AP4010 Introduction to Nuclear Science
Homework 6: Due 9 November, 2004.

NOTE: There will be no class on Tuesday, November 2 because of Election Day. Our next class will be November 9.
QUIZ 2: Our second open-book, open-notes quiz will be in-class on November 9. The primary subject of the quiz is nuclear structure and introductory quantum mechanics.

Question 1

Using Fig. 2.9 from our textbook (p. 48, Lilley), justify the nuclear spin and parity (+ for even, and − for odd) for the following nuclei:

$$
\begin{array}{ccc}
{}^{3}_{2}\mathrm{He}(\tfrac{1}{2}+) & {}^{4}_{2}\mathrm{He}(0+) & {}^{27}_{13}\mathrm{Al}(\tfrac{5}{2}+) \\
{}^{28}_{14}\mathrm{Si}(0+) & {}^{38}_{18}\mathrm{Ar}(0+) & {}^{41}_{19}\mathrm{K}(\tfrac{3}{2}+) \\
{}^{63}_{29}\mathrm{Cu}(\tfrac{3}{2}-) & {}^{65}_{29}\mathrm{Cu}(\tfrac{3}{2}-) & {}^{64}_{30}\mathrm{Zn}(0+)
\end{array}
$$

Question 2

What is the minimum photon energy required to dissociate ^{2}H (deuterium)?

$$\gamma + {}^{2}\mathrm{H} \rightarrow n + p$$

Assume the binding energy to be 2.224589 MeV (and don't forget to conserve total energy and momentum).

Question 3

Fig. 1.5 (p. 13) from the textbook is attached showing a curve of lowest mass isobars, sometimes called the *Segré chart*, (*i.e.* the most stable combination of neutrons, N, and protons, Z, for a given atomic number, $A \equiv Z + N$.)

For each combination of N and Z, the nucleus mass is approximately given by a so-called "semi-empirical mass formula", or SEMF. The SEMF is

$$\begin{aligned} M(Z,N) &= Zm_H + Nm_n - B(Z,N)/c^2 \\ B(Z,N) &= a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta \end{aligned}$$

where $a_v = 15.56$ MeV, $a_s = 17.23$ MeV, $a_c = 0.7$ MeV, and $a_a = 23.28$ MeV and $\Delta \sim 12/A^{1/2}$ MeV. Also, $m_H c^2 = 938.8$ MeV and $m_n c^2 = 939.6$ MeV.

Part a

For each value of A, there is a minimum M corresponding to the most stable isobar. If the ratio of N to Z is not equal to this minimum, a radioactive decay can occur. What type of radioactive decay reduces M without changing A?

Part b

Use the semi-empirical mass formula (below) to derive an analytical expression for the most stable ratio of N/Z as a function of A. This formula is: $N/Z = 0.98 + 0.015A^{2/3}$.

[*Hint:* To find the minimum mass as Z is changed keeping A constant, you need first express the SEMF in terms of Z and A only. Also, explain why or why not Δ can be ignored in your estimated formula. After finding the lowest energy for each A, substitute $Z \to A/(1 + N/Z)$ and solve for the quantity (N/Z).]

Question 4

Heavy nuclei (*i.e.* large $A \sim 150$) become unstable to alpha decay even though they may be stable to beta decay.

Estimate, for large values of A, the maximum nucleus size that remains stable to alpha decay processes.

Hint: To form your estimate, you should estimate the change of mass (or the change of energy) upon release of an α particle:

$$\begin{aligned} \Delta_\alpha M &= M(Z,N) - M(Z-2,N-2) - m_\alpha \\ &= 2(m_H + m_n) - m_\alpha - \frac{1}{c^2}[B(Z,N) - B(Z-2,N-2)] \\ &\approx 2(m_H + m_n) - m_\alpha - \frac{4}{c^2}\frac{dB(A/2.4, 0.58A)}{dA} \end{aligned}$$

In the above formula, $N/Z \approx 1.4$ was assumed. This approximation (see from Question 3) means that $Z \approx A/(1+1.4)$ and $N \approx 1.4A/(1+1.4)$. The expression for $B(Z = A/2.4, N = 0.58A)$ is

$$B(Z = A/2.4, N = 0.58A) \approx a_V A - a_s A^{2/3} - 0.17 a_c A^{5/3} - a_a 0.027A$$

The mass difference that appears above is $2(m_H + m_n) - m_\alpha = 28.8$ MeV/c^2.

When $\Delta_\alpha M > 0$, then the nucleus is unstable to alpha decay. When $\Delta_\alpha M < 0$, then the nucleus is stable.

[Please note: you need only *estimate* the maximum α-stable value of A.]

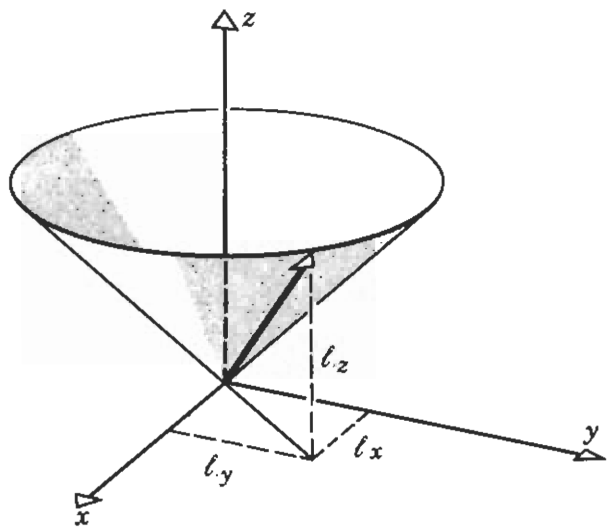

Figure 1: The vector $\vec{l}$ precesses rapidly around the z axis, so that l_z stays constant, but l_x and l_y are variable.

Question 5

Fig. 1 (above) illustrates a model of the convention in quantum mechanics. The z component of the angular momentum is quantized with the m_l quantum number. m_l can take the integer values $m_l = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm l$.

What are the possible angles between the angular momentum vector, $\vec{l}$, and the z-axis for $l = 1$ and $l = 2$?

Question 6

For a two-dimensional box (with infinite-potential boundaries), how do the energy levels and level degeneracies change when the box dimensions change from a by a (*i.e.* square) to a by $2a$ (*i.e.* rectangular)?