

# Naturalizing Deontic Logic: indeterminacy, diagonalization, and self-affirmation\*

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It is an appealing idea that deontic modality is a modality of the open future, and that the indeterminacy of the open future is the key, within natural language, to understanding the deontic modal puzzles that form the traditional subject-matter of deontic logic.<sup>1</sup> Richmond Thomason makes this suggestion in the closing section of his article “Indeterminist Time and Truth-Value Gaps” (1970):

I mentioned ... several ways in which the present theory can yield accounts of the relation of time to other things, by enriching the formal language so as to include a vocabulary for expressing these things ... [M]any of the puzzles that deontic logicians have encountered in formalizing the notion of obligation [and permission] arise in part from [this] interaction. (Thomason, 1970, pg. 280)

“Indeterministic ... logic,” Thomason goes on to conjecture, “is particularly attractive for this purpose in view of the traditional position that obligations presuppose freedom.”<sup>2</sup>

In this essay, I follow Thomason’s call, taking seriously his remark about the freedom of agents. On this “active” brand of indeterminism—what I will call **agential indeterminism**—one sees positions in formal models not as mere places one might be, or as real estate one might eliminate via a conversational update, but as a space of places an agent might *move to*. This view backgrounds certain questions (like the question of the *act-prior* chance that something is choiceworthy) in favor of certain other questions (like the *act-posterior* chance that something is choiceworthy). It backgrounds a certain kind of power—for example, a speaker’s power to change the conversational scoreboard—in favor of an agent’s raw power to move her body, thereby changing facts about the world which the conversational scoreboard is constrained to reflect. In Stalnaker (1999a)’s celebrated example of a goat walking into a roomful of speakers, an agent in the context of agential indeterminism bears better comparison to the *goat* than to the speakers who come to presuppose that the goat is present.

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<sup>1</sup>For a characterization of deontic logic in this style—as a branch of modal logic research driven by a proprietary batch of natural language puzzles—see, for example, McNamara (2010, §4).

<sup>2</sup>*op. cit.*, pg. 281.

In this setting, I pull together three well-studied strands of indeterminism—Thomason’s own settledness operator ( $\blacksquare$ ), the modal base of Kratzer (1981, 1991b)’s analysis of modals, and Stalnaker (1999a)’s notion of diagonal acceptance—to argue for **two theses** governing a deontic logic for natural language.

The first thesis makes a claim about postsemantic truth, and is couched in terms Stalnaker’s dagger ( $\dagger$ ) operator.<sup>3</sup> Where  $W(c)$  is a set of worlds not excluded by the totality of facts that obtain at an act-prior context  $c$ , the first thesis says

**Thesis 1.**  $\phi$  is true at  $c$  iff for any  $u, v \in W(c)$ :  $u, v \models \blacksquare \dagger \phi$ .

This “box-dagger” combination is familiar, having been cast as a marker of “a priori truth” (Stalnaker *op. cit.*, pg. 83). Here, however, the role of  $\blacksquare \dagger$  will be to mark a restricted kind of a priority: that which obtains prior to *choice*. The thesis is important because it helps to model how the quantum of control an agent exercises over her actions translates into a degree of control over the *propositional content* of statements—including deontically modalized statements—that concern her future self.

The second thesis addresses what it is for an act to be *permissible*—in the toy object language I introduce in §2, for the sentence  $\lceil \text{MAY } p \rceil$  to be true—when agents exercise partial control over their obligations as well as their bodily movements. The second thesis is

**Thesis 2.**  $\text{MAY } p := \blacklozenge(p \wedge \diamond p)$

where  $\blacklozenge$  is historical possibility—the dual of Thomason’s  $\blacksquare$  operator—and  $\diamond$  is a normal modal diamond which reflects deontic accessibility from the point of view of a single possible world in a Kripke frame. This proposal, while unusual in the context of modal logic, has a decision-theoretic pedigree; its motivation snaps into focus when seen from a perspective on which agents *choose* which world(s) to occupy. I treat Theses 1 and 2 syntactically, with the goal of setting up a sound and complete proof system for a two-dimensional deontic modal logic that it at least as strong as K.

It’s a bit abstract, however, to mass troops under the banner of indeterminism without having an application—one of the “many of the puzzles... deontic logicians have encountered”—in mind. From this collection, I choose the phenomenon known as *free choice permission*, the apparent ability of (here, deontically-) modalized sentences like

- (1) You may have coffee or tea.  
 $\text{MAY}(p \text{ OR } q)$

to entail the stronger-than-expected conjunction of modals:

- (2) You may have coffee and you may have tea.  
 $\text{MAY}(p) \wedge \text{MAY}(q)$

<sup>3</sup>The  $\dagger$  operator is also present in van Fraassen (1977) (as the ‘*wahr*’ operator) and in Lewis (1973, §2.8), who in turn cites Vlach (1973). A time-indexed settledness operator—more faithful aspects of tense-talk which are backgrounded here—is discussed in a related context by MacFarlane (2008) via King (2003).

Ultimately, my explanation of this stronger-than-expected entailment works just in case it is *within your control*, with respect to the open future, whether you have coffee or tea. As I argued in Fusco (2015b), this approach also provides, with no extra assumptions, a solution to Ross’s Paradox (Ross, 1941), a second puzzle in the traditional cabinet of deontic curiosities. To argue directly for my account of Free Choice and Ross, though, would be to get ahead of myself. In this paper, I attack from the rear, marshaling general considerations under the banner of indeterminism.

## 1 Thesis 1: Agential Indeterminism

As I will understand it, agential indeterminacy is nonepistemic and future-oriented. It concerns what is (and what is to become) actual. I take it this kind of indeterminacy is familiar from the standpoint of rational agency. As Stalnaker (1976) describes it:

a theory of rational action...contains implicitly an intuitive notion of **alternative possible courses of events**... [A] rational agent [...] considers various **alternative possible futures**, knowing that the one to **become actual** depends in part on his choice. (Stalnaker, pg. 81, emphasis added)

Stalnaker’s talk here of a future *becoming actual* fits naturally within a picture of future-oriented agential choice. If it is now future-contingent, for example, whether you will accept or decline a bet, that is because both a betting-branch and a decline-to-bet-branch are compatible with—or “overlap” at—the present moment.

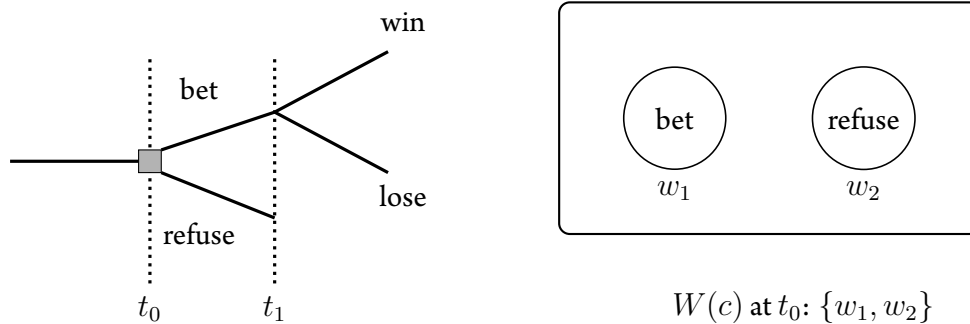


Figure 1: Agential future contingents as branching histories (left) and as overlapping worlds (right)

Following Thomason, this kind of indeterminacy is often framed in terms of branching histories (Belnap et al., 2001). To bring the terminology in line with that used by Stalnaker, Kaplan, and Lewis, however, I will talk of **possible worlds** instead of histories, thinking of a world as a maximally decided course of contextually available choices.<sup>4</sup> A **proposition** is function from these possible worlds to truth-values.

<sup>4</sup>The reader will note that these worlds are thus not maximally specific from the point of view of additional sentences (like *win* and *lose* on the left of Figure 1), which describe events outside the agent’s control. Hence the complement of a deontic modal in the toy language explored here will always functionally express a relation between an agent and an *action*. For the natural appeal of this idea, see, *inter alia*, Schroeder (2011); Horty (2001); Geach (1982); von Wright (1951).

Also following the terminology of Lewis, I'll speak generally of assigning truth-values to sentences relative to a context  $c$  and an index  $i$  (Lewis, 1980). A **context** is a concrete setting in which a speech act takes place, while an **index** is a formal object—usually an  $n$ -tuple of parameters—relative to which semantic values are recursively defined. (In our opening gambit, the index consists only of a possible world.) On the traditional picture, context has two roles: first, it is invoked directly in the understanding of context-sensitive vocabulary. Second, context is taken to identify a unique index—what Lewis calls “the index of the context” (pg. 86)<sup>5</sup>—which plays a distinguished role in evaluating token speech acts: a sentence  $\phi$  is true at a context  $c$  just in case it is true at  $i_c$ , the index of  $c$ .

Against this background, the primary modeling commitment of indeterminism is simple to state. It is just that there is no *unique* world of the context. Returning to the betting example, any world in which the agent takes the bet and any world in which the agent declines it are worlds which exclude one another, but—if betting and declining are truly future contingents—neither is excluded by the totality of facts that obtain at her context of choice. Hence it makes no sense to speak of  $w_c$ —the “world of the context”. *A fortiori*, there is no (unique) index of the context,  $i_c$ —contra the traditional picture. The best we can do is appeal to a set  $W(c)$ , the *worlds* of the context.

This modeling commitment is typically taken to underwrite intuitions about sentential truth. Our pretheoretic intuitions concerning *accurate* statements about the future—for example, that it wouldn't (yet) be accurate to say you will take the bet, and also not accurate to say that you won't, while it *would* be accurate to say that the bet was offered—are sensitive to contingency with respect to  $W(c)$ .<sup>6</sup> On Thomason's supervaluationist proposal, the notion of truth at a context *simpliciter* is *settled* truth: that is, truth at *every* world in  $W(c)$ .

In the logic, it's helpful to add an operator,  $\blacksquare$ , for settledness (*op. cit.*, pg. 275), which quantifies universally over the worlds in  $W(c)$ .  $\blacksquare$  syntactically relates **settled consequence**—the notion the Indeterminist thinks is important for tracking pretheoretic truth and its preservation—to an ecumenical notion of consequence  $\vDash$  which is neutral between them:  $\psi$  is a settled consequence of  $\phi$  just in case  $\blacksquare\phi \vDash \blacksquare\psi$ . An (agential) Indeterminist and an (agential) Determinist can agree on what follows from what in a formal language containing  $\blacksquare$ . This is the sense in which their disagreement is *post-semantic*, rather than semantic (MacFarlane, 2013).<sup>7</sup>

## 1.1 Diagonalization

If a Bayesian agent entertains some space of worlds  $w_1$ - $w_n$  as possible, and a trustworthy interlocutor tells her that  $\phi$ , she will aim to update her possibility space by eliminating from  $w_1$ - $w_n$  all worlds where the proposition expressed by  $\phi$  is false.

$w_1$	$w_2$	$w_3$	...
T	F	T	...

<sup>5</sup>In Kaplan's terminology, this is the *circumstance* of the context. See e.g. (Kaplan, 1989b, pg. 549) for the existence of such a circumstance, and pg. 552 for its distinguished role in evaluating token speech acts.

<sup>6</sup>Why accuracy instead of *truth*? We leave open the possibility of Relativism.

<sup>7</sup>I use “Indeterminist” here as someone who holds that the formal notion of settled truth is important either for pretheoretic notions of truth or pretheoretic notions of accuracy.

A background commitment of Stalnaker (1999a)’s discussion of **diagonal truth** is the following obstacle to this Bayesian task: sometimes, the answer to the question of *which proposition is expressed by  $\phi$*  itself depends on world-historical facts—such as who is speaking (to value the indexical “I”), and who is being addressed (to value the indexical “you”). Schematically, where  $y$  is a contextual candidate for the worldly facts, the proposition expressed by a sentence  $\phi$  is in fact:

$$\lambda w . y, w \models \phi \quad (\alpha)$$

Hence relative to each contextually available candidate—by convention, arrayed along the vertical axis—for the actual world, the semantics supplies a *potentially distinct* “horizontal” function from worlds  $w$  to the truth-values T and F.

	$w_1$	$w_2$	$w_3$	...
$w_1$	T	F	T	
$w_2$	F	F	F	
$w_3$	F	F	T	

But then, in order to locate which update to make, the agent needs to know which world she is in—and this is exactly what the Bayesian agent does not know to begin with.

Diagonalization is a strategy for confronting these cases. Stalnaker proposes that in such a predicament, an utterance of  $\phi$  can be rationally reinterpreted as:

*What is said in  $S$ ’s utterance of  $[\phi]$  is true, where the definite description, what is said in  $S$ ’s utterance of  $[\phi]$  may be a nonrigid designator—a description that refers to different propositions in different worlds (Stalnaker, 1999a, pg. 82).*

In each world  $w \in \{w_1 \dots w_n\}$ , the reinterpreted propositional concept is thus

$$\lambda w . w, w \models \phi \quad (\beta)$$

This reinterpretive move reduces the -arity of semantic values, eliminating the dependence of truth value on the problematic free parameter  $y$ .

To illustrate, suppose you find yourself in Lewis Carroll’s Wonderland, with an advanced case of philosopher’s amnesia. Alice stands before you. At the beginning of the interaction, three worlds are possible:  $w_1$ , where Alice stands before Tweedledum,  $w_2$ , where Alice stands before Tweedledee, and  $w_3$ , where Alice stands before the White Rabbit. Alice points at you and says, “you are human.” The proposition expressed by “you are human”, while necessarily true if true at all, is different in each world.<sup>8</sup> In  $w_1$ , the content is *Tweedledum is human*; in  $w_2$ , the content is *Tweedledee is human*; in  $w_3$ , the content is *The White Rabbit is human*.

Therefore, if you don’t know which world you are in, you don’t know which proposition to update on. Moreover, in this particular case, you cannot nontrivially update on *any* of the

<sup>8</sup> Here I assume, for expository purposes, the necessity of origins. See Kripke (1980).

	$w_1$	$w_2$	$w_3$	...
$w_1$	T	T	T	
$w_2$	T	T	T	
$w_3$	F	F	F	

2D matrix for  $\lceil \text{You are human} \rceil$

	$w_1$	$w_2$	$w_3$	...
$w_1$	T	T	F	
$w_2$	T	T	F	
$w_3$	T	T	F	

2D matrix for  $\lceil \dagger \text{You are human} \rceil$

Figure 2: In  $w_1$ , the addressee is Tweedledum; in  $w_2$ , the addressee is Tweedledee; in  $w_3$ , the addressee is The White Rabbit.

possible propositions expressed (any row of the left matrix in Figure 2) because all candidate propositions are either “true everywhere”—eliminating nothing—or “false everywhere”—which would eliminate everything, leaving you in an absurd “empty” state. Updating on the *diagonalized* proposition, by contrast, is both nontrivial and intuitively correct: it eliminates from Bayesian consideration only world  $w_3$ , where Alice’s addressee is the White Rabbit.

In the logic, it’s helpful to add an operator,  $\dagger$ , for the operation which converts  $(\alpha)$  to  $(\beta)$ , visualizable in Stalnaker’s matrix diagrams as *diagonalization* (see also [Segerberg, 1973](#); [Vlach, 1973](#); [van Fraassen, 1977](#)):

$$y, w \models \dagger\phi \text{ iff } w, w \models \phi \quad (\gamma)$$

In the Wonderland case, the upshot of diagonalization is indeed a proposition true in  $w_1$  and  $w_2$ , but false in  $w_3$ .

Syntactically, the  $\dagger$  operator relates **diagonal consequence**—the notion the Diagonalizer thinks is important for tracking the interpretation of propositions expressed under conditions of uncertainty—to a neutral notion of consequence  $\models$ :  $\psi$  is a diagonal consequence of  $\phi$  just in case  $\dagger\phi \models \dagger\psi$ . Once again, making the operator explicit facilitates the neutrality of the resulting logic. A Diagonalizer and a Non-diagonalizer can agree on what follows from sentences in a formal language containing  $\dagger$ , though they may disagree about how those notions hook up to pretheoretic notions like *what is said* and *what is learned*. In a “deterministic” context in which  $W(c)$  is a singleton,  $\dagger\phi$  and  $\phi$  are equivalent, and diagonalization is vacuous.<sup>9</sup>

## 1.2 A Marriage of Diagonalization and Indeterminism

The indeterminacy quarantined by the settledness operator  $\blacksquare$  and the indeterminacy of update potential quarantined by the diagonalization operator  $\dagger$  form a natural pair; where both content and truth matter, there is a straightforward argument from the need for each to the need for the other.

The first argument is aimed at nonepistemic indeterminists, of which the Thomasonian is a prominent example. In reaching for the notion of settled truth, the Indeterminist suppresses the problematic demand for a unique world of the context. But where we want to

<sup>9</sup> More carefully, the semantic value of  $\dagger\phi$  is the semantic value of  $\phi$ : if the contextually relevant set  $W$  is a singleton, then  $y = w$  for all  $y, w \in W$ . Hence  $y, w \models \dagger\phi$  iff  $y, y \models \dagger\phi$  iff  $y, y \models \phi$ . Hence for all  $y, w \in W$ :  $y, w \models \dagger\phi$  iff  $y, w \models \phi$ .

know which propositions are expressed by sentences, an additional maneuver is needed to avoid the this demand’s resurgence at the level of *propositional content*: both the semantic and post-semantic roles played by the presumptive world of the context must be suppressed. The guiding insight, as in Stalnaker’s examples, is that we sometimes evaluate statements about the (agentially) open future as (settled) true (or settled false) even when their *contents* are future-contingent.

An example focused on this point will be illustrative. In the mold of *The Importance of Being Earnest* (Cumming, 2008; Wilde, 2009), consider an oracle telling Gwendolyn:

- (3) In the future, you will marry a man named ‘Ernest’. **He** will bring you every happiness.

Suppose that the oracle’s predictions are infallible. (3) might be true in context although there are two people named ‘Ernest’ Gwendolyn could—for all that is settled by  $W(c)$ —freely marry: let us suppose they are Ernest Worthing (in world  $w_W$ ) and Ernest Montcrieff (in world  $w_M$ ). Discourse (3) may thus be determinately true in a context  $c$  even though it is indeterminate whether the directly referential “He” in the second sentence refers to Worthing or Montcrieff.<sup>10</sup>

	$w_W$	$w_M$
$w_W$	T	F
$w_M$	F	T

Figure 3: “He will bring you every happiness” in (3).

Leveraging the perspective of  $\{w: w \in W(c)\}$ , we can simply observe that the sentence is interpretable as (settled) true in  $c$  as long as, for every  $w \in W(c)$ , each  $w$ -local referent of “he” goes on to bring Gwendolyn every contextually relevant happiness. This parallels Stalnaker’s recommendation for extracting an update potential from the sentence asserted in the Wonderland case, even though one cannot discern which (undiagonalized) proposition Alice’s utterance expresses.

A second argument for marrying settled truth and diagonal truth goes in the “right-to-left” direction; it is aimed at the epistemic Diagonalizer, of the sort who accepts the analysis in the Wonderland case. The Diagonalizer believes that situations which trigger diagonalization are cases where quasi-Bayesian agents’ knowledge leaves open more than one candidate for the facts relevant to the content of a sentence  $\phi$ . Agential indeterminism is the view on which the context *itself*, due to the agent’s quantum of control, leaves open more than one candidate for the facts relevant to  $\phi$ ’s content. But where the context itself is constrained

<sup>10</sup>Does this mean that  $\blacksquare\ddagger$  combination will turn out to be a *monster*, expressly forbidden by Kaplan (1989b)? It depends on how “monster” is evaluated when there is no single world of the context. The strong ban on monsters comes from the idea that directly referential terms, like indexicals (including “he”), have their referents [using Kaplan’s terms] “directly loaded” into the proposition expressed by an embedded sentence (see e.g. Kaplan, 1989a, pg. 569). But such referents can be directly loaded by a given world  $y$  within the context set  $W(c)$  without every world in  $W(c)$  having to load the same objects.

to fall short, knowledge (common or uncommon) is not possible. A commitment to thoroughgoing agential indeterminism will therefore trigger diagonalization for discourses like (3). The move to settled truth and the move to diagonal truth go together.

### 1.3 Moving to a lexicon

I’ve made a quick argument for the inclusion of  $\blacksquare$  and  $\dagger$  in a logic of agential indeterminacy. These operators allow us relate different notions of truth and consequence to one another, forming a framework within which the Diagonalizer and the Nondiagonalizer, as well as the Determinist and the Indeterminist, can express their (dis)agreements—and study the formal implications of their views—in a common object language.

With only  $\blacksquare$  and  $\dagger$ , though, it should be emphasized that the lexicon of the language itself is a *tabula rasa*. The  $\blacksquare$  and  $\dagger$  operators are included in the vocabulary of the formal fragment for conceptual reasons; they are not hypothesized to correspond to lexical items.<sup>11</sup>

How, then, should we start to flesh out a lexicon? First, modal operators should reflect rigidity phenomena in natural language (such as the rigidity of “He” in (3); whichever suitor Gwendolyn marries, we ought to be able to follow up (3) with “Necessarily, he is human”). This motivates the inclusion of the “*A*” (“actually”) operator and a variety of natural language modals  $\Box_n$  of the familiar kind (*must*, *ought*, *necessarily*, etc.) Importantly, “*A*” is rigid in the scope of natural-language modals:  $A\phi \equiv \Box A\phi$  is an axiom of the standard modal logic of these operators (Crossley & Humberstone, 1977, pg. 14, 24).<sup>12</sup>

Thus it is natural to place  $\dagger$  in the context of the following standard (schematic) two-dimensional entries:

$$\begin{aligned} \Box_n. y, w \models \Box_n \phi & \text{ iff } \text{ for all } v \text{ s.t. } wR_nv: y, v \models \phi. \\ & \dots \text{ where } \Box_n \text{ is some natural language modal (must, ought, etc.)} \\ A. y, w \models A\phi & \text{ iff } y, y \models \phi \end{aligned}$$

There are two possibilities for the “settled” operator  $\blacksquare$ , one bidimensional and one unidimensional:

$$\begin{aligned} 1D. y, w \models \blacksquare \phi & \text{ iff } \text{ for all } v \in W(c): y, v \models \phi \\ 2D. y, w \models \blacksquare \phi & \text{ iff } \text{ for all } v \in W(c): v, v \models \phi \end{aligned}$$

<sup>11</sup> Lewis emphasizes this point with regard to Stalnaker’s  $\dagger$  operator, remarking that even though “Stalnaker speaks of reinterpreting sentences in certain contexts so that they express their diagonal... [the] horizontal-diagonal ambiguity is very unlike ordinary sorts of ambiguity... [because] it is **neither syntactic nor lexical**” (Lewis, 1980, pg. 94, emphasis added). Likewise, Thomason does not suggest that any *word* or *morpheme* in the natural language lexicon is equivalent to the settledness operator  $\blacksquare$ , nor that it occupies some tacit place in syntax. Belnap, Perloff and Xu (2001) add that there are ways of bringing out settledness intuitions in natural language—see, for example, their suggestion that one might win a bet that *p* without winning a bet that it’s settled that *p* (op. cit., pg. 160)—but do not claim that this is systematically traceable to particular natural language lexical items, in the way that e.g. “ought” in English seems to have a firm grip on articulating a boxlike modality.

<sup>12</sup> For arguments, see, *inter alia*, Crossley & Humberstone (1977, pgs. 11-13), Kripke (1980), Gregory (2001, pgs. 58-61)



I suggest using (1D); this makes explicit the separate conceptual contributions of Indeterminism and Diagonalization, and assimilates settledness to the standard modal operators in  $(\Box_n)$ . Our postsemantic notion is:

$$\phi \text{ is true at } c \quad \text{iff} \quad \text{for all } w \in W(c): w, w \vDash \phi$$

which is equivalent to our first thesis:

**Thesis 1.**  $\phi$  is true at arbitrary  $u, v \in W(c)$  iff  $u, v \vDash \blacksquare \dagger \phi$ .

Preservation of this type of truth gives rise to a corresponding notion of **settled-diagonal consequence**, which I will notate with  $\Vdash$ :  $\phi \Vdash \psi$  iff  $\blacksquare \dagger \phi \vDash \blacksquare \dagger \psi$ .<sup>13</sup>

## 2 Thesis 2: Deontic Concepts

I’ve encouraged a certain kind of viewtaking on models—the agential view—and advertised a notion of consequence that, I think, fits naturally within it. On this picture, we look at the worlds along the axes of matrix models as a-historically accessible. None of those worlds are ruled out by what is actual, and truth at a context is naturally modeled as settled-diagonal truth.

Work remains to be done, however, in bringing the agential viewpoint into contact with the resources of *deontic* models—and, ultimately, into contact with the deontic logicians’ “many modal puzzles”. What we want are models capable of representing how the content of our talk of **future obligations**—not just **future facts**—might depend on what we (freely) choose. Returning to the discourse in (3), for example, Gwendolyn might well be able to choose which suitor named “Ernest” to marry. But as Thomason points out, with this freedom comes (the possibility of) moral responsibility—perhaps, in Gwendolyn’s case, certain duties of loyalty or care. Modeling the interaction of obligation and choice involves upgrading historical matrix models into full-blown modal models—models, in particular, that can support a deontic sub-modality.

I therefore begin in this section with a review of deontic modal logic in *one* dimension, as it is studied in Kripke semantics for modal logic. The ultimate goal is to “lift” a Kripke model, deontic accessibility relation and all, into two dimensions as in Figure 4, uniting the matrix model and the Kripke model points of view.

<sup>13</sup> It’s worth emphasizing a formal point here which is relevant to Fusco (ms). Axiomatizations of two-dimensional semantics most often feature the operator  $A$ , rather than  $\dagger$ . Given this, it is worth asking how different the roles of  $A$  and  $\dagger$  are. After all, diagonal consequence can also be defined  $A$ -prefixed consequence, and indeed this is how it is standardly done in the literature (for example, with the notion of *real world consequence* in Davies & Humberstone, 1980).

It is true that  $A\phi \vDash A\psi$  iff  $\dagger\phi \vDash \dagger\psi$ , so long as the two worlds in the index are drawn from the same domain  $W$ . However,  $A$  cannot play  $\dagger$ ’s conceptual or lexical role. As noted above,  $A$  is rigid with respect to the “box” modalities, while  $\dagger$  is not: hence (given a 1D treatment of  $\blacksquare$ )  $\blacksquare A\phi \vDash \blacksquare A\psi$  is not the same as  $\blacksquare \dagger\phi \vDash \blacksquare \dagger\psi$ . By the Crossley & Humberstone axiom  $\lceil A\phi \equiv \Box A\phi \rceil$ , the former is just diagonal consequence again, while the *latter* preserves a kind of supertruth—the thing the Diagonalizing Indeterminist is interested in. Thus only  $\dagger$  can play a contributing role in relating  $\Vdash$  to the ecumenical notion of consequence,  $\vDash$ , the general notion of consequence on which e.g. a completeness proof can be built.

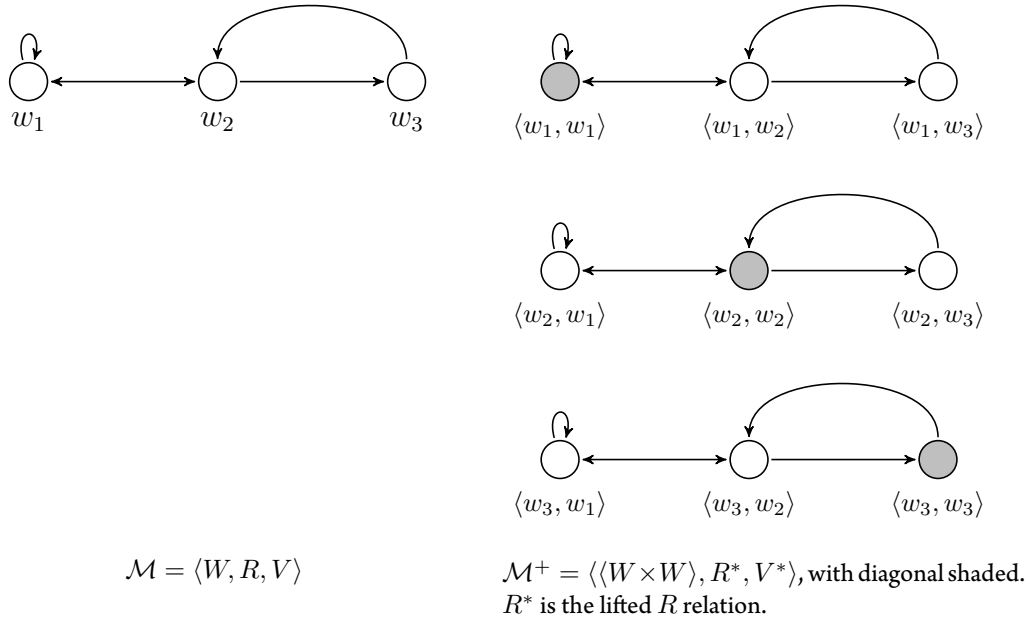


Figure 4: A Kripke model (left) lifted into two dimensions (right).

## 2.1 Towards a Deontic Logic for an agentially open future

In a typical Kripke semantics for modal logic, sentences are true or false at worlds in a model  $\mathcal{M}$ , such as the model in Figure 5.  $\mathcal{M}$  is a tuple  $\langle W, R, V \rangle$  consisting of a domain  $W$ , valuation  $V$ , and a binary accessibility relation  $R$  on worlds in the domain. If the accessibility relation is deontic,  $w$ 's being  $R$ -related to  $v$  represents the fact that  $v$  is deontically ideal by the lights of  $w$ .

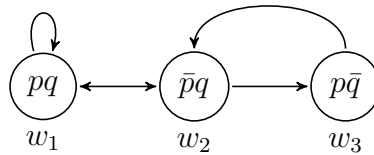


Figure 5: A one-dimensional modal model  $\mathcal{M}$ .  $\mathcal{M}, w_1 \models \Box q, \Diamond p$

In what has become known as Standard Deontic Logic (“SDL”), the distinguishing characteristic of the accessibility relation is that it fails to be reflexive: a world need not see *itself* as deontically ideal. However, the accessibility relation is taken to be *serial*: every world sees *some* world as ideal, hence some options as choiceworthy on moral grounds.

The standard semantic hypothesis, in this framework, is that OUGHT records world-bound necessity with respect to  $R$ , and ‘may’ records its dual—world-bound deontic possibility.

**Conventional Wisdom 1** (worldbound obligation and permission in SDL)

(1-a)  $\ulcorner \text{OUGHT } p \urcorner$  is true at  $w$  just in case  $w \models \Box p$ .

(1-b)  $\ulcorner \text{MAY } p \urcorner$  is true at  $w$  just in case  $w \models \diamond p$ .

If, in a Kripke model, the seriality of  $R$  is the only constraint we take to govern deontic accessibility, the resulting logic is KD (McNamara, 2010, §2.1).

Our project is to study deontic logic under the *indeterminist* banner, where indeterminism involves seeing a set of worlds as chooseable. The natural opening move here is just the one undertaken in §1: a move to seeing context as providing some set  $W(c)$ , rather than a particular world-history  $w_c$ . This spurs two complimentary upgrades: first, in the model, we include an equivalence relation  $R_{\blacksquare} \supseteq R_{\square}$ , whose image for any world in  $W(c)$  is  $W(c)$  itself. The deontic accessibility relation  $R_{\square}$  illustrated in e.g. Figure 2 is now a serial, deontic *sub*-modality of the S5 relation  $R_{\blacksquare}$ , which tells us what is permissible and obligatory at different worlds between which the agent can choose.

**Definition 1.** A bimodal deontic Kripke model  $\mathcal{M}$  is a tuple  $\langle W, R_{\blacksquare}, R_{\square}, V \rangle$  where  $R_{\blacksquare}$  is an equivalence relation and  $R_{\square}$  is a serial submodality of  $R_{\blacksquare}$ .

The second upgrade is in the object language, where we link  $R_{\blacksquare}$  to the settledness operator  $\blacksquare$  and its dual  $\blacklozenge$ .

**Postulate 1** ( $R_{\blacksquare}$  and  $R_{\square}$ ).  $\blacklozenge p \rightarrow \blacksquare p$

**Postulate 2** ( $R_{\blacksquare}$  an equivalence relation).

$T_{\blacksquare}$ )  $\blacksquare p \rightarrow p$

$5_{\blacksquare}$ )  $\blacklozenge p \rightarrow \blacksquare \blacklozenge p$ .

This leaves room for a version of “*ought* implies *can*” on which the relevant “can” is genuinely historical. Those working in a natural-language setting will immediately see Kratzer (1991b)’s modal base—a body of information which represents, as Kratzer puts it, the conversational background against which modal talk is situated—in Postulate 1’s implicit claim that deontic ideality is constrained to choose between worlds which are circumstantially available to the agent.<sup>14</sup>

Making the upgrades in Postulates 1 and 2—seeing historical accessibility as an equivalence relation, but deontic accessibility as nontrivially worldbound—leaves room for future contingency of obligations, as well as future contingency of facts.<sup>15</sup> Gwendolyn’s predicament from (3) illustrates one such case. For another, more tailored to the second thesis I want to establish in this paper, we consider

**Nice Choices at the Spa.** Aromatherapy [=  $p$ ] or body-wrap [=  $q$ ]—which is it to be? You believe that, whichever you choose, you will be very glad you

<sup>14</sup>To see that Postulate 1 is the starting point of Kratzer’s own semantic project, see e.g. Kratzer (1991b), Defn. 1, reading  $f(w)$  as  $\{v : wRv\}$ .

<sup>15</sup>Given that historical modality is an alethic modality, this move follows the precedent of Williamson (2013) in treating alethic modalities in S5.

chose it. Mid-aromatherapy, the aromatherapy will seem self-evidently superior [to the body-wrap]. Mid-body-wrap, the body-wrap will seem self-evidently superior [to the aromatherapy]. (Hare & Hedden, 2016, pg. 3)

**Choosing Childbearing.** You face a choice between conceiving a child early in your life [=  $p$ ] or a different child significantly later [=  $q$ ]. You believe that your values will be transformed by the choice you make. In particular, because the choice you make will be a necessary condition for the existence of a person you will love, you will affirm *that* choice over any other. There is no single psychological standpoint that values both of these potential persons to equal degree. [Parfit, 1984, pg. 360-361; Paul, 2014, Ch. 3; Paul, 2015.]

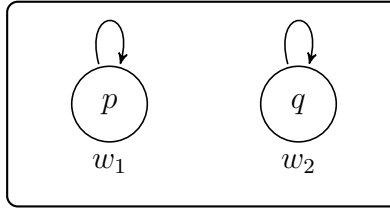


Figure 6: A model  $\mathcal{M}_{\text{nice}}$  for Self-affirming Choices. Arrows represent the (local) image of  $R_{\square}$ , while the box represents the (global) image of  $R_{\blacksquare}$ .

In decision problems with the structure of *Nice Choices at the Spa* and *Choosing Childbearing*—which I will generically refer to as “Self-affirming Choices”—deontic accessibility depends on what the agent chooses to do. In particular, she ought to do precisely whatever she chooses to do. The facts are illustrated in the Kripke model in Figure 6. In Figure 6, the  $R_{\square}$ -relation is a serial submodality of  $R_{\blacksquare}$ . Moreover,  $R_{\square}$  is maximally opinionated: every world sees exactly one option—either  $p$ , or  $q$ —as ideal. Since  $p$  and  $q$  are mutually exclusive, Conventional Wisdom 1 holds that exactly *one* atomic ‘may’-statement—either  $\ulcorner \text{MAY } p \urcorner$  or  $\ulcorner \text{MAY } q \urcorner$ —holds at each maximally determinate world in the model.

We, however, are Indeterminists, and the natural indeterminist perspective to take on the model is that the agent is not *already* located at either world. We want to know is what is permissible from the *act-prior* perspective the agent occupies before she chooses. The key observation is that since the agent can choose where to go, she has *two* permissible options, not just one. Our desideratum is that for each world  $w$  in  $\mathcal{M}_{\text{nice}}$ :  $\mathcal{M}_{\text{nice}}, w \models \text{MAY}(p) \wedge \text{MAY}(q)$ .

Achieving this desideratum will make permissibility’s formal profile, in an indeterminist framework, different from that of obligation and (factual) truth. Following Thomason, we held in §1 that factual truth is **supertruth**: a fact-describing statement like “You were offered a bet” is settled true at  $W(c)$  when it is true at each individual world in  $W(c)$ . Likewise, the natural notion of obligation is **superobligation**: in these bimodal Kripke models, a statement like “the agent is obligated to do  $p$ ” is settled true at  $W(c)$  when it is true at each individual world in  $W(c)$ : this is when, from the agential view, the worldbound obligation to do  $p$  is *inescapable*, no matter what point one moves to occupy in the model. But

for permission, the worldbound point of view simply does not scale up. Figure 6, and the existence of Self-affirming Choices more generally, shows that “superpermissibility” is *not* a necessary condition for the intuitive notion of permissibility to hold at a context.  $p$  and  $q$  are permissible *without* being superpermissible.

I wish to take seriously, in the object language, the ramifications of this point. To do so, we need a different lexical entry for ‘MAY’, which we can compose out of  $\blacklozenge$ , our quantifier over the modal base, and  $\diamond$ , our operator expressing worldbound deontic permissibility. What each option in a Self-affirming Choice has is the property that it is ideal from the point of view of the  $R_{\square}$ -relation *on the condition that the agent performs it*. An option  $p$  is hence intuitively permissible with respect to a contextually supplied modal base  $W(c)$  if the agent can constrain the future so that  $p$  is both true and deontically ideal from the perspective she will occupy after she chooses.

Given the simplifying assumptions in play here, this condition’s holding at  $W(c)$  is equivalent to its holding at a single world  $v \in W(c)$ . This gives us Thesis 2:

**Thesis 2.**  $\text{MAY } p := \blacklozenge(p \wedge \diamond p)$

At the end of this section, I provide an axiomatization of the logic that results from putting Conventional Wisdom (1-a), Postulates 1-2 and Thesis 2 into the framework of (1 Dimensional) Standard Deontic Logic. First, though, I want to make a quick case for a property which simplifies that task. Self-affirming Choices illustrates that deontic ideality can depend on choice—in particular, that ideality can be *correlated* with choice. But giving a clean axiomatization of the logic will depend on whether deontic ideality can also be *anti-correlated* with choice. This would be the obverse of a Self-affirming Choice—call it a “Nasty Choice”—where one ought (from a local perspective) to do the opposite of whatever one actually does (Figure 7):

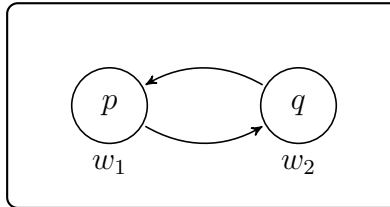


Figure 7: A model  $\mathcal{M}_{\text{nasty}}$  with negative correlation between act and negative deontic status, ruled out by Shift-Reflexivity of  $R_{\square}$ .

Is such a thing conceptually possible? To this, I suggest, following Vorobej (1982), that the answer is “no”. An extra condition governing modal accessibility relations, Shift (or Secondary) Reflexivity, would rule out the possibility of Nasty cases on the grounds that it is a conceptual truth that moral accessibility cannot self-undermine: in particular, if a world  $w$  is in the image of the deontic accessibility relation, then it must be deontically accessible to itself.<sup>16</sup>

<sup>16</sup>In more detail:  $R$  is shift-reflexive in a Kripke frame  $\langle W, R \rangle$  iff  $\forall w, v \in W, (wRv \rightarrow vRv)$ .

Why treat correlation between an act and a positive normative status (as in Figure 6) differently from correlation between an act and a negative normative status (as in Figure 7)? The answer, I think, is simply that positive normative status can be grounded in the will. On this limited “deontic voluntarist” view, it is possible for something  $[p]$  to be better (in the deontic sense) than something else  $[q]$  simply because the agent chooses it on *nonnormative* grounds.<sup>17</sup> In this, the normativity whose logical bones are modeled by the  $R_{\square}$ -relation is flexible enough to recognize a source of structure outside itself. For the difference shift-reflexivity makes, the key point is that where normativity is correlated with the will, that correlation is *positive*. *Inter alia*, such a view respects the asymmetric conceptual possibility of *permissive consent*—a morally explanatory notion par excellence in contemporary work on sexual autonomy and bodily integrity.<sup>18</sup> The unifying theme of these applications is moral indeterminacy prior to choice. In such areas, agents do not navigate a totally independently contoured moral landscape; rather, the moral landscape is contoured by, and finds itself in conformity with, their choices.

The left column of Table 1 axiomatizes the interaction of the historical and deontic modalities discussed in this section, using the traditional axiom names in the modal logic literature. As we say in the industry, it is sound and complete for the corresponding class of frames.<sup>19</sup> The right column reorganizes the same content in terms closer to a naturalized lexicon, by using (i) “MUST” and “CAN” for the historical box and diamond, (ii) the super-obligation hypothesis about “OUGHT” and (iii) Thesis 2 for “MAY”.

$T_{\blacksquare}) \blacksquare p \rightarrow p$	• MUST $p \rightarrow p$
$5_{\blacksquare}) \blacklozenge p \rightarrow \blacksquare \blacklozenge p$	• CAN $p \rightarrow$ MUST (CAN $p$ )
$Z) \blacklozenge p \rightarrow \blacklozenge p$	• MAY $p \rightarrow$ CAN $p$
$D_{\square}) \square p \rightarrow \blacklozenge p$	• OUGHT $p \rightarrow \neg$ OUGHT $\neg p$
$U_{\square}) \square(\square p \rightarrow p)$	• OUGHT(OUGHT $p \rightarrow p$ )
	• OUGHT $p \rightarrow$ MAY $p$
	• OUGHT $p \rightarrow$ CAN $p$

Table 1: Axioms for a normal modal logic.

What it means for  $\phi$  to be valid in this setting is that, for any bimodal serial shift-reflexive deontic Kripke model  $\mathcal{M}$ ,  $\mathcal{M} \models \phi$  holds. When we move into two dimensions to study

<sup>17</sup>I take the term, and the relevant scope of its limits, from Chang (2013).

<sup>18</sup>See, for example, Dougherty (ms); Pallikkathayil (2011, §2).

<sup>19</sup>In more detail: let  $1\mathcal{D}$  be the normal modal logic which contains all tautologies of propositional logic, the K axioms for  $\blacksquare$  and  $\square$ , all the axioms in the left column of Table 1, and is closed under the rules Modus Ponens and Necessitation for  $\blacksquare$  and  $\square$ . All the axioms of  $1\mathcal{D}$  are *canonical* in the sense of Blackburn et al. (2002, Ch. 4). Soundness and completeness for the class of frames defined by  $1\mathcal{D}$  follows immediately by the Sahlqvist Completeness Theorem (Blackburn et al. *op. cit.*, Theorem 4.42.)

lifted Kripke models  $\mathcal{M}^+$  and the settled-diagonal consequence relation  $\models$ , we have the result that: for any  $\phi$  is in the one-dimensional fragment of the lexicon,  $\mathcal{M} \models \phi$  entails that  $\mathcal{M}^+ \models \blacksquare \dagger \phi$ , and hence entails that  $\mathcal{M}^+ \models \phi$ .

### 3 Deontic Puzzles

This ends my brief for Theses 1 and 2. As advertised, I now want to move briefly on to the disjunction-involving puzzles of deontic logic. Here is our data for disjunction as it naturally occurs under “ought” and “may”:

- (4) a. You may have coffee or tea.  
 $\text{MAY}(p \text{ or } q)$
- b. [*felt entailment to:*] You may have coffee, and you may have tea.  
 $(\text{MAY } p) \wedge (\text{MAY } q)$  (Kamp, 1973)
- (5) a. You ought to post the letter.  
 $\text{OUGHT } p$
- b. [*felt lack of entailment to:*] You ought to post the letter or burn it.  
 $\text{OUGHT}(p \text{ or } q)$  (Ross, 1941)

Schematically:

(Free Choice)	$\text{MAY}(\phi \text{ OR } \psi) \Rightarrow \text{MAY } \phi \wedge \text{MAY } \psi$
(Ross)	$\text{OUGHT } \phi \not\Rightarrow \text{OUGHT}(\phi \text{ OR } \psi)$

Table 2: Data

Within deontic logic, the literature on these puzzles is enormous and richly varied. My way into them will be idiosyncratic, leveraging the indeterminist considerations I marshalled above.

With the schematic data in Table 2 in mind, consider an objection, from a Determinist, to the argument I made for Thesis 2. I §2 I claimed that both options are choiceworthy in Self-affirming Choices such as *Choosing Childbearing* and *Nice Choices at the Spa*, and used this claim to motivate the claim that Thesis 2 captures permissibility in bimodal deontic Kripke models. But perhaps my reasoning was specious on metaphysical grounds—that is, from the very opening move according to which we should see context as selecting a mere set  $W(c)$  of candidates for actuality. A determinist might push back as follows:

You have claimed that both acts are choiceworthy in Nice Choices, but in fact only one act is. Indeed, I can tell you which act! Let “Julius”, in each case, rigidly denote **the act that you will actually perform**. In both *Nice Choices at the Spa* and *Choosing Childbearing*, Julius is the unique choiceworthy act. It is your ignorance concerning which act you will actually perform that creates the *mere illusion* that you have two permissible options.

Julius is a “cheesy mode of presentation” (Dorr et al., 2014) of one of the available acts  $\{p, q\}$  in Figure 6. The view under consideration is that we can think of “Julius” as determinately referring to one of  $\{p, q\}$  at the relevant index of the context—that is, the time that the agent deliberates—though naturally, as she considers her choice, deliberation crowds out prediction and she does not know which act she refers to.<sup>20</sup> (Comparison: we can stipulate that “Brutus” refers to the last green patch in a green-to-blue sorites series. According to epistemicism (Williamson, 1996), “Brutus” will refer whenever I am faced with the sorites, though I can never know, of any patch that I am looking at, that “Brutus” refers to *it*. Thus I cannot guide actions by Brutus as a mode of presentation.) The Determinist’s claim is that this maneuver can defeat the intuition that both options are choiceworthy in a “Nice Choices” model.

If the reader has the suspicion that there is something fallacious about the determinist’s “epistemicist” view of actuality—the one appealed to in defining Julius as the act that will *actually* be performed—I, of course, agree. But given the conceptual availability of diagonalization, one need not embrace a fatalist view of actuality to consider the ramifications of introducing a denoting concept like “Julius” into deontic Kripke models, just as one need not embrace an epistemicist view of Gwendolyn’s future to interpret discourse (3). The definite description associated with “Julius” is:

(6) the act I [the agent] will perform

rigidifying this description, as the Determinist suggests, we get

(7) the act I [the agent] will **actually** perform

Julius is a useful (if cheesy) mode of presentation. Being told, in a decision situation regarding spa treatments, childbearing, or the exercise of sexual autonomy that you may perform Julius is tantamount to being told that you may do whatever it is you actually do. How many things are you *thereby* permitted to do, in such a case? A Determinist says: just one. An Indeterminist, tapping the indexical view of actuality,<sup>21</sup> says instead: however many things you *can* do, which is a question of how many distinct acts there are in the modal domain  $W(c)$ . For the Indeterminist, permission to do one thing—Julius—becomes de facto permission to do many things, because the permission you count as having, at a world in  $W(c)$ , moves in step with what your future self does in  $w \in W(c)$ .

That is, in essence, my account of free choice permission. The semantic claim is that a disjunction “ $p$  or  $q$ ”, when embedded in modal environments, functions, with respect to the disjuncts  $p$  and  $q$ , like “Julius” functions: it denotes, at a world  $w \in W(c)$ , whichever disjunct— $p$ , or  $q$ —is  $w$ -actually true. This semantic value for disjunctions comes from the classic literature on disjunctive questions (Groenendijk & Stokhof, 1982; Lewis, 1982). Since its definition makes tacit recourse to a rigidified notion of actuality, as in (7), we require two dimensions to model its semantic value.

<sup>20</sup>For the “deliberation crowds out prediction” thesis in decision theory, see Levi (1997) and ensuing literature.

<sup>21</sup>Lewis (1986, Ch. 1).



Here is a two-dimensional matrix for “ $p$  or  $q$ ” with respect to the two worlds of the “Nice Choice” in Figure 6.

	$p$	$q$
$w_1$	T	T
$w_2$	T	F
$w_3$	F	T
$w_4$	F	F

	$w_1$	$w_2$	$w_3$	$w_4$
$w_1$	T	T	T	F
$w_2$	T	T	F	F
$w_3$	T	F	T	F
$w_4$	T	T	T	F

Figure 8: 2D matrix for  $\lceil p \text{ OR } q \rceil$  (right), given the world-types  $w_1$ - $w_4$  (left).

We see that disjunction is equivalent to Boolean join of  $p$  and  $q$  under the dagger operator—that is, on the diagonal. This fact about disjunction is fully general, giving rise to a modal logic on which classical inference rules hold throughout the nonmodal fragment (Fusco, 2015b, Lemma 1), but the rule of disjunction introduction is blocked in the scope of deontic modal operators. This rules out the Ross inference from (5a) to (5b) (*op. cit.*, pg. 15). Free Choice is provable in this framework given that each embedded disjunct of e.g. (4a) is  $R_{\blacksquare}$ -accessible to the agent (*op. cit.*, Theorem 1).

In Fusco (ms), I prove completeness for the two-dimensional deontic logic sketched here, with recourse to the “lifted” canonical structures in Figure 4. The proof leverages the gloss on settled-diagonal truth provided by Thesis 1, and the paraphrase of permissibility provided by Thesis 2. Completeness sheds light on whether the nonclassical behavior of the non-Boolean disjunction in Figure 8 has contours which match the empirical phenomenon of disjunction under deontic modality, and whether it can manage this—as I say it can—without intolerable violence to our other instincts about what follows from what.

## 4 Conclusion

In this paper, I have worked, in a Thomasonian vein, with a *nonepistemic* modal base that represents options that are choosable to an agent. I used it to motivate the idea that the agent has control over what is permissible to her future self; this is the key both for motivating Thesis 2 and for seeing the “Julius”-type concept described above as licensing—via diagonalization—a kind of “double vision” with respect to how many things an agent may permissibly do at an act-prior context. This, I claim, is the key to understanding free choice permission. In closing, I want to bring the agentially indeterministic semantics for deontic modals sketched here into brief contact with the much larger recent literature on the *epistemic*, or *information* sensitivity of deontic modals—foregrounded, for example, in the proposed solution to the widely discussed “Miners’ Puzzle” of MacFarlane & Kolodny (2010). MacFarlane & Kolodny argued, on the basis of that puzzle and in the vein of Kratzer’s previous work, that deontic modals are sensitive to an informational modal base  $i$ . Moreover, following Kratzer (1991a) and Yalcin (2007), MacFarlane & Kolodny argued that  $i$  could be shifted by the antecedents of indicative conditionals.

Naturally, only confusion results from losing sight of the difference between  $p$ ’s being contingent or settled in the the *epistemic* sense relevant to MacFarlane & Kolodny’s project, and  $p$ ’s being contingent settled in the fundamentally *nonepistemic* sense modeled here by

the Thomasonian  $\blacksquare$  and  $\blacklozenge$ . But none of the considerations I have advanced in this paper are incompatible with deontic modals' ultimately displaying sensitivity to *both* kinds of modal background. Indeed, sophisticated treatments of our temporal talk, such as Beaver & Condoravdi (2012), invoke complex models in which both epistemic and nonepistemic indeterminacy are represented.<sup>22</sup>

For the interaction of modals and conditionals, a move parallel to the Kratzer-Yalcin-MacFarlane & Kolodny move can easily be made in the present framework, by adding to the index a shiftable parameter  $h$  whose context-initialized value is  $W(c)$ ; indeed, that is what I do in Fusco (2015a). This is isomorphic to adding a shiftable parameter over the domain of the  $S5$  accessibility relation  $R_{\blacksquare}$ ; the deontic submodality of  $R_{\blacksquare}$  axiomatized in Table 1 can be seen as a generalization of a deontic selection function (MacFarlane & Kolodny *op. cit.*, pg. 131) which—because of the possibility of Self-affirming Choices—permits different choice-worlds to differ on what is deontically ideal.

It is worth flagging, though, that on empirical grounds—when we look at natural language—we must proceed with caution in assuming the corresponding parallelism in interaction. Future-directed conditionals, especially those conversationally relevant to deliberation and action, are their own beasts, with their own distinctive behavior with regard to nonepistemic settledness.<sup>23</sup> As Dummett (1964)'s well-known discussion of fatalism highlights, speakers sometimes evaluate consequents of such conditionals as if their antecedents are settled true, and sometimes not; this gives us two readings, one fatalist and tautological, the other *non-fatalist* and *non-tautological*, of e.g. (8):

- (8) If I'm going to be killed, then even if I take precautions, I will be killed.  
 [ if  $p$  ] [ [ if  $q$  ]  $p$  ]  
 (Dummett, 1964, pg. 346)

Given that speakers find (8) both attractive *and* repellent, Dummett was dismissive of the idea that a semantics of conditionals should attempt to categorically resolve the question of whether the fatalist reading of e.g. (8) is built into the lexicon.<sup>24</sup>

To this, I will simply add that it seems easy to resist evaluating indicative consequents as if their antecedents are (nonepistemically) settled true, when we consider future-directed conditionals with *deontically modalized* consequents. This happens in (9):

<sup>22</sup>It is true that, in my argument in §1, I appealed to the simple assumption that the open future cannot be known—implicitly, to the idea that  $\blacklozenge p$ 's truth is incompatible with  $p$  being known—thus sidestepping exotic possibilities raised in the literature on crystal balls (Lewis, 1971; Hall, 1994) and on divine foreknowledge (Stump & Kretzmann, 1985; Rhoda et al., 2006).

<sup>23</sup>DeRose (2010) points out that, *inter alia*, it is difficult to tell whether such “conditionals of deliberation” are best classified as counterfactuals or indicatives. This may be relevant to whether speakers are sensitive to a “backtracking” reading of (8).

<sup>24</sup>To quote:

[conditionals like (8)] are clearly correct on many interpretations of “if”; and I do not propose to waste time by inquiring whether they are correct on “the” interpretation of “if” proper to well-instructed users of the English language. (Dummett *op. cit.*, pg. 246)

- (9) (Even) if you don't apologize, you ought to.  
 $[if \neg p] \text{ OUGHT } p$

(9) is naturally interpretable as true, without intuitively conflicting with the idea that OUGHT implies CAN.

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