NOV 7, 2004

# AP4010 HOMEWORK #6 (QUIZ #2 PREPARATION)

## PROBLEM #1

| | | |
|---|---|---|
| $^{3}He$ | (2p, 1n) | n: $(1s_{1/2})^1$ |
| $^{4}He$ | (2p, 2n) | CLOSED SHELL: NO SPIN |
| $^{27}Al$ | (13p, 14n) | p: $(1d_{5/2})^{-1}$ "HOLE IN OUTER P SHELL" |
| $^{28}Si$ | (14p, 14n) | CLOSED $1d_{5/2}$ SHELL: NO SPIN |
| $^{38}Ar$ | (18p, 20n) | CLOSED/PAIRED STATES: NO SPIN |
| $^{41}K$ | (19p, 22n) | p: $(1d_{3/2})^{-1}$: "HOLE IN OUTER PROTON SHELL" |
| $^{63}Cu$ | (29p, 34n) | p: $(2p_{3/2})^1$ EXTRA PROTON IN $p_{3/2}$ |
| $^{65}Cu$ | (29p, 36n) | p: $(2p_{3/2})^1$ NEUTRONS PAIRED IN $1f_{5/2}$ STATES |
| $^{64}Zn$ | (30p, 34n) | CLOSED SHELLS |

## PROBLEM #2

$E_\gamma \rightsquigarrow \text{o} \longrightarrow \text{o}$ $\qquad \gamma + D \to n + p$

THE MINIMUM ENERGY REQUIRED TO DISSOCIATE D WILL BE WHEN THE REACTION PRODUCTS HAVE NO PERPENDICULAR MOMENTUM. THEN CONSERVATION LAWS REQUIRE

$$E_\gamma - 2.224589 = K.E. = E_n + E_p$$

$$E_\gamma / c = M_n v_n + M_p v_p$$

CONSIDER CASE #1: BOTH N+P MOVE FORWARD AT SAME SPEED. THEN, $K.E. \approx \frac{(MV)^2}{M}$

WITH $(MV) = \frac{1}{2}\left(\frac{E_\gamma}{c}\right) \therefore KE \approx \frac{E_\gamma^2}{4Mc^2}$

CONSIDER CASE #2: ONE REMAINS STATIONARY AND THE OTHER PRODUCT MOVES FORWARD. THEN $K.E. \approx \frac{(MV)^2}{2M}$

WITH $(MV) \approx \frac{E_\gamma}{c} \therefore KE \approx \frac{E_\gamma^2}{2Mc^2}$

THEREFORE, MINIMUM HAS BOTH NEUTRON + PROTON MOVING FORWARD

$E_\gamma(\min) = 2.224589 + E_\gamma^2/(4\times 940) \approx 2.2259 \text{ MeV}$

## QUESTION #3

### PART A

BETA DECAY CHANGES $N/Z$ WITHOUT CHANGING $A$

### PART B

MINIMUM MASS: $\left.\dfrac{\partial M}{\partial Z}\right|_{A=\text{CONSTANT}} = 0$

$$\frac{\partial M}{\partial Z} = \frac{\partial}{\partial Z}\left[ Z m_H + (A-Z) m_N - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} \right]$$

WE DO NOT NEED TO CONSIDER PAIRING ENERGY SINCE THIS JUST ADJUSTS MINIMUM FOR EVEN-EVEN AND ODD-ODD...

$$\frac{\partial M}{\partial Z} = m_H - m_N + 2 a_c \frac{Z}{A^{1/3}} - 4 a_a \frac{(A-2Z)}{A} = 0$$

$$(m_H - m_N - 4 a_a) + \left( \frac{2a_c}{A^{1/3}} + \frac{8 a_a}{A} \right) \frac{A}{1 + \left(\frac{N}{Z}\right)} = 0$$

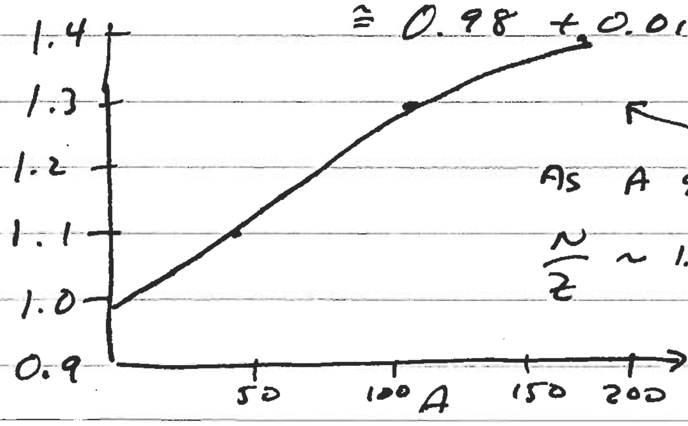
SOLVING FOR $N/Z$ ...

$$\frac{N}{Z} + 1 = \frac{2 a_c A^{2/3} + 8 a_a}{m_N - m_H + 4 a_a}$$

$$= \frac{8 a_a}{m_N - m_H + 4 a_a} - 1 + \frac{2 a_c}{m_N - m_H + 4 a_a} A^{2/3}$$

$$\cong 0.98 + 0.015 A^{2/3}$$

COULOMB TERM




AS $A$ GETS LARGE,

$\frac{N}{Z} \sim 1.4 \rightarrow$ FEWER PROTONS FOR A GIVEN ISOBAR GIVES BETA STABILITY

QUESTION #4

$$\Delta \approx 28.8\ \text{MeV} - 4\left(a_v - \frac{2}{3} a_s A^{-1/3} - \frac{5}{3} 0.17\, a_c A^{2/3} - 0.027 a_a\right)$$

$\alpha$-DECAY IS UNSTABLE ABOVE THRESHOLD $\Delta \simeq 0$

$$\Delta \approx 0 \approx (28.8 - 4a_v + 4\ 0.027\ a_a) + \frac{4}{3}\left(\frac{a_s}{A^{1/3}} + 0.85\, a_c A^{2/3}\right)$$

COULOMB TERM
DE-STABILIZES
$\alpha$-DECAY AT HIGH A

SOLVING FOR CRITICAL A...

$$\Delta \approx 0 \approx -31\ \text{MeV} + \frac{46}{A^{1/3}} + 0.81 A^{2/3}$$

or $\quad 0.81 A = 31 A^{1/3} - 46$

$$A = 38 A^{1/3} - 57$$

TRY...

$A \sim 150$ ?

$$A = 38(150)^{1/3} - 57 = 145$$

$A \sim 130$ ?

$$A = 38(130)^{1/3} - 57 = 135$$

$A \sim 140$ ?

$$A = 38(140)^{1/3} - 57 = 140 \quad \text{YES!}$$

WHEN $A > 140$ (APPROXIMATELY), THEN NUCLEI WHICH ARE $\beta$-STABLE BECOME UNSTABLE TO $\alpha$-DECAY.

## QUESTION #5

THE EXPECTED LENGTH OF THE ANGULAR MOMENTUM VECTOR IS FOUND FROM $\langle \ell^2 \rangle = \hbar^2 \ell(\ell+1)$. THE EXPECTED LENGTH OF THE Z-COMPONENT OF THE ANGULAR MOMENTUM IS $\langle \ell_z \rangle = \hbar m$.

THEREFORE, THE EXPECTED ANGLE OF THE ANGULAR MOMENTUM VECTOR WITH RESPECT TO THE Z-AXIS IS:

$$\text{EXPECTED ANGLE} = \sin^{-1}\left(\frac{\langle \ell_z \rangle}{\sqrt{\langle \ell^2 \rangle}}\right) = \sin^{-1}\left(\frac{m}{\sqrt{\ell(\ell+1)}}\right)$$

FOR $\ell = 1$, $m = -1, 0, 1$ AND $\measuredangle = (-45^\circ, 0, +45^\circ)$

FOR $\ell = 2$, $m = -2, -1, 0, +1, 2$ AND $\measuredangle = (-54.7^\circ, -24.1^\circ, 0, 24.1^\circ, 54.7^\circ$

## QUESTION #6

SQUARE: $E(n_x, n_y) = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}(n_x^2 + n_y^2)$

RECTANGLE: $E(n_x, n_y) = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2}(n_x^2 + n_y^2/4)$ WITH BOX LENGTH IN Y-DIRECTION TWIC

WHEN THE BOX IS <u>NOT</u> SYMMETRIC, MOST DEGENERACIES DISSAPPEAR. DEGENERATE STATES OCCUR WHEN STATES WITH <u>DIFFERENT</u> QUANTUM NUMBERS HAVE THE <u>SAME</u> ENERGY. FOR A SQUARE BOX, THIS OCCURS, FOR EXAMPLE, AT (2,1) AND (1,2) AND AT (1,3) AND (3,1).

THE DEGENERACY IS MOSTLY <u>BROKEN</u> WHEN THE BOX SIDES ARE NOT EQUAL. THE FIRST FEW ENERGY STATES FOR ▭ ARE

$E \sim 5/4,\ 2,\ 13/4,\ 17/4,\ 5,\ 5,\ \frac{25}{4},\ \frac{29}{4},\ 8,\ \frac{37}{4},\ 10,\ \frac{41}{4} \ldots$

$(n_x, n_y) \sim (1,1), (1,2)\ (1,3)\ (2,1)\ (1,4), (2,2), (2,3), (1,5), (2,4), (3,1), (3,2), (2,5) \ldots$