Factoring Disjunction out of Deontic Modal Puzzles*

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Abstract

Ross's puzzle (Ross, 1941) and the paradox of Free Choice Permission (Kamp, 1973), puzzles involving disjunction under deontic operators, have received wide discussion in recent work in natural language semantics.

First, I contrast the opposed modal views—call them the "box-diamond" theory and EU theory—that form two poles of the contemporary debate. The opposition between them is underwritten by distinct, well-developed conceptions of what it is for an action to be good. I present an axiomatization of obligation and permissibility—of 'ought' and 'may'—that is *neutral* between the two theories. Adding in the interpretation of 'or' as Boolean union—that is, as the relevant kind of propositional fusion—we get the received dialectic in the literature between the two theories on explaining Ross and FCP. Factoring out this assumption, we get a picture of how far apart the two theories are as theories of value, with no questions begged about the semantics of sentential disjunction.

1 Introduction

In this paper I will discuss two puzzles. The first is Ross's Puzzle (Ross, 1941): from a premise like

(1) Alice ought to call her mother.
$Ought(C)$

one may not, it seems, infer

(2) Alice ought call her mother or rob the bank.
$Ought(C \text{ or } R)$

...despite the fact that disjunction introduction in the scope of 'ought' is valid on many semantic theories of 'ought' and 'or'. Call this

*Preprint of May 2014. Final version to appear in the Proceedings of the 12th International Conference in Deontic Logic and Normative Systems (DEON), Springer LCNS, 2014.

(Ross) $Ought(\phi) \not\Rightarrow Ought(\phi \text{ or } \psi)$

The second puzzle is the corresponding one for 'may' instead of 'ought'. From

(3) Alice may take the bus.
$May(B)$

it seems unreasonable to infer that

(4) Alice may take the bus or hijack a car.
$May(B \text{ or } H)$

The failure of the inference from (3) to (4) has a better-known positive half: the paradox Free Choice Permission (von Wright, 1969), by which (4) (but not (3)) seems to entail (5):

(5) Alice may take the bus and Alice may hijack a car.
$May(B) \wedge May(H)$

Call this:

(FC) $May(\phi \text{ or } \psi) \Rightarrow May\ (\phi) \wedge May\ (\psi)$

In this paper, I shall focus mostly on the weaker, negative datum, which we can call (FC-):

(FC-) $May(\phi) \not\Rightarrow May(\phi \text{ or } \psi)$

This data has, in the literature, primarily been interpreted as bearing on the interpretation of deontic modals rather than bearing on the interpretation of 'or'.[1] As I will explain in §3, it makes tempting an expected utility (EU) approach to these operators. My goal in this paper is to contrast this strategy with an approach to (Ross) and (FC-) from a less-examined angle: a revisionary semantics for disjunction.

First, I contrast the opposed modal views—call them the "box-diamond" theory and EU theory—that form the two poles of the debate about these natural language modals. The opposition between them is underwritten by distinct, well-developed conceptions of what it is for an action to be permissible. I present an axiomatization of obligation and permissibility—of 'ought' and 'may'—that is *neutral* between EU and box-diamond theories: both assign the same truth-conditions when applied to prejacents that describe actions which are *basic* in the

[1] See, for example, Portner (2010); Cariani (2011); von Fintel (2012), and von Wright himself (von Wright, 1969).

relevant model, but different truth-conditions when applied to prejacents that are multiply realizable in the model.

Adding in the interpretation of 'or' as Boolean union—that is, as the relevant kind of multiple realizability—we get the received dialectic in the literature between the two theories on explaining (Ross) and (FC-). Factoring out this assumption, we get a picture of how far apart the two theories are as theories of value, with no questions begged about disjunction. In the rest of the paper, I take up this position to lay out a theory of 'or' in a 2-dimensional semantic framework that both camps should be able to agree to; these are conditions under which a semantics for 'or' *could* block disjunction introduction by the lights of either side, since it is framed from a neutral standpoint. Finally, I respond to an argument from Cariani (2011) that raises a challenge for revisionary theories of disjunction.

2 The Standard Modal Theory versus Disjunction

Let us begin with a standard modal approach to deontic operators. To simplify our demands on the model—to avoid making stipulations, in particular, about accessibility relations—I consider a language without iterated modalities.

Definition 1 (Well-Formed Formulas and Models)**.** *Let* `prop` *be a set of propositional well-formed formulas such that, if* $\phi, \psi \in$ `prop`*, so is* $\ulcorner \phi$ *or* $\psi \urcorner$*. Any* $\phi \in$ `prop` *is a well-formed formula. For any* $\phi \in$ `prop`*,* $\ulcorner Ought(\phi) \urcorner$*,* $\ulcorner May(\phi) \urcorner$ *are also well-formed formulas.*

A model $\mathcal{M}$ *is a tuple* $\langle W, OPT, V \rangle$*, where* W *is a nonempty set of possible worlds,* $OPT : W \rightarrow \mathcal{P}(W)$ *is a function from a given world* w *to a sets of worlds (worlds 'deontically ideal' from the point of view of* w*), and* $V : \texttt{wff} \times W \rightarrow \{0, 1\}$ *is a recursive valuation function on well-formed formulas in* `prop`*.*

It will also be convenient to speak of the intension $I(\phi)$ of a sentence ϕ, defined in terms of the extensional valuation function, V:

Definition 2 (Intensions)**.** $I(\phi) = \{w' : V_{\mathcal{M}}(\phi, w') = 1\}$.

The standard deontic modals quantify existentially and universally, respectively, over worlds in $OPT(w)$.

Definition 3 (Quantificational Modals)**.** $\mathcal{M}, w \vDash May(\phi)$ *iff* $\exists w' \in OPT(w)$*:* $w' \in I(\phi)$*.*
$\mathcal{M}, w \vDash Ought(\phi)$ *iff* $\forall w' \in OPT(w)$*:* $w' \in I(\phi)$*.*

What counts as deontically ideal relative to a world w—which worlds are in $OPT(w)$—may be context-sensitive; this sensitivity may include, but perhaps not be limited to, what is *known* at w.[2]

[2]The premise semantics of Angelika Kratzer (Kratzer, 1981) is a generalization of this theory, according to which a set of premises determines what counts as good, where these premises may be inconsistent. The

It is a result of the semantic entries for the quantificational modals that they are *Upward Closed*:

(Consequence)	$\phi \vDash \psi$ iff, for any model $\mathcal{M}$ and any $w \in W_{\mathcal{M}}$, if $\mathcal{M}, w \vDash \phi$, then $\mathcal{M}, w \vDash \psi$.
(Upward Closure (UC))	An operator O is upward-closed just in case, if $\phi \vDash \psi$, then $O(\phi) \vDash O(\psi)$.
((UC) for Deontic Modals)	If $\phi \vDash \psi$, then $Ought(\phi) \vDash Ought(\psi)$ and $May(\phi) \vDash May(\psi)$.

This result is discomfited by (FC-) and (Ross), where embedded 'or' introduction seems to be blocked. For on a Boolean 'or', $\phi \vDash (\phi$ or $\psi)$. In terms of I, the intension $I(\phi$ or $\psi)$ of a disjunction, relative to any world w, is just $I(\phi) \cup I(\psi)$; it is a *multiply realizable* outcome that obtains in any ϕ-world and in any ψ-world. To the extent to which (Ross) and (FC-) strike us as problematic inferences, whatever is wrong with them must be explained in the pragmatics, rather than in the semantics. At first blush, this doesn't look hard to do for 'ought' (see, for example, Wedgwood (2006), Follesdal & Hilpinen (1971)). Kratzer & Shimoyama (2002) take a similar approach to the prima facie the more complex case of 'may': disjunction introduction under 'may' gives rise via a *second-order* implicature to the effect that both are permissible. My purpose here is not to weigh in on these projects, but simply to point out that such views must appeal to pragmatic resources to explain the failure of embedded disjunction introduction, given this consequence in the semantics.

3 EU to the Rescue?

A much different reaction to the data in (FC-) and (Ross) is to use it to overturn the standard modal operator semantics, and to give new entries for 'ought' and 'may' that respects these inferences as semantic.

This route models 'ought' and 'may' as reflecting the notions of obligatoriness and permissibility that are found in Expected Utility Theory. Expected Utility Theory enjoins an agent perform the act with the highest expected utility, or one of these options, when there are ties.

Definition 4 (EU Models and Expected Utility)**.** *An EU-model*[3] $\mathcal{M}$ *is a tuple* $\langle W, Pr, Val, Act, V\rangle$

result is that worlds may be *ordered* by context, according to how many premises they satisfy. Modulo the Limit Assumption (Lewis, 1973), it will still be the case that 'ought' is a univeral quantifier, and 'may' is an existential quantifier, over a modal base, which can be characterized as follows: any world in the modal base satisfies more premises than any world outside the modal base. For Kratzer's discussion of the Limit Assumption, see Kratzer (1981), §3.

[3]There are many expected utility models in the literature; the simplified one I present here most closely follows Goble (1996).

such that W is a nonempty set of possible worlds and Pr is a probability function on $\mathcal{P}(W)$; for any $w \in W_{\mathcal{M}}$, Val_w is a function $\mathcal{P}(W) \to \mathbb{N}$ which, at a world w, takes a proposition p to a natural number (the utility of p, relative to w);[4] *$Act \subseteq \mathcal{P}(W)$ is a set of available acts (closed under union), and V is a valuation function on well-formed formulas.*

Where $I(\phi) \in Act_{\mathcal{M}}$, $EU_w(I(\phi))$ is the expected utility of $I(\phi)$: $\sum(Pr(w_j|I(\phi)) \cdot Val_w(w_j)$ for all $w_j \in W_{\mathcal{M}}$.

The expected utility of ϕ is maximal *relative to an EU-model $\mathcal{M}$ and world $w \in W_{\mathcal{M}}$ iff $I(\phi) \in Act_{\mathcal{M}}$ and $\neg\exists q \in Act_{\mathcal{M}} : EU_w(q) > EU_w(I(\phi))$.*

Definition 5 (EU Modality)**.** *$\mathcal{M}, w \vDash May(\phi)$ iff $I(\phi) \in Act_{\mathcal{M}}$ and $EU_w(I(\phi))$ is maximal. $\mathcal{M}, w \vDash Ought(\phi)$ is true iff $I(\phi) \in Act_{\mathcal{M}}$ and $I(\phi) = \bigcup\{p \in Act_{\mathcal{M}} : EU_w(p)$ is maximal$\}$.*

EU modals have a swift take on the negative data in (Ross) and (FC-): the problematic inferences are not semantically valid. Whereas the quantificational modals are upward-closed, the EU notion of permissibility is *downward closed*: if $\phi \vDash \psi$, then $May(\psi) \vDash May(\phi)$. Since the expected utility of a multiply realizable option p is the (probability-weighted) average of its realizations, p's EU will be maximal only if the EU of all its realizations is also maximal.[5] Interpreting Boolean 'or' as multiple realizability, we get the result that, for example, if it is EU-permissible to have coffee or tea, then both the coffee option and the tea option must be EU-permissible.

$$\text{if } \phi \vDash (\phi \text{ or } \psi), \text{ then } May(\phi \text{ or } \psi) \vDash May(\phi)$$

Because EU permissibility is downward entailing, and EU optimality entails the EU permissibility of any option, Boolean disjunction introduction is also blocked in the scope of EU-'Ought.' From the EU point of view, given Boolean disjunction, we get (Ross), (FC-) and the positive datum (FC) all in one go.

4 Does Natural Language Semantics Reflect EU Permissibility?

The ease with which the EU modals account for the puzzles of disjunction under modals raises a natural question: has anyone ever embraced these views? To my knowledge, no one has embraced both EU modals as a package, but they have appeared individually in the literature as a response to our puzzles.

(EU-'May') is EU permissibility imported directly into the object language: if a proposition p is permissible and multiply realizable in context, then every realization, or every *way*, of doing p must be permissible. Such a notion of permissibility—*strong permissibility*—was proposed by von Wright (1969), who, in turn, was originally motivated by the Free Choice

[4]It is point familiar from decision theory that an individual's preferences should be modeled by a *family* of such functions, unique only up to positive affine transformation von Neumann & Morgenstern (1944). I abstract from this detail here.

[5]I ignore zero-probability options here.

puzzle.[6] von Wright argued that sometimes, what it means to say "you may ϕ" to someone is to give him or her permission to ϕ "in every way." In this vein, EU theory can be seen as an extensive exploration and formal development of von Wright's notion of strong permissibility—the notion of permissibility which, to von Wright's ear, was simply manifest in (some) natural language uses of 'may.' Von Wright did not, however, have anything like this to say about 'ought.'

For hints of an inclination towards (EU-'Ought'), we can look to Goble (1996), Lassiter (2011) and Cariani (2011). Lassiter notes simply that $EU(\phi) \geq \theta$ does not imply $EU(\phi \vee \psi) \geq \theta$, where θ is some threshold for expected utility (26). Cariani's semantics for 'Ought(ϕ)' requires that $I(\phi)$ be an option in context and that every atomic act in $I(\phi)$ be above some 'benchmark' of permissibility that is accessible in the metalanguage. This is enough to block disjunction introduction in the scope of 'ought,' on roughly the same grounds as a more straightforward *EU* semantics would: the failure of the inference is explained by way of holding that the introduced disjunct is not (strongly) permissible. The main difference is whether this type of permissibility *is* (von Wright) or *isn't* (Cariani) identified explicitly with same brand of permissibility that provides the semantics for the object-language 'May.'

Is it true that there is a downward-entailing notion of permissibility that is active in the semantics of our deontic talk? The theory has some drawbacks, which I'll canvas here, first for an object-language theory of 'ought' (for Goble, Lassiter, and Cariani) and then for an object-language theory of 'May' (von Wright).

4.1 'Ought' as Requiring Strong EU-Permissibility

If the truth of $\ulcorner Ought(\phi) \urcorner$ at a model requires the EU-permissibility if $I(\phi)$, ϕ cannot be the prejacent of a true "ought" claim unless every more fine-grained act which is a way of carrying out ϕ is *EU*-permissible. Disjunctive cases aside, is this claim plausible?

The first drawback concerns an analogy with decision-making—it doesn't seem like we use a principle like this in deciding what to do. But this raises doubts about whether it could really be a hidden feature of what we *ought* to do. Call this problem (Means-Ends); we do not limit ourselves to actions such that *every* way of carrying them out is permissible.

To illustrate this, consider the case of

> (Professor Punctual.) Professor Punctual is invited to review a book on whose subject matter he is the world's foremost expert. If Punctual accepts the invitation and writes the review, the book will receive a high-quality assessment—this is the best possible outcome. If Punctual accepts and does not write, the delay will constitute an injustice to the author and an embarrassment for the journal. If Punctual declines the invitation, another, less-qualified person will write a mediocre review. Finally, Professor Punctual is dutiful. He indefatiguably fulfills his commitments in a timely manner.

[6]See, for example, (von Wright, 1969, pg. 26).

It seems perfectly normal for Professor Punctual to accept, and overwhelmingly natural to say that he *ought* to accept. However, there is a salient way of accepting the invitation to write the review that would bring about the worst possible outcome (this is obviously a feature Punctual's case shares with the case of his better-known co-author, Professor Procrastinate (Jackson & Pargetter, 1986).) If Strong Permissibility is really a necessary condition on the truth of "ought" claims, "Professor Punctual ought to accept" is false. But this doesn't seem right; it doesn't seem to be a necessary condition on the truth of $\ulcorner Ought(\phi) \urcorner$ that *every* way of ϕ-ing is permissible.

Is it possible that, because of Punctual's punctuality in w, the option of accepting and failing to write is not represented in the model's set of acts at w? Consider dialogues with *fronted* alternatives:

(6) a. May I bring some wine to the party?
    b. No—the host is allergic. But you ought to bring something.

On a straightforward application of (EU-'Ought'), this dialogue is inconsistent. In (6), the possibility of bringing wine to the party is explicitly raised and classified as impermissible. But then it seems that it cannot be true that *every visible way* of bringing something to the party is above benchmark. Yet by (6-b), "You ought to bring something to the party" is true.

4.2 Strong 'May'

The problems for (EU-'May') mirror the problems for the strong permission theory of 'Ought.' It seems we can construct Professor cases in which it is true that

(7) Punctual may accept.

but it is false that

(8) Punctual may accept and fail to write.

So it seems, as much as in the 'ought' case, that the requirement *that every way of accepting be permissible* is too strong. Even when we temper this claim with the proviso that it is only the *represented* or *salient* ways of accepting that must be permissible, we can generate cases with fronted alternatives:

(9) a. May I bring some wine to the party?
    b. No—the host is allergic. But you may bring *something*.

Deliberatively, as well, (Means-Ends) resurfaces for the 'may' case: it is implausible that we take this piecemeal approach to action, at each earlier moment minimizing the harm we can do at some later moment: rather, we often undertake actions which will make things go much worse, if we fail to follow through. Since this is a pervasive feature of the kinds of actions we *do* undertake, it is hard to believe that the model for our deontic talk would tell us that we *may* not do such things.

My interest, in the rest of this paper, is in isolating an argument for blocking embedded disjunction introduction that doesn't rely on 'Ought' and 'May' being downward entailing—in fact, is compatible with their being *upward*-entailing. The EU theorist has a shorter way home, of course. But if I can do this, I can offer someone tempted by the EU modals a way to get the data without having to bite the bullets in (Punctual), (Fronted Alternatives), and (Means-Ends). (FC-) and (Ross) can, perhaps, be had for less.

5 Another Route

The first thing to do is to isolate what the two competing theories of 'Ought' and 'May' have in common. Both begin with the notion of a fine-grained possibility: fine-grained, that is, with respect to their relevant models. For modal logic, a fine-grained possibility is a *possible world*; for EU theory, this is an atomic act in the set *Act*. Some of these possibilities are good, according to the model, and some are not; call the good ones P-states ('P' for *permissible*.) The two theories are different in how they interpret the normative status of multiply realizable possibilities—how they interpret the information that one's action will place one within a *set* of fine-grained options, some which are P-states and some of which are not-P ($\overline{P}$) states.

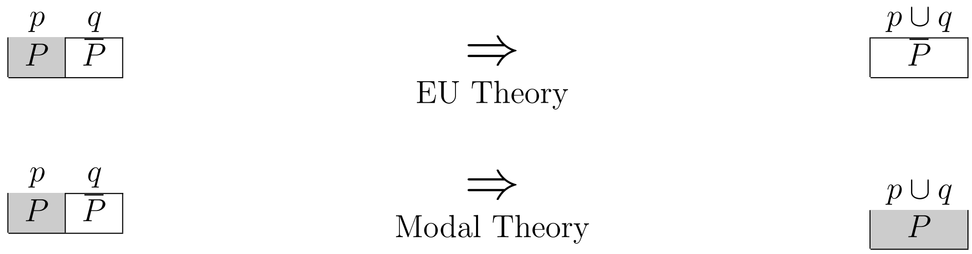
Definition 6 (P-States in EU Theory and Deontic Logic)**.**
EU Theory.
(Base Case). If q is an atomic act such that $EU_w(q)$ is maximal in $\mathcal{M}$, then q is an atomic P_w-state. Otherwise it is an (atomic) $\overline{P}_w$-state.
(Recursive Clause). Any union of P_w-states and $\bar{P}_w$-states is a $\overline{P}_w$-state.

Classic Deontic Logic.
(Base Case). If w' is a possible world such that $w' \in OPT(w)$, then $\{w'\}$ is an atomic P_w-state. Otherwise it is an (atomic) $\overline{P}_w$-state.
(Recursive Clause). Any union of P_w-states and $\overline{P}_w$-states is a P_w-state.

Visually, under union, an *EU* theory sees the $\overline{P}$ status as infective: it takes any multiply realizable option to $\overline{P}$, since averaging maximal and non-maximal expected utilities will always result in a lower-than-maximal expected utility.[7] The modal theory is more forgiving: it interprets the P status as *modal compatibility* with the best outcome(s), and if a proposition p is modally compatible with the best outcome(s), then so is any superset of p.

From this perspective, both semantic theories endorse the following semantics for 'Ought' and 'May':

[7]Once again, I ignore the case of zero-probability propositions.



Observation 1 (Common Core)**.** *For any EU model or modal model $\mathcal{M}$ and any $w \in W_{\mathcal{M}}$, $\mathcal{M}, w \vDash May(\phi)$ iff $I(\phi)$ is a P_w-state in $\mathcal{M}$.*
$\mathcal{M}, w \vDash Ought(\phi)$ iff (i) $I(\phi)$ is a P_w-state in $\mathcal{M}$, and (ii) no proposition p disjoint from $I(\phi)$ is such that p is a P_w-state in $\mathcal{M}$.

This factorization of the views is convenient, because it divides them into a shared starting point (the basic notion of a P-state and its relation to the object language) and two nonequivalent notions of how the status of being a P-state propagates up under propositional union. According to the Boolean 'or', disjunction *just is* propositional union. So if we add Boolean 'or' to this picture, we get the received dialectic: (Punctual), (Fronted Alternatives) and (Means-Ends) on one side of a sharp divide, and (FC-) and (Ross) on the other—such that we cannot interpret both sets of inferences in terms of semantic consequence.

6 Disjunction in 2 Dimensions

Let us (i) keep the common core of 'ought' and 'may' axiomatized according to P-states, and (ii) reject the Boolean idea that

$$\phi \vDash (\phi \text{ or } \psi).$$

There are many frameworks which reject unrestricted disjunction introduction (for example, linear logic and relevance logic). What I propose to explore here, though, is fleshing out (ii) by going to a 2-dimensional semantics, as in Kaplan (1989); Davies & Humberstone (1980).

According to a 2-dimensional semantics, the interpretation function V on sentences ϕ in the language must be evaluated relative to *two* worlds in $W_{\mathcal{M}}$, a world-as-actual (call this 'y') and an evaluation world (call this 'x'). So instead of

$$V_{\mathcal{M}}(\phi, w) \in \{0, 1\}$$

we have

$$V_{\mathcal{M}}(\phi, x, y) \in \{0, 1\}$$

Now, the intension of a sentence ϕ is once again a set of worlds, but this set must be relativized to y, the world-as-actual; instead of

$$I(\phi) = \{w' : V_{\mathcal{M}}(\phi, w') = 1\}$$

we have

$$I(\phi, y) = \{w' : V_{\mathcal{M}}(\phi, w', y) = 1\}$$

for a well-formed formula ϕ and $x, y, w \in W$.[8]

The relativity of $I(\phi)$ to a world $y \in W$ allows us to model the idea that ϕ might express different intensions at different possible worlds. For example, if, in w_1, Alice called her mother, but in w_2, Alice forgot to call her mother, we might like to say that "It ought to be that Otto does what Alice actually did" is true in w_1 and false in w_2, in virtue of the fact that "Otto does what Alice actually did" expresses a different intension in w_1 than it does in w_2. Intuitively, w_1 and w_2 differ, not in respect of what is morally required at each, but in virtue of what is expressed by "what Alice actually did" in each.

The simplest upgrade of our deontic modals to a two-dimensional system will reflect the sensitivity of intensions to the world-as-actual.

Definition 7 (2D Modals)**.** *$\mathcal{M}, x, y \vDash May(\phi)$ iff $\{w' : V(\phi, w', y) = 1\}$ is a P_x-state.*

$\mathcal{M}, x, y \vDash Ought(\phi)$ iff (i) $\{w' : V(\phi, w', y) = 1\}$ is a P_x-state, and (ii) no proposition p disjoint from $\{w' : V(\phi, w', y) = 1\}$ is such that p is a P_x-state.

With these new points of evaluation, we distinguish two relevant notions of consequence, which I will call *diagonal* ($\vDash_D$) and *unrestricted* ($\vDash$), respectively:

Definition 8 (Notions of Consequence)**.** *For any well-formed formulas ϕ and ψ:*
$\phi \vDash_D \psi$ iff for all $w \in W_{\mathcal{M}}$, if $\mathcal{M}, w, w \vDash \phi$, then $\mathcal{M}, w, w \vDash \psi$.
$\phi \vDash \psi$ iff for all $x, y \in W_{\mathcal{M}}$, if $\mathcal{M}, x, y \vDash \phi$, then $\mathcal{M}, x, y \vDash \psi$.

Following a common strain in 2D semantics, let us assume that it is *diagonal* that most closely approximates intuitive consequence relations between natural language sentences.[9]

With all this on board, the non-Boolean 'or' we need, I suggest, is just an 'or' such that Disjunction Introduction is valid at diagonal points, but not at nondiagonal points.

Proposal 1 (A Non-Boolean 'or')**.** *$\phi \vDash_D (\phi \text{ or } \psi)$, but $\phi \nvDash (\phi \text{ or } \psi)$.*

[8]Going forward, I implicitly retain the idea that the intension of ϕ relative to a point of evaluation in the model is both (i) a set of possible worlds and (ii) the only notion of compositional semantic value that embeds under deontic modals. This contrasts with an *inquisitive semantics* approach to Free Choice Permission and Ross's Paradox in the vein of Aher (2012); Ciardelli & Aloni (2012); Ciardelli et al. (2013), and Roelofsen (2013).

[9]See, for example, the corresponding notion of validity in Kaplan (1989, pg. 547), and the notion of *real world validity* in Davies & Humberstone (1980).

6.1 Putting It All Together

I claimed above that a non-Boolean semantics for 'or' could offer an explanation of (FC-) and (Ross) that both theories of $\ulcorner Ought(\phi)\urcorner$ and $\ulcorner May(\phi)\urcorner$ could accept. The relevant feature of both theories is that, in 2 dimensions, each requires the semantic value of the embedded formula ϕ to be evaluated at nondiagonal points, but only requires the modalized sentences $\ulcorner Ought(\phi)\urcorner$ and $\ulcorner May(\phi)\urcorner$ to be evaluated at diagonal points.

> $\phi \vDash_D (\phi \text{ or } \psi)$ iff, for any $\mathcal{M}$ and $w \in W_{\mathcal{M}}$,
> if $V_{\mathcal{M}}(\phi, w, w) = 1$, then $V(\ulcorner\phi \text{ or } \psi\urcorner, w, w) = 1$.

> $May(\phi) \vDash_D May(\phi \text{ or } \psi)$ iff, for any $\mathcal{M}$ and $w \in W_{\mathcal{M}}$,
> if $\{w' : V_{\mathcal{M}}(\phi, w, w') = 1\}$ is a P_w-state in $\mathcal{M}$, then $\{w' : V_{\mathcal{M}}(\ulcorner\phi \text{ or } \psi\urcorner, w, w') = 1\}$ is a P_w-state in $\mathcal{M}$.

On the non-Boolean 'or', the inference from $\phi \vDash_D (\phi \text{ or } \psi)$ to $May(\phi) \vDash_D May(\phi \text{ or } \psi)$ fails, since $V(\phi, w, w') = 1$ does not entail $V(\ulcorner\phi \text{ or } \psi\urcorner, w, w') = 1$. The inference to $Ought(\phi) \vDash_D Ought(\phi \text{ or } \psi)$ fails for the same reason. The 2D deontic modals are upward closed—but only when we consider prejacents ϕ and ψ such that ψ is a *general* consequence, and not merely a *diagonal* consequence, of ϕ. We can preserve upward closure and still block disjunction introduction; we just have to flesh out a 2-dimensional non-Boolean 'or,' which coincides with Boolean 'or' at diagonal points, but departs from it off the diagonal.

6.2 A Comparison: "I am here now"

What would it look like to have a logic in which $(\phi \text{ or } \psi)$ is a diagonal, but not an unrestricted, consequence of ϕ? Disjunction introduction will pattern with cases in which it is valid to introduce a disjunct *outside* the scope of an upward-entailing intensional operator O, but not *inside* its scope. The status of disjunction introduction—the inference from ϕ to $(\phi \text{ or } \psi)$—will be an *a priori contingent* inference, in the sense of Evans (1977). It is like one's knowledge of the truth of the sentence

(10) I am here now.
IHN

Since (10) is true at all diagonal points, conjoining it with any sentence will preserve truth at a diagonal point. We might call an inference rule that reflects this fact '$\wedge\ IHN$'-Introduction: from any ϕ, conclude $(\phi\ \wedge\ IHN)$.

For example, if 2+2 = 4, then 2+2=4 and I am here now; but from the fact that it is (metaphysically) necessary that 2+2=4, it does not follow that it is (metaphysically) necessary that (2+2=4 and I am here now), since it is not metaphysically necessary that I am here now.

'$\wedge$ IHN' Introduction	$\dfrac{\phi}{\phi \wedge IHN}$	$\dfrac{O(\phi)}{O(\phi \wedge IHN)}$
	valid	invalid
'or ψ' Introduction	$\dfrac{\phi}{\phi \text{ or } \psi}$	$\dfrac{O(\phi)}{O(\phi \text{ or } \psi)}$
	valid	invalid

Disjunction introduction—an 'or ψ' rule—works the same way. For example, if I am mailing the letter, it follows that I am mailing it or burning it; but from the fact that I *ought* to mail the letter, it does not follow that I ought to mail it or burn it. This is the perspective we can begin to get from the semantics of the non-Boolean 'or.'

7 Coda: Is "Blaming Disjunction" Too General?

In this paper, I've given an overview of the debate over disjunction within the scope of deontic modals, and sketched the ground for a semantic explanation of the data which jettisons the Boolean 'or.' I've merely laid a groundwork, of course, for I still haven't even begun to offer an explanation of (FC)—the positive inference for which the failure of in-scope disjunction introduction is merely the negative half. However, what we've done already accomplishes something: it is compatible with upward closure for the modals, and it begins to explain how it is that disjunction introduction might be *unimpeachable*, but also *unembeddable*.

In closing, I'd like to consider an objection, advanced by Cariani (2011), to my approach to (FC-) and (Ross) via disjunction (an approach Cariani calls a "BD" approach, for "blame disjunction.") Cariani's claim bears direct quotation: BD accounts are too general, because they

> do not predict that deontic modals and epistemic modals should give rise to disanalogous predictions. In fact they naturally predict the opposite—that an epistemic 'must' taking scope over a disjunction should pattern in the relevant respects with a deontic 'ought' in the same position. (21)

It would be bad, I think, if this outcome were predicted by the approach I just sketched. But it isn't predicted, as should by now be clear. Epistemic modals, whatever their precise semantics is, should generate a logic in which sentences true at all diagonal points—the *a priori truths*—are axioms. This is just to say that, for example,

(11) $\Box_e$(I am here now)

should be true at any diagonal point (with '$\Box_e$' marking that the relevant necessity is epistemic) just as its unembedded prejacent should be.

It is a point familiar from Kaplan's own remarks that we can capture what is distinctive about a priori truths by looking at what is true at every diagonal point (see, for example, (Kaplan, 1989, pg. 509).) The most natural way of marking these a priori truths in the object language is with epistemic necessity operators, and indeed a 'monstrous' approach to them—where one quantifies over diagonal points, rather than points that are constant in one of the two dimensions—has been proposed as the distinctive feature of epistemic operators by Perry & Israel (1996); Weatherson (2001), and others. It is epistemically necessary that I am here now, but it is not deontically necessary; it could well be permissible for me to be elsewhere. That is just the pattern we recapitulate on our nascent semantics for 'or': 'or'-introduction is predicted to be valid in the scope of upward-entailing epistemic operators—including, of course, epistemic 'must'—but not valid in the scope of our deontic operators, 'ought' and 'may.'

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