APPH 6101 Plasma Physics I
Midterm Exam
Submit your solutions before noon, Monday, Oct. 31, 2005.

This is an open book, open note, open anything exam. I intend for the exam to be completed in about 2 hours of work, but I suspect that some of you may take longer. (That's ok, but not required.) Since this is an open-book exam, full credit is given **only** *if you show your work and explain your reasoning. Please write clearly so that I can understand your solutions.* Each question counts for the same fraction of the total exam score. Please, on your honor, do not speak with any other student, colleague, or scientist, either within or outside Columbia University. You are on your honor to do this exam **entirely** by yourself.

Question 1

Describe trapped particle motion in large-aspect ratio tokamak geometry and show that the particle motion is described by an orbit shaped like a "banana" when viewed on a poloidal plane. Also give the ratios between the bounce and magnetic drift frequencies as compared to the cyclotron frequency.

Background: This is a well-known problem in toroidal magnetic physics described in textbooks about tokamaks. (You do not need to reference these other textbooks, but it is allowed.) Your starting point must be a formulation of the magnetic field. In the "large-aspect-ratio" limit, the magnetic field is nearly toroidal. There is a weak poloidal field such that the magnetic field makes a helical trajectory as it goes around the plasma torus. The geometry that we'll use for the magnetic field is approximately cylindrical, with $(\rho,\ \theta)$ representing the minor coordinates from the major radius of the torus. In this coordinate system, the toroidal field must decrease with radius, and

$$B_t(\rho,\ \theta) = B_0 \frac{R_0}{R_0 + \rho\cos\theta} = \frac{B_0}{1+\epsilon\cos\theta}$$

where $\epsilon \equiv \rho/R_0 \ll 1$. When $\theta = 0$, the field line is on the "outside" of the torus, and the toroidal field is relatively weak. When $\theta = \pi$, the field line is on the "inside" of the torus, and the toroidal field is stronger. This variation of the strong toroidal field causes (1) particle trapping when $v_{||}/v_{\perp}$ is sufficiently small and (2) magnetic drifts. As $\epsilon \to 0$, the poloidal field depends only upon ρ. If the plasma current density is a constant within the plasma, then $B_p(\rho) = \mu_0 \rho J/2\pi$. The "safety factor" is the ratio of the number of times that the magnetic field line goes the "long-way" around the torus to the number of times that the field line goes the "short-way" around the torus. The symbol q is used to describe the safety factor, and it's given by $q(\rho) = \rho B_0/R_0 B_p(\rho)$. For a constant J, the safety factor is constant, $q = 2\pi B_0/\mu_0 J$, and $B_p(\rho) = \epsilon B_0/q \ll B_0$. Therefore, the total magnetic field is $\mathbf{B} = \hat{\phi} B_t + \hat{\theta} B_p$, where $\hat{\phi}$ is the unit vector in the toroidal direction and $\hat{\theta}$ is the unit vector in the poloidal direction.

With the magnetic field defined, you should consider the constants of particle motion. Assume that the particle's gyroradius is small compared with the size of of the tokamak (or the drift orbit.) Then, the magnetic moment, μ, and particle energy, E, are conserved.

Because the tokamak is axisymmetric, the total (canonical) angular momentum is also conserved (as described in Section 3.7 of the textbook.) You need to know the component of the vector potential in the direction of symmetry, *i.e.* the toroidal direction, $A_\phi = \hat{\phi} \cdot \mathbf{A}$. For the approximations used above,

$$A_\phi \approx \frac{\rho^2}{2} \frac{B_0}{R_0 q} .$$

Question 2

Describe the conditions when light can propagate through *and be guided by* a "channel" through a plasma. In other words, imagine an infinite uniform plasma with a straight channel of dimension a across it's cross-section where the plasma density within the channel is different from the plasma density outside channel. When does the channel act like an "optical fiber" and guide the light for long distances?

Question 3

Consider a uniformly magnetized and a fully-ionized plasma made from carbon. There would be six times the density of electrons than of ions (but the plasma would still be approximately charge-neutral.)

Describe the Alfvén wave, the electron whistler wave, and the ordinary wave in this plasma. How do these waves (in a fully-ionized carbon plasma) compare to the same waves in a plasma made from singly-ionized carbon having the same mass density (*i.e.* the same density of carbon)?

Question 4

Part A. Write the equations for the first three moments of the particle distribution function given by

$$f(\mathbf{x},\ \mathbf{v},\ t) = n\,\delta(v_x - V_0)\delta(v_y)\delta(v_z)$$

where $(\mathbf{x},\ \mathbf{v})$ are expressed in cartesian coordinates, V_0 is a constant, and where you should assume that there is no magnetic force field.

Part B. Assume that initially the ions are described by $f_i = n\delta(v_x)\delta(v_y)\delta(v_z)$ and the electrons are described by the distribution function given in Part A. Describe the linear electrostatic waves that exist in this plasma.