9/15/2005
APPH 6101

HW #1 SOLUTIONS

QUESTION #1

(A) $n = 10^{10}\ cm^{-3}$, so $n^{-\frac{1}{3}} = 4.6 \times 10^{-4}\ cm$

IF $T_e = 1eV$, $K.E. = \frac{3}{2} n k T_e = 1.5 \times 10^{10}\ eV/cm^3$

Avg POTENTIAL ENERGY $\Rightarrow$ $P.E. \simeq \frac{e^2 n}{4\pi \epsilon_0 \langle r \rangle}$

OR $P.E. = e\left(\frac{e}{4\pi\epsilon_0 \langle r \rangle}\right) n$

$\frac{e}{4\pi\epsilon_0\langle r \rangle} \sim 3.2 \times 10^{-6}$ VOLT $\therefore$ $P.E. \simeq 3.2 \times 10^{4}\ eV/cm^3$

$\frac{P.E.}{K.E.} \sim 2.1 \times 10^{-6}$ (VERY SMALL!)

(B) $\frac{4}{3}\pi \lambda_D^3 n = 5.2 \times 10^{3}$

(C) $1\ mTorr = 3.5 \times 10^{13}\ cm^{-3}$

$\therefore \frac{n}{n_{Hg}} \sim 2.8 \times 10^{-4}$ (VERY WEAKLY IONIZED!)

(D) $I_{PROBE} = 75 \times 10^{-6}\ Amp = e n C_e \sqrt{\frac{m_e}{m_i}} A_{Probe}$

WITH $C_e \sim 4.2 \times 10^{7}\ cm/sec$,

$n = 6.7 \times 10^{9}\ cm^{-3}$

QUESTION #2

$$\Phi = \frac{Q}{4\pi\epsilon_0 r} e^{-r/\lambda_D}$$

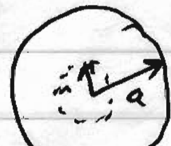
$$\rho = n_0 e\left(1 - e^{e\Phi/kT}\right) \simeq -\frac{n_0 e^2}{kT}\Phi \simeq -\frac{\epsilon_0}{\lambda_D^2}\Phi$$

$$Q_{PLASMA} = \int_0^\infty 4\pi r^2 dr \frac{\rho}{\epsilon_0} = -\int_0^\infty 4\pi r^2 dr \frac{\Phi}{\lambda_D^2}$$

LET $x \equiv r/\lambda_D$, THEN $Q_{PLASMA} = -Q\int_0^\infty x\,dx\,e^{-x} = -Q$ Q.E.D.

QUESTION #3

(A)



GAUSS'S LAW $\iint_S d\bar{A}\cdot\bar{E} = \iiint_V dv\, \rho/\epsilon_0$

OR $4\pi r^2 E = \frac{e n_0}{\epsilon_0}\frac{4\pi}{3} r^3$

$$E = \frac{e n_0}{\epsilon_0}\left(\frac{r}{3}\right)$$

$$W_E = \int_0^a dr\, 4\pi r^2 \frac{\epsilon_0}{2} E^2$$

$$= \frac{2\pi e^2 n_0^2}{9\epsilon_0}\int_0^a dr\, r^4 = \frac{2\pi e^2 n_0^2}{45\,\epsilon_0} a^5$$

$$K.E. = \left(\tfrac{3}{2} n k T\right)\left(\tfrac{4\pi}{3} a^3\right)$$

$$\frac{W_E}{K.E.} = \frac{2\pi}{45}\frac{1}{2\pi}\left(\frac{e^2 n_0}{\epsilon_0 k T}\right) a^2 = \frac{1}{45}\frac{a^2}{\lambda_D^2} \gg 1 \quad !!$$

(B) $\frac{\partial \Phi}{\partial r} = -E \Rightarrow \Phi(r) = -\frac{e n_0}{\epsilon_0}\frac{r^2}{6}$

(C) $W_E = K.E.$ WHEN $a^2 = 45\lambda_D^2$ OR

$$\boxed{a = 6.7\lambda_D}$$

FOR $\lambda_D = 0.005$ cm, $a = 0.3$ mm

WHEN IS THE $W_E = K.E.$ IN A TORUS? LIKE PROF. PEDERSEN'S EXPERIMENT.