AP 4010 INTRO NUC SCIENCE

HW # 5 SOLUTIONS

PROBLEM # 2.5

ENERGY SHIFT DUE TO SPIN-ORBIT COUPLING:

$$\Delta E \propto \langle \bar{l} \cdot \bar{s} \rangle \propto \begin{cases} \frac{1}{2} l \hbar^2 & \text{FOR } j = l + \frac{1}{2} \\ -\frac{1}{2}(l+1)\hbar^2 & \text{FOR } j = l - \frac{1}{2} \end{cases}$$

BUT THE NUMBER OF STATES IS $2j+1$

$$\text{\# OF STATES} \begin{cases} 2(l+1) & j = l + \frac{1}{2} \\ 2l & j = l - \frac{1}{2} \end{cases}$$

$\therefore$ (ENERGY CHANGE) $\times$ (NUMBER OF STATES) =

$$= \begin{cases} \frac{1}{2} l \times 2(l+1) = l(l+1) \\ -\frac{1}{2}(l+1) \times 2l = -l(l+1) \end{cases}$$

SO CHANGE IN ENERGY LEVEL, WEIGHTED BY NUMBER OF STATES, IS INDEPENDENT OF SPIN-ORBIT COUPLING

PROBLEM # 2.6

a) A STATE WITH UP TO 16 NUCLEONS WOULD HAVE 16 PROTONS OR 16 NEUTRONS WITH

$16 = 2(2j + 1)$ = NUMBER OF STATES

$\therefore j = 7/2$

SO $j = l \pm \frac{1}{2}$, WE MUST HAVE $l = 3$ ODD or $l = 4$ EVEN ← THUS $l = 3$, $j = 7/2$

PROBLEM # 26 (CONT.)

b) SHELL MODEL WORKS WELL WITH ODD-EVEN OR EVEN-ODD NUCLEI. THE UNPAIRED NUCLEI GIVES THE ANGULAR MOMENTUM AND PARITY OF THE NUCLEI.

$^{7}Li \; (Z=3, N=4) \rightarrow (1P_{3/2})^{1}_{Z} \quad l=1 \;\; j=\frac{3}{2} \quad$ ODD

$^{11}B \; (Z=5, N=6) \rightarrow (1P_{3/2})^{3}_{Z} \quad l=1 \;\; j=\frac{3}{2} \quad$ ODD

$^{31}P \; (Z=15, N=16) \rightarrow (2S_{1/2})^{1}_{Z} \quad l=0 \;\; j=\frac{1}{2} \quad$ EVEN

$^{39}K \; (Z=19, N=20) \rightarrow (1d_{3/2})^{3}_{Z} \quad l=2, \;\; j=\frac{3}{2} \quad$ EVEN

$^{59}Co \; (Z=27, N=32) \rightarrow (1f_{7/2})^{7}_{Z} \quad l=3 \;\; j=\frac{7}{2} \quad$ ODD

$^{127}I \; (Z=53, N=74) \rightarrow (2d_{5/2})^{3}_{Z} \quad l=2, \;\; j=5/2 \quad$ EVEN

THE NEUTRON LAST (CLOSED/PAIRED) SHELLS ARE:

$(1P_{3/2})^2 \; ; \; (1P_{3/2})^4 \; ; \; (2S_{1/2})^2 \; ; \; (1d_{3/2})^4 \; ;$

$(2P_{3/2})^4$ AND $(1h_{11/2})^4$

PROBLEM # 27

$^{17}O \; (Z=8, N=9)$ GROUND STATE $=$ $(1S_{1/2})^2(1P_{3/2})^4(1P_{1/2})^2$ (P)

$(1S_{1/2})^2(1P_{3/2})^4(1P_{1/2})^2(1d_{5/2})^1$ (N)

$l=2 \quad j=5/2 \quad$ EVEN

TO GET ODD PARITY, WE PROMOTE/EXCITE ANOTHER NEUTRON!

PROBLEM # 7.7 (CONT.)

THE EASIEST POSSIBILITIES ARE

$(1p_{1/2})^{-1} (1d_{5/2})^2_{I=0}$ — THE TWO NEUTRONS PAIR SO "HOLE" IN $1p_{1/2}$ GIVES $\ell = 1 \quad j = \frac{1}{2}$ ODD

$(1p_{1/2})^{-1} (1d_{5/2})^2_{I=2}$ — HERE THE TWO PAIRED NEUTRONS FORM ANGULAR MOMENTUM WITH $I = 2$. THE <u>TOTAL</u> ANGULAR MOMENTUM IS $j = 2 + \frac{1}{2} = 5/2 \quad \ell = 3$ ODD

$(1p_{3/2})^{-1} (1d_{5/2})^2_{I=0}$ — HERE WE PROMOTE A NEUTRON FROM THE $1p_{3/2}$ SHELL. $j = 3/2 \quad \ell = 1$ ODD

THE ANSWER IN THE TEXTBOOK GIVES ADDITIONAL POSSIBILITIES.

PROBLEM 7.8

a) $E = I(I+1)\, \hbar^2 / 2\mathcal{I}$

$$\frac{E}{I(I+1)} = \frac{0.093}{2\times3} \quad \frac{0.309}{4\times5} \quad \frac{0.641}{6\times7} \quad \frac{1.084}{8\times9}$$

$$= 0.0155 \quad 0.01545 \quad 0.01526 \quad 0.0151$$

b) $$\mathcal{I} = \frac{\hbar^2}{2\ 0.015\ \text{MeV}} = \frac{41.8\ u\ \text{MeV fm}^2}{2\ \ 0.015\ \text{MeV}} = 1400\ u\ \text{fm}^2$$ NOT BAD!

CLASSICAL $\mathcal{I} = \frac{2}{5} MR^2 = \frac{2}{5} A (1.25 A^{1/3})^2 = \frac{2}{5} 180^{5/3} (1.25)^2 = 3600$ ABOUT $\frac{1}{2}$

PROBLEM #2.9

$$\text{ENERGY} = \frac{1}{2} I \omega^2$$

$$\text{ENERGY} = I(I+1)\hbar^2/2I$$

$$\therefore \quad \omega^2 = \frac{2E}{I} = \frac{2E^2}{3\hbar^2} \qquad E = 0.093 \text{ MeV}$$

$$I = 2$$

$$\omega = \sqrt{\frac{2}{3}}\frac{E}{\hbar} = \sqrt{\frac{2}{3}} \frac{0.093}{0.658\times10^{-21}} = 1.15\times10^{20} \frac{\text{RAD}}{\text{SEC}}$$

SPEED OF 50 MeV NUCLEON IS

$$V = 1.389\times10^{7} \frac{\text{m}}{\text{sec}} \sqrt{\frac{E(\text{MeV})}{\mu}} = 9.8\times10^{7} \text{ m/s}$$

$$\text{TRANSIT TIME} \sim \frac{2R}{V} \sim \frac{2 \quad 1.25\,(180)^{1/3}}{9.8\times10^{7} \quad 10^{15}} = 1.44\times10^{-22} \text{ SEC}$$

SO $\omega T \sim 0.0165 \ll 1$ ROTATION IS VERY SLOW!

PROBLEM #2.10

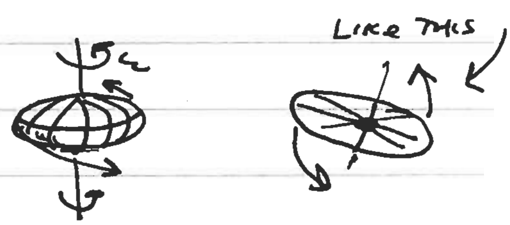
ΔE = TRANSISTION ENERGY (TWO GAMMA RAYS)

$$= 4\hbar^2/I$$

$$I = \frac{4\hbar^2}{\Delta E} \sim \frac{4 \quad 41.8 \text{ MeV u fm}^2}{0.05 \text{ MeV}} = 3344 \text{ u fm}^2$$

$$\text{RIGID SPHERE} = \frac{2}{5} m R^2 = \frac{2}{5}\, 152\,(1.25)^2 (152)^{2/3} = 2706$$

EQUAL TO 2:1 ASPECT RATIO



Problem #2

Angular velocity = $\omega = \frac{\ell\hbar}{I} = \frac{5}{2}\frac{(\ell\hbar)}{AR^2} = \frac{5}{2}\frac{(\ell\hbar)}{1.4^2 A^{5/3}}$

Kinetic energy = $\frac{1}{2}\omega^2 I = \frac{1}{2}\frac{(\omega I)^2}{I} = \frac{5(\ell\hbar)^2}{4AR^2}$

$$= \frac{5}{4}\frac{(\ell\hbar)^2}{1.4^2 A^{5/3}}$$

Current rotating axis = $\frac{\omega Z}{2\pi}$

(In the last answer, the factor of "2π" comes from

$$\text{Current} = \frac{Z}{\frac{4}{3}\pi R^3}\,\omega \underbrace{\int_0^R\int_0^\pi r\sin\theta\, r\,dr\,r\,d\theta}_{\frac{2}{3}R^3} = \frac{\omega Z}{2\pi}$$)

Typical values:

$Z = 32 \quad A = 64 \quad R = 5.6\,\text{fm} \quad \ell = 1$

$\hbar = 0.658\times10^{-21}$

$\hbar c = 197$ MeV fm

$\hbar^2 = 41.8$ MeV u fm^2

then $\omega = \frac{5}{2}\frac{\hbar}{64(5.6)^2} \Rightarrow$

$\hbar\omega = 0.052$ MeV $\quad \omega \Rightarrow 8\times10^{19}$ rad/sec

$\frac{1}{2}\omega^2 I = 0.026$ MeV

$\frac{\omega Z}{2\pi}e = 68$ Amp (roughly independent of A)

AP 4010 Homework 5 (Solutions)

Energy_AMU	931.494	*MeV*
Mass_Alpha	3727.379	*MeV*
Mass_Hydrogen	938.783	*MeV*
Mass_Neutron	939.565	*MeV*
av	15.56	*MeV*
as	17.23	*MeV*
aa	23.28	*MeV*
ac	0.7	*MeV*

Formula =
av*$A-as*$A^(2/3)-ac*$Z^2/$A^(1/3)-aa*($N-$Z)^2/$A+2*$D

Problem 3

Separation Energy = (m_product + m_n) - m = B - B_prod
Separation Energy = (m_product + m_H) - m = B - B_prod

	A	Z	N	Pairing	Binding	Del Z	Del N	New Pairing	Product New Binding	Separation
7Li	7	3	4	0	39.3	**0**	**-1**	**-1**	31.0	**8.3**
91Zr	91	40	51	0	787.4	**0**	**-1**	**1**	780.6	**6.8**
236U	236	92	144	1	1790.7	**0**	**-1**	**0**	1782.7	**8.0**
20Ne	20	10	10	1	160.5	**-1**	**0**	**0**	150.5	**10.0**
55Mn	55	25	30	0	481.0	**-1**	**0**	**1**	473.9	**7.1**
197Au	197	79	118	0	1551.4	**-1**	**0**	**1**	1547.2	**4.2**

From Appendix F... (Actual & not the same as the SEMF)

Predicted from SEMF

	Mass	Product	Mass	Separation	SEMF	SEMF
7Li	16004	**6Li**	15122	**7.2**	15996	16188
91Zr	-94355	**90Zr**	-95296	**7.2**	-90428	-91758
236U	45562	**235U**	43923	**6.5**	45230	45142
20Ne	-7560	**19F**	-1597	**12.8**	-7365	-4478
55Mn	-61950	**54Cr**	-61115	**8.1**	-60798	-61001
197Au	-33448	**196Pt**	-35065	**5.8**	-24917	-28191