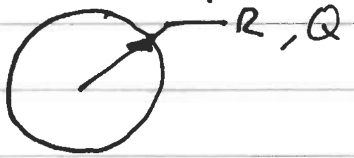
14 2007

AP 4010 INTRO NUCLEAR SCIENCE

HOMEWORK # 4 - SOLUTIONS -

PROBLEM 2-1



A SPHERE OF UNIFORM CHARGE DENSITY
RADIUS = R, CHARGE = Q

WHAT IS THE ELECTROSTATIC ENERGY?

TWO WAYS: (#A) INTEGRATE ELECTRIC FIELD ENERGY
(#B) COMPUTE WORK REQUIRED TO ASSEMBLE SPHERE

(#A) ELECTRIC FIELD ENERGY = $\iiint dv \, \frac{\epsilon_0 E^2}{2}$

GAUSS'S LAW GIVES ELECTRIC FIELD:

$$\iint d\bar{a} \cdot \bar{E} = \iiint dv \; \rho/\epsilon_0$$

SINCE THE PROBLEM IS SPHERICALLY SYMMETRIC, THESE INTEGRALS ARE 1-DIMENSIONAL WITH $\bar{E}$ IN THE RADIAL DIRECTION

$$\text{ENERGY} = \int_0^\infty 4\pi r^2 dr \, \frac{\epsilon_0 E^2}{2}$$

$$4\pi r^2 E_r = \begin{cases} \frac{r^3}{R^3}\frac{Q}{\epsilon_0} & r < R \\ Q/\epsilon_0 & r > R \end{cases}$$

PROBLEM 2.1 (CONT.)

THUS,

$$\text{ENERGY} = \int_0^R 4\pi r^2 dr \frac{\epsilon_0}{2}\left(\frac{r Q}{4\pi R^3 \epsilon_0}\right)^2 + \int_R^\infty 4\pi r^2 dr \frac{\epsilon_0}{2}\left(\frac{Q}{4\pi r^2 \epsilon_0}\right)^2$$

$$= \frac{Q^2}{8\pi\epsilon_0}\left[\frac{1}{R^6}\int_0^R r^4 dr + \int_R^\infty \frac{dr}{r^2}\right]$$

$$= \frac{Q^2}{4\pi\epsilon_0}\left[\frac{1}{5R} + \frac{1}{R}\right] = \frac{Q^2}{4\pi\epsilon_0}\frac{3}{5}\frac{1}{R}$$

(#B) THE METHOD USED TO CALCULATE THE ENERGY OF A SPHERE IN THE TEXT BOOK WAS TO SUM THE ENERGY REQUIRED TO "ASSEMBLE" THE SPHERE:

$$\text{ENERGY} = \int_0^Q \frac{q\, dq}{4\pi\epsilon_0 r} \quad \text{WHERE} \quad q = \left(\frac{r}{R}\right)^3 Q$$

$$\text{OR} \quad r = R\left(\frac{q}{Q}\right)^{1/3}$$

AS MORE CHARGE IS ADDED, THE RADIUS OF THE SPHERE INCREASES SO THAT THE CHARGE IS UNIFORMLY DISTRIBUTED WITHIN SPHERE

$$\text{ENERGY} = \int_0^Q \frac{q\, dq}{4\pi\epsilon_0 R (q/Q)^{1/3}} = \frac{Q^{1/3}}{4\pi\epsilon_0 R}\int_0^Q q^{2/3}\, dq$$

$$= \frac{3}{5}\frac{Q^2}{4\pi\epsilon_0 R}$$

PROBLEM # 2.1 (CONT)

NOW, CONSIDER THE ISOBARS $^{41}_{20}Ca$ and $^{41}_{21}Sc$

ENERGY DIFFERENCE ≈ DIFFERENCE IN ELECTROSTATIC ENE

$$\Delta E = 350.420 - 343.143 = 7.28\ \text{MeV}$$

$$\Delta E = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\frac{1}{R}\left((21)^2-(20)^2\right) = \frac{3}{5}\frac{e^2}{4\pi\epsilon_0}\frac{1}{R}\,41$$

BUT $\frac{e^2}{4\pi\epsilon_0} = 1.44\ \text{MeV fm}$, SO $7.28\ \text{MeV} = \frac{3}{5}\,1.44\,\frac{41}{R}\ \text{MeV}$

$\therefore\ R = 4.87\ \text{fm}$!!

PROBLEM # 2.2

SINCE A=46 IS EVEN, THERE ARE TWO EVEN-EVEN NUCLEI THAT ARE STABLE TO β-DECAY: ^{46}Ca AND ^{46}Ti

PROBLEM # 2.3

$$\text{MASS}(A,Z) = Z m_H + (A-Z)m_n - B(A,Z)$$

WHERE

$$c^2 B(A,Z) = A_V A - A_S A^{2/3} - \frac{A_c Z^2}{A^{1/3}} - \frac{A_A (A-2Z)^2}{A} \pm \ldots$$

$$\left.\frac{\partial\,\text{MASS}}{\partial Z}\right)_{A=\text{CONST}} = m_H - m_n + \left[\frac{2a_c Z}{A^{1/3}} - \frac{4a_a(A-2Z)}{A}\right]\frac{1}{c^2}$$

SOLVE FOR Z_{min} ...

$$m_n - m_H = \frac{2}{Ac^2}\left[\left(a_c A^{2/3} + 4a_a\right)Z - 2a_a A\right] \quad \text{OR}$$

$$Z = \frac{Ac^2}{2}\left[\frac{m_n - m_H + 4a_a/c^2}{a_c A^{2/3} + 4a_a}\right] \sim 43.8$$

$^{101}_{44}Ru$

PROBLEM #2

SCHRÖDINGER'S EQ WITHIN 1D, 2D, 3D BOXES

1D: $E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

2D: $E\psi = -\hbar^2/2m\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right)$

3D: $E\psi = -\hbar^2/2m\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right)$

LET EACH BOX SIDE BE OF LENGTH "a":

$\psi(x) = N\sin\left(\frac{m\pi}{a}x\right)$

$\psi(x,y) = N\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right)$

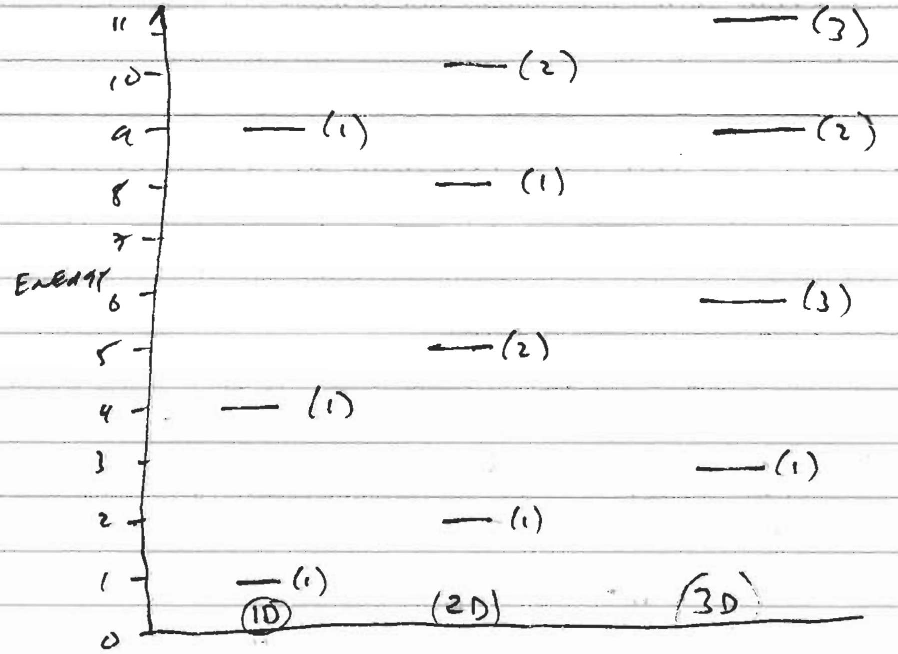
$\psi(x,y,z) = N\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}y\right)\sin\left(\frac{p\pi}{a}z\right)$

$\text{ENERGY} = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\left(m^2\right)$

$= \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\left(m^2 + n^2\right)$

$= \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}\left(m^2 + n^2 + p^2\right)$

(EXAMPLE, LET $a = 4\,fm$, $m = 1\,Amu$
$\hbar^2 = 41.8\ Amu\ MeV\ fm^2 \quad \therefore \frac{\hbar^2}{2m}\frac{\pi^2}{a^2} = 12.9\,MeV$) (1)



(...) = DEGENERA[TE]
"EQUAL
ENERGY"
STATES

PROBLEM #3

CONSIDER TWO "MANUFACTURED" NUCLEI:

$^{257}Fm \rightarrow {}^{257}Es + \beta^+ + \gamma$ BETA

$\rightarrow {}^{253}Cf + \alpha$ α - DECAY

$^{258}Md \rightarrow {}^{258}Fm + \beta^+ + \gamma$ BETA

$\rightarrow {}^{254}Es + \alpha$ α - DECAY

WE USE THE SEMF TO SHOW THE ENERGY RELEAS IS ABOUT (0.11, 7.7) FOR ^{257}Fm AND (2.7, 8.2) FOR ^{258}Md. CLEARLY, α-DECAY WILL OCCUR MUCH MUCH MORE FREQUENTLY THAN β^+ DECAY.

^{257}Fm IS A KNOWN α EMITTER WITH A 30 MINUTE HALF LIFE

^{258}Md HAS A 2 MONTH HALF-LIFE BEFORE IT α-DECAYS

AP 4010 Homework 5 (Solutions)

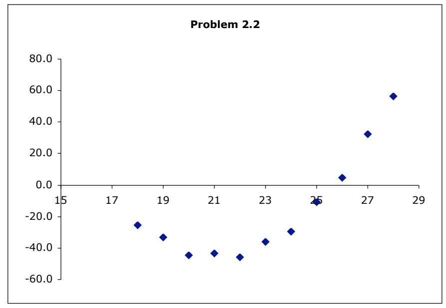
Energy_AMU	931.494	*MeV*
Mass_Alpha	3727.379	*MeV*
Mass_Hydrogen	938.783	*MeV*
Mass_Neutron	939.565	*MeV*
av	15.56	*MeV*
as	17.23	*MeV*
aa	23.28	*MeV*
ac	0.7	*MeV*

Formula =
av*$A-as*$A^(2/3)-ac*$Z^2/$A^(1/3)-aa*($N-$Z)^2/$A+2*$D

Problem 2.2

A 46 *AMU*

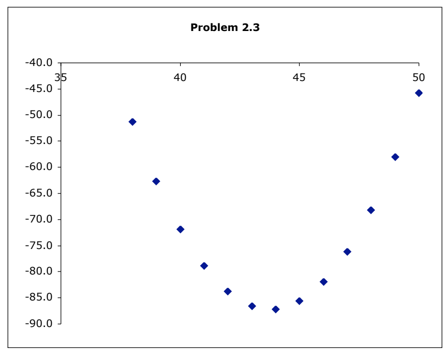
	Z	N	Pairing	Binding	Rel Mass
	18	28	1	382.7	-25.5
	19	27	-1	389.6	-33.2
Stable	**20**	**26**	**1**	**400.2**	**-44.6**
	21	25	-1	398.3	-43.5
Stable	**22**	**24**	**1**	**400.0**	**-45.9**
	23	23	-1	389.2	-35.9
	24	22	1	382.0	-29.5
	25	21	-1	362.4	-10.6
	26	20	1	346.3	4.7
	27	19	-1	317.7	32.4
	28	18	1	292.8	56.6



Problem 2.3

A 101 *AMU* **Z (Min)** 43.79

	Z	N	Pairing	Binding	Rel Mass
	38	63	0	836.8	-51.3
	39	62	0	847.3	-62.7
	40	61	0	855.7	-71.8
	41	60	0	862.0	-78.9
	42	59	0	866.1	-83.8
	43	58	0	868.1	-86.5
Most Stable	**44**	**57**	**0**	**867.9**	**-87.2**
	45	56	0	865.6	-85.6
	46	55	0	861.2	-82.0
	47	54	0	854.5	-76.1
	48	53	0	845.8	-68.2
	49	52	0	834.9	-58.1
	50	51	0	821.9	-45.8



Problem 3

A	Z	N	Pairing	Binding	Rel Mass
257	100	157	0	1907.1	88.91
258	101	157	-1	1910.8	92.53

Beta Decay (A = 257)

A	Z	N	Pairing	Binding	Rel Mass	
257	95	162	0	1902.2	97.80	
257	96	161	0	1905.0	94.13	
257	97	160	0	1907.0	91.41	
257	98	159	0	1908.0	89.63	
257	**99**	**158**	**0**	**1908.0**	**88.80**	**(Most Stable= Einsteinium)**
257	*100*	*157*	*0*	*1907.1*	*88.91*	
257	101	156	0	1905.3	89.97	
257	102	155	0	1902.5	91.97	
257	103	154	0	1898.8	94.92	

Beta Decay (A = 258)

A	Z	N	Pairing	Binding	Rel Mass	
258	95	163	-1	1905.8	102.18	
258	96	162	1	1910.5	96.72	
258	97	161	-1	1911.3	95.20	
258	98	160	1	1914.1	91.62	
258	99	159	-1	1912.9	91.98	
258	**100**	**158**	**1**	**1913.8**	**90.29**	**(Most Stable= Fermium)**
258	*101*	*157*	*-1*	*1910.8*	*92.53*	
258	102	156	1	1909.8	92.73	
258	103	155	-1	1904.9	96.85	

Alpha Decay (A = 257)

A	Z	N	Pairing	Binding	Rel Mass	Total Mass	
257	100	157	0	1907.1	88.91	239483	
253	98	155	0	1885.5	79.78	235748	
					Alpha Mass	**3727**	
					Difference (Energy Release)	**7.7**	*MeV*

Alpha Decay (A = 258)

A	Z	N	Pairing	Binding	Rel Mass	Total Mass	
258	101	157	-1	1910.8	92.53	240418	
254	99	155	-1	1889.7	82.92	236682	
					Alpha Mass	**3727**	
					Difference (Energy Release)	**8.2**	*MeV*