Dutch-Booking Indicative Conditionals*

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Abstract

Recent literature on Stalnaker's Thesis, which seeks to vindicate it from Lewis (1976)'s triviality results, has featured linguistic data that is prima facie incompatible with Conditionalization in iterated cases (McGee 1989, 2000; Kaufmann 2015; Khoo & Santorio, 2018). In a recent paper (2021), Goldstein & Santorio make a bold claim: they hold that these departures light the way to a new, non-conditionalizing theory of rational update.

Here, I consider whether this new form of update is subject to a Dutch book. On the official, invariantist version of the theory, I show that the answer is "yes". On a competing, contextualist theory of indicative conditionals (Bacon, 2015), the answer is "no", for reasons that have familiar connections to the limits of textbook Bayesianism. After presenting a concrete case, I explore the dialectical ramifications. The upshot is some hard choices for theories that seek to save the linguistic phenomena.

1 Introduction

Suppose I say to you, of a fair, six-sided die:

(1) If it doesn't come up 1, it'll come up even.

What do you suppose is the probability of (1)? A natural answer is 3/5; in a probability distribution Pr wherein each face of the die is given equal probability, this is the conditional probability $Pr(\text{even} \mid \neg 1)$.[1]

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[1] Where $Pr(\cdot|E)$ is equal to $Pr(\cdot \wedge E)/Pr(E)$ whenever the latter is defined. The equation $Pr(A|E) = Pr(A \wedge E)/Pr(E)$ is called the *Ratio Formula*.

Suppose now that you learn that the antecedent of (1) is true: the die didn't come up 1. What is your subjective probability in the consequent of (1), viz. (2)?:

(2) The die came up even.

Again, it seems that a reasonable answer is Pr(even | $\neg$ 1), viz., 3/5.

Simple intuitions like these illustrate the attractions of Conditionalization and Stalnaker's Thesis (henceforth ST). Conditionalization is a claim about the correct probability to assign a sentence after a learning event.[2] It says that for any agent with credence $Pr(\cdot)$ at time t, if the agent learns exactly E, her new credence function should be $Pr(\cdot|E)$. (ST) is a claim about the semantics of indicative conditionals (like (1)). It says that the probability of a conditional $A \to B$, or $Pr(A \to B)$, is equal to the conditional probability $Pr(B|A)$.

This paper is about the force exerted on the semantics of indicative conditionals—in respect of which (ST) has been a longstanding goal—by Dutch Books, which dramatize a well-known argument for Conditionalization. Recent literature on indicatives highlights linguistic data that is prima facie incompatible with (ST) in iterated cases (McGee, 1989, 2000; Kaufmann, 2015; Khoo & Santorio, 2018; Khoo, 2020). In a new paper (2021), Simon Goldstein & Paolo Santorio make a bold claim: they hold that these departures light the way to a new, non-conditionalizing theory of rational update.

Here, I consider whether this new form of update is Dutch bookable. On the official, invariantist version of Goldstein & Santorio theory, I show that the answer is "yes". On a competing, contextualist theory of indicatives (Bacon, 2015), the answer is "no", for reasons that have familiar, but often-overlooked, connections to the limits of textbook Dutch Book (henceforth DB) arguments. After discussing a concrete case, I consider some objections, and dip a toe into the ramifications.

[2] I speak here of objects in the range of the probability function Pr as *sentences* rather than propositions, or as elements of a σ-algebra, for reasons that will become clear below.

2 Puzzlement

We begin with a fresh set of intuitions about the fair, six-sided die—only this time we will also look at right-nested conditionals.[3] Intuitively, the probability of (3) is 1/4:

(3) If the die does not land (two or four), then it will land six.

This is the familiar conditional probability $Pr(6 \mid \neg(2 \vee 4))$. In addition—again, intuitively—the probability of (4) and (5) are each 1/2:

(4) The die will land even.

(5) The die will land odd.

Suppose now that we know we'll learn whether the die landed even or odd. I say to you:

(6) If the die lands even, then (if it does not land (two or four), it will land six).

What do you suppose is the probability of (6)? A natural answer is 1: (6) is certainly true.

In addition,

(7) If the die lands odd, then (if it does not land (two or four), it will land six).

appears to be certainly *false*: it has an apparent probability of 0. Summarizing:

	form	$Pr(\cdot)$
(3)	$A \to B$	1/4
(4)	C	1/2
(5)	D	1/2
(6)	$C \to (A \to B)$	1
(7)	$D \to (A \to B)$	0

Table 1: Intuitive probability assignments

[3] This example is Goldstein & Santorio (2021)'s own.

2.1 A Dutch Book

The intuitions in Table 1 defy Conditionalization, and we can Dutch book them.

What, exactly, is a DB? Suppose we have an agent who has credence $Cr(\phi)$ in ϕ. We assume that $Cr(\phi)$ thus determines her fair price for a standard bet that ϕ, and that she is willing to buy or sell any number of standard bets if she regards the price as fair or better.[4] A DB theorem is a biconditional of the following form: there will be a system of fair bets in which the agent is bound to lose money, in any possible world, **iff** $Cr(\cdot)$ fails to satisfy conditions F. A DB argument presents a DB theorem, and draws from it the conclusion that agents' systems of belief should have feature(s) F. Lewis and Teller (Teller, 1973) are credited with establishing that there is a DB if one doesn't update $Cr(\cdot)$ by conditionalization.

Suppose an agent ("you") has the credences in Table 1. To frame the Dutch book, we note the following: (i) there is an upcoming learning event (corresponding to (4) and (5)) in which each of two mutually exclusive, jointly exhaustive events gets probability 1/2; (ii) you have current credence *one-quarter* in $A \rightarrow B$; (iii) you think (from (7)) that ¬*even* (viz., D) is *not compatible* with $A \rightarrow B$.

Let Bets 1 and 2, listed below, be offered to you by a shady Dutchman on the basis of this information. The claim is that you will accept both as strictly fair.

Bet 1[5]
Pays $0 if ¬*even*
Pays -$25 if (*even* and $A \rightarrow B$)
Pays $25 if (*even* and $\neg(A \rightarrow B)$)

> Gloss: classically, the facts described entail that this bet has an expected utility of 0. By (iii), you believe only *even* is compatible with $A \rightarrow B$. *even* has probability 1/2, and $A \rightarrow B$ has probability 1/4. So $Pr($*even* $\wedge (A \rightarrow B)) = Pr($*even* $\wedge \neg(A \rightarrow B)) = 1/4$. ✓

Bet 2
Pays $5 if *even*
Pays -$5 if ¬*even*

[4] A typical standard bet on ϕ: costs nothing; (i) pays the agent +$\$Cr(\neg\phi)$ if ϕ, and (ii) pays the agent -$\$Cr(\phi)$ if $\neg\phi$. Expected utility is 0.
A standard *conditional* bet on $(A \rightarrow B)$ costs the agent +$\$k$ and (i) pays the agent +$1 if $(A \wedge B)$ and (ii) pays the agent $0 if $(A \wedge \neg B)$ and (iii) refunds the agent's premium $\$k$ if $(\neg A)$; see §3 below.

[5] Although Bet 1 pays (or costs, respectively) a nonzero amount only if a conjunction obtains, it has the structure of a classic *conditional bet*, given *even* (de Finetti, 1937).

Gloss: by your lights, it's 50-50! ✓

Now we turn to (6) and (7). You believe that: in the first Bet 2 case (viz., *even*), you will come to believe $A \to B$ has probability 1 (viz., $Cr_{even}(A \to B) = 1$), and in second case, you will come to believe $A \to B$ has probability 0 (viz., $Cr_{\neg even}(A \to B) = 0$).

Leveraging this, the Dutchman can offer a final bet, Bet 3, at a later time, just in case you learn *even*:

Bet 3 (offered iff *even*)
Pays \$15 if $A \to B$
Pays -\$35 if $\neg(A \to B)$

Gloss: since $Cr_{even}(A \to B) = 1$, Bet 3, if offered, is valued at a *sure* \$15. ✓

even: $A \to B$	*even*: $\neg(A \to B)$	$\neg$*even*
Bet 1: -\$25	Bet 1: +\$25	Bet 2: -\$5
Bet 2: +\$5	Bet 2: +\$5	**Total:** -\$5
Bet 3: +\$15	Bet 3: -\$35	
Total: -\$5	**Total:** -\$5	

Table 2: The Dutch Book ("you" lose)

However events proceed from this series of bets, you will have lost \$5 (Table 2).

2.2 Caveat

Having presented a Dutch Book argument for (3)-(7), I now want to add a loophole. As emphasized by Moss (2012), a limit on DB arguments concerns context sensitivity in (constituents of) the betting propositions. The prima facie issue can be approached like this: Conditionalization entails that once you learn something, you remain sure of it forever.[6] But it would be absurd, upon learning "today is Thursday", to be certain of *that* forever—for example, to be sure of it tomorrow, and the next day, and the day after that…Likewise it would be absurd, if one is .75 confident that it's Thursday, to continue to be .75 confident that it's Thursday tomorrow, and the next day, etc.

[6] If $Cr(\phi) = 1$, then $Cr(\phi|\psi) = 1$ for all ψ s.t. $Cr(\phi|\psi)$ is defined.

In response, Moss commits herself to a thesis along the following lines:[7]

> (SURROGATE) one always has a *context-invariant* way of expressing any content one learns or assigns credence to. It is (only) this *context-invariant content* which is governed by Conditionalization.

In the "Today is Thursday" case, for example, the agent can convert:

(8) Today is Thursday

to e.g.

(8') Beatrix is Thursday

where 'Beatrix' is a special term, introduced—by the agent herself, if necessary—to *context-invariantly* denote the day on which the sentence (or corresponding thought) is tokened.[8] Unlike (8), (8') expresses a content the agent is never rationally required to alter her credence in, unless she gains (or loses) what Moss calls "genuine information".[9]

On the (SURROGATE) view, agents like us are *sensitive* to context-sensitivity—in a way that shows up in our rational betting behavior. No rational agent will use her credences to bet on a *sentence* ϕ if (s)he knows that the context might change before the sentence is assessed for truth. Rather, she will insist on betting on some context-insensitive *surrogate* for ϕ instead—if necessary, inaugurating nonce terms like "Beatrix". If no surrogate is available, the agent simply does not fall under the purview of the DB theorem.

[7] This is my distillation of Moss's PROXY principle (*op cit.*, §1).

[8] See Moss's analogous discussion of the name "Dr. Demonstrative" (*op. cit.*).

[9] Connoisseurs of two-dimensional semantics will note that it is a *contingent a priori truth* that (8) and (8') are equivalent. Equivalently—semantically ascending—it is an a priori truth that *'Beatrix' denotes today*.

3 Contextualism About Indicative Conditionals

That was (a bit about) context-sensitivity in general. How does its analogue, Contextualism, apply to indicative conditionals?

In the present setting, Contextualism about indicatives has two important components. First: the proposition expressed by a conditional $A \rightarrow B$ is sensitive to the epistemic context in which it is uttered. Let this be represented by a *global evidence parameter*, E. One can make the relevant sensitivity explicit by subscripting the connective $\rightarrow$ with E:

(9) If A, then B.
$A \rightarrow_E B$

In a different evidential context E', "if A, then B" may very well express a different proposition—which we can annotate $A \rightarrow_{E'} B$—instead.

The second component of the contextualist view concerns what happens when conditionals like (9) are embedded, specifically under right-nesting (that is, in sentences of the form $\ulcorner \kappa \rightarrow (A \rightarrow B) \urcorner$). It says that in such a context, $A \rightarrow B$ is interpreted not relative to E, but relative to a state we can call $E + \kappa$, which is the global evidence E updated with κ.[10, 11]

In a starting epistemic context E, we must therefore beware of a potential ambiguity in the interpretation of the nested conditional $\kappa \rightarrow (A \rightarrow B)$:

(10) If κ, then (if A, then B).
a. $\kappa \rightarrow_E (A \rightarrow_E B)$
b. $\kappa \rightarrow_E (A \rightarrow_{E+\kappa} B)$

[10] This does some justice to the Ramsey Test, the intuition that speakers evaluate $\ulcorner p \rightarrow q \urcorner$ by "adding p hypothetically to their stock of knowledge and arguing on that basis about q" (Ramsey, 1931). I intend κ to be read as not itself a conditional.

[11] NB these two components of what I've called "contextualism" are divorceable. On a classic Kaplanian contextualist view (Kaplan, 1989), for example, attitude verbs like *believes* are sensitive to content but not to character. As a result, for the kinds of context-sensitive sentences Kaplan was concerned with, precisely the opposite (of the analogue of the second component) holds. For discussion, see e.g. Yalcin (2007, pg. 1009-1013).

3.1 Back to the Example

In applying Contextualism to our DB, it will be helpful to think about constraints on any proposition—any set of points in the range of the probability function Pr—that could be expressed by (3) (repeated below):

(3) If the die doesn't come up 2 or 4, then it will come up 6.
$\neg$(2 or 4) $\rightarrow$ 6.

Let 'H' be a label for this proposition. Where in the domain of Pr—at least, at the time the agent is initially contemplating (3)—is H true? Letting A be (3)'s antecedent and B be its consequent, we certainly want H to be true at AB worlds, and we want H to be false at $A\overline{B}$ worlds (Table 3):

die comes up...	antecedent (A)	consequent (B)	H (= $A \rightarrow B$)
1	T	F	F
3	T	F	F
5	T	F	F
6	T	T	T
			Ratio of AB-worlds to A worlds: 1:4.

Table 3: Where the proposition expressed by (3) is true and false: easy cases

This leaves the question of where H is true in "antecedent-false" worlds: worlds where the die comes up 2 and worlds where the die comes up 4. If we want to keep the natural idea behind (ST), we will need to extend the ratio 1/4—that is, *one* true case out of *four* possibilities—into the 'antecedent-false' region of logical space. Since, intuitively, nothing distinguishes between die-landing-2-worlds and die-landing-4-worlds insofar as proposition H is concerned, we do so such that $Pr(H|2) = Pr(H|4) = 1/4$.[12] This gives us Figure 1 (after Khoo & Santorio, 2018, pg. 50), where the shaded region represents H and the green-boxed region is the antecedent-false region:

Different updates of Pr will assign different propositions to the conditional $A \rightarrow C$. Just as we would expect systematic change in the proposition expressed by "today is Thursday" to evolve as the days pass, there is systematic change in the proposition expressed by "if A, then C" as information evolves: $\ulcorner A \rightarrow_E C \urcorner$, $\ulcorner A \rightarrow_{E'} C \urcorner$, $\ulcorner A \rightarrow_{E''} C \urcorner$, etc.

The contextualist will hold that this systematic evolution in the proposition expressed is key to understanding what happens to (3) in the betting puzzle. While it can be true that

[12] There is a better argument for this than brute intuition: according to the classic conditional bet (footnote 4, above) the same premium is returned whether the die comes up 2 or 4.

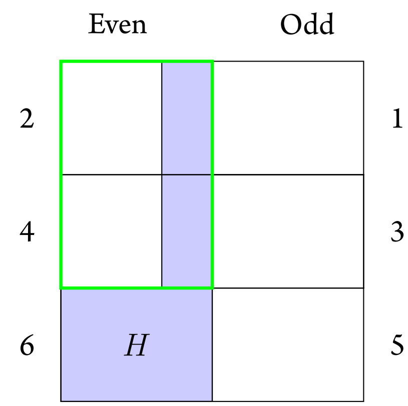

Figure 1: Proposition H. The ratio of shade to white *inside* the green-boxed region (1:4) is equal to the ratio of shade to white outside the green-boxed region.

$Pr_E(\neg(2 \vee 4) \rightarrow_E 6)) = Pr_E(H) = 1/4$, and H can, in keeping with the law of total probability, be divided along the partition {*even, odd*}, this probability need not be equal to the weighted average of the probabilities of two *other* propositions, which we can write as $(\neg(2 \vee 4) \rightarrow_{E+even} 6))$ and $(\neg(2 \vee 4) \rightarrow_{E+odd} 6))$. These are the two propositions the conditional might express after the learning event (depending on whether the agent learns *even* or *odd*). Returning to the ambiguity in (10), it is the (a)-schema which the norm of Conditionalization governs.[13] But it is the (b)-schemas which guide our intuitions about (6) and (7).

Indeed, when it comes to the Dutch book sketched above, we can use the Contextualist framework to go a bit further, and argue that the agent can *make* money off the bets offered by the bookie. Let's think again about the candidate propositions to be expressed by (3) at the later time t^+ (after learning whether the die came up even or odd). Call these H_2 and H_3:

H_2: $(\neg(2 \vee 4) \rightarrow_{E+even} 6))$

H_3: $(\neg(2 \vee 4) \rightarrow_{E+odd} 6))$

H_2 is the proposition expressed by (3) at the later time—more precisely, at the more informed information state—the agent occupies just in case she is offered Bet 3.

[13]Why? Where the agent stands to learn the true proposition in $\{B, \neg B\}$ at t, the Law of Total Probability says that $Pr(\phi) = Pr(\phi|B)Pr(B) + Pr(\phi|\neg B)Pr(\neg B)$. But to the Contextualist, the Table 1 intuitions show only that $Pr(\phi) \neq Pr(\psi|B)Pr(B) + Pr(\chi|\neg B)Pr(\neg B)$, with (at least potentially) $\chi \neq \psi \neq \phi$; this is no violation of any law of probability.

Naively, H_2 is equivalent to *even*;[14] hence $Pr^+_{even}(\neg(2 \vee 4) \rightarrow_{E+even} 6)) = Pr^+_{even}(\textit{even}) = 1$. It follows that $H \subsetneq H_2$: there is a region of logical space where H_2 is true even though H is false (green region of Figure 2).

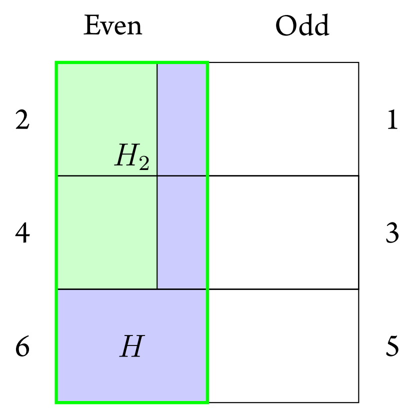

Figure 2: Contextualism and two propositions expressed by (3).

In this region,

(i) $(\neg H$ and Even) is true,

(ii) (Even) is true, and

(iii) (H_2) is true

As the reader can thus verify, in *this* region, the agent can *win* all three of her bets in the original Dutch book. By (i), she wins $25 on Bet 1; by (ii), she wins $5 on Bet 2; and by (iii), she wins $15 on Bet 3.

4 Another Way?

In the previous section, we saw that, according to Contextualism, what's tricky about the series of bets in §1.1 is that it targets speakers' failure to recognize an instance of context-sensitivity in their language. As such, the bookie's offer might be a way of parting a careless agent from her money. But it is not a genuine Dutch book.

[14]This intuition is underwritten by the validity, in Goldstein & Santorio's system, of *Import-Export* (IE): $A \rightarrow (B \rightarrow C) \equiv (A \wedge B) \rightarrow C$. More on IE below.

The second view on the semantics of indicative conditionals that I'll consider is Goldstein & Santorio's *invariantist* view.[15] This is a sequence-based semantic view; it is designed to assign intuitive probabilities to conditional sentences by means of a system on which conditionals express fine-grained contents—contents that cut finer than (sets of) possible worlds. We will then raise the question of the series of bets in §1.1 with regard to this second theory.

4.1 Semantic Details

On Goldstein & Santorio's semantics, indicative conditionals take truth-values at *paths*, which are sequences of worlds drawn from the domains of probability spaces (Goldstein and Santorio call them *epistemic spaces*). In essence, the approach treats the underlying epistemic space, like the one in Figures 1-2, as an *urn* from which worlds are drawn without replacement; the relative frequencies for the truth of different Boolean propositions in the underlying space contribute a natural assignment of probabilities to the resulting sequences. A (simple) conditional "$q \to r$" is true at a path, or sequence, if r is true at the first q-world in that sequence.[16]

Two details of the compositional semantics are helpful for understanding the relationship between paths and worlds. First, a "descriptive" (nonconditional) sentence, like *the die comes up even*, is true at a path p just in case it is true at the *first* world in the path. Further worlds in the path capture information relevant to evaluating arbitrary conditionals: the conditional $A \to B$ is true at at a path p just in case B is true at p updated with A (written $p + A$):

where p_i is the i-th world in p:

- $[\![q]\!]^p = 1$ iff $p_1(q) = 1$.

where
(i) $p + A$ is the largest member of $\{p' \leq p \mid \forall p'' * p' \text{ then } p'' \in [\![A]\!]\}$;
(ii) $p' \leq p$ iff p' is a subsequence of p;
(iii) $p'' * p'$ iff p'' is a permutation of p':

- $[\![A \to B]\!]^p = 1$ iff $[\![B]\!]^{p+A} = 1$.

Goldstein & Santorio (2021, §4)

[15] For their explicit endorsement of invariantism and rejection of Contextualism, see Goldstein & Santorio (2021, §3.3).

[16] Goldstein & Santorio do not themselves use the urn analogy, but Wójtowicz & Wójtowicz (2021) do, suggesting that a sequence of draws from an urn provides the natural setting for a simple "[indicative] conditional game" (§4) whose win-loss conditions mirror the intuitive truth-conditions codified by (ST).

It will be helpful consider the conditional (3) ("if $\neg(2 \vee 4)$, then 6") here. Restricting our attention to worlds where the antecedent of (3) is false (and abbreviating a world where the die comes up i as w_i), the content expressed by the conditional is true at e.g. all the paths in the left column below, and false at all the paths in the right column:

$\langle w_2, w_6, w_1, w_3, w_4, w_5 \rangle$	$\langle w_2, w_1, w_6, w_3, w_4, w_5 \rangle$
$\langle w_2, w_6, w_1, w_3, w_5, w_4 \rangle$	$\langle w_2, w_1, w_3, w_6, w_5, w_4 \rangle$
$\langle w_2, w_6, w_1, w_4, w_5, w_3 \rangle$	$\langle w_2, w_1, w_4, w_5, w_6, w_3 \rangle$
$\langle w_2, w_6, w_1, w_4, w_3, w_5 \rangle$	$\langle w_2, w_1, w_4, w_3, w_5, w_6 \rangle$
$\langle w_2, w_6, w_1, w_5, w_3, w_4 \rangle$	$\langle w_2, w_3, w_6, w_1, w_5, w_4 \rangle$
$\langle w_2, w_6, w_1, w_5, w_4, w_3 \rangle$	$\langle w_2, w_3, w_1, w_6, w_5, w_4 \rangle$
...	...

These sequences fill in the region of logical space occupied by '2' in Figures 1-2. A quarter of these sequences will satisfy the truth-conditions for the simple conditional (3), in keeping with (ST).

It remains to officially assign probabilites to spaces of paths. From a prior $Pr(\cdot)$ over worlds W—which Goldstein & Santorio call the "*proto*-epistemic space"—we induce a probability assignment $C(\cdot)$ on a 'lifted' space of paths as follows:[17]

> Where $U = \langle W, Pr \rangle$ is a probability distribution Pr over W, an **epistemic space** $E = \langle P, C \rangle$ is a lift of U (notation: $\uparrow U$) iff
>
> - P is the set of all paths of worlds in W;
> - C assigns a probability to every member of $\wp(P)$ s.t., where $p[w_1, \ldots, w_n] = p[1, \ldots, n] = \{p \mid p_1 = 1, \ldots, p_n = n\}$:
>   - $C(p[w]) = Pr(w)$
>   - $C(p[1, \ldots, n]) = C(p[1, \ldots, n-1]) \times \frac{Pr(n)}{Pr(W - \{1, \ldots, n-1\})}$.
>
> (Goldstein & Santorio, 2021, §6, Definitions 4-6).

The reader can verify that the last clause above, which specifies $C(p[1, \ldots, n])$ in terms of $C(p[1, \ldots, n-1])$, corresponds to the probability of drawing the next member of sequence of objects from the underlying space without replacement. For example, if there are two white, two red, and two green balls in an urn, the probability of ⟨ *first white, then red, then white again, then green* ⟩ is equal to the probability of ⟨ *first white, then red, then white again* ⟩ times the probability of drawing one of two green balls from an urn with no (remaining) whites and just one (remaining) red. Goldstein & Santorio call an epistemic space E

[17]Here I follow Goldstein and Santorio's exposition, which assumes initially that W is finite.

well-behaved iff there exists some proto-epistemic space U such that $E =\uparrow U$ (*op. cit.*, Definition 7).

We are now in a better position to look at update. These Goldstein-Santorio epistemic spaces—sets of paths P paired with an induced initial probability function C_0—are updated in a way that is distinct from conditionalization. $E = \langle P, C \rangle$'s update with descriptive A—written $E +_{HC} A = \langle P^*, C^* \rangle$—subjects $p \in P$ to an operation which removes every $\neg A$-world from p. Goldstein and Santorio call this update operation *hyperconditionalization* (*op. cit.*, §5.2).

The distinctive feature of hyperconditionalization is path-collapse: two paths which were distinct before the removal of A-worlds might be the same after the removal of A-worlds. Consider the following set P of paths, consisting of worlds which make the (mutually exclusive) descriptive propositions A, B, and C true:

$$\{\langle w_B, w_A, w_C \rangle, \langle w_A, w_B, w_C \rangle, \langle w_B, w_C, w_A \rangle\}$$

After update with $\neg A$, all three paths in the set will give their probability mass to the collapsed path $\langle w_B, w_C \rangle$: $C_A(\{\langle w_B, w_C \rangle\}) = 1$. By contrast, if updating P with $\neg A$ had merely removed paths p from P s.t. $[\![A]\!]^p = 1$—as conditionalization on $\neg A$ removes worlds w from W such that $[\![A]\!]^w = 1$—the first and third paths in P would not have been affected by the update at all.[18]

When update of lifted epistemic spaces goes by hyperconditionalization, update and lifting commute in the descriptive fragment of the language (Goldstein & Santorio *op. cit.*, Theorem 3).[19] A hyperconditionalizing operation by which epistemic space E evolves to $E + A$ for factual A can hence be seen in two ways: either by hyperconditionalizing E's domain on A, or by *conditioning* the underlying proto-epistemic space U on the factual content of A and then lifting $U + A$ to $E + A$. The Stalnaker's Thesis-like result that hyperconditionalizing and the semantic clauses achieve, is

Theorem 1 (Goldstein & Santorio, 2021, §6.2)
If A and B are descriptive and E is well behaved, then $C^E(A \to B) = C^E(B|A)$.

[18] See Goldstein and Santorio's discussion of a similar example in *op. cit.* §5. Goldstein & Santorio note that this probability transfer makes hyperconditionalization a form of *imaging* (Lewis, 1981; Gärdenfors, 1982).

[19] Khoo's view of update is similar: all rational update takes place by conditionalization on a space of worlds BEL, but this space can be "lifted" at any time into a space of sequences by the operation $\Uparrow$. By (an analogue of) Goldstein & Santorio's commutativity theorem, hyperconditionalizing directly between the lifted prior and the lifted posterior and reversing the $\Uparrow$ lift (called $\Downarrow$) achieves the same results.

As a result of Goldstein & Santorio's Theorem 1, embedding under a conditional antecedent achieves the same result as conditionalizing whenever A and B express descriptive propositions (like *odd* or *even*) and the epistemic space is well-behaved. However, when, as in (6), B is a conditional—hence, looks at the tail of a path p as well as the first member p_1—conditional embedding *still* tracks the *sui generis* notion of update embodied by hyperconditionalizing.

Indeed, Goldstein & Santorio argue, from here, that they can get precisely the intuitions in Table 1 without equivocating on the content of (3). By the definition of a lifted proto-epistemic space, we know that a prior over the faces of the die that respects the Principle of Indifference will result in a lifted epistemic space that respects a corresponding Principle of Indifference over the set of *all* paths of length six. These paths p are permutations of $\{w_1, w_2, w_3, w_4, w_5, w_6\}$. Half of the paths make *even* true in virtue of beginning with a world w_i where i is even; we also saw that 1/4 of these paths make H true. However, when the space is hyperconditionalized on the content *even*, any H-path p is converted into some p' which is a permutation of $\{w_2, w_4, w_6\}$. Any such p' will make H ("$\neg(2 \vee 4) \rightarrow 6$") true, so the probability of H is 1 in the posterior epistemic space. Because of path-collapse, the upper-bound constraint set by the Ratio Formula does not apply.[20]

4.2 A Conclusion

Where does this leave the Dutch Book of §2? On the view presented above, there is a *single* content expressed by H, true at some paths in some lifted epistemic spaces, and false at others. So the hyperconditionalizer's response *cannot* be contextualist in flavor: that is, to disambiguate between distinct propositions expressed by (3) at differently informed states. But as far as I can see, that is the only maneuver that prevents the credal assignments in Table 1 from being subject to a genuine Dutch book. The invariantist faces a *guaranteed loss* if she bets in accordance with hyperconditionalization; on Goldstein & Santorio's view, then, we are Dutch-bookable as a bare and direct result of our semantic competence with indicatives. This seems damning.

5 Objections and Replies

In closing, I would like to (i) sketch a family of objections on behalf of Goldstein & Santorio to my pessimistic conclusion, and (ii) say a few words about how the present argument relates to McGee (1989), a preeminent paper in the literature on Dutch books and

[20] By this, I mean: the constraint that conditioning on A can raise the probability of an event C by a maximum of $\frac{1}{Pr(A)}$. By the Ratio Formula for conditional probability (ftn. 1 above), $Pr(C|A) = Pr(C \wedge A)/Pr(A)$. Since $Pr(C \wedge A)$ cannot exceed $Pr(C)$, $Pr(C|A)$ is upper-bounded by $Pr(C)/Pr(A)$. Finally, since $Pr(A) \leq 1$, the right-hand side multiplies $Pr(C)$ by some number $\frac{1}{Pr(A)}$ which (because $Pr(A)$ is a probability) is greater than or equal to 1.

the probabilities of indicatives.

I begin with the objections. An objector may hold that, despite invariantism, the agent in question has reason not buy the individual bets in the §1.1 package in a way that reflects her credences. This is because it is difficult, if not impossible, to establish whether the individual bets (essentially, bets on sentences containing conditionals) are won or lost. Establishing payout conditions may be argued to be *impossible* because it is not a *descriptive*, or *worldly*, matter whether the betting propositions π_i are true or false. This point, can, of course, be made quite precise in the context of Goldstein & Santorio's semantics. The core of the objections, in short, is that the metaphysics implicit in the path-dependent approach gives us a reason to break with the long tradition in decision theory (via Ramsey and de Finetti[21]) of measuring degrees of belief by betting odds.

5.1 A Flaw in the DB?

In the context of the present DB, an objector sympathetic to this argument can make a particularly pointed observation.[22] Recall Bet 1, which is called-off if *odd*, wins \$25 if (*even* and $\neg$(3)), and loses \$25 if (*even* and (3)).

(3) If the die does not land (two or four), then it will land six.
$A \rightarrow B$

	probability of (3) when the first path-world is w_i	expected payout of Bet 1 when the first path-world is w_i
w_2	3/4	+\$12.50
w_4	3/4	+\$12.50
w_6	0	-\$25.00

Table 4: Given only worldly knowledge,
one can *lose* money on Bet 1, but never win.

At all *odd* worlds, Bet 1 is called off. At w_6, Bet 1 certainly loses. But at w_2 and w_4, whether the conditional $A \rightarrow B$ is true is path-dependent. This means the worldly parameters of the semantics are *never* enough to settle whether the bet is won, even though they are *sometimes* enough (at w_6) to settle that the bet is lost! This asymmetry—for the objector—gives the agent a reason to balk at buying bets like Bet 1, even when their expected utility, as assigned by the Goldstein-Santorio credences, is fair or better. From a strictly "worldly" point of view, the objector can argue, Bet 1's settlement conditions really look more like this:

[21]Ramsey (1931); de Finetti (1937).

[22]Thanks to an anonymous referee for pressing me to consider the issue in this form.

w_1	w_2	w_3	w_4	w_5	w_6
\$0	?	\$0	?	\$0	loss

Table 5: Bet 1 at worlds.

Obviously, if the agent declines to buy Bet 1, then the Dutch Book argument, which relies on the purchase of *all* of Bets 1-3, is defanged.

It is true that, as I have presented the DB in §1.1, worldly facts can only establish that the agent has lost Bet 1, and never that she has won it. The question is whether this asymmetry is accidental. Can one make a *symmetric* Dutch book against the Goldstein-Santorio agent, with worldly *win* conditions as well as worldly *loss* conditions?

Yes. Here is the schematic picture of a different bet, not part of the previous Dutch Book, where there are worldly win and worldly loss conditions:

w_1	w_2	w_3	w_4	w_5	w_6
win		lose			
+\$35	?	-\$35	\$0	\$0	\$0

Table 6: Bet 1′ at worlds

Such a bet is symmetric in that, where G is the relevant betting proposition, it is possible for the world to settle whether the bet on G is won (viz. it is won in w_1) and also whether the bet on G is lost (it is lost in w_3). We hence proceed to build a second Dutch Book for Goldstein & Santorio around a bet with this payoff schema. (As I go on to explain below, a further objection can be raised against the second book on metaphysical grounds—but this further objection has less to do with an intuitive notion of metaphysical symmetry, and I will discuss it independently.)

On, then, to a second Dutch Book. We begin anew with a six-sided die. Our betting conditional $\ulcorner A \rightarrow B \urcorner$ is:

(11) If $\neg 2$, then 3.
$A \rightarrow B$

We will consider betting on this conditional before and after the agent learns whether *low* (where *low* = $\{w_1, w_2, w_3\}$).

This time, though, I will stipulate that the faces of the die are *not* equiprobable on the agent's prior. Instead, they bear the probabilities listed in the table below. These are illustrated proportionally in the accompanying diagram, with *low* in the left column and *high* in the right. Once again, a shaded box encloses the antecedent-false region of logical space.[23]

world	$Cr(\cdot)$
w_1	1/6
w_2	1/9
w_3	2/9
w_4	1/6
w_5	1/6
w_6	1/6

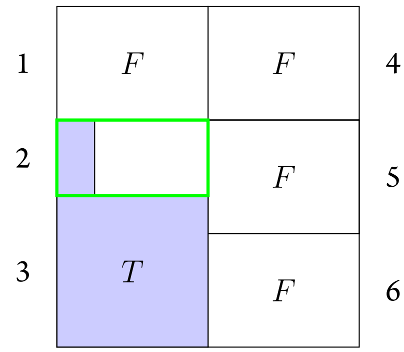

Figure 3: A Revised Dutch Book.

As the table illustrates, the initial probability of (11) is 1/4.[24]

Once again, we have

(12) The die lands low.
low

(13) The die lands high.
high

Despite the adjustment of ratios in the *low* region, the outcomes associated with (12) and (13) remain equally likely.

Our right-nested conditionals, relevant to the betting proposition G, are:

(14) If low, then if $\neg 2$, then 3.
low $\rightarrow (A \rightarrow B)$

[23]Perhaps this would be better illustrated with six colors of balls in an urn, but I stick here to the terminology of a weighted die for consistency. For my own thinking about the case, I found it helpful to think of the space as a 36-point space with worldly properties in the following proportions: w_1: 6, w_2: 4, w_3: 8, w_4: 6, w_5: 6, w_6: 6.

[24]Recall that Goldstein-Santorio credences coincide with Stalnaker's Thesis credences on all noniterated cases. Hence $Cr(\neg 2 \rightarrow 3) = Cr(3 \mid \neg 2) = \frac{Cr(3)}{Cr(\neg 2)} = \frac{2/9}{8/9} = \frac{1}{4}$.

(15) If high, then if $\neg 2$, then 3.
high $\rightarrow (A \rightarrow B)$

We call these Bets $1'$, $2'$, and $3'$ (with Bet $1'$ reflecting payoff schema in Table 6):

Bet $1'$
Pays \$0 if $\neg$*low*
Pays -\$35 if ($A \rightarrow B$ and *low*)
Pays +\$35 if ($\neg(A \rightarrow B)$ and *low*)

Bet $2'$
Pays \$2.50 if *low*
Pays -\$2.50 if $\neg$*low*

Bet $3'$ (offered iff *low*)
Pays \$30 if $A \rightarrow B$
Pays -\$40 if $\neg(A \rightarrow B)$

The agent's reason for accepting Bet $2'$ is straightforward: *low* and *high* are still equiprobable in the new distribution. For Bet $3'$, note that, once the Goldstein-Santorio agent updates on *low*, the probability of $(A \rightarrow B)$—viz., $(\neg 2 \rightarrow 3)$—is $\frac{4}{7}$.[25] The probability of its negation is thus $\frac{3}{7}$. The +\$30/-\$40 payoffs on Bet $3'$ flatfootedly reflect these odds to yield a bet with an expected utility of 0.

Finally, assuming the agent is probabilistically coherent, she will accept the odds on the conjunctive Bet $1'$ at the earlier time. First, $Cr(A \rightarrow B) = \frac{1}{4}$, on Goldstein & Santorio's semantics. Since $Cr(A \rightarrow B) = \frac{1}{4}$, the probability of the conjunction (($A \rightarrow B$) and *low*) is upper-bounded by $\frac{1}{4}$, and the probability of the conjunction ($\neg(A \rightarrow B)$ and *low*) is lower-bounded by $Cr(\textit{low}) - \frac{1}{4} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. The agent's expected loss on Bet $1'$ is hence bound to be smaller than -\$$35(\frac{1}{4})$ and her expected gain is bound to be bigger than +\$$35(\frac{1}{4})$. So the overall expected utility of the bet is fair or positive.[26]

The guaranteed loss can again be laid out in tree form:

[25] Using reasoning that is valid for Goldstein & Santorio: $\textit{low} \rightarrow (\neg 2 \rightarrow 3) \equiv (\textit{low} \wedge \neg 2) \rightarrow 3 \equiv (1 \vee 3) \rightarrow 3$. Probability is hence $\frac{Cr(3)}{Cr(1 \vee 3)} = \frac{2/9}{2/9 + 1/6} = \frac{4}{7}$.

[26] The shading in the figure illustrates $(A \rightarrow B)$ visually in similar fashion to Figures 1-2: in the antecedent-false region (viz., outside of the 2-region) the ratio of truth-to-falsity is 1:4. Hence within the antecedent-false region the same proportion is reproduced (which is why a quarter-sliver of the 2-region is shaded, while the rest is white). Indeed this gives $(A \rightarrow B)$ a probability of exactly $1/4$.

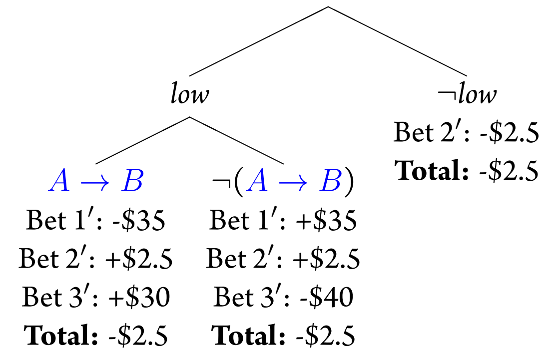

Table 7: Sure loss.

If we want a sure-loss package all of whose bets the agent regards as *strictly* favorable, of course, we can pay the agent $\$\frac{\epsilon}{3}$, where ϵ is tiny, to take each of the bets. The sure loss would then be $\$(2.50-\epsilon)$.

5.2 Further Reservations?

The foregoing suggested that the problem with the original Dutch book was that Bet 1 could never win at worlds, and could only lose. That specific concern does not apply to the revised DB with Bets 1′-3′. But I indicated above that both DBs are subject to a metaphysical reservation that is somewhat different from the first, narrow asymmetry concern raised above. It is to this worry that I now turn.

To see the issue in the present case, we can look again at the Figure 3 diagram, focusing on the left column (throughout which *low* is true). Here, Bet 1′ loses in the shaded region and wins in the white region. But on the new prior, *w_3 has a higher probability than w_1*, so the "worldly" region throughout which Bet 1′ loses exceeds the "worldly" region throughout which Bet 1′ wins. Given that the win-loss amounts are equal ($\$\pm35$), doesn't this give the agent a reason to reject the new Bet 1′? More generally, one seeking to block the DB might hold that a bet's payoff should count towards its overall expected utility *only* if it's *a strictly worldly matter* whether it is won or lost—where, once again, this is a "worldly" matter only if the conditional's truth-value is settled by the *first* world of the relevant sequence in the lifted epistemic space.[27] Call this interpretation of the metaphysical reservation "stringency", and a Goldstein-Santorio agent who triages bets according to this norm a "stringent agent". Although Bet 1′ wins at some world(s), is would be rejected by this more demanding metaphysical requirement. Is the stringent Goldstein-Santorio agent therefore in the clear?

As previewed, I think the stringency requirement is objectionable on independent grounds; exploring why will help us to clarify the technical issues at stake. The reasoning in virtue of which Bet 1′ is fair is reproduced below:

[27] Again, I am grateful to an anonymous referee.

...assuming the agent is probabilistically coherent, she will accept the odds on the conjunctive Bet $1'$ at the earlier time. $Cr(A \rightarrow B) = \frac{1}{4}$, on Goldstein & Santorio's semantics. Since $Cr(A \rightarrow B) = \frac{1}{4}$, the probability of the conjunction $((A \rightarrow B)$ and *low*) is upper-bounded by $\frac{1}{4}$, and the probability of the conjunction $(\neg(A \rightarrow B)$ and *low*) is lower-bounded by $Cr(\textit{low}) - \frac{1}{4} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. The agent's expected loss on Bet $1'$ is hence bound to be smaller than -\$$35(\frac{1}{4})$ and her expected gain is bound to be bigger than +\$$35(\frac{1}{4})$. So the overall expected utility of the bet is fair or positive.

This argument is quite austere. It relies on (i) the intuition underlying Stalnaker's Thesis (to get $Cr(A \rightarrow B) = \frac{1}{4}$), (ii) the Law of Total Probability (to split $Cr(\textit{low})$ into $Cr(\textit{low} \wedge (A \rightarrow B)) + Cr(\textit{low} \wedge \neg(A \rightarrow B)))$, and (iii) the operationalization of these credences via betting odds. Which one is being rejected by the stringent view, and why?

The answer must be (iii), as we can see by returning to the logical space diagrams. Recall that Goldstein & Santorio's strategy is to partition the antecedent-false region of logical space into True and False regions in a way that mimics the proportions of truth to falsity in the antecedent-true region, thus maintaining the overall ratio of truth-to-falsity in W that is mandated by Stalnaker's Thesis. In the antecedent-false region, doing this of necessity requires going beyond the first parameter of the sequence, so this is where we tap the "non-factual" part of the lifted epistemic space. Consider the conditional $\neg 2 \rightarrow 3$, before (left, in pink) and after (right, in violet) the space W is updated on *low*.

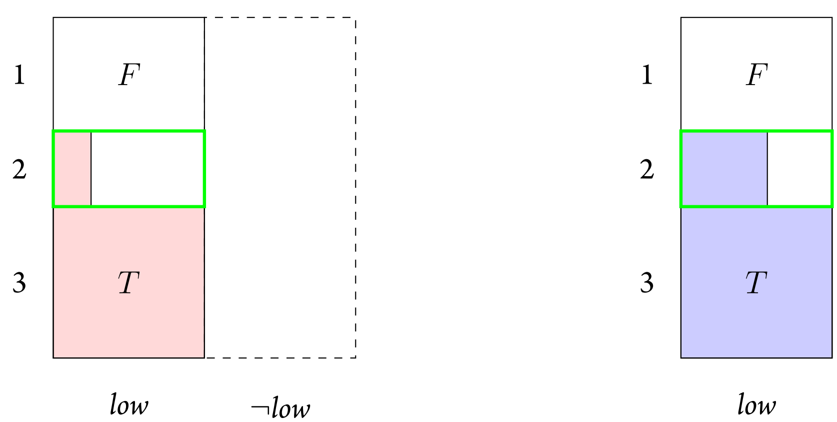

Figure 4: Proposition G up close, at two times.

As these pictures illustrate, the proportion of the antecedent-false (viz., die-lands-2) region in which the proposition denoted by the conditional is true grows, from less than .5 to more than .5. It has to, since it must occupy $\frac{1}{4}$ of the volume of this region before updating on *low* and $\frac{4}{7}$ of this region afterwards.[28]

According to the stringent theorist, the whole of the antecedent-false region of a conditional $A \to B$ *never* plays a role in that agent's expected utility calculations: it is strictly in this region where payouts are not resolved in a "worldly" manner, so the region as a whole always has *de facto* expected utility 0 for her. This agent thus functionally ignores any "creep" in how the betting proposition is demarcated throughout this region. But it is precisely the creep in this region which makes it impossible to regard the agent with Santorio-Goldstein credences as assigning credence to the *same* proposition (a fixed portion of logical space) across time; hence it is precisely the creep in this region on which the incoherence of the Santorio-Goldstein assignment of credences turns.

The proper response to the stringent theorist, then, is not to present another Dutch book, but to ask: in virtue of *what* is it correct to say that the agent assigns credence 1/4 (or 2/7)[29]) to proposition G (the colored region), given that she apparently only bets on the region as if it were de facto equivalent to w_1 (viz., to a region with probability mass 2/9)? More generally, in virtue of what is it correct to say that the agent regards the proposition denoted by a conditional $(A \to B)$ as *distinct* from the proposition denoted by the conjunction $(A \wedge B)$? The latter region, after all, is the only region that she *treats* as one in which the conditional is true, given her betting behavior. Given the present dialectic, I am skeptical that there is a satisfying answer to this question.[30]

5.3 More General Still

The foregoing is addressed to the case of an agent who is wary of a token individual bet (Bet 1 or 1′) for fear that it is more likely to lose, given what can be settled by the worldly facts, than it is to win, given the same proviso. But what if the wary agent's policy is simply to decline bets with *any* nonworldly payout conditions at all? This entails the agent will decline Bets 1 and 1′, and much more besides. As glossed above, it is not possible to Dutch-book such a agent; she is betting-indistinguishable from a conditionalizer, and a conditionalizer cannot be Dutch-booked.

[28]Recall that the posterior on $(\neg 2 \to 3)$, after learning *low*, is $\frac{4}{7}$: $Cr_{low}(\neg 2 \to 3) = Cr((low \wedge \neg 2) \to 3) = Cr((1 \vee 3) \to 3) = \frac{Cr(3)}{Cr(1 \vee 3)} = \frac{2/9}{2/9+1/6} = \frac{12/54}{12/54+9/54} = \frac{12}{21} = \frac{4}{7}$.

[29]In the posterior space, the shaded region has volume $\frac{2}{7}$ because $\frac{2}{7} = \frac{2}{9} + \frac{1}{9}(\frac{4}{7})$. In the prior space, NB that $\frac{1}{4} = \frac{2}{9} + \frac{1}{9}(\frac{1}{4})$.

[30]At least, one that is available to Goldstein & Santorio. A good answer to the question might come from de Finetti (1937), who departs from Goldstein & Santorio at the present juncture in assigning to the antecedent-false region of logical space a third truth-value, N. Since De Finetti declines to parcel the antecedent-false region into the True and the False, he is not under an obligation to adjust the T/F-ratios of the respective areas throughout this region to keep them in step with the ratios dictated by Stalnaker's Thesis. In uniformly assigning N to the troublesome region and excluding it from probability calculations, then, De Finetti avoids the creep. (Of course, the resulting system is nothing like Goldstein & Santorio's; it has a trivalent Kleene logic (Egré et al., 2021), with all its attendant complications and puzzles.)

It is difficult to address this response without delving into the reasons behind the blanket prohibition on bets that carve along nonfactual lines. The objector is now taking up a position that is quite radical. To avoid the Dutch book, this position apparently concedes that the agent must be such that the extension of her credences to conditionals is totally behaviorally inert. The makes it obscure why the numbers assigned to these sentences should be thought of as *credences* at all. The blanket policy will have the consequence, for example, that agents will reject Elga (2010)'s "great" series of bets, as in the following package (after Elga, pg. 4):

Bet A: if $(A \to B)$, you lose \$10. Otherwise, you win \$15.
Bet B: if $(A \to B)$, you win \$15. Otherwise, you lose \$10.

According to the noninterventionist view currently on the table, the Goldstein-Santorio agent should decline this package. The agent thus apparently loses out on a guaranteed \$5.[31] To be sure, passing up this guaranteed gain may not be regarded as so bad as embracing a guaranteed loss. But that the blanket policy recommends this is a reflection of urgency of the question just raised, which is why, on this view, one should regard the Goldstein-Santorio numbers as credences—and thus regard hyperconditionalization as a form of *updating* those credences—in the first place. The envisioned agent behaves exactly like an agent with credences restricted to a Boolean fragment of the language which is conditional-free, and over which she is a pure conditionalizer.

It could be that what is envisioned here is a challenge to the underlying metatheory. I take it that that would be to retort that, precisely because the *cases* in the Dutch book partition logical space along nonfactual lines, it just *isn't true* that e.g. one would lose \$5 (or \$2.50) for sure in the Dutch book—even though it *is* true that one loses \$5 (or \$2.50) *in every case*(!). The objector could be envisioning betting agents who *refuse to pay* a package, for instance, unless the bettors can establish which case they are in. One cannot conclude, in transacting with such agents, that a bet on an arbitrary instance of LEM, say $\ulcorner \phi \vee \neg\phi \urcorner$ for \$5, is equal in value to the following bet package:

Bet C: pays \$5 iff ϕ
Bet D: pays \$5 iff $\neg\phi$

Is the LEM bet equivalent to the package, or not? I'm not sure how to settle the issue (the browbeating of bookies aside). But it is suggestive to observe that considerable ingenuity is

[31]One *could* bite this bullet. In the vagueness literature, Schiffer (2003), for example, argues that, in a borderline case, we are rationally required to adopt credence zero in both e.g. *O'Leary is bald* and its negation. The attitudes rationality requires towards borderline cases are thus subject to different norms of credence than those of classical probability theory, and Schiffer's envisioned rational agent is apparently obligated to decline Elga's series (for a betting proposition like *O'Leary is bald*). But in the present dialectic, where the policy is being embraced *for the purpose of avoiding the sure monetary loss*, this seems to me a clear cost.

expended in classic Dutch Book arguments to ensure that the target doesn't just lose money in each situation, but loses the *same* amount of money in each situation. Christensen, for example, stresses that the bookie's arbitrage strategy is *a priori* (Christensen, 1996, pg. 457; 2004, pg. 121). This stance presupposes that one does not *have* to establish what case one is in to collect on the sure loss. So it should not matter whether it is (im)possible, as a factual matter, to do so.

5.4 Echoes of Hume

The foregoing discussed, and responded to, three ways of developing a metaphysical objection to the Dutch book against Goldstein & Santorio. In wrapping up the discussion, I want to raise a methodological point as well, regarding how the metaphysical scruples underlying the objection sit awkwardly with other areas of decision theory.

Take, for example, Humeanism about practical reason. Humeans hold that "factual" matters cannot tell us what to *prefer*. They thus recognize a metaphysical distinction—to wit, a distinction between the factual and the *motivational*—that parallels the one which, on the Goldstein-Santorio view, drives factual and conditional content apart. Still, for Humeans and non-Humeans alike, a simple arbitrage argument is standardly taken to demonstrate that one ought not to have intransitive preferences.

Familiarly, the arbitrage itself goes something like this: if an agent prefers A to B to C to A, I can trade her three times, charging her e.g. a penny each time, so that she winds up with what she began with (say, A), and is three pennies poorer. This quick, widely-accepted argument may obscure the fact that the exact form the arbitrage takes depends on other assumptions about what the agent values. Suppose an objector retorts, as against the arbitrage argument, that a penny actually has *negative* utility to her, so that—given the exact form of the procedure as stated—she is actually *richer* for having had cyclic preferences! This is a dialectically available move.[32] But in response, the bookie can run the arbitrage procedure again, only this time using *negative pennies*, or something else which has positive value for the agent *so long as* pennies have negative value. Once again, then, we have sure-loss contract argument that works by cases. The person who grants that they owe the bookie \$5 if $(A \rightarrow B)$, and that they owe the bookie \$5 if $\neg(A \rightarrow B)$, but won't pay up because the bookie can't prove (on factual grounds alone) which of $(A \rightarrow B)$ or $\neg(A \rightarrow B)$ is the case is like the preference-inconsistent agent who grants that either a penny is better than the status quo, or the status quo is better than a penny, but won't concede a sure loss by arbitrage because 'tis not contrary to reason (on factual grounds alone) to prefer penny to a negative penny (or vice-versa).[33] I think it is fair to say, then, that a parallel metaphysical

[32] See Hájek on the "Czech book" argument (Hájek, 2008, pg. 796), a Dutch Book-in-reverse procedure according to which an incoherent agent is guaranteed to wind up richer by taking bets she perceives as strictly fair.

[33] The dialectical situation here—that the form of arbitrage depends on the agent's other preferences, or,

objection is typically not taken to be convincing an arbitrage context, even for theorists—like Humeans—who *do* countenance the metaphysical underdetermination of one domain of facts by another.

5.5 Tracking the use of Dutch Books in McGee 1989

Finally, a few words about McGee (1989). Dutch books appear twice in McGee's paper, in importantly different roles. The first time, McGee uses a betting argument to extend a given probability function $Pr(\cdot)$ over Booleans to simple conditionals $\ulcorner q \to r \urcorner$ and their Boolean compounds *only*. This extension thus tells us nothing about right-nested conditionals such as (6) and (7), and is unaffected by the Dutch book I have presented in this paper. The main theorem of this part of McGee's paper is his Theorem 1. The second part is an extension of $Pr(\cdot)$ to *all* of $\mathcal{L}$—including the right-nested conditionals. The main theorem here is Theorem 5. My concern is not that McGee is susceptible to the Dutch Book I have presented; rather it is that the considerations here raise a problem for a dialectical conclusion he draws regarding Theorem 5:

> This theorem vindicates the claim that the conditional represents the way we should change our beliefs when we acquire new evidence. (508)

To clarify, it will help to anatomize McGee's article (Table 4).

Part	Goal	Main Theorem	DB Ancestor
I	extend $Pr(\cdot)$ without right-nesting	Theorem 1	conditional bet table (Ramsey, DeFinetti)
II	extend $Pr(\cdot)$ with arbitrary right-nesting	Theorem 5	Lewis-Teller diachronic DB (1973)

Table 4: uses of Dutch books in McGee (1989)

In the first part, McGee begins with a conditional bet on $A \to B$, where A and B are Boolean. Table 5, discussed by Ramsey and de Finetti, is sufficient to show that k must be $\$Pr(B|A)$ on pain of a Dutch Book.[34]

what comes to the same thing, that she must "post" her odds and preferences in advance—is a familiar one in the Dutch Book literature. See e.g. Skyrms (1993) for this point in re Conditionalization, and van Fraassen (1989) for the same in re Reflection.

[34]We can argue via the following expected utility calculation. Since buying this bet costs $\$k$:

1. $k = [Pr(AB) \times 1 + Pr(A\bar{B}) \times 0] + [Pr(\bar{A}) \times k]$
2. $k = [Pr(AB)] + [Pr(\bar{A}) \times k]$
3. $k = [Pr(A)Pr(AB|A)] + [Pr(\bar{A}) \times k]$

payoff	A	B	$A \to B$
$\$1$	T	T	T
$\$0$	T	F	F
$\$k$	F	T	(refund)
$\$k$	F	F	(refund)

Table 5: premium = $\$k$

In his §3-4, McGee extends a coherent, Dutch book-proof assignment of probabilities to Boolean compounds of conditionals on the basis of a similar bet payoff tables. Theorem 1 reads as follows:

> **Theorem 1**. Let Pr be a real-valued function defined on the sentences of $\mathcal{L}$, and let $\mathcal{K}$ be the fragment of $\mathcal{L}$ consisting of Boolean combinations of factual sentences and simple conditionals of the form $(A \to B)$ with $Pr(A) \neq 0$. Suppose that we have an agent who regards $Pr(\phi)$ as a fair price for a standard bet that ϕ, for each sentence ϕ of $\mathcal{K}$, and who is willing to buy or sell any number of standard bets if she regards the price as fair or better, where the bet settlement conditions conform to the Additivity Principle as well as to the principles that bets on tautologically equivalent sentences should have the same settlement conditions, that a bet that $(F \wedge (F \to G) \wedge \psi)$ should have the same settlement conditions as a bet that $(F \wedge G \wedge \psi)$, and that a standard bet that $((A_1 \to B_1) \wedge (A_2 \to B_2) \wedge \cdots \wedge (A_n \to B_n))$ should be
>
> - won $($payoff $\$1)$ if, for each i: $(A_i \wedge B_i)$
> - lost $($payoff $\$0)$ if, for some i: $(A_i \wedge \neg B_i)$
> - called off $($premium refund$)$ if, for each i: $\neg A_i$.
>
> Then **there will be a finite system of fair bets in which the agent is bound to lose, no matter what, iff the agent's system of personal probabilities fails to satisfy the following conditions:**
>
> 1. the Independence Principle;[35]

4. $k = [Pr(A) \times (\frac{Pr(AB)}{Pr(A)})] + [Pr(\bar{A}) \times k]$
5. $k = [Pr(A) \times Pr(B|A)] + [Pr(\bar{A}) \times k]$
6. $k - [Pr(\bar{A}) \times k] = Pr(A) \times Pr(B|A)$
7. $k(1 - Pr(\bar{A})) = Pr(A) \times Pr(B|A)$
8. $k \times Pr(A) = Pr(A) \times Pr(B|A)$. Hence $k = \$Pr(B|A)$. ✓

[35]"Assuming C is truth-functionally incompatible with each of $A_1 \ldots A_n$ and that none of the A_i's have

2. the standard laws of probability w.r.t. the Boolean connectives;
3. $Pr(F \wedge (F \rightarrow G)) \equiv (F \wedge G)) = 1$
   (McGee, 1989, pg. 501)

Then there's a pivot to the second part, which relies on Import-Export (IE), the principle $(A \wedge B) \rightarrow C \equiv A \rightarrow (B \rightarrow C)$ to extend probability assignments to right-nested conditionals.[36] Its main theorem, Theorem 5, relies on the definition of a "standard probability function" (below), which incorporates, as McGee notes, a "strong version" of IE (in C7):

> **Definition: Standard Probability Function** A standard probability function for $\mathcal{L}$ is a probability function Pr, defined on the sentences of $\mathcal{L}$, such that, if we define a function Pr_H, for each factual H, by
>
> $$Pr_H(\phi) = Pr(H \rightarrow \phi)$$
>
> Then the following conditions will be satisfied:
>
> C1. The Independence Principle
> C2. If $Pr(H) \neq 0$, then Pr_H obeys the standard laws of probability.
> C3. $Pr((A \wedge (A \rightarrow B)) \equiv (A \wedge B)) = 1$.
> C4. $Pr_H((A \rightarrow \phi_1) \wedge (A \rightarrow \phi_2) \wedge \cdots \wedge (A \rightarrow \phi_n)) = Pr_{H \wedge A}(\phi_1 \wedge \phi_2 \wedge \cdots \wedge \phi_n)$.
> C5. $Pr(((A \wedge B) \wedge C) \rightarrow \phi) = Pr((A \wedge (B \wedge C)) \rightarrow \phi)$.
> C6. $Pr(\phi) = Pr_T(\phi)$
> C7. $Pr_H((A \rightarrow (B \rightarrow C)) \equiv ((A \wedge B) \rightarrow C)) = 1$.
> C8. If $Pr(H) = 0$, then $Pr_H(\phi) = 1$.
>
> (McGee, 1989, pg. 504, highlight added)

I have highlighted McGee's identification of the subscripting notation with the object language conditional in the statement of this definition. The same identification, also highlighted, appears in the statement of the main theorem, below:

probability 0, we have $Pr(C \wedge (A_1 \rightarrow B_1) \wedge (A_2 \rightarrow B_s) \wedge \cdots \wedge (A_n \rightarrow B_n)) = Pr(C) \times Pr((A_1 \rightarrow B_1) \wedge (A_2 \rightarrow B_2) \wedge \cdots \wedge (A_n \rightarrow B_n))$" (493). NB this principle describes the payoffs in Table 5, where $C = \neg A$, $A_1 = A$, and $B_1 = B$.

[36] See, for example, pg. 503. McGee's syntax does not allow left-nesting.

> **Theorem 5.** Let $A_1, A_2, \ldots, A_n$ be an exclusive and exhaustive system of factual sentences, each of which represents a complete description of a possible course of experience for our agent between now and time t. Suppose that the standard probability function [see above] Pr represents the agent's beliefs now, that the standard probability function Pr_i represents the beliefs that the agent will have at time t should A_i occur, and that $Pr(A_i) \neq 0$. Suppose further that these beliefs are reflected in the agent's betting behavior, with the bet settlement conditions described in Theorem 1 for the fragments of $\mathcal{L}$ described in Theorem 1. Then our agent will be **immune to a finite Dutch book** iff $Pr_i = Pr_{A_i}$, that is, iff $Pr_i(\phi) = Pr(A_i \rightarrow \phi)$ for each i and ϕ. (McGee, 1989, pg. 508, highlight added)

Immediately after the stating Theorem 5, McGee concludes: draws the conclusion quoted above: that the conditional "represents the way we should change our beliefs when we acquire new evidence."

> This theorem vindicates the claim that the conditional represents the way we should change our beliefs when we acquire new evidence. (508)

My diagnosis is that this extension of the subscripting notation to the full object language reserves the subscripting notation for a notion that is not defensible via a DB (except for simple (non-right-nested) conditionals, as guaranteed by his Theorem 1).

It is a claim of equivocation on the relevant subscripting notation which I shall now defend. I do so by revisiting McGee's diagnosis of Lewis (1976)'s Triviality proof.

Mock-up of Lewis's Proof[37]

1. $Pr(A \rightarrow B)$ = (by Law of Total Probability (LTB))
2. $Pr(B \wedge (A \rightarrow B)) + Pr(\neg B \wedge (A \rightarrow B))$ = (by the Ratio Formula)
3. $Pr(A \rightarrow B|B)Pr(B) + Pr(A \rightarrow B|\neg B)Pr(\neg B)$ = (by ST)
4. $Pr(B \rightarrow (A \rightarrow B))Pr(B) + Pr(\neg B \rightarrow (A \rightarrow B))Pr(\neg B)$ = (by IE)
5. $Pr((B \wedge A) \rightarrow B)Pr(B) + Pr((\neg B \wedge A) \rightarrow B)Pr(\neg B)$ = (by ST)
6. $Pr(B|(B \wedge A))Pr(B) + Pr(B|(\neg B \wedge A))Pr(\neg B)$ = (by algebra)

[37] See Lewis (1976, pg. 300). As it is written in his paper, the proof is more compressed than the version I give here.

7. $1 \cdot Pr(B) + 0 \cdot Pr(\neg B)$ = (by algebra)
8. $Pr(B)$ ✓

What (from McGee's point of view) is wrong with Lewis's proof? A key assumption, at steps 3-4, is that $Pr(\phi|A) = Pr(A \to \phi)$ holds in generality: that is, even for the complex conditional $A \to (B \to C)$. McGee acknowledges that this cannot be the case (pg. 490).[38] Having established the *inequality*

$$Pr(\phi|A) \neq Pr(A \to \phi) \tag{1}$$

we have a choice with regard to the subscripted notation $Pr_A(\phi)$: which notion is *subscripting* going to track? The two sides of the inequality gives us two obvious candidates:

a. Ratio Formula construal: $Pr_A(\phi) = Pr(\phi|A)$, where the latter is defined by the Ratio Formula as $\frac{Pr(A \wedge \phi)}{Pr(A)}$.

b. Object Language construal: $Pr_A(\phi) = Pr(A \to \phi)$, where the latter obeys C1-C8 in McGee's definition of a Standard Probability function.

Because McGee accepts the inequality (1) in order to block Lewis's proof, (a) and (b) are mutually exclusive. By the highlighted portions of Theorem 5 and the definition of a Standard Probability Function, McGee goes for (b). But in the *proof* of Theorem 5 (*op. cit.*, pg. 536), he cites the *Lewis-Teller* DB result, which uses (a)—the Ratio Formula construal—instead. It is indeed (only) this construal that is Dutch-book proof.[39] It is this gap which our Dutch Book exploits.

Having established, then, that $Pr_A(\phi) = Pr(A \to \phi)$ (via Theorem 5), that $Pr(A \to \phi) \neq Pr(\phi|A)$ (via Lewis-Teller), *and* that the DB defends $Pr_A(\phi) = Pr(\phi|A)$ as the way to update, the negation of McGee's gloss on Theorem 5 follows: the conditional, as described in axioms C1-C8, *does not* represent the way agents should change their beliefs when they acquire new evidence.

[38] "The obvious generalization [of Stalnaker's Thesis], that [it] should hold for all conditionals, simple *or complex*, has not worked out" (*op. cit.*, emphasis added). McGee calls the thesis to be generalized *Adams's Thesis* after Adams (1965, 1975), who explicitly rejected its application to iterated conditionals.

[39] See Teller *op. cit.*, pg. 220 for (a). For the converse Dutch Book theorem for Conditionalization, see Skyrms (1987).

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