine these predictions.

Since we only have discrete brackets given by the nine possible switch points in the list, we take two different approaches to estimating this model. First, we estimate OLS and IV specifications with errors clustered at the individual level, where we approximate the subject’s MRS by calculating the midpoint between the ratios of the later over the earlier payment at which the subject switches from choosing the late to choosing the early payment.\(^{38}\) The MRS for individuals who within a decision set always choose the earlier payment may lie anywhere on the interval \((6, \infty)\), and for those who always choose the later payment, it may be anywhere on \((0, 0.75)\). To account for those observations we chose several plausible endpoint values and checked the robustness of our estimates to these variants. The regression results reported here use 0.708 as the lowest and 8 as the highest MRS.\(^{39}\) This approach cannot account for (surveyor or subject) errors within a choice list, and we exclude inconsistent choice lists in which there is more than one switch.

For our second approach we assume that the (latent) MRS is a linear function of the financial variables as above plus an additive logistic error term. In each of the (up to) 24 binary MPL choices the subject makes, the probability of choosing the later payment is given by the probability that the MRS is lower than the ratio of the later to the earlier payment. This can be used to construct a conditional log likelihood and to estimate the inverse standard deviation of the error term and the coefficient on the financial variable in the MRS. The conditional likelihood method can accommodate person fixed effects and inconsistent choices (see appendix F for details).

### 3.1 Income and Preference Shocks and MRS

We first examine the relationship of MRS and income. Columns (1), (2), and (5) of table 6 report a significant negative relationship between total income and MRS, with column (5) reporting the conditional logit estimates. This rejects narrow bracketing or fixed interest rates, and supports a model with credit constraints.

\(^{38}\)In the case of preference shocks we instrument spending on the adverse event with a dummy for the event occurrence, see below.

\(^{39}\)0.708 is the next lower switch point if the MPL had included the choice between 450 CFA earlier vs. 300 CFA later.
Table 6: Effect of income (in US$100) on $MRS_t$, total income in columns (1), (2), and (5), and by income source in columns (3), (4), and (6).

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
<th>CL</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total income</strong></td>
<td>-0.176</td>
<td>+0.187</td>
<td>*</td>
<td>-0.23</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.095)</td>
<td></td>
<td>(0.102)</td>
<td></td>
</tr>
<tr>
<td><strong>Labor income</strong></td>
<td>0.018</td>
<td>-0.005</td>
<td>-0.112</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.115)</td>
<td>(0.116)</td>
<td>(0.120)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Nonlabor income</strong></td>
<td>-0.310</td>
<td>-0.299</td>
<td>-0.306</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.255)</td>
<td>(0.262)</td>
<td>(0.286)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;endogenous&quot;</td>
<td>-0.414</td>
<td><strong>-0.415</strong></td>
<td><strong>-0.396</strong></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.146)</td>
<td>(0.152)</td>
<td>(0.190)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;exogenous&quot;</td>
<td>-0.414</td>
<td><strong>-0.415</strong></td>
<td><strong>-0.396</strong></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.146)</td>
<td>(0.152)</td>
<td>(0.190)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/(error SD)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.908</td>
<td><strong>0.909</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.043)</td>
<td>(0.043)</td>
</tr>
<tr>
<td><strong>Ind FE</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Time FE</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>2484</td>
<td>2484</td>
<td>2484</td>
<td>2484</td>
<td>13152</td>
</tr>
</tbody>
</table>

*Standard errors clustered at the individual level (in parentheses).  
Significance levels + p<0.10, * p<0.05, ** p<0.01.

Within this model, the coefficients we report may in fact underestimate the effect of exogenous income changes on MRS, if households are able to affect their income to some degree in response to shocks. Our model predictions regarding the relation between income shocks and MRS continue to hold under mild conditions when labor supply is endogenous (see section 1.7.1), but an endogenous component to total income leads to a downward bias in the estimates.

In a partial solution we therefore classify each separately recorded income source according to the level of control that the household likely has over that source. We break out labor income, where we include all income-earning activities that require labor input from the household; working in one’s own business (in activities such as making and selling items, or driving a taxi), working for a piece-rate or time-dependent pay, or working for a regular salary. Non-labor income includes important income sources like rent paid to the household or transfers from other households. Since the timing and variation of non-labor income is likely not entirely exogenous to the household’s consumption needs either, we further categorize government and military payments, rent paid to the household, and loans paid back to the household by others as most likely to be “exogenous”, that is, less likely to be determined by a choice that the household made. By comparison, payments received from a RoSCA (tontine), sales revenues, and transfers from other households might be actively requested or generated by the household according to their consumption smoothing needs, and are thus to some degree “endogenous”.

Our model predicts that the negative relationship between MRS and income will be
strongest for the most “exogenous” income sources. Columns (3), (4), and (6) of table 6 estimate the effect of income split into its different sources.\textsuperscript{40} The results support this assumption: whereas the effect of labor income on MRS is small and insignificant, non-labor earnings have a larger effect, and for those income sources least under the household’s control, the effect is strongest and significant at the 1% level.\textsuperscript{41} Note that the reported coefficients in the CL estimates must be be rescaled by $\sigma$, the standard deviation of the error, giving an effect size of -0.436 for exogenous non-labor income in column (6) (see appendix F).

Since this approach does not use truly exogenous income variation, we carry out some robustness checks. First, we examine the correlation of the three income categories with the occurrence of adverse events and find that the correlation is overall low, but highest for labor income (0.052), followed by endogenous non-labor income (0.020) and finally exogenous non-labor income (-0.003). Only the correlation with labor income is significant (at the 1% level), consistent with the idea that labor income responds endogenously to consumption needs. Second, the degree to which the MRS correlates with income may be driven by the (lack of) overall variation of the different income types, rather than different degrees of endogeneity. However, the frequencies of positive income observations in the three categories suggest that, due to many zero-income observations, the variation in non-labor income is lower than in labor income (Figure 2, appendix D). Last, note that, to the extent that endogeneity remains an issue for the income-MRS relationship, it suggests that the coefficient of -0.411 (-0.436) \textit{underestimates} the true effect of exogenous income shocks.

We use spending on adverse events to test the effect of preference shocks on MRS. We assume that the occurrence of an adverse event is exogenous, and that expenditure on the event – repairing or replacing an item, paying for healthcare – acts essentially like a negative income shock, by reducing the amount of money available for other consumption (see section 1.6.3).

The first two columns of table 7 show that the occurrence of an adverse event has a significant positive effect on the MRS. This is again supportive of our model with credit

\textsuperscript{40}In previous versions we also reported the effects of income from each source separately. The results are very similar and omitted here for brevity.

\textsuperscript{41}As discussed in section 2, it is possible that income is systematically under-reported in our sample. In the simple case in which all income is under-reported by a constant multiplier $\alpha < 1$, this will have the effect of increasing the estimated coefficient of the relationship between income and MRS by $\frac{1}{\alpha}$. In other words, the relationship between MRS and reported income overstates the relationship between MRS and true income. However, standard errors will also be scaled accordingly, so that our hypothesis tests are correct. If there is additionally variance in the degree to which income is underreported, this will bias our tests towards insignificance, assuming this noise is uncorrelated with MRS.
constraints over no constraints or narrow bracketing. The next two columns show that the effect remains significant at the 10% level when using event expenditure as the independent variable. These results are echoed in the conditional likelihood estimates in the last two columns. The CL procedure estimates $\frac{1}{\sigma}$ to be 0.898 and 0.897, yielding estimated effects of 0.271 and 0.477 of an adverse event and $100 adverse event spending, respectively.

Similar to the issue of endogenous labor supply above, the amount spent on an adverse event may be correlated with marginal consumption utility through the household’s choice of how to respond to the event. The household can for example reduce expenditure by doing their own repairs instead of hiring someone (see section 1.7.1). This may occur in periods where the household’s marginal utility to consumption is high, perhaps because credit constraints are binding or there was a concurrent negative income shock. We therefore instrument for spending on adverse events with the indicator variable for the occurrence of such an event, in order to estimate the (local) average treatment effect (columns (5) and (6), first stage results in table 12, appendix E). The IV approach is valid if the occurrence of an adverse event is random and affects marginal consumption utility only through its effect on what is spent on the event. If the adverse event also increases the marginal value of other consumption independently, the IV coefficient overestimates the effect of adverse event spending. The OLS and IV estimates can therefore be seen as respectively lower and upper bounds on the true effect. The results suggest that the simple OLS substantially underestimates the impact of exogenously imposed adverse event expenditure onto MRS.

A concern the reader might have at this point is that the observed correlations are not due to the failure of narrow bracketing, but rather that preference vary over time, and
that this on the one hand changes experimentally measured MRS, and on the other affects households’ financial choices. However, if the correlation of MRS and financial variables is due to the household’s response to changes in preferences, then it should be strongest for the *endogenous* components of income and spending. This is the opposite of what we see in our data.

### 3.2 Savings and MRS

The results on income and preference shocks rule out narrow bracketing and the no-constraints version of our model. In this section we test whether there is a relationship between savings and MRS, which distinguishes the partial-constraints from the complete-constraints model.

The theory relates the stock of savings $s_{it}$ to MRS through the function $R(s_{it})$. Our data does not contain good measures of the savings stock of the household, but we can construct a test of this relationship based on flow savings (the difference between income and consumption). If our model is correct, then a Taylor expansion of $R'$ around $s_{it} - 1$ links last-period MRS, the change in savings, and current MRS:

$$
R'_it(s_{it}) = R'_it(s_{it} - 1) + R''_it(s_{it} - 1)(s_{it} - s_{it} - 1) + \frac{1}{2} R'''_it(s_{it} - 1)(s_{it} - s_{it} - 1)^2 + o_{it}
$$

$$
MRS_{it} = MRS_{it} - 1 + R''_it(s_{it} - 1)(\Delta s_{it}) + R'''_it(s_{it} - 1)\left(\frac{1}{2}(\Delta s_{it})^2 + \eta_{it}\right)
$$

where $\Delta s_{it} = y_{it} - c_{it}$ represents the flow of savings in $t$, and the person and time indices on $R$ account for variation in financial conditions between periods and individuals (see section 1.7.3). The term $\eta_{it}$ includes the approximation error $o_{it}$; if $R_{it}$ differs from week $t - 1$ to $t$, for example due to a change in credit market conditions, then $MRS_{it} - 1 = R'_it(s_{it} - 1)$ and $\eta_{it}$ also contains the difference $R'_it(s_{it} - 1) - R'_it(s_{it} - 1)$ (see section 1.7.3).

Now consider the regression equation

$$
\tilde{\Delta}MRS_{it} = \lambda_1 \tilde{\Delta}s_{it} + \lambda_2 \left[0.5 \tilde{\Delta}s_{it}^2\right] + \alpha_i + \gamma_t + \epsilon_{it}.
$$

$\tilde{\Delta}MRS_{it}$ and $\tilde{\Delta}s_{it}$ denote values we observe in our data. $\tilde{\Delta}MRS_{it}$ is the difference in observed experimental trade-offs between period $t - 1$ and $t$, and is measured with error $r_{it}$, which will enter $\epsilon_{it}$ along with $\eta_{it}$ above. The fixed effects $\alpha_i$ and $\gamma_t$ capture any period-specific or individual-specific trends in $R'$. If there are no such trends, these coefficients will be zero.\(^{42}\)

Measured flow savings $\tilde{\Delta}s_{it} = \tilde{y}_{it} - \tilde{c}_{it} = \Delta s_{it} + \tilde{y}_{it} - \tilde{c}_{it} - 1$ may contain \(^{42}\)Because measured flow savings are on average negative, estimating the equation without any constant will bias the estimate of $\lambda_1$ if such trends are present. Note also that an individual fixed effect can capture changes in experimental decisions over time that are not related to the financial market, due for example
measurement error in income and consumption, $z_y^y$ and $z_c^c$, respectively.

We first use this regression to test our model against the null hypothesis that the households in our sample are subject to complete credit constraints. In the complete-constraints case, $MRS_{it} = \frac{u'(y_{it})}{E[\beta u'(y_{it+1})]}$ in each period, there are no savings, and (true) flow savings equal zero. Measured flow savings $\tilde{\Delta}s_{it} = z_y^y - z_c^c = z_{it}$ therefore represent only measurement error. We would expect $\lambda_1$ to be zero under this model. By contrast, if our model holds, we expect $\lambda_1 < 0$, since $R$ is concave.

Table 8 reports the regressions of $\tilde{\Delta}MRS_{it}$ onto linear and squared flow savings terms with different specifications for the intercept (note that the lag in the dependent variable means we have here only two weeks of data). All four regressions show a significant negative relationship between flow savings and measured MRS, i.e. $\hat{\lambda}_1 < 0$. The individual fixed effects reduce power, but increase the absolute size of the coefficient on linear savings.

Under the complete-constraints model, the estimate of $\lambda_1$ will not be significantly different from zero, under the assumption that $z_{it} \perp \tilde{\Delta}MRS_{it}$. We do however expect $MRS_{it}$ to vary with (true) income, so if $z_{it}$ is non-classical measurement error and correlated with true income or spending, this assumption may be violated. Could such a correlation be responsible for our negative estimate for $\lambda_1$? Our survey asked individuals to report income and spending separately. A plausible source of a correlation between income or spending and error $z_{it}$ is therefore differential under- or over-reporting; and since reported flow savings are on average negative, and reported income has a lower variance than reported spending, this would imply that true income is negatively correlated with flow savings (for example, $\tilde{y}_{it} = \alpha_y^y y_{it}$ and $\tilde{c}_{it} = \alpha_c^c c_{it}$ so that $z_{it} = (\alpha_y^y - \alpha_c^c) y_{it}$ and $\alpha_y^y < \alpha_c^c$ on average). Since MRS is negatively correlated with income in the full constraints model, this error structure would introduce a positive bias and predict $\hat{\lambda}_1 > 0$ if $\lambda_1 = 0$.

Another type of bias could occur if the measurement error in MRS, $r_{it}$, were directly correlated with $z_{it}$, in other words, whenever subjects underreport income relative to consumption, they also systematically overreport their MRS (and vice versa), perhaps
to changing levels of trust in the surveyors by our sample subjects. This corresponds to a time trend in (non-classical) measurement error.

43 Under the no-constraint and narrow bracketing models $\lambda_1 = 0$ as well: with narrow bracketing, experimental MRS is unrelated to any outside financial variables by assumption, and in the no-constraints case, the interest rate is unaffected by savings, and $R' = 0$ and $R'' = 0$.

44 The exception would be if flow savings $\tilde{\Delta}s_{it}$ and the unobserved variation in MRS, $\epsilon_{it}$, are strongly positively correlated. See below for an additional discussion on potential error correlations.

45 Column (1) is a simple OLS and therefore allows only for a common constant time trend in MRS (restricting all $\alpha_i$ to be equal and $\gamma_t = 0$). Columns (3) and (4) relax the common trend assumption and include individual fixed effects $\alpha_i$. The constant terms (or individual fixed effects) are significant in each regression, indicating there is a time trend that needs to be accounted for. Allowing the time trend to change over time does not change results significantly (columns (2) and (4)).
Table 8: Savings (flows) and $MRS_t - MRS_{t-1}$.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings (I-E)</td>
<td>-0.182 *</td>
<td>-0.180 *</td>
<td>-0.392 +</td>
<td>-0.391 +</td>
</tr>
<tr>
<td></td>
<td>(0.0865)</td>
<td>(0.0868)</td>
<td>(0.220)</td>
<td>(0.222)</td>
</tr>
<tr>
<td>1/2 Savings^2</td>
<td>-0.00639</td>
<td>-0.00598</td>
<td>0.0247</td>
<td>0.0247</td>
</tr>
<tr>
<td></td>
<td>(0.0268)</td>
<td>(0.0268)</td>
<td>(0.0778)</td>
<td>(0.0778)</td>
</tr>
<tr>
<td>Time FE</td>
<td>yes</td>
<td></td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Ind FE</td>
<td></td>
<td>yes</td>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>Observations</td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
</tr>
</tbody>
</table>

Standard errors clustered at the individual level (in parentheses).
Significance levels $+ p<0.10$, $* p<0.05$, $** p<0.01$, $*** p<0.001$.

Figure 1: Proportion of delayed payment choices (300 CFA) in each binary decision, by experimental payment today and split into households below and above median (flow) savings.

Due to a mistaken perception of being more constrained. However, we judge this source of bias to be fairly implausible under complete credit constraints, because subjects with no ability to save or borrow should realize that it is not actually possible that they consumed an amount different from their income. Moreover, the way we collected our income and consumption data – by asking household members individually for their income from different sources, as well as their spending on different types of items – makes it unlikely that the difference between total reported consumption and income (i.e. $z_{it}$) is even salient to the subject.

In summary, our results suggest that there is a real relationship between savings and $MRS_t - MRS_{t-1}$. But note that this cannot be driven by perceived consumption $\hat{c}_{it}$. For example, a plausible effect could be that subjects who perceive the household’s consumption to be lower than what it really was will also overestimate the marginal utility of additional consumption. This would again lead to a positive correlation of flow savings and MRS.
MRS, supporting the partial-constraints model over the three alternatives. Figure 1 provides supplementary evidence from the raw data, showing that across weeks and households, below-median flow savings are associated with a more frequent choice of the earlier payment in each MPL decision.

The results in table 8 suggest a significant curvature of \( R (\lambda_1 < 0) \) that is constant across the range of possible savings (\( \lambda_2 \) is small and not significantly different from zero). Table 13 in appendix E shows that the results are similar when including a cubed savings term. This is consistent with “soft” credit constraints, rather than a constant interest rate with a hard credit limit, which would imply that there is only one (minimum) level of savings where the MRS responds strongly to shocks. Note that the coefficients in table 8 may underestimate the average curvature of \( R \), if in addition to common time trends, \( R \) is subject to individual- and period-specific shocks and subjects are dissaving (see section 1.7.3). However, even if there is such variation in individual effective interest rates, our results show that it cannot be large enough to conceal the average curvature of \( R \). Lastly, note that without the experimental MRS data, we would need to derive information about the shape of \( R \) from financial information alone. This would require estimating the shape of \( u \) along with the shape of \( R \), and deriving variables on both sides of the equation – MRS and savings – from the same consumption data.

3.3 The Relative Importance of Income and Preference Shocks

We find that measured MRS varies with financial parameters in ways predicted by a model of partial intertemporal smoothing. As discussed in section 1.6.3, we can now use the relationship between consumption and MRS to understand the relative importance of income and preference shocks. Columns (1) to (3) of table 9 show that the relationship between spending and MRS is positive and significant in our data: higher current expenditure is related to greater impatience. This tells us that high realizations of spending are primarily the result of preference shocks that cannot be smoothed, and are in fact associated with lower levels of “utility-relevant” net consumption and higher marginal utility. From equation (7), the preference shocks households experience are bounded by

\[
Var(\rho_t) > \frac{\partial n_t}{\partial w_t} Var(w_t).
\]

Using a crude (and low) estimate of the marginal propensity to consume\(^{47}\) of 0.38, and noting that the variance of cash on hand at period start is at least as high as the variance

\(^{47}\)The MPC is obtained from regressing changes in spending on changes in income, controlling for week fixed effects. 0.38 is likely a lower bound on the true value.
of income, the coefficient sign implies that the standard deviation of $\rho$ is bounded below at 72.77 (using the empirical standard deviation of weekly income of 92.96, see table 5); the true number is likely higher. This implies that preference shocks explain more than 46% of the variance of measured household expenditure.

Equation (7) continues to hold if we apply it to specific categories of expenditure, by replacing $\rho$ with category-specific shocks and $\frac{\partial n}{\partial w}$ with the propensity to consume in this particular category. Thus, we can study which types of preference shocks affect measured MRS the most. Different categories of expenditure may differ both in the size of shocks they are subject to, and the effect of these shocks on spending, given by the category-specific propensity to consume and the degree to which the household can smooth the shocks through the financial market.

The remaining columns of table 9 show the results of such an exercise. Social events are significantly negatively correlated with MRS. This suggests that variation in this type of spending is driven by income, and there are few “shocks”. Similarly, gifts and personal expenditure – which includes “luxury goods” like cigarettes or tea and phone credit – have large and negative (but not significant) coefficients, again pointing to high income elasticity and little “shock-driven” spending. Utility bills and rent and large purchases

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>CL</th>
<th>OLS</th>
<th>OLS</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Spending</strong></td>
<td>0.203*</td>
<td>0.180*</td>
<td>0.167+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>(0.089)</strong></td>
<td>(0.092)</td>
<td>(0.091)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Food/Necessities</strong></td>
<td>-</td>
<td>-</td>
<td>-6.91+</td>
<td><strong>0.672</strong></td>
<td><strong>0.623</strong></td>
<td><strong>0.620</strong></td>
</tr>
<tr>
<td><strong>(0.157)</strong></td>
<td>(0.158)</td>
<td>(0.240)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adv. event expenses</strong></td>
<td>-</td>
<td>-</td>
<td>-0.274+</td>
<td><strong>0.269</strong></td>
<td>+0.513***</td>
<td><strong>0.269</strong></td>
</tr>
<tr>
<td><strong>(0.151)</strong></td>
<td>(0.146)</td>
<td>(0.182)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Large purchases</strong></td>
<td>-</td>
<td>-</td>
<td>0.0507+</td>
<td>0.0496</td>
<td>0.209</td>
<td>-</td>
</tr>
<tr>
<td><strong>(0.379)</strong></td>
<td>(0.386)</td>
<td>(0.299)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bills and Rent</strong></td>
<td>-</td>
<td>-</td>
<td>-0.104-</td>
<td>-0.179</td>
<td>0.093</td>
<td>-</td>
</tr>
<tr>
<td><strong>(0.407)</strong></td>
<td>(0.412)</td>
<td>(0.438)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Gifts and Donations</strong></td>
<td>-</td>
<td>-</td>
<td>-0.254-</td>
<td>-0.372</td>
<td>0.873</td>
<td>-</td>
</tr>
<tr>
<td><strong>(0.761)</strong></td>
<td>(0.760)</td>
<td>(0.729)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Personal Expenditure</strong></td>
<td>-</td>
<td>-</td>
<td>-0.487-</td>
<td>-0.754</td>
<td>-1.288</td>
<td>-</td>
</tr>
<tr>
<td><strong>(0.763)</strong></td>
<td>(0.772)</td>
<td>(0.870)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Social Events</strong></td>
<td>-</td>
<td>-</td>
<td>-0.919-</td>
<td>-0.977</td>
<td>-0.729</td>
<td>-</td>
</tr>
<tr>
<td><strong>(0.478)</strong></td>
<td>(0.474)</td>
<td>(0.476)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>1/(error SD)</strong></td>
<td>0.913***</td>
<td>-</td>
<td>-</td>
<td>0.904***</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>(0.044)</strong></td>
<td>(0.044)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ind FE</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Time FE</strong></td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>2418</td>
<td>2418</td>
<td>12688</td>
<td>2439</td>
<td>2439</td>
<td>12848</td>
</tr>
</tbody>
</table>

*Standard errors clustered at the individual level (in parentheses).*

*Significance levels + p<0.10, * p<0.05, ** p<0.01, *** p<0.001.*
have the lowest correlation with MRS, implying that households are able to smooth this (planned) variation in spending well.

By contrast, as we have seen earlier, spending due to adverse events has a significant positive coefficient in the regression. Remarkably, however, the strongest positive relationship is between MRS and spending on food and household essentials. This indicates that variation in demand for basic household goods is driven by negative preference shocks, rather than, for example, splurging on a good meal after a successful day at work. Shocks in this category could come from seasonal price fluctuations, fixed and transaction costs of acquiring certain items (e.g. travel to the market), spoiled, spilled, or soiled foods, spending on unexpected guests, and so on. The size of the coefficient is likely a consequence of the fact that basic consumption needs are unresponsive to income, and that greater spending needs in this category are more difficult to delay, and possibly less predictable, than other shocks. An additional reason may lie in the traditional organization of Malian families, where women are expected to cover household needs from their weekly allowance and request additional money only in exceptional cases, making these expenses look like an exogenous shock from the perspective of the household head.

3.4 Discussion

3.4.1 Robustness

As a test of the robustness of our results, we carry out two additional checks. First, we report a set of regressions that include both income shocks and preference shocks, and then all sources of income and spending simultaneously (table 14 in appendix E). This controls for any covariance in income and preference shocks, which could bias the individual estimated effects of these variables onto MRS. The results are broadly robust to this specification change, and the effect sizes remain the same.

Second, in order to test if there are important nonlinearities, we include quadratic terms in the estimations, shown in appendix E. The coefficient sizes and signs suggest that the main correlations hold as predicted. Although the coefficients on the individual variables are not significant, F-tests show that the income shock variables remain jointly significant in all estimations. Event spending is not significant anymore (note that we do not have enough instruments for both linear and quadratic event spending). F-tests for the inclusion of all the quadratic terms cannot reject that they are jointly insignificant, except in the CL estimates. We interpret these results to mean that the main predictions of the model are robust, but that any potential nonlinear effects are not strong enough to be reliably estimated in this relatively short panel.
3.4.2 Interest Rates and Average MRS

As with many other experimental studies (see Frederick et al. [2002] for a survey), measured MRS in our survey is higher than what can plausibly be explained by external interest rates alone: the mean MRS is 4.7 and the median 4.5. The high average MRS is partially driven by the group of 238 subjects who chose the early payment in every single MPL decision. It could be that this subset of individuals is facing a binding borrowing constraint. Another possibility is that they did not engage with the question, or that they operate under a decision-making heuristic that is not described by our model. If we exclude subjects who make the same choice in all 24 MPL decisions (either the late or the early payment always), the mean MRS falls to 3.45 and the median to 1.75. Yet these numbers still imply an annual interest rate at the higher end of the spectrum reported by Frederick et al. [2002]. As suggested by Collins et al. [2009], part of the reason may be one-off (non-monetary) transaction costs of borrowing or saving that drive up the effective interest rate on small payments. A second possible explanation is that our subjects attach a significant probability to future payments not being made, either because the surveyor does not return, or because the subjects themselves become unavailable for interview. A constant hazard rate of survey interruption acts like an additional discount factor and shifts all measured MRS upwards. The model predictions for the effect of financial shocks on changes to the MRS are not affected by the inclusion of such an adjustment. Our key finding remains that external financial changes affect the experimental decisions of at least a proportion of subjects.

4 Relationship with the Literature and Conclusion

From a theory standpoint, the closest paper to ours is Pender [1996]. He considers an integrated model of choices over experimental payments in two periods in different credit environments, including where interest rates increase with borrowing. Our model extends this work in a number of ways. First, it allows for both preference shocks and endogenous labor responses, which our empirical results suggest are important. Second, it shows that the relationship between credit market conditions and measured MRS holds both when

\footnote{Repeating the analysis on the effect of shocks on MRS and the correlation between flow savings and MRS for this subsample strengthens the results considerably. We do not report these results, because the exclusion of subjects whose measured MRS is stable through the entire panel biases us towards finding the effects our model predicts.}

\footnote{Given the political instability of the area and frequent flooding during the rainy season such a hazard rate is not implausible. There are also more ‘behavioral’ explanations related to small amount of time between sooner and later payments. Frederick et al. [2002] document that shorter time horizons are associated with higher observed discount rates. However, note that in our case this cannot be due to a fixed cost associated with delayed payment (as in the model of Benhabib et al. [2010]) as we also observe high MRS in decision B.}
experimental payments are not actively arbitraged and when there is quasi-hyperbolic discounting. Third, by considering the choices of a quasi-hyperbolic discounter over multiple time periods, we demonstrate that time-inconsistent choices do not well identify present-biased consumers when there is some access to credit and no narrow bracketing. Finally, the applications we consider in sections 1.6 and 3.3 are, to our knowledge, novel. From an empirical perspective, Pender’s identification strategy, which is based on experimental data only, requires that (large) experimental payments themselves affect the household’s credit market position, whereas our model and empirics consider the role of exogenous financial shocks on experimental choices.

Another theoretical contribution is Cubitt and Read [2007], who consider the implications of a model of arbitrage with two distinct interest rates for borrowing and saving. Similar to Pender, their predictions derive from the assumption that subjects apply different interest rates to experimental payments at different points in time, but they do not consider the impact of financial shocks. As discussed in section 1.6.1, Andreoni and Sprenger [2012] estimate a parameter that captures background consumption, but identification in their model of experimental choices requires changes to the MRS that cannot be arbitraged. None of these papers explore what can be learned about consumption smoothing, credit constraints or financial shocks from MRS experiments.

On the empirical side, several papers investigate the variation of individual’s time preference measures over time (see Chuang and Schechter [2015] for an overview). On balance, papers that study the relationship of experimental time preference measures with external financial conditions offer support for the general idea that the latter can influence the former. None of these papers make predictions about, or separately study, the relationship of MRS with individual-level variation in the inputs into the household’s financial decision problem, i.e. financial shocks, spending and savings. They therefore cannot perform the range of tests we use to learn about the household’s financial decision-making.

Four studies correlate time preferences with some measure of subjects’ outside financial situation in a range of populations and find a relationship.\textsuperscript{50}

Harrison et al. [2005] is the first longitudinal study of time preferences and outside financial conditions. They ask a sample of 97 subjects a variety of questions on perceived changes in their financial situation. The coefficients on these survey measures are jointly insignificant, although one particular question regarding the general economic situation in the country is significantly related to the measured discount rate.\textsuperscript{51} Their data does

\textsuperscript{50} In related work, Meier and Sprenger [2010] study whether credit constraints and future liquidity are correlated with time preference in cross sectional, rather than panel data. They find no evidence for such a relationship.

\textsuperscript{51} Turning to the economic conditions in the country as a whole, would you say that at the present
not contain objective financial information and no source of exogenous variation. Krupka and Stephens [2013] report data collected in the Seattle and Denver Income Maintenance Experiments from the 1970s, a panel data set of 1194 subjects that contains income and labor supply information as well as hypothetical questions designed to elicit the participants’ discount rates. They find that changes in weekly income and monthly hours worked (as well as inflation rates) are correlated with changes in the discount rate, which they interpret as a response to real interest changes in line with our model predictions. Again, this study does not have the rich data required to perform the tests we report in this paper, and also relies on hypothetical, rather than actualized payments. Concurrent with our own study, Carvalho et al. [2016] and Ambrus et al. [2015] also report a relationship between financial variables and measured MRS. Carvalho et al. use the natural variation in liquidity caused by the pay cycle to examine the effect on a number of cognitive and behavioral measures in a sample of low income US households. They show that before-payday participants behave as if they are more present-biased in choices over monetary rewards but not over non-monetary real effort tasks, supporting a liquidity-constraint interpretation. Ambrus et al. find that only subjects without a stable background income exhibit present bias.

Three other studies do not find a correlation between financial variables and measured discount rates. Meier and Sprenger [2015] survey a sample of 250 individuals at a tax filing center in Massachusetts and find no relationship between time preferences and demographic or income changes. Similarly, in Chuang and Schechter’s sample of 49 households from Paraguay, income changes between 2007 and 2009 have no significant impact on (hypothetical) time preference measures in the two survey rounds. The findings in these papers may be different from ours due to the smaller sample size and the fact that events from the past year have already entered subjects’ intertemporal optimization, or do not reflect income shocks (but, for example, endogenous or permanent income changes), and thus do not affect intertemporal trade-offs between current and future payoffs. By comparison, our measures track the financial situation of an individual over the same time frame for which time preferences were elicited and separate out financial shocks.

Giné et al. [2018] perform an experiment in rural Malawi to examine the revision of intertemporal choices. Subjects were first asked to allocate money across an intertemporal budget (in the manner of Andreoni and Sprenger [2012]), and were then offered to change their initial allocation later. The authors find no significant effect of measured shocks – a death in the family or “unexpected income shortfalls” – on the subject’s choice to revise. This is surprising given our findings, but the authors also state that there were few deaths time economic conditions are better or worse than they were X months ago?"
(2% of households) and income shortfalls were generally small, so that the presence of undetected large effects cannot be rejected given the wide standard errors.

Lastly, some indirect support for our model comes from Halevy [2015]. He reports time $t$ preferences between payments at $t$ and $t + 1$ and between payments at $t + 1$ and $t + 2$, and time $t + 1$ preferences over payments at $t + 1$ and $t + 2$. This allows the categorization of non-stationary time preferences into those that violate time invariance (changing relative value of immediate and delayed payments) and time consistency (changing relative valuations of payments that occur in $t + 1$ and $t + 2$). While time inconsistency could be a consequence of present bias, a quasi-hyperbolic narrow bracketer should still make time-invariant choices. The author reports significant violations of time invariance, consistent with temporary liquidity constraints.

The body of literature confirms that narrow bracketing is violated under fairly general conditions. While the majority of cited papers find some covariation of financial conditions with MRS, the detailed financial data in our study allows us to go further and verify a range of predictions of the model for the relationship of MRS with different financial variables at the individual level. We find some causal evidence by addressing the potential endogeneity of labor and some types of non-labor income and employing an IV strategy to bound the effect of exogenous preference shocks due to adverse events. Moreover, using our predictions for savings and the effect of shocks on future MRS, we can distinguish between different types of credit constraints. Given the empirical support for our “soft constraints” model, we propose novel applications for MRS experiments in the study of intertemporal decision making, for example to test credit constraints and informal insurance models, pointing to new uses for detailed financial and MRS panel data. In a demonstration of the advantages of this approach, we quantify preference shocks from the overall spending-MRS correlation and show that they explain almost half of all expenditure variation in our population.
References


(Appendix for online publication)

A Proof of Propositions in Section 1.4

Proof of Proposition 1: Case 1: Suppose that $a_0$ must be consumed right away, while $a_1$ is added to the stock of wealth in the next period. Then the decision maker strictly prefers $a_0$ over $a_1$ if

$$u(w_t - s_t^* + a_0, \rho_t) + \beta \delta E_t V(y_{t+1} + R(s_t^*) > u(w_t - s_t^*, \rho_t) + \beta \delta E_t V(y_{t+1} + R(s_t^*) + a_1)$$

or

$$\frac{u(w_t - s_t^* + a_0, \rho_t) - u(w_t - s_t^*, \rho_t)}{a_0} > \frac{a_1 \beta \delta [E_t V(y_{t+1} + R(s_t^*) + a_1) - E_t V(y_{t+1} + R(s_t^*))]}{a_1}$$

(10)

The fractions on the left and right converge to the derivatives of $u$ and $\beta \delta E_t V$, respectively, as $a_0$ and $a_1$ converge to zero. Now take any sequence of payments such that both transfers converge to zero, while their ratio stays constant at $\hat{R}$. Since equation 3 holds in the limit, and using the fact that $V'(w_{t+1}^*) = d_{t+1} u'(c(w_{t+1}, \rho_{t+1}), \rho_{t+1})$ (see Harris and Laibson [2001]), there must be some nonzero lower bound $a_0$ (and associated $a_1$) where inequality (10) also holds. Finally, using equation (2) to equate MRS with the derivative of $R$ gives the desired result.

A similar argument shows that $a_1$ is preferred if

$$\frac{u'(c_t^*, \rho_t)}{\beta \delta V'(w_{t+1})} < \frac{a_1}{a_0}.$$ 

Case 2: Suppose the decision maker can re-optimize their savings choice after receiving $a_0$. Preferences between monetary payments can then be established via an arbitrage argument. Consider a decision maker that would like to change their consumption in $t$ by $c(a_1)$ if promised $a_1$ in $t + 1$. Wealth in period $t + 1$ will then be $R(s_t^* - c(a_1)) + a_1$. Now if the decision maker received $a_0$ instead, one possible strategy would be to change consumption by the same amount $c(a_1)$ and have $R(s_t^* - c(a_1) + a_0)$ in $t + 1$. These two strategies now provide exactly the same consumption in period $t$, and differ only in the amount of money available in period $t + 1$. Thus, receiving $a_0$ must be strictly preferred to $a_1$ if

$$R(s_t^* - c(a_1) + a_0) > R(s_t^* - c(a_1)) + a_1$$

or

$$\frac{R(s_t^* - c(a_1) + a_0) - R(s_t^* - c(a_1))}{a_0} > \frac{a_1}{a_0}$$

(11)

Constructing a sequence as above and noting both that the fraction on the left converges to the derivative of $R$ and that equation (3) holds in the limit, there must be a non-zero $a_0$. 

51
below which inequality (11) holds and the earlier payment is preferred. Parallel arguments show that the later payment is strictly preferred if \( R'(s_t) < \frac{a_t}{a_0} \).

**Proof of Proposition 2:** Consider a decision maker in period \( t \) who is given a surprise choice between \( b_1 \) in \( t + 1 \) and \( b_2 \) in \( t + 2 \). We will proceed by first approximating the value to the decision maker of receiving \( b_2 \) in period \( t + 2 \) with the value of receiving an appropriately discounted amount in period \( t + 1 \) upon learning \( w_{t+1} \), which we call \( t(b_2 : w_{t+1}) \). We will then examine the “approximation error” that results from replacing \( b_2 \) with this transfer. As we will show, this “error” is equal to zero unless the decision maker is time inconsistent and the interest rate varies with savings. In other cases we can determine the direction of this error as long as the decision maker is not too present biased.

Let \( s(b_2 : w) \) be the savings rate in period \( t + 1 \) if the decision maker has wealth \( w \) in that period and anticipates receiving a transfer of \( b_2 \) in period \( t + 2 \). We define the equivalent transfer to \( b_2 \) as

\[
t(b_2 : w) = R^{-1}(R(s(b_2 : w)) + b_2) - s(b_2 : w)
\]

Note that this is exactly the discounted value of \( b_2 \) in period \( t + 1 \) if the interest rate is constant.

The decision maker at time \( t \) will prefer the earlier payment \( b_1 \) if

\[
\beta \delta E_t V(w_{t+1} + b_1) > \beta \delta E_t V(w_{t+1} + t(b_2 : w_{t+1})) + E_t O(b_2 : w_{t+1}), \quad (12)
\]

where \( O \) is the approximation error. If we let \( W(\cdot) \) equal the value of wealth in period \( t + 2 \) from the perspective of the period \( t \) self, we can write

\[
O(b_2 : w_{t+1}) = \beta \delta u (w_{t+1} - s(b_2 : w_{t+1}), \rho_{t+1}) + \beta \delta^2 E_{t+1} W(y_{t+2} + R(s(b_2 : w_{t+1})) + b_2) - \beta \delta u (w_{t+1} + t(b_2 : w_{t+1}) - s(0 : t(b_2 : w_{t+1}) + w_{t+1}), \rho_{t+1}) - \beta \delta^2 E_{t+1} W(y_{t+2} + R(s(0 : t(b_2 : w_{t+1}) + w_{t+1})))
\]

where \( s(0 : t(b_2 : w_{t+1}) + w_{t+1}) \) is savings in period \( t + 1 \) with wealth \( w_{t+1} \) and transfer \( t(b_2 : w_{t+1}) \) in \( t + 1 \) but no transfer in period \( t + 2 \). We will return to the nature of \( O \) below. For now, note that this implies that the earlier payment will be preferred to the latter payment if

\[
\beta \delta E_t \frac{V(w_{t+1} + b_1) - V(w_{t+1})}{b_1} > \frac{b_2}{b_1} \left[ \beta \delta E_t \frac{V(w_{t+1} + t(b_2 : w_{t+1})) - V(w_{t+1})}{b_2} + E_t O(b_2 : w_{t+1}) \right].
\]
Note that
\[
\lim_{b_2 \to 0} \frac{t(b_2 : w)}{b_2} = \lim_{b_2 \to 0} \frac{R^{-1}(R(s(b_2 : w)) + b_2) - s(b_2 : w)}{b_2} = \frac{1}{R'(s(0 : w))}
\]
and so, holding the ratio of payments \(\frac{b_2}{b_1}\) constant, letting \(b_1 \to 0\) gives:

\[
\beta \delta E_t V'(w_{t+1}) > \frac{b_2}{b_1} \left[ \beta \delta E_t \left( \frac{V'(w_{t+1})}{R'(s(w_{t+1}))} \right) + E_t O_b(b_2 : w_{t+1}) \right]
\]

where \(O_b(b_2 : w_{t+1}) = \lim_{b_2 \to 0} \frac{O(b_2 : w_{t+1})}{b_2}\) and we use \(s(w_{t+1})\) as a shorthand for \(s(0 : w_{t+1})\).

Using that \(\beta \delta V'(w_{t+1}) = d_{t+1}u'(c_{t+1})\) and \(R'(s(w_{t+1})) = MRS_{t+1} = \frac{w'(c_{t+2}, \rho_{t+2})}{d_{t+2}w'(c_{t+2}, \rho_{t+2})}\) gives

\[
\frac{E_t \left[ d_{t+1}u'(c_{t+1}, \rho_{t+1}) \right]}{E_t \left[ d_{t+1}d_{t+2}u'(c_{t+2}, \rho_{t+2}) + O_b(b_2 : w_{t+1}) \right]} > \frac{b_2}{b_1}.
\]

Using equivalent arguments to those deployed in the proof of Proposition 1, this is enough to conclude the existence of a lower bound \(\bar{b}_1\) such that for all pairs of payments \(b_1 < \bar{b}_1, b_2 = b_1 \hat{R}\), the earlier payment will be preferred if \(\hat{R}\) is below the left hand side of the above inequality. An equivalent argument shows that the decision maker prefers \(b_2\) if the opposite inequality holds.

We now discuss the nature of \(O_b(b_2 : w_{t+1})\) (for convenience we will suppress the preference parameter \(\rho\)). In order to do so we add and subtract to \(O(b_2 : w)\) the value of the status quo, i.e. receiving no transfers:

\[
\beta \delta u(w - s(0 : w)) + \beta \delta^2 E_{t+1} W(y + R(s(0 : w))).
\]

Doing so allows us to write

\[
\frac{O(b_2 : w)}{b_2} = \beta \delta u(w - s(b_2 : w)) - u(w - s(0 : w))
\]

\[
-\beta \delta \frac{u(w + t(b_2 : w)) - s(0 : w + t(b_2 : w))) - u(w - s(0 : w))}{b_2}
\]

\[
+ \beta \delta^2 E_{t+1} W(y_{t+2} + R(s(b_2 : w)) + b_2) - E_{t+1} W(y + R(s(0 : w)))
\]

\[
- \beta \delta^2 E_{t+1} W(y_{t+2} + R(s(0 : w + t(b_2 : w))) - E_{t+1} W(y + R(s(0 : w)))
\]

\[
- \beta \delta^2 E_{t+1} W(y_{t+2} + R(s(0 : w + t(b_2 : w))) - E_{t+1} W(y + R(s(0 : w)))
\]

53
Taking the limit as $b_2$ goes to zero gives

$$
\beta \delta \frac{u'(c(w))}{R'(s(0 : w))} \left[ -1 - R'(s(0 : w))s_b(0 : w) + s_w(0 : w) \right] \\
+ \beta \delta^2 E_{t+1} W'(y_{t+2} + R(s(0 : w))) \left[ 1 + R'(s(0 : w))s_b(0 : w) - s_w(0 : w) \right]
$$

where $s_w$ and $s_b$ are respectively the marginal propensity to save from current income and the future transfer $b$. This means that

$$
O_b(b_2 : w_{t+1}) = \left[ \beta \delta^2 E_{t+1} W'(y + R(s(0 : w))) - \frac{\beta \delta u'(c(w))}{R'(s(0 : w))} \right] \left[ 1 + R'(s(0 : w))s_b(0 : w) - s_w(0 : w) \right]
$$

The approximation error will therefore be zero if either of the two bracketed terms is equal to zero. If $\beta = 1$, the agent is dynamically consistent, and $W$ will be equal to the value of money in period $t + 2$ from the perspective of $t + 1$. Thus, the Euler equation for the $t + 1$ agent ensures that the first term is equal to zero.

If $\beta < 1$, we can explore the nature of the error term by deriving expressions for $s_b$ and $s_w$ using the SHEE. Note that the SHEE must continue to hold in response to infinitesimal changes in income in the current period. The derivatives of the left and right hand side of the SHEE in response to a change in $w$ must be equal, and so (suppressing the transfer term in savings $s(0 : w) = s(w)$)

$$
(1 - s_w(w)) u''(c(w)) = s_w(w) R''(s(w)) E [d(w') u'(c(w'))] \\
+ s_w(w) [R'(s(w))]^2 E [d'(w') u'(c(w')) + d(w') c'(w') u''(c(w))]
$$

where $w' = y + R(s(w))$. This implies

$$
s_w(w) = \frac{u''(c(w))}{u''(c(w)) + R''(s(w)) E [d(w') u'(c(w'))] + [R'(s(w))]{E}[d'(w') u'(c(w')) + d(w') c'(w') u''(c(w'))]}
$$

Similarly, taking derivatives with respect to an anticipated increase in $y$ (or equivalently, $b_2$) in all states of the world gives

$$
-s_b(w) u''(c(w)) = s_b(w) R''(s(w)) E [d(w') u'(c(w'))] \\
+ (R'(s(w))s_b(w) + 1) R'(s(w)) E [d'(w') u'(c(w')) + d(w') c'(w') u''(c(w'))]$$
We show that savings realizations are patient than without the bias. This will be the case if

\[ \text{where} \]

We begin by proving the two predictions in section 1.5 that are not proved in the text.

\[ B \text{ Proofs for the Claims in Section 1.5} \]

The key term for the evaluation of \( O_b(b_2 : w_{t+1}) \) is

\[ 1 + R'(s(w))s_b(w) - s_w(w). \]

Plugging in the above expressions gives

\[ 1 - \frac{[R'(s(w))]^2E[d'(w')u'(c'(w')) + d(w')c'(w')]}{R''(s(w))E[d'(w')u'(c'(w')) + d(w')c'(w')]} \]

\[ \text{Notice first that} \]

\[ R''(s(w)) = 0 \text{ when there are no credit constraints, so that} \]

\[ O_b(b_2 : w_{t+1}) = 0. \]

Thus, the approximation error is equal to zero if either there is no present bias, or if interest rates are not a function of savings \( s \).

Outside these cases, it is possible to sign the term \( O_b(b_2 : w_{t+1}) \) under mild conditions. Note that \( R''(s(w)) \) and \( u''(c(w')) \) are negative, and \( d(w'), c'(w') \), and \( u'(c(w')) \) are positive, so as long as \( d'(w')u'(c'(w')) \) is small in magnitude, the numerator and denominator will both be negative, and the whole fraction will be less than 1. Thus the expression (14) is positive and less than 1.

Next, consider the term \( \beta \delta^2 E_{t+1}W'(y + R(s(w))) - \frac{\beta \delta^2 u'(c(w))}{R'(s(w))} \). Recall that if the agent were time consistent, this would equal zero. A present-biased decision maker will consume more in period \( t + 1 \) than is desirable from the perspective of the agent in period \( t \), meaning that the marginal utility of consumption will be lower, and so this expression will be positive. Thus it must be the case that \( O_b(b_2 : w_{t+1}) > 0 \). Looking at the expression in 13, the presence of a positive \( O_b(b_2 : w_{t+1}) \) term will reduce the left-hand side, meaning that it will reduce the break-even value of \( \frac{b_2}{b_1} \) — in other words, it will make Decision B look more patient than without the bias.

\[ \square \]

B Proofs for the Claims in Section 1.5

We begin by proving the two predictions in section 1.5 that are not proved in the text.

Prediction (Income shocks and MRS): Consider a decision maker who holds savings from the previous period \( s_{t-1} \) and has preference parameter \( \rho_t \). For any two possible income realizations \( y_t, y'_t \) and associated \( MRS_t, MRS'_t \), \( y_t > y'_t \) implies \( MRS_t < MRS'_t \).

Proof. We show that savings \( s_t \) must increase with with \( y_t \) and thus reduce \( R'(s_t) \). Con-
Consider two optimal savings levels \( s \) and \( s' \) at wealth \( w_t > w'_t \). It must be the case that

\[
\begin{align*}
    u(w_t - s, \rho) + \beta \delta EV(R(s) + y_{t+1}) & \geq u(w_t - s', \rho) + \beta \delta EV(R(s') + y_{t+1}) \\
    u(w_t - s, \rho) - u(w_t - s', \rho) & \geq \beta \delta EV(R(s') + y_{t+1}) - \beta \delta EV(R(s) + y_{t+1})
\end{align*}
\]

If \( s \leq s' \), the left hand side is positive: it is the increase in instantaneous utility from reducing savings from \( s' \) to \( s \). Moreover, by the concavity of \( u \) we have

\[
\begin{align*}
    u(w'_t - s, \rho) - u(w'_t - s', \rho) & > u(w_t - s, \rho) - u(w_t - s', \rho) \\
    \Rightarrow u(w'_t - s, \rho) + \beta \delta EV(R(s) + y_{t+1}) & > u(w'_t - s', \rho) + \beta \delta EV(R(s') + y_{t+1})
\end{align*}
\]

contradicting the optimality of \( s' \) for \( w'_t \). Thus, \( s > s' \) and \( R'(s) < R'(s') \). \( \square \)

**Prediction (Preference shocks and MRS):** Consider a decision maker with cash on hand \( w_t \). For any two realizations of the preference shock \( \rho_t, \rho'_t \) and the associated \( MRS_t, MRS'_t \), \( \rho_t < \rho'_t \) (and therefore \( \frac{\partial u(c, \rho)}{\partial c} < \frac{\partial u(c, \rho')}{\partial c} \) for all \( c \)) implies \( MRS_t < MRS'_t \).

**Proof.** Assume that preference parameters \( \rho \) and \( \rho' \) are such that \( \frac{\partial u(c, \rho)}{\partial c} < \frac{\partial u(c, \rho')}{\partial c} \) for all \( c \). Let \( s \) and \( s' \) be the optimal savings rates for the two parameters respectively (for given \( w_t \)). If \( s \leq s' \) (i.e. the consumer saves more when current-period marginal utility is higher) it must be the case that

\[
\begin{align*}
    u(w_t - s, \rho') - u(w_t - s', \rho') & > u(w_t - s, \rho) - u(w_t - s', \rho) \\
    \geq \beta \delta EV(R(s') + y_{t+1}) - \beta \delta EV(R(s) + y_{t+1}) \\
    \Rightarrow u(w_t - s, \rho') + \beta \delta EV(R(s) + y_{t+1}) & > u(w_t - s', \rho') + \beta \delta EV(R(s') + y_{t+1}),
\end{align*}
\]

a contradiction. Thus, \( s > s' \) and \( R'(s) < R'(s') \). \( \square \)

We next discuss the conditions under which these predictions mean that MRS will exhibit covariance with the financial variable of interest of the appropriate sign. In each case, the object of interest is the expectation of a sample covariance between a variable \( x \) (which could be income shocks, preference shocks, spending, etc.) and MRS, calculated from a \( T \) length sample. That is, we consider

\[
E \left( T^{-1} \sum_{t=1}^{T} (MRS_t - \overline{MRS}) (x_t - \overline{x}) \right),
\]

where \( \overline{MRS} = T^{-1} \sum_{t=1}^{T} MRS_t \) and \( \overline{x} = T^{-1} \sum_{t=1}^{T} x_t \).
We will show the following: if \( x_t \) is positively (negatively) related to \( MRS_t \) for any (fixed) realization of the other state variables,\(^{53}\) as in the propositions above, we can conclude that the related sample covariance will be weakly positive (negative) in expectation, provided that

1. the distribution for the initial savings stock \( s_0 \) and the parameterization of the model described in section 1.1 is such that, at any subsequent time \( t \) the distributions of MRS and \( x \) conditional on the remaining state variables have finite second moments; and

2. the magnitude of the covariance between \( MRS_t \) and \( x_t \) is larger than between \( MRS_t \) and \( x_s \) for any \( s \neq t \), that is

\[
|E \left[ (MRS_t - E(MRS)) \left( x_t - E(x) \right) \right] | \geq \\
|E \left[ (MRS_t - E(MRS)) \left( x_s - E(x) \right) \right] |
\]

for any \( t > 0 \) and \( s \neq t \).

Whether or not this second condition holds will of course depend on the underlying parameters of the model. However, this formulation has an intuitive interpretation - financial variables must be more strongly related to MRS in the concurrent period than in future or lagged periods. This holds true for the simulations we run on various parameterized versions of the model in appendix C.

Note that this covariance is zero for \( s > t \) if income and preference shocks are iid, so this assumption requires only that MRS is more strongly related to current than to lagged values of these variables.

First, note that

\[
E \left( T^{-1} \sum_{t=1}^{T} (MRS_t - MRS) \left( x_t - \bar{x} \right) \right) = \\
E \left( T^{-1} \sum_{t=1}^{T} MRS_t \left( x_t - \bar{x} \right) \right) - E \left( T^{-1} \sum_{t=1}^{T} MRS \left( x_t - \bar{x} \right) \right) = \\
E \left( T^{-1} \sum_{t=1}^{T} MRS_t \left( x_t - \bar{x} \right) \right),
\]

so we can focus on the expression in which only \( x \) is centered.

---

\(^{53}\)The state variables are the preference shock, the income shock, and last-period savings.
We will assume that MRS and $x$ are negatively related and show that the sample covariance is negative in expectation. The argument for a positive relation and positive covariance is entirely analogous.

Fix some $1 \leq \tau \leq T$. In order to prove the result, it is sufficient to show that, for any such $\tau$

$$E[MRS_\tau (x_\tau - \bar{x})] \leq 0$$

as the sample covariance is simply the sum of such expectations across all $\tau$.

Note that

$$E[MRS_\tau (x_\tau - \bar{x})] = E \left[ MRS_\tau \left( x_\tau - T^{-1} \sum_{t=1}^{T} x_t \right) \right]$$

$$= \frac{1}{T} \sum_{t=1}^{T} E[MRS_\tau (x_\tau - x_t)]$$

It is enough therefore to show that $E[MRS_\tau (x_\tau - x_t)]$ is weakly negative for every $\tau$. If $t = \tau$ then we have $E[MRS_\tau (y_\tau - y_\tau)] = 0$. Otherwise, add and subtract $E(x)$:

$$E[MRS_\tau (x_\tau - x_t)] = E[MRS_\tau (x_\tau - E(x))] - E[MRS_\tau (x_t - E(x))]$$

Is the expectation of $x_\tau$. The first term is the covariance of $MRS_\tau$ and $x_\tau$. Using the law of iterated expectations we have

$$E[MRS_\tau (x_\tau - E(x))] = E[E(MRS_\tau (x_\tau - E(x)) \mid z_\tau)]$$

where $z_\tau$ is the value of the remaining state variables at time $\tau$. By assumption, we know that for any $z_\tau$, $MRS_\tau$ is a monotonically decreasing function of $x_\tau$. This is sufficient to guarantee that the conditional covariance is weakly negative under assumption (1) of finite second moments for MRS and $x$ (see for example Schmidt [2003]). The unconditional covariance, as the expectation of weakly negative numbers, must also be weakly negative.

The second term is the covariance between $MRS_\tau$ and $x_t$ for $t \neq \tau$ which, by assumption (2), is smaller in magnitude than the first term. Thus the difference between the two must also be weakly negative.
C  Model Extensions

C.1 Endogenous Labor Supply

While some income variation is exogenous and beyond the control of the household, the household can likely also endogenously change their income in response to shocks, most obviously by adjusting the time or effort spent on working.

In order to capture endogenous labor supply, we extend our model in the following way. In addition to non-labor income $y_t$, in each period the consumer has one unit of leisure, any fraction of which she can either consume (as leisure $l_t$) or sell at wage rate $W$. Utility is now given by

$$u(c_0, \rho_0, l_0) + \beta E_0 \sum_{t=1}^{\infty} \delta^t u(c_t, \rho_t, l_t),$$

and current-period resources are $y_t + W(1-l_t) + R(s_{t-1})$.

Define the instantaneous function $v(e_t, \rho_t)$ to be the utility of net spending $e_t$, defined as any expenditure above labor earnings $W(1-l_t)$ in period $t$, assuming that the leisure-labor trade-off has been made optimally, i.e.

$$v(e_t, \rho_t) = \max_{c_t, l_t} u(c_t, \rho_t, l_t)$$

subject to

$$c_t - W(1-l_t) = e_t.$$

We can think of this as the instantaneous utility obtained if the decision maker adjusts their savings by the amount $e_t$ in period $t$, and therefore rewrite the dynamic problem purely in terms of choice of $e_t$:

$$v(e_0, \rho_0) + \beta E_0 \sum_{t=1}^{\infty} \delta^t v(e_t, \rho_t)$$

such that

$$e_t = w_t - s_t$$
$$w_t = y_t + R(s_{t-1})$$
$$w_0$$ given.

We showed in section 1.5 that exogenous income shocks (i.e. changes in $y_t$) are negatively related to $MRS_t$. The argument holds equivalently if the function $v$ is concave in its first
argument. Conditions on the utility function $u$ which guarantee that this is the case are well known: for example that $u$ is concave in both $c$ and $l$, and that consumption and leisure are both normal goods (see Bordley [1995]). Similarly, the result pertaining to the relationship between preference shocks and MRS goes through if we replace the original assumption, that an increase in $\rho_t$ everywhere increases the derivative of $u$ with respect to $c$, with the assumption that it everywhere increases the derivative of $v$ with respect to $e$. Finally, savings remain negatively related to MRS by identity.

By contrast, endogenous increases in income (i.e. in $W(1-l_t)$) will be positively related to MRS. For example, a preference shock that increases the marginal utility from consumption will lead to an increase in endogenous (labor) income as well as a decrease in savings and higher measured MRS. Similarly, households will partly compensate an exogenous drop in income by increasing their labor supply. This implies that sources of income which are more under the household’s control will be less negatively or even positively related to MRS. Moreover, the endogenous income response has an attenuating effect on the relationship between $MRS_t$ and total income, but also between $MRS_t$ and preference shocks. We will discuss these issues more in section 3.

C.2 Serially Correlated Income and Preference Shocks

Our baseline model assumes that income and preference shocks are independent over time. There are, however, circumstances in which this may be unrealistic, especially for income – for example, poor business conditions in one week may be predictive of bad conditions in the following week, lowering income today as well as expected income tomorrow.

Do our results hold up in the case of correlated shocks? For the relationship between savings and MRS the answer is yes, as the two are linked by an identity. For the relationship between shocks and MRS, the answer depends on the degree of correlation. Consider first the question of whether income shocks are negatively related to MRS. In the case of the partial constraints model, this boils down to the question of whether the marginal propensity to consume from the shock is less than one: if so, then a positive income shock will lead to increased savings, and so a fall in MRS. If not, then the shock will lead to a decrease in savings and so an increase in MRS, the opposite of our prediction.

The question of when correlated income shocks lead to an increase in savings has been studied in macroeconomics (see for example Uribe and Schmitt-Grohé [2017]). Broadly speaking, the answer is that as long as the shock raises current income more than permanent income, savings will increase, because the decision maker wishes to move resources into the future. Thus, if an AR(1) income process has a parameter of less than one, a

---

54 Assuming that changes are driven by shocks to income and preferences, and not shocks to the wage rate.
positive shock will increase savings, as current income increases more than lifetime income. However, if the parameter is greater than one, a shock will increase permanent income more than present income, and the decision maker will wish to reduce savings. In the case of quadratic utility, exponential discounting and constant interest rates, this result can been readily established analytically (see Uribe and Schmitt-Grohé [2017] chapter 2). Maliar and Maliar [2004] show that the same result holds in the special case of quasi-hyperbolic discounting and exponential utility with no credit constraints.

Outside these simple cases, and in particular in the presence of partial credit constraints, analytical results can no longer be obtained.\textsuperscript{55} We therefore numerically simulate our more complex model to show that the same intuition hold. These simulation results are based on the following parameterization of the model. Utility is assumed to be of the Constant Relative Risk Aversion (CRRA) form. We abstract from preference shocks:

$$u(c, \rho) = u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}$$

Log income is assumed to follow an autoregressive process.

$$\ln(y_t) = \mu \ln(y_{t-1}) + \epsilon_t$$

Where $\epsilon_t$ is distributed independently from a normal distribution with standard deviation $\sigma$. We use an AR(1) process in logged income in order to ensure that income is positive. The interest rate function is given by

$$R(s) = a - ae^{-bs}.$$  

$R(s)$ is concave, with $R(0) = 0$ and a savings ceiling $a$ (note that $R(s) \to a$ as $s \to \infty$). $b$ determines the curvature of $R$.

This means that the model has seven parameters: $\gamma$ (curvature of the utility function), $\mu$ (persistence of income shocks), $\sigma$ (standard deviation of income shocks), $a$ and $b$ (curvature of the interest rate function), $\beta$ (present bias) and $\delta$ (discount rate).

This model can be solved numerically using Euler equation iteration. There are two state variables: income $y$ and savings $s$. The numeric approach approximates the optimal policy (i.e. consumption) on a discrete grid of values for the two state variables. This in turn allows us to approximate the marginal propensity to consume (MPC) from income shocks at each point on this grid.

\textsuperscript{55} Uribe and Schmitt-Grohé [2017] (chapter 4) use numerical simulations to derive similar results for the case of upward-sloping interest rates and capital accumulation, for example.
In Table 10, we report the maximal MPC across all grid points for various different parameterizations of the model. The key question is whether the MPC is always less than one: if so, then savings will rise with increases in income, as they do in the iid case, meaning that income and MRS are negatively related. Each row varies the degree of income autocorrelation, given by \( \mu \). Each column reports results for different configurations for the other six variables, as described below:

- Baseline: \( \gamma = 2, \sigma = 0.01, a = 1.05, b = 0.98, \beta = 0.97, \delta = 0.99 \)
- High risk aversion: As Baseline, apart from \( \gamma = 4 \)
- High income variation: As Baseline, apart from \( \sigma = 0.04 \)
- High credit constraints: As Baseline, apart from \( a = 0.94 \) and \( b = 1.06 \)
- High present bias: As Baseline, apart from \( \beta = 0.95 \)
- High discounting: As Baseline, apart from \( \delta = 0.9 \)

The final row reports the average correlation between income and savings across 100 repetitions of 20-period samples, drawn from the stationary distribution of the model under each configuration. In all cases the resulting correlation is strongly positive.

Table 10: Maximal MPC and simulated correlation of income and savings from parameterized model.

<table>
<thead>
<tr>
<th>Inc. pers. ( \mu )</th>
<th>Baseline</th>
<th>Risk aversion ( \gamma = 4 )</th>
<th>Variance ( \sigma = 0.04 )</th>
<th>Fin. constraints ( a = 0.94, b = 1.06 )</th>
<th>Present bias ( \beta = 0.95 )</th>
<th>Discounting ( \delta = 0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65</td>
<td>0.78</td>
<td>0.67</td>
<td>0.78</td>
<td>0.79</td>
<td>0.80</td>
<td>0.85</td>
</tr>
<tr>
<td>0.70</td>
<td>0.81</td>
<td>0.70</td>
<td>0.80</td>
<td>0.82</td>
<td>0.83</td>
<td>0.88</td>
</tr>
<tr>
<td>0.75</td>
<td>0.84</td>
<td>0.74</td>
<td>0.83</td>
<td>0.84</td>
<td>0.85</td>
<td>0.91</td>
</tr>
<tr>
<td>0.80</td>
<td>0.87</td>
<td>0.78</td>
<td>0.86</td>
<td>0.87</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>0.85</td>
<td>0.90</td>
<td>0.83</td>
<td>0.89</td>
<td>0.91</td>
<td>0.92</td>
<td>0.98</td>
</tr>
<tr>
<td>0.90</td>
<td>0.94</td>
<td>0.88</td>
<td>0.93</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>0.95</td>
<td>0.98</td>
<td>0.95</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Maximal MPC in simulations
Average estimated correlation between income and savings (100 simulations)

| 0.95 | 0.55 | 0.58 | 0.58 | 0.50 | 0.48 | 0.55 |

Baseline: \( \gamma = 2, \sigma = 0.01, a = 1.05, b = 0.98, \beta = 0.97, \delta = 0.99 \). \( \mu \) refers to the persistence of income shocks. Each column represents a different parameterization of the model as described in the text.

Our results show that, for a wide range of parameterizations, the marginal propensity to consume is less than one (meaning that savings increase as a result of income shocks) and that the average sample correlation between income and saving is positive.
C.3 Shocks to the Returns Function \( R_i \)

So far we have assumed that the interest rate faced by the household is determined only by the level of savings through \( R \). However, interest rates may vary for other reasons, for example if investment opportunities available to the household change over time.

Shocks to \( R \) that are observed by the household introduce a new force that can lead to a relationship between savings and MRS. To see this, consider the no-constraints version of the model with a period-specific interest rate, in which

\[
MRS_t \equiv \frac{u'(c(w_t, \rho_t), \rho_t)}{E_t [d_{t+1}u'(c(w_{t+1}, \rho_{t+1}), \rho_{t+1})]} = (1 + r_t).
\]

Here, (for example) a positive shock to the interest rate in period \( t \) will lead to an increase in MRS. In order to determine the resulting change in savings, we can take the derivative of \( (1 + r_t)E_t [d_{t+1}u'(c(w_{t+1}, \rho_{t+1}), \rho_{t+1})] \) – i.e. the marginal value of savings – with respect to a change in \( r \) (holding \( s_t \) constant). If this derivative is positive (negative), then \( u'(c(w_t, \rho_t), \rho_t) \) must rise (fall) in order to maintain the Euler equation, meaning that consumption must fall (rise) and savings rise (fall) as a result of the change. The relevant derivative is

\[
E_t [d_{t+1}u'(c_{t+1}, \rho_{t+1})] + (1 + r_t)s_tE_t \left[d_{t+1}u''(c_{t+1}, \rho_{t+1}) \frac{\partial c_{t+1}}{\partial w_{t+1}} + \frac{\delta d_{t+1}}{\delta w_{t+1}} u'(c_{t+1}, \rho_{t+1}) \right].
\]

The first expectations term captures the mechanical effect of the increase in the interest rate on the marginal value of savings (the substitution effect of making saving more attractive than immediate consumption), and it is always positive. The second term captures the impact of the change in wealth caused by higher interest rates, first through the impact on future marginal utility (the income effect), and second through the change in the discount factor in the quasi-hyperbolic model (where \( \beta < 1 \)). With exponential discounting, the second expectation will be negative, as \( \frac{\delta d_{t+1}}{\delta w_{t+1}} \) equals zero, the discount rate and marginal propensity to consume are positive, and the utility function is concave. Thus, if the household is in debt and savings are negative, savings and measured MRS will be positively related; both the substitution and income effect drive the household to reduce current consumption. If savings are positive, then the relation may be positive or negative, depending on whether the income or substitution effect dominates. Higher savings tend to make a negative relationship more likely. With quasi-hyperbolic discounting, the wealth effect depends in addition on the effect of wealth on the marginal propensity to consume and therefore the discount factor.
D  Distribution of Experimental Choices and Income

Table 11: Weekly distribution of experimentally elicited MRS in decision A.

<table>
<thead>
<tr>
<th>Implied MRS at interval midpoint</th>
<th>week 1</th>
<th>week 2</th>
<th>week 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Percent</td>
<td>Number</td>
</tr>
<tr>
<td>0.71</td>
<td>53</td>
<td>6.4%</td>
<td>38</td>
</tr>
<tr>
<td>0.80</td>
<td>5</td>
<td>0.6%</td>
<td>1</td>
</tr>
<tr>
<td>0.93</td>
<td>22</td>
<td>2.7%</td>
<td>25</td>
</tr>
<tr>
<td>1.1</td>
<td>122</td>
<td>14.7%</td>
<td>122</td>
</tr>
<tr>
<td>1.35</td>
<td>61</td>
<td>7.4%</td>
<td>74</td>
</tr>
<tr>
<td>1.75</td>
<td>58</td>
<td>7.0%</td>
<td>93</td>
</tr>
<tr>
<td>2.5</td>
<td>53</td>
<td>6.4%</td>
<td>56</td>
</tr>
<tr>
<td>4.5</td>
<td>54</td>
<td>6.5%</td>
<td>47</td>
</tr>
<tr>
<td>8</td>
<td>402</td>
<td>48.4%</td>
<td>415</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>830</td>
<td>100</td>
<td>871</td>
</tr>
</tbody>
</table>

Distribution of MRS in decision A, ratio of earlier to later payment at interval midpoint. Assuming value 0.708 for individuals who always choose the late payment and 8 for those who always choose the earlier payment. Inconsistent choices excluded.

Figure 2: Distribution of different types of income by source, excluding person-weeks with zero income in each category. Labor income has the most non-zero observations, followed by exogenous non-labor income. The overall amounts of non-labor income tend to be smaller.

E  Additional Regression Results and Robustness Checks

E.1 Additional Results

Table 12 reports the first stage of the instrumental variables regression in table 7.
Table 13: MRS change over time as a function of (flow) savings, third-order Taylor expansion.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings (I-E)</td>
<td>-0.231</td>
<td>-0.228</td>
<td>-0.593</td>
<td>-0.592</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td>(0.135)</td>
<td>(0.328)</td>
<td>(0.330)</td>
</tr>
<tr>
<td>1/2 Savings^2</td>
<td>-0.0240</td>
<td>-0.0234</td>
<td>-0.0861</td>
<td>-0.0860</td>
</tr>
<tr>
<td></td>
<td>(0.0341)</td>
<td>(0.0344)</td>
<td>(0.134)</td>
<td>(0.134)</td>
</tr>
<tr>
<td>1/3 Savings^3</td>
<td>0.00705</td>
<td>0.00695</td>
<td>0.0317</td>
<td>0.0317</td>
</tr>
<tr>
<td></td>
<td>(0.00819)</td>
<td>(0.00822)</td>
<td>(0.0286)</td>
<td>(0.0287)</td>
</tr>
<tr>
<td>Time FE</td>
<td>yes</td>
<td></td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Ind FE</td>
<td>yes</td>
<td></td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
<td>1462</td>
</tr>
</tbody>
</table>

Standard errors clustered at the individual level (in parentheses).
Significance levels + p<0.10, * p<0.05, ** p<0.01, *** p<0.001.

Table 12: Instrumental variables regression with individual fixed effects (left) and individual and time fixed effects (right). First and third columns reproduce results from table 7, second and fourth columns show first stage regressions.

<table>
<thead>
<tr>
<th></th>
<th>Individual FE</th>
<th>Individual and time FE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MRS (A)</td>
<td>Adv. event expenses</td>
</tr>
<tr>
<td>Adv. event (0/1)</td>
<td>0.161 ***</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Adv. event expenses</td>
<td>1.707 *</td>
<td>(0.789)</td>
</tr>
<tr>
<td>Observations</td>
<td>2467</td>
<td>2467</td>
</tr>
</tbody>
</table>

Standard errors clustered at the individual level (in parentheses).
Significance levels * p<0.05, ** p<0.01, *** p<0.001.

Table 13 repeats the regressions from Table 8, but includes a cubed savings term. The first-order effect of savings is slightly stronger than with only squared savings included.

E.2 Robustness Checks

Table 14 shows MRS regressed on all income and spending components. The two columns titled “IV” again instrument for adverse event spending.

Exogenous income is significantly and negatively correlated with the MRS elicited from decision A (MRS_t) in every specification. Adverse event spending is positively and significantly correlated with MRS, except in the OLS regressions where ‘other spending’ is included. This specification does not change the coefficient sizes, but does lead to somewhat larger standard errors, possibly due to collinearity. Event spending remains significantly related to MRS in the IV specification, even when other spending is included.
Table 14: Income and spending effects on $MRS_t$.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
<th>OLS</th>
<th>IV</th>
<th>IV</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor income</td>
<td>-0.185</td>
<td>-0.189</td>
<td>-0.153</td>
<td>-0.159</td>
<td>-0.262</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.142)</td>
<td>(0.143)</td>
<td>(0.163)</td>
<td>(0.142)</td>
<td>(0.136)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlabor income</td>
<td>-0.330</td>
<td>-0.321</td>
<td>-0.268</td>
<td>-0.265</td>
<td>-0.316</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;endogenous&quot;</td>
<td>(0.251)</td>
<td>(0.258)</td>
<td>(0.261)</td>
<td>(0.270)</td>
<td>(0.282)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonlabor income</td>
<td>-0.409 * -0.409 *</td>
<td>-0.382 ** -0.384 **</td>
<td>-0.378 ** -0.380 **</td>
<td>-0.379 *</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;exogenous&quot;</td>
<td>(0.142)</td>
<td>(0.149)</td>
<td>(0.125)</td>
<td>(0.133)</td>
<td>(0.128)</td>
<td>(0.134)</td>
<td>(0.171)</td>
</tr>
<tr>
<td>Other spending</td>
<td>0.268 * 0.245 + 0.192 0.177 0.215 +</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td>(0.131)</td>
<td>(0.141)</td>
<td>(0.132)</td>
<td>(0.119)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adv. event expense</td>
<td>0.252 + 0.233 + 0.251 0.222 1.683 + 1.562 * 0.390 *</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.145)</td>
<td>(0.139)</td>
<td>(0.182)</td>
<td>(0.183)</td>
<td>(0.865)</td>
<td>(0.873)</td>
<td>(0.199)</td>
</tr>
<tr>
<td>1/(error SD)</td>
<td>- - - - - - 0.916 **</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.044)</td>
<td></td>
</tr>
<tr>
<td>Ind FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2540</td>
<td>2540</td>
<td>2390</td>
<td>2390</td>
<td>2289</td>
<td>2289</td>
<td>12608</td>
</tr>
</tbody>
</table>

Standard errors clustered at the individual level (in parentheses).
Significance levels + p<0.10, * p<0.05, ** p<0.01.

Table 15 shows the estimation results for exogenous shocks when including quadratic terms. This preserves the expected coefficient signs, but renders the coefficients individually insignificant. However, we can carry out F-tests (likelihood ratio tests) for the joint significance of the terms relating to each type of shocks, and these show that the coefficients on non-labor exogenous income are reliably jointly significant at the 1% level in all specifications. By contrast, adverse event spending is not significant in the OLS specifications and exhibits some concavity. The coefficients suggest that the effect on MRS is strongest at about $322 weekly expenses and falls after. This may be a result of the endogeneity of the amount spent and an implicit selection effect: only households that are not liquidity constrained can afford to spend several hundred dollars out of pocket on an unexpected event in a given week. Unfortunately we do not have two independent instruments for the linear and the squared terms and therefore cannot carry out IV regressions. The coefficient sizes and signs in the conditional likelihood specification echo those of the OLS estimates but the shock variables are not significant. However, unlike in the OLS estimates, likelihood ratio tests for the coefficients on the quadratic terms show that they jointly contribute significantly (at the 10% level) to improving the conditional likelihood model fit.
Overall, these results suggest that the sample is not quite large enough to deliver the required power for including higher order terms. Nonetheless, the effects are qualitatively robust and the simple linear estimates do not appear to mask important nonlinear effects.

## F Conditional Likelihood Estimation

Recall that the decision maker chooses \( a_1 \) over \( a_0 \) if

\[
\frac{u'(c_t, \rho_t)}{E_t \left[ d(w_{t+1}, \rho_{t+1})u'(c_{t+1}, \rho_{t+1}) \right]} \leq \frac{a_1}{a_0}
\]

and \( a_0 \) over \( a_1 \) otherwise. The left-hand side is the MRS.

Consider decision set A. For each decision maker \( i \) in period \( t \) we observe 8 decisions \( k = 1, \ldots, 8 \). Let \( a_0^k = 50k \) and \( a_1 = 300 \), so that low \( k \) corresponds with a high MRS. We model the set of decisions A in a given week as a latent discrete choice problem, separately
for each binary choice:

\[ Prob \left( i \text{ chooses } a_1 \text{ over } a_k^0 \text{ in } t \right) = P \left( MRS_{it} + \epsilon_{itk} \leq \frac{a_1}{a_k^0} \right) \]

Assuming that the error term has a logistic distribution with scale parameter \( \sigma \), the probability is

\[ P \left( \epsilon_{itk} \leq \frac{a_1}{a_k^0} - MRS_{it} \right) = \frac{\exp \left( \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{it} \right) \right)}{1 + \exp \left( \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{it} \right) \right)}. \]

Without an error term, we would by necessity expect decisions to be monotonic, that is, there is a single switch point within a set of choices, and each set of choices pins down \( MRS_t \) up to an interval (of varying size across \( k \)). We would like to estimate the \( MRS_{it} \) as a function of an individual fixed parameter and the effect of financial variables between different periods using conditional likelihood estimation (Chamberlain 1984).

Denote the choice of \( a_1 \) over \( a_k^0 \) in \( t \) by \( y_{itk} = 1 \) (0 otherwise). Assume for the moment that there are only two levels of \( k \) and two periods. Now consider for example the probability that \( y_{i11} = y_{i12} = 1 \) but \( y_{i21} = 0 \) and \( y_{i22} = 0 \), given by

\[
\frac{\exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{i1} \right) \right] \exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{i1} \right) \right]}{\prod_{t=1}^{2} \prod_{k=1}^{2} \left\{ 1 + \exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{it} \right) \right] \right\}}
\]

Then the probability that \( y_{i11} = y_{i12} = 1 \) conditional on \( \sum_{t,k} y_{itk} = 2 \) is

\[
\frac{\exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{i1} \right) \right] \exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{i1} \right) \right]}{\sum_{t,k|\sum_{t,k} y_{itk}=2} \exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{i1} \right) \right] \exp \left[ \frac{1}{\sigma} \left( \frac{a_1}{a_k^0} - MRS_{i1} \right) \right]}
\]

where \( MRS_{it} = \alpha_i + \lambda X_{it} + \gamma_t \). Since \( \frac{\alpha_i}{\sigma} \) is constant, this term cancels from all the exponents. This construction works equally with three periods and eight choices \( k \), and all possible choice combinations. Denoting \( m_{itk} = \frac{1}{\sigma} \frac{a_1}{a_k^0} - \beta X_{it} - \tilde{\gamma}_t \) where \( \beta = \frac{\lambda}{\sigma} \) and \( \tilde{\gamma}_t = \frac{\gamma_t}{\sigma} \), the log conditional likelihood is

\[
\ln L = \sum_{i=1}^{N} \left\{ \sum_{t,k} y_{itkm_{itk}} - \ln \left( \sum_{\{d_{tk}\} \in B_{itk}} \exp \left[ \sum_{t,k} d_{tk}m_{itk} \right] \right) \right\}
\]

where \( B_{itk} \) contains all indicator sets such that \( \sum d_{tk} = \sum y_{itk} \). This is equivalent to
estimating a conditional logistic regression with person fixed effects and period dummies, where we can in addition cluster the error terms at the person level. For computational reasons and given the short panel, it is assumed that the error variance $\sigma$ is constant across the population. We parameterize the MRS as a linear or quadratic function of the financial variables as in the main text. We exclude any subjects who in any set of decisions switch between early and late payments more than three times. This does not occur frequently, but we assume that these participants have not understood the experimental decisions well enough to contribute meaningfully to the MRS estimation.