

The Basic Lecture

The basic lecture is held every Thursday during the regular term of the universities in the physics building of Frankfurt University, Campus Riedberg, Room .101 at 11:00 am.

19. Oct. 2006	C. Blume	Experiments	ppt
26. Oct. 2006	C. Blume	Global Observables	ppt
2. Nov. 2006	C. Blume	Strangeness and Heavy Flavor	ppt
9. Nov. 2006	C. Blume	Photons and neutral Mesons	ppt
16. Nov. 2006	-	no lecture - Quark Matter Conference	-
23. Nov. 2006	H. Appelshäuser	HBT 1	
30. Nov. 2006	H. Appelshäuser	HBT 2	
7. Dec. 2006	H. Appelshäuser	HBT 3	
14. Dec. 2006	H. Stöcker	Nuclear matter	ppt
21. Dec. 2006	H. Stöcker	?	ppt
11. Jan. 2007	M. Gyulassy	So, What's the sQGP?	material: pdf , 1 , 2 , 3 , 4
18. Jan. 2007 Jan.11,2007	M. Gyulassy	So, What's the sQGP, part 2?	

So, What's a sQGP?



So, What's a sQGP? Quiz

Please choose at least one from the list

1. I don't know
2. I only know how detectors work
3. I know some of its signatures
4. It's a bird, its a plane, its Superman
5. It's ah, ah, ah

**sQGP one of the new forms of QCD matter
discovered at RHIC Au+Au @200A GeV**

The Current Theoretical and Experimental Case for

- 1. str**o**ngly coupled Quark Gluon Plasma = sQGP**
- 2. Color Glass Condensate = CGC**

M.Gyulassy and L. McLerran

Nucl.Phys.A750:30-63,2005.

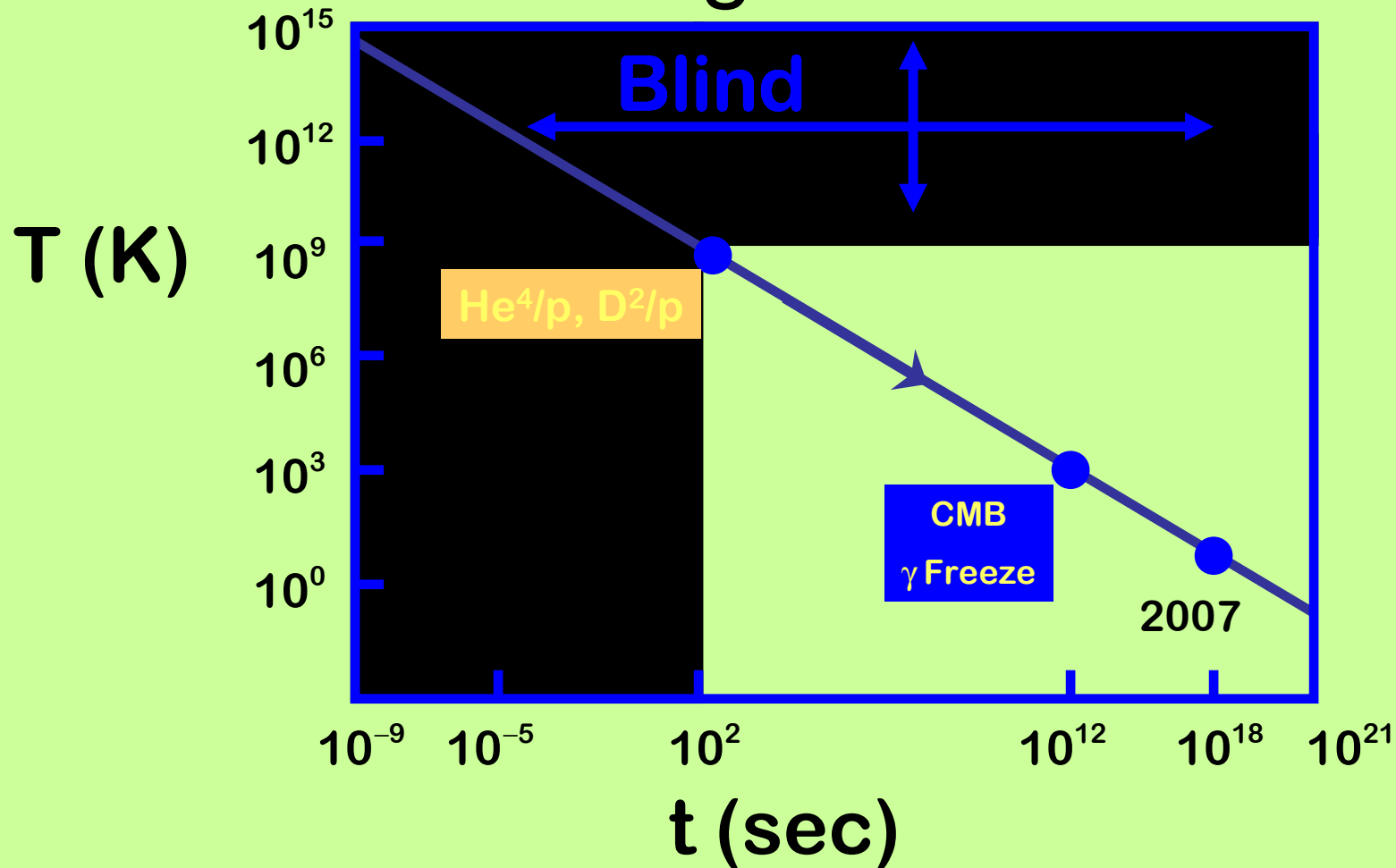
e-Print Archive: [nucl-th/0405013](http://arxiv.org/abs/hep-th/0405013)

See also in NPA750 H.Stocker, E.Shuryak,

J.P.Blaizot, X.N.Wang, B.Mueller

Big Bang

Nuclear Freeze-out blinds cosmologists to $t < 3$ minutes



← We are blind to $t < 20$ min

→ We can observe only this side of time

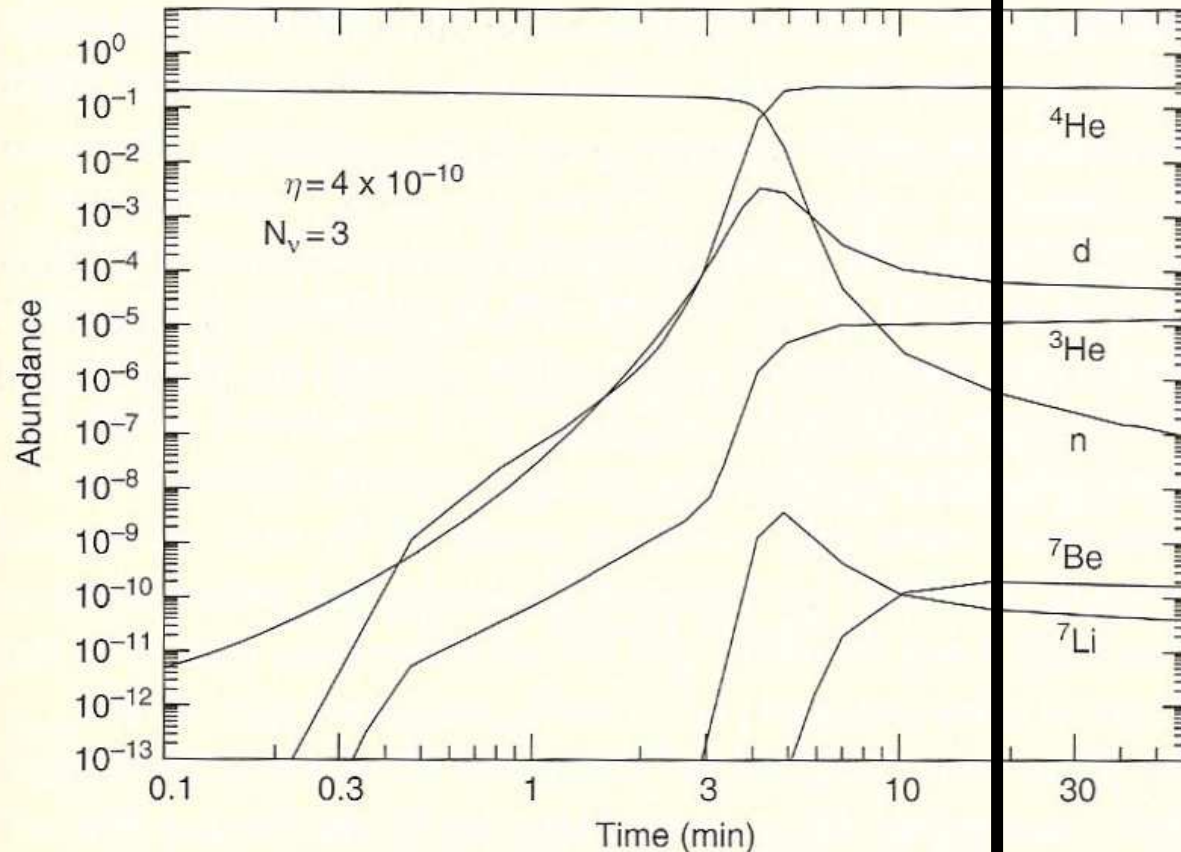
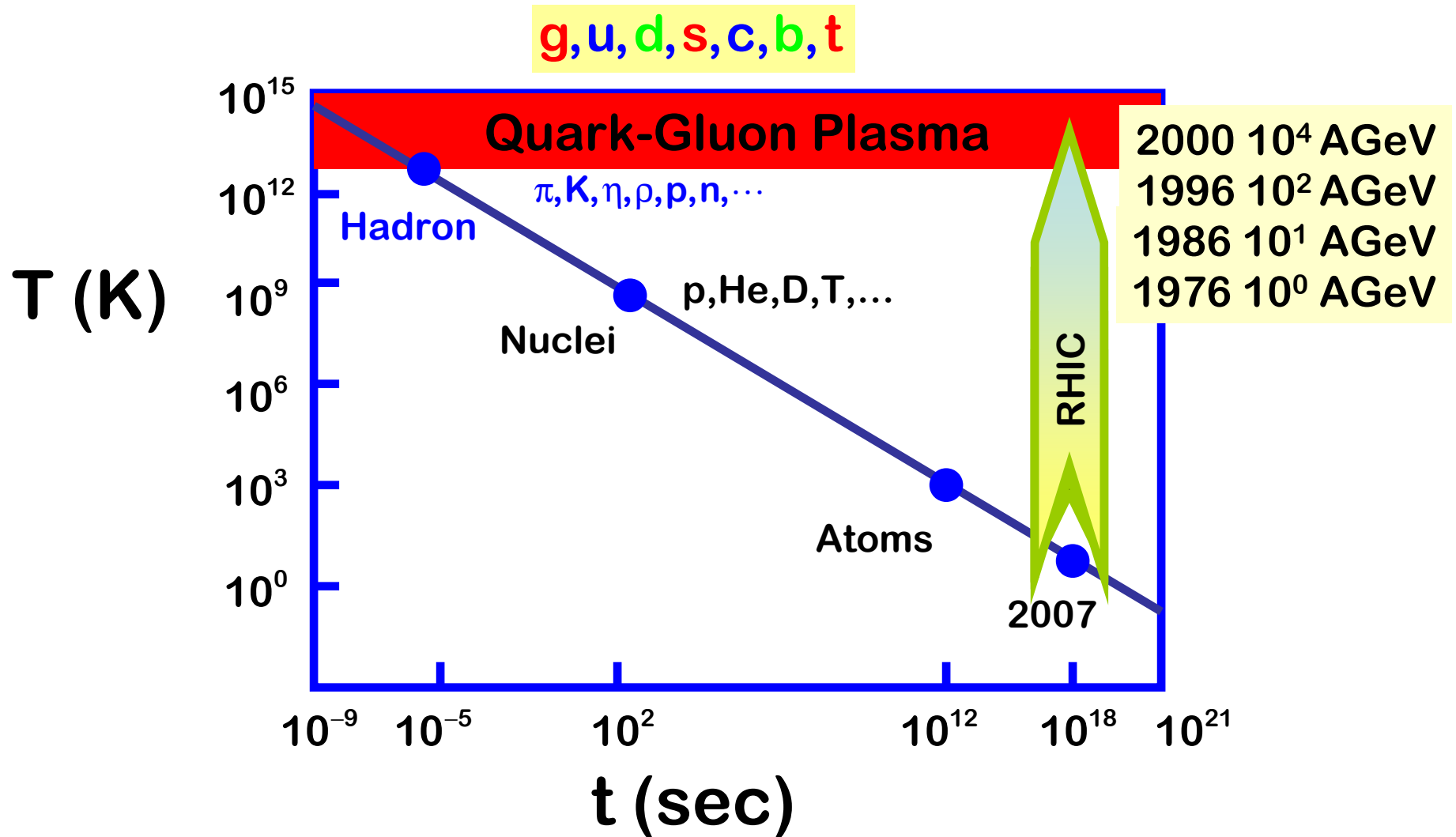


Fig. 8.10. The mass fraction of ^4He and the number abundance (relative to H) of the other light elements during the first hour after the Big Bang. Note that $\eta = n_b/n_\gamma$ and N_v denote the baryon-to-photon ratio and the number of light neutrino species, respectively. Adapted from Schramm and Turner (1998).

Quark-gluon plasma: From big bang to little bang.

[K. Yagi](#) , [T. Hatsuda](#) , [Y. Miake](#)

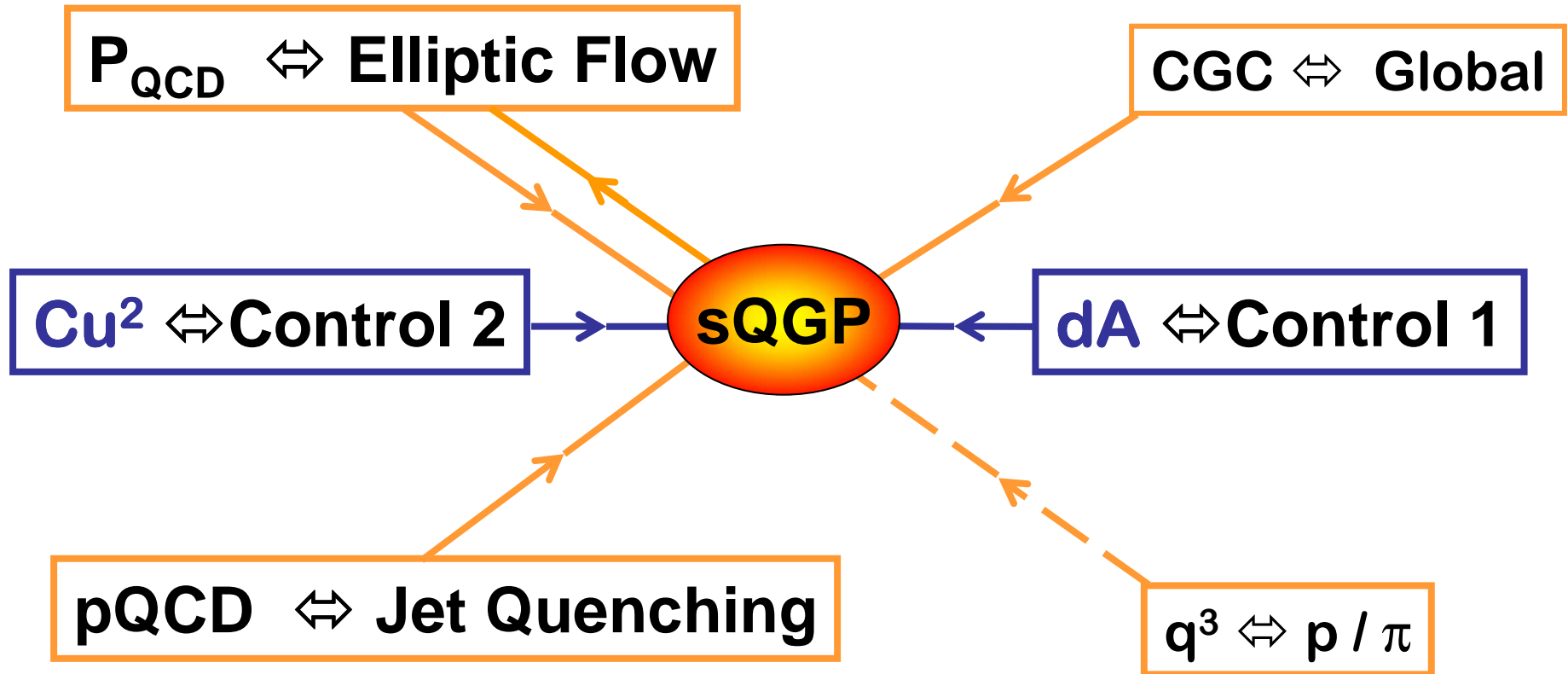
Beyond the QCD Plasma borders with A+A



**** Hadronic Freeze-out at $T=170$ MeV Blinds RHIC observers to sQGP at $t < 3$ fm/c in A+A**

Empirical Evidence at RHIC that points to the Discovery of **sQGP** and CGC

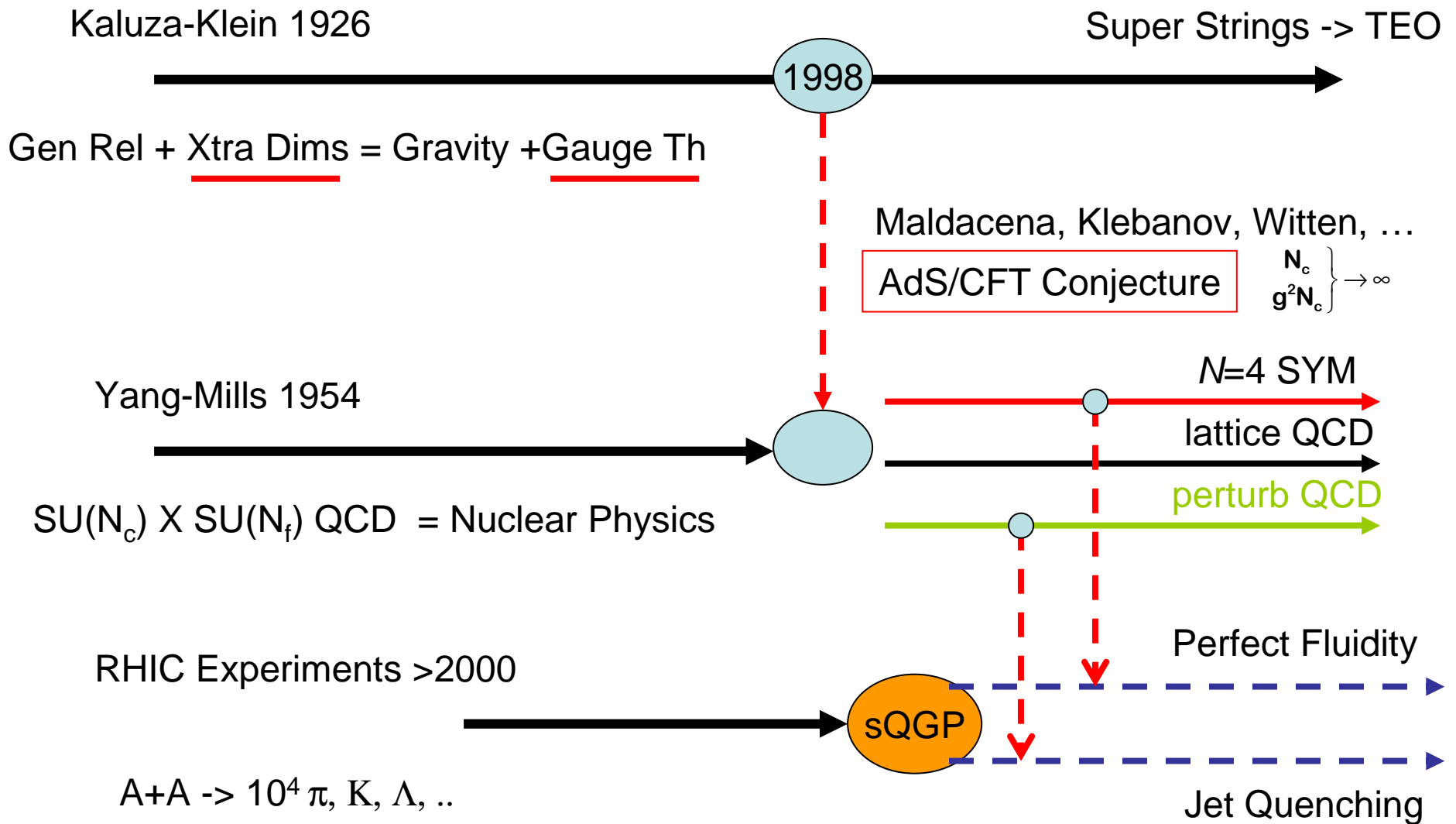
strongly coupled



$$\text{sQGP} = P_{\text{QCD}} + p\text{QCD} + \text{dA} + Q_s + q^3 + \dots$$

$$\text{CGC} = Q_s(y, A)$$

sQGP at Cross Roads of Physics in 2007



De-Confined massless gluon and quarks degrees of freedom

Screened Yukawa Interactions: $V_{q\bar{q}}(r) = \alpha_s(T) \frac{e^{-\mu(T)r}}{r}$ $\xrightarrow[\mu=gT]{T \rightarrow \infty} 0$

Chiral $SU(2)_L \times SU(2)_R$ Symmetry: Ambidextrous quarks

u, d, g

$$\langle \bar{\Psi}\Psi \rangle_{T>T_c} \approx 0$$

$$T = T_c \sim 150-170 \text{ MeV}$$

π, ρ, N

$$\langle \bar{\Psi}\Psi \rangle_{T>T_c} \approx -\Lambda_{\text{QCD}}^3$$

Confined g, q into Colorless White Massive Hadrons

Anti-screened String Interaction: $V_{q\bar{q}}(r) = -\frac{\alpha}{r} + \kappa r \rightarrow \infty$

Broken Chiral Symmetry: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{\text{isospin}}$

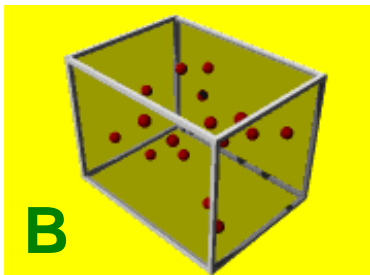
1975 QCD Prediction new form of Matter

Collins, Perry
Baym, Chin
Freedman,
McLerran,
Shuryak, ...

Because QCD is Asymptotically *Free*

$$P_{\text{QCD}}(T) \xrightarrow{T \gg \Lambda_{\text{QCD}}} P_{\text{SB}}(T)$$

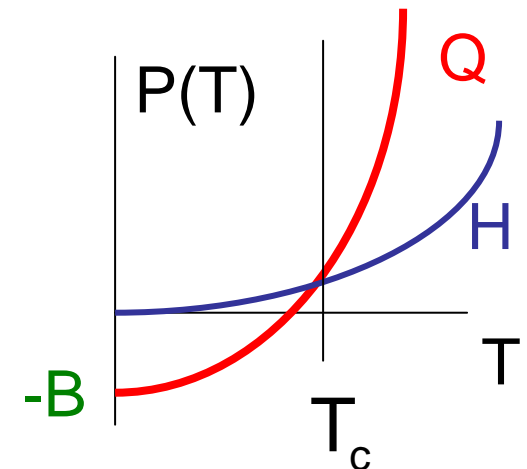
Ideal (Stefan-Boltzmann) Pressure at $g \rightarrow 0$



$$P_{\text{SB}}^{\text{QCD}}(T) = \left(\underbrace{2_s \times 8_c}_{\text{gluons}} + \frac{7}{8} \times \underbrace{2_s \times 3_c \times 2_{q\bar{q}} \times n_f}_{\text{quarks}} \right) \frac{\pi^2 T^4}{90} - \underbrace{B}_{\text{vac}}$$

$$P^{\text{H}}(T) = \left(\underbrace{3_{\text{iso}}}_{\text{pions}} + \underbrace{O(e^{-M/T})}_{\rho, \omega, \dots} \right) \frac{\pi^2 T^4}{90}$$

$$T_c = \left(\frac{B}{K_Q - K_H} \right)^{1/4} \approx \Lambda_{\text{QCD}} \approx 150 - 200 \text{ MeV}$$



The **wQGP** = weakly coupled QGP at $T \gg T_c$

$$P_{SB}(T, n_f = 2) = \left(\underbrace{16}_{\text{gluons}} + \underbrace{21}_{\text{quarks}} \right) \frac{\pi^2 T^4}{90} \approx 4 T^4$$

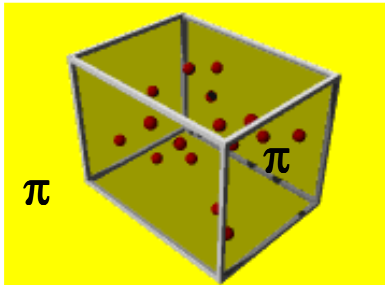
$$\varepsilon_{SB}(T, n_f = 2) = 3 P_{SB}(T, n_f = 2) \approx 12 T^4$$

$$\sigma_{SB}(T, n_f = 2) = \frac{\varepsilon_{SB} + P_{SB}}{T} \approx 16 T^3$$

$$P_{wQGP}(T) = P_{SB}(T) \{1 + a_2 g^2(T) + a_3 g^3(T) + \dots\}$$

$$\sigma_{wQGP}(T) = \frac{\partial P_{wQGP}}{\partial T}$$

$$\varepsilon_{wQGP} = -P_{wQGP} + T \sigma_{wQGP} = T^2 \frac{\partial (P_{wQGP} / T)}{\partial T}$$



π

Pion Gas Equation of State $T < T_c$

Chiral Perturbation

(Leutwyler, 1989):

$$k < f_\pi = 93 \text{ MeV}$$

$$P_H/P_{SB} = 1 + \frac{T^4}{36f_\pi^4} \ln \frac{\Lambda_p}{T} + O(T^6), \quad (3.52)$$

$$\varepsilon_H/\varepsilon_{SB} = 1 + \frac{T^4}{108f_\pi^4} \left(7 \ln \frac{\Lambda_p}{T} - 1 \right) + O(T^6), \quad (3.53)$$

$$s_H/s_{SB} = 1 + \frac{T^4}{144f_\pi^4} \left(8 \ln \frac{\Lambda_p}{T} - 1 \right) + O(T^6), \quad (3.54)$$

$$P_{SB}^\pi(T) = 3_{iso} \frac{\pi^2 T^4}{90}$$

with $d_\pi=3$,

where P_{SB} , ε_{SB} and s_{SB} are the leading order Stefan–Boltzmann EOS given in Eqs. (3.38)–(3.40), and $\Lambda_p (= 275 \pm 65 \text{ MeV})$ is a quantity related to the pion–pion scattering.

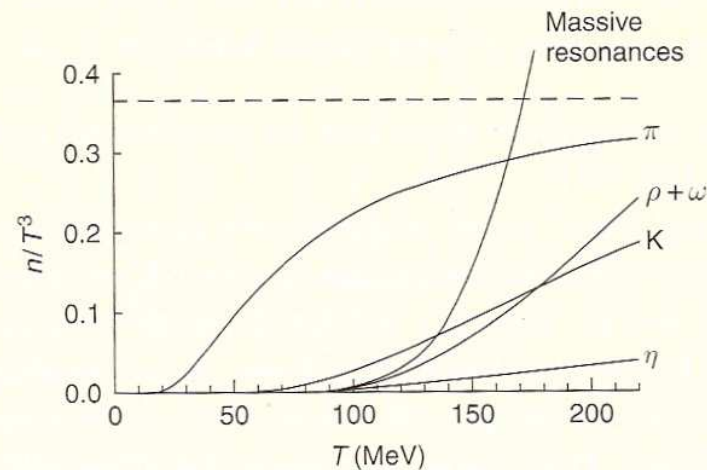


Fig. 3.3. The number density of hadrons, $n(T)$, calculated from the Bose–Einstein and Fermi–Dirac distributions at finite T . The dashed line is a contribution from massless pions. The figure is adapted from Gerber and Leutwyler (1989).

MIT Bag Model Equation of State

Special case: $M(T)=0$, $B(T)=B$ limit of the effective massive quasiparticle model

Displaced Stefan-Boltz $\left\{ \begin{array}{l} P_{Bag}(T) = d_Q \frac{\pi^2}{90} T^4 - B \\ \epsilon_{Bag}(T) = 3P_{Bag}(T) + 4B \end{array} \right.$

Latent heat $L = T_c [\sigma(T_c + \epsilon) - \sigma(T_c - \epsilon)] = 4B$

B is the Binding energy per unit volume of the physical Non-Perturbative Vacuum

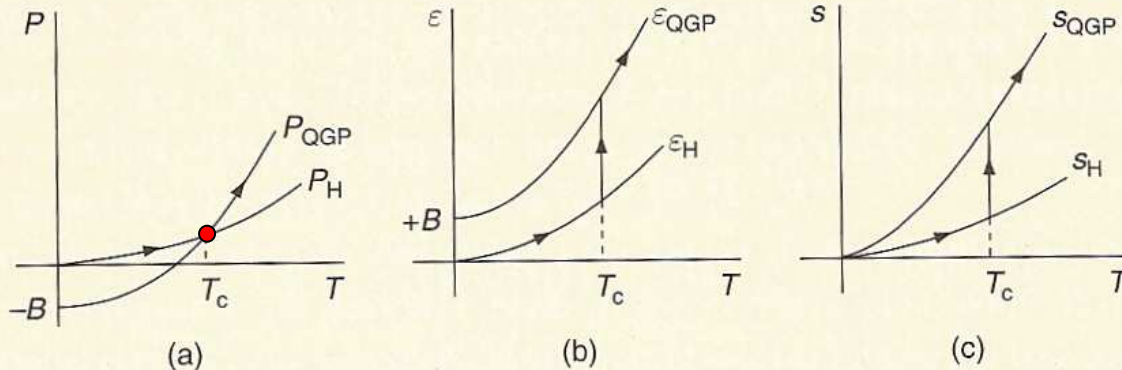


Fig. 3.2. The equations of state in the bag model at finite T with zero chemical potential: (a) the pressure, (b) the energy density, (c) the entropy density. The arrows show how the system evolves as an adiabatic increase of T .

$$T_c = \left(\frac{90}{\pi^2} \frac{B}{d_Q - d_H} \right)^{1/4}$$

$$d_Q \approx 37, \quad d_\pi = 3$$

$$B \approx \Lambda_{QCD}^4$$

“Quark-gluon plasma: From big bang to little bang”

K. Yagi , T. Hatsuda , Y. Miake, Cambridge Press 2005

Continuous Finite Width “Crossover Transition” in Bag Model

$$\sigma(T) = f(T)\sigma_H(T) + (1 - f(T))\sigma_Q(T)$$

$$f(T) = \{1 + \tanh[(T - T_c)/\Delta T_c]\}/2$$

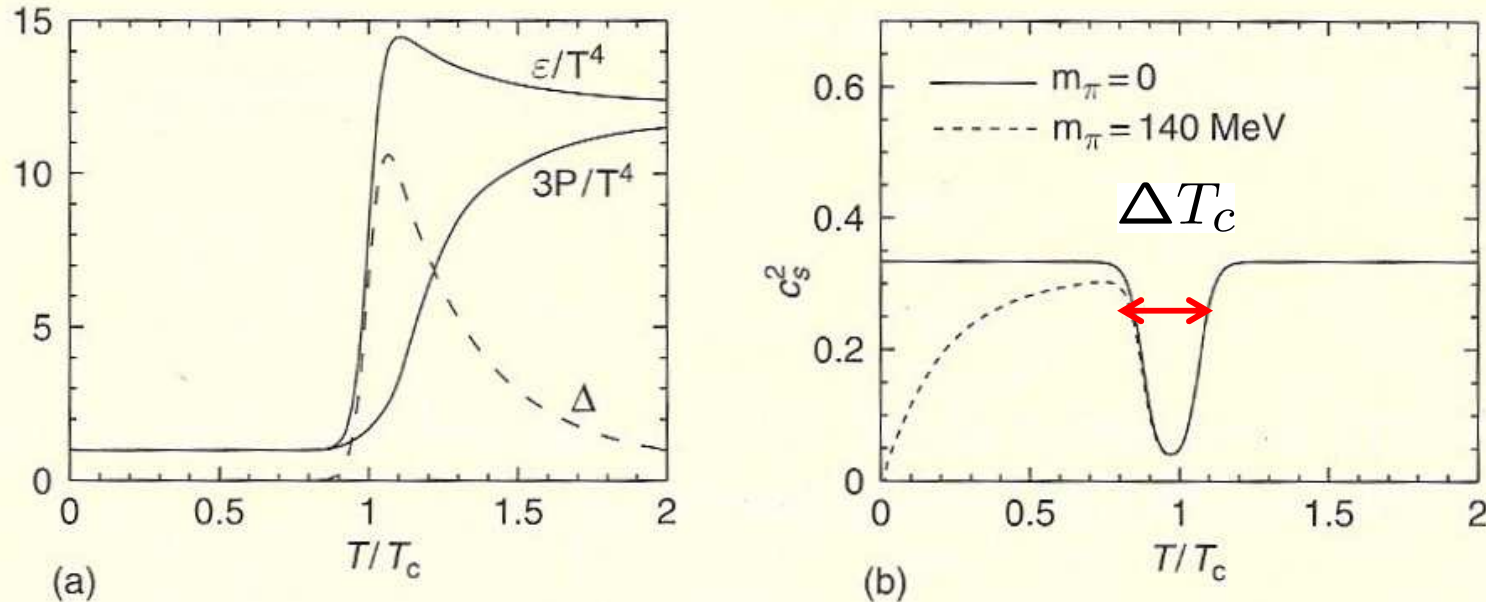
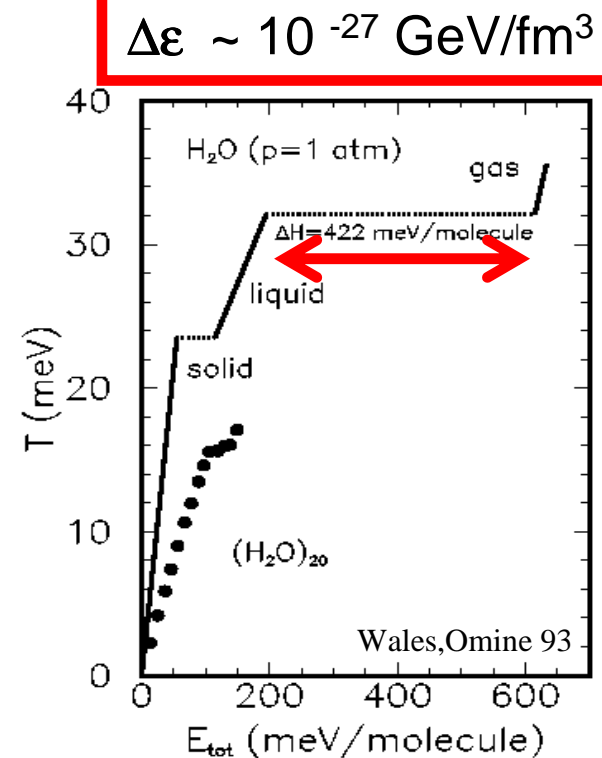
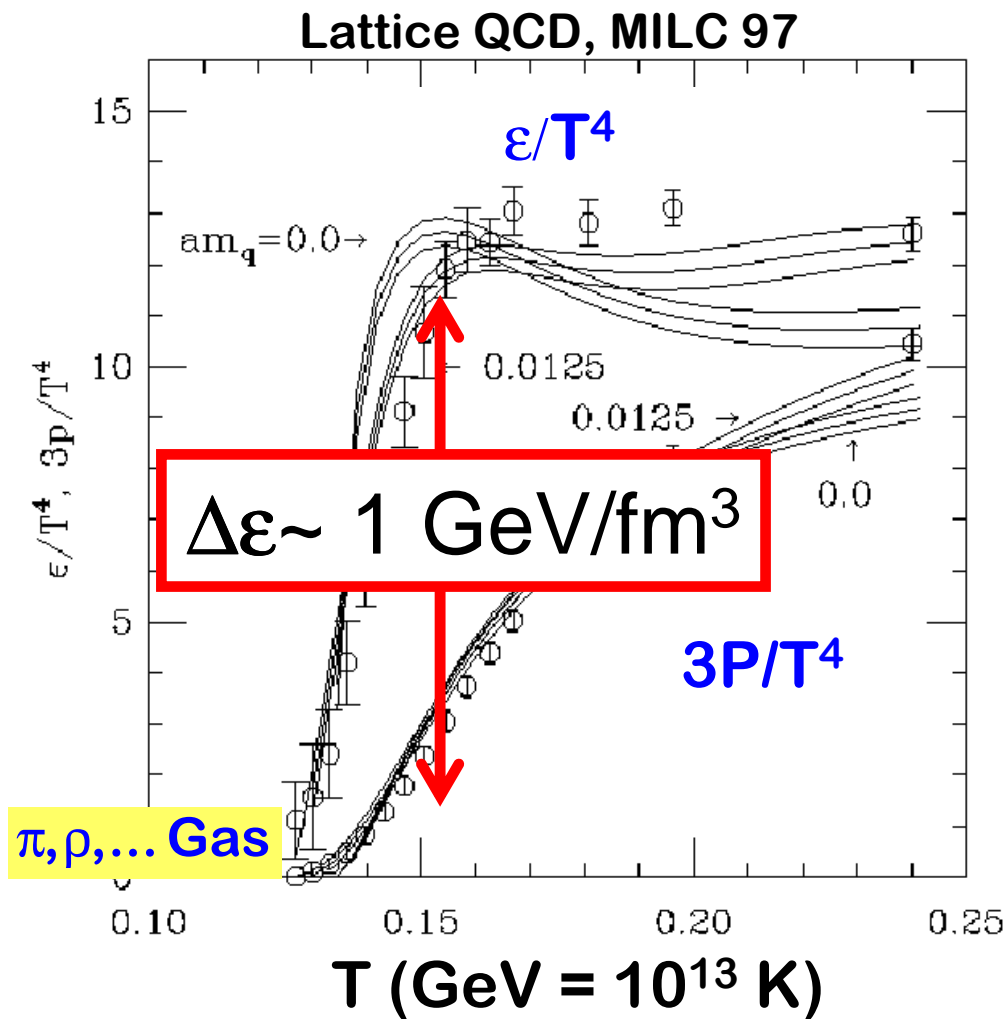


Fig. 3.4. (a) ε/T^4 and $3P/T^4$ obtained from the parametrized entropy density, Eq. (3.60), with $\Gamma/T_c = 0.05$ and $N_f = 2$; $\Delta \equiv (\varepsilon - 3P)/T^4$ is shown by the dashed line. (b) Sound velocity squared as a function of T with the same parametrization for the entropy. The figures are adapted from Asakawa and Hatsuda (1997).

QCD Equation of State



Caloric curve of bulk water at atmospheric pressure (line) and of a water clusters consisting of 20 H_2O molecules predicted³⁶ by molecular dynamics calculations (dots).

$$(150 \text{ MeV})^4 = \frac{1}{20} \frac{\text{GeV}}{\text{fm}^3} = 10^{34} \text{ Pa} = 10^{29} \frac{\text{Kg}}{\text{cm}^2}$$

What's a **s**QGP? Part 2

- Review of Stefan-Boltzmann sbQGP
- **Weakly coupled wQGP** & perturbative QCD
 - Zero radius of convergence about $g=0$
 - The Linde disease at $O(g^6)$
- Effective mass Quasi-particle QGP
 - Phenomenology
 - Hard Thermal Loop and variational QGP/HTL
 - Dynamic Quasiparticle Model: $\gamma(T) \sim M(T)$

Equation of State of **Hot** QCD matter Seems perturbative for $T > T_c$

Lattice QCD

F.Karsch et al, PLB 478 (00) 477

$16^3 \times 4$ improved action and staggered q

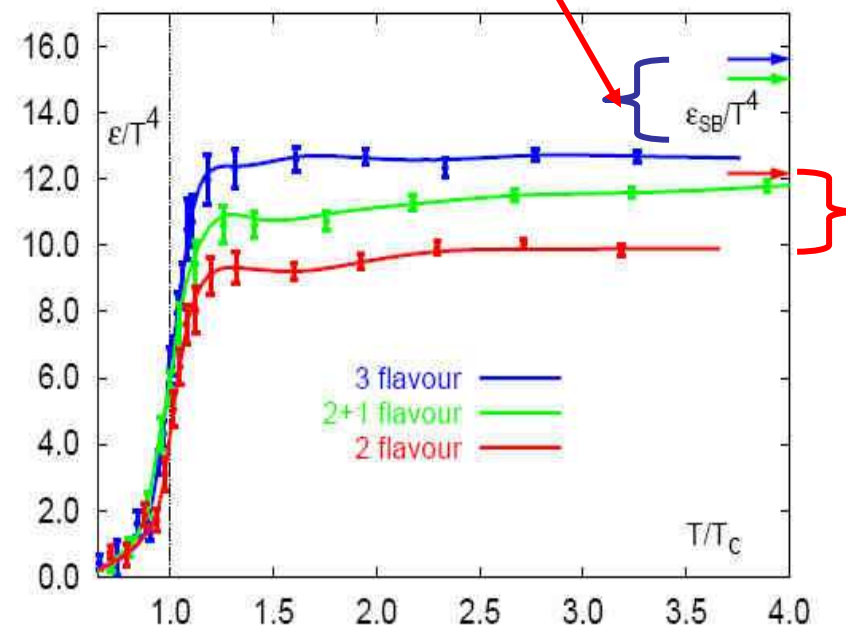
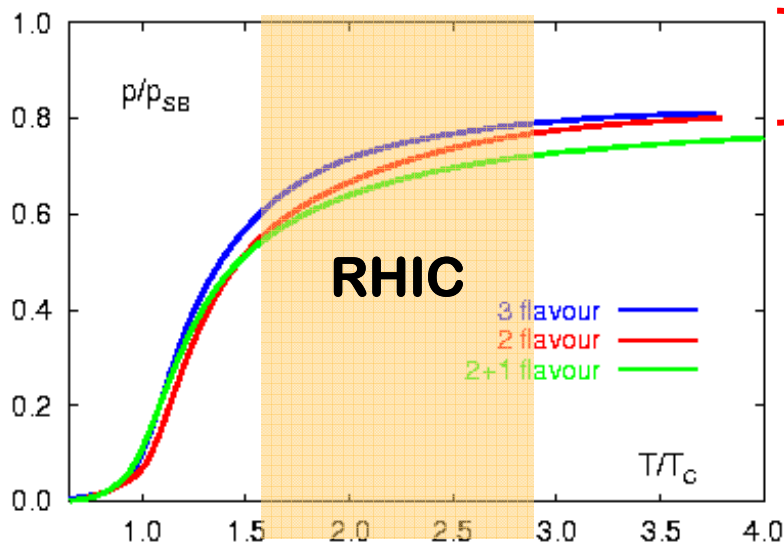
$M_{u,d} \sim T/4$, $M_s \sim T$

Rapid cross-over

$T_c \sim 160\text{-}170 \text{ MeV} \sim 2 \cdot 10^{12} \text{ K}$

$$\Delta \varepsilon \sim 1 \frac{\text{GeV}}{\text{fm}^3} \sim 10^{35} \text{ Pa}$$

Only 20% deficit in P , ε and S from SB limit



However weak coupling perturbative expansion fails

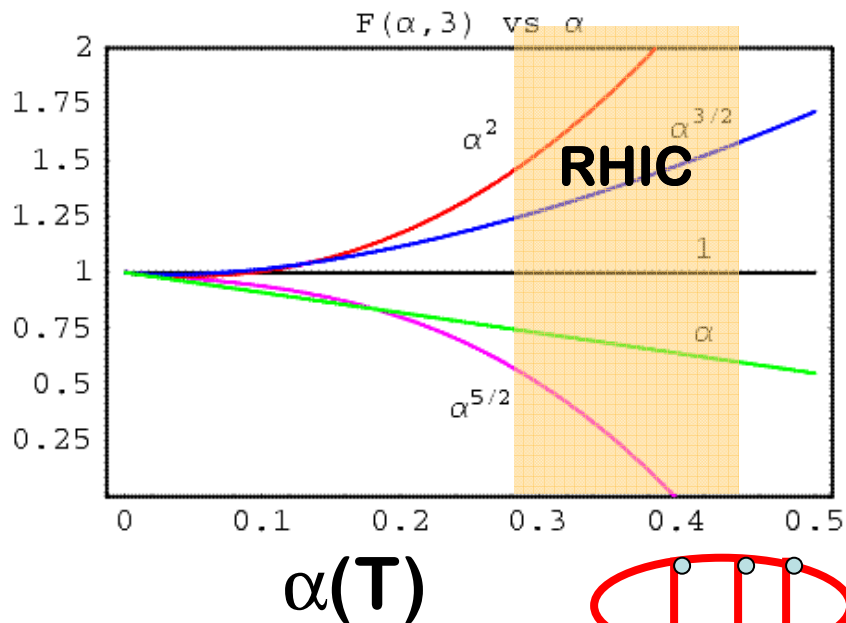
High Temperature QCD Perturbation Theory

(E.Braaten and A. Nieto, PRL 76 (96) 1417)
(C.Zhai and B. Kastening, PRD 52 (95) 7232)

Quark – Gluon Plasma Pressure

$$P = \frac{\pi^2}{90} T^4 \left(2_s 8_c + \frac{7}{8} 2_s 3_c 2 n_f \right) F(\alpha, n_f)$$

$$F(\alpha, 3) = 1 - 0.9\alpha + 3.3\alpha^{3/2} + (7.1 + 3.5\log \alpha)\alpha^2 - 20.8\alpha^{5/2} + \infty\alpha^3 + \dots$$



Linde's "Infrared Barrier"

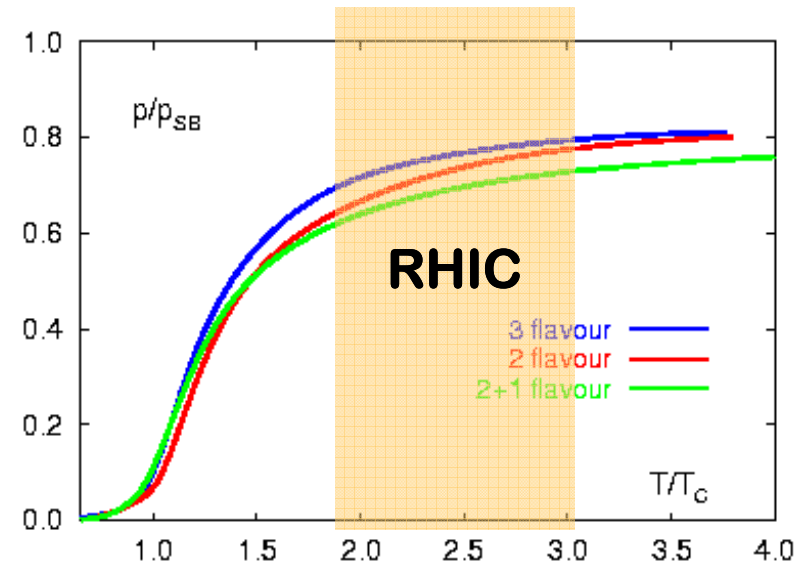
Lattice QCD

F.Karsch et al, PLB 478 (00) 477

$16^3 \times 4$ improved gauge and staggered q

$M_{u,d} \sim T/4$, $M_s \sim T$

QGP@RHIC \neq wQGP



The **wQGP** = weakly coupled QGP at $T \gg T_c$ (slide 2)

$$P_{\text{wQGP}}(T, n_f) = P_{\text{SB}}(T, n_f) \{1 + a_2 g^2(T, n_f) + a_3 g^3(T, n_f) + \dots\}$$

$$a_2 = -\frac{10}{4\pi^2} \left(\frac{12+5n_f}{32+21n_f} \right) \approx -0.075$$

$$a_3 = +30 \left(\frac{6+n_f}{24\pi^2} \right)^{3/2} \left(\frac{32}{32+21n_f} \right) \approx +0.081$$

But $|a_n| = \infty$ for $n \geq 6$!

Perturbation theory fails for orders g^n with $n > 5$
Due to singular infrared chromo-magnetic arising from $m_g = 0$

All the contributions to the pressure involving odd powers of g in (4.5) (as well as part of those involving even powers) are coming from the soft sector. Inserting the leading-order value (4.2) for m_E gives the QCD pressure up to and including order $g^4 \ln g$; to obtain all the terms to order g^5 , next-to-leading order corrections to the m_E -parameter have to be obtained by a matching calculation as given in reference [136]. The result is known in closed form [133, 134, 136], but we shall quote here only the case of SU(3) with N_f quark flavours and numerical values for the various coefficients:

wQGP:

$$\begin{aligned}
 P = \frac{8\pi^2}{45} T^4 & \left\{ \left(1 + \frac{21}{32} N_f\right) - \frac{15}{4} \left(1 + \frac{5}{12} N_f\right) \frac{\alpha_s}{\pi} + 30 \left[\left(1 + \frac{1}{6} N_f\right) \left(\frac{\alpha_s}{\pi}\right) \right]^{3/2} \right. \\
 & + \left\{ 237.2 + 15.97 N_f - 0.413 N_f^2 + \frac{135}{2} \left(1 + \frac{1}{6} N_f\right) \ln \left[\frac{\alpha_s}{\pi} \left(1 + \frac{1}{6} N_f\right) \right] \right. \\
 & \quad \left. - \frac{165}{8} \left(1 + \frac{5}{12} N_f\right) \left(1 - \frac{2}{33} N_f\right) \ln \frac{\bar{\mu}}{2\pi T} \right\} \left(\frac{\alpha_s}{\pi}\right)^2 \\
 & + \left(1 + \frac{1}{6} N_f\right)^{1/2} \left[-799.2 - 21.96 N_f - 1.926 N_f^2 \right. \\
 & \quad \left. + \frac{495}{2} \left(1 + \frac{1}{6} N_f\right) \left(1 - \frac{2}{33} N_f\right) \ln \frac{\bar{\mu}}{2\pi T} \right] \left(\frac{\alpha_s}{\pi}\right)^{5/2} + \mathcal{O}(\alpha_s^3 \ln \alpha_s) \Big\}. \quad (4.5) \\
 & + \alpha^3 \left(c_6 \ln \frac{1}{\alpha} + C_\infty \right).
 \end{aligned}$$

Not calculable !

The coefficient of the $\alpha_s^3 \ln \alpha_s$ term, the last in the pressure at high T and vanishing chemical potential that is calculable completely within perturbation theory, has recently been determined as [137, 152]

$$\begin{aligned}
 P \Big|_{g^6 \ln g} &= \frac{8\pi^2}{45} T^4 \left[1134.8 + 65.89 N_f + 7.653 N_f^2 \right. \\
 & \quad \left. - \frac{1485}{2} \left(1 + \frac{1}{6} N_f\right) \left(1 - \frac{2}{33} N_f\right) \ln \frac{\bar{\mu}}{2\pi T} \right] \left(\frac{\alpha_s}{\pi}\right)^3 \ln \frac{1}{\alpha_s}. \quad (4.6)
 \end{aligned}$$

(slide 4)

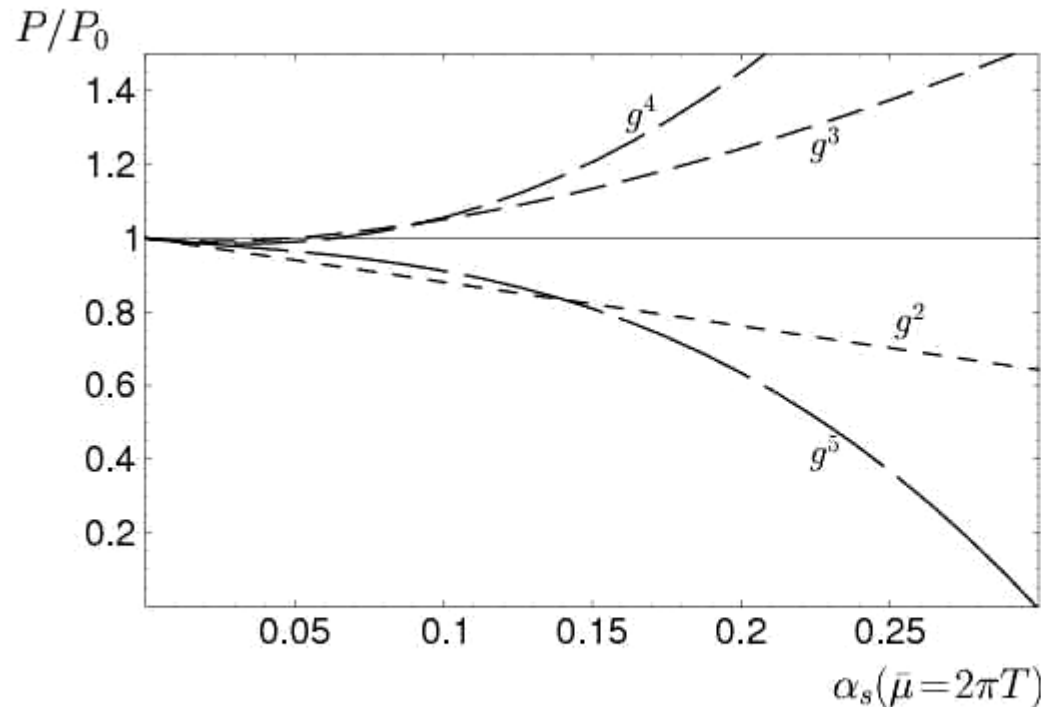


Figure 5. Strictly perturbative results for the thermal pressure of pure glue QCD normalized to the ideal-gas value as a function of $\alpha_s(\bar{\mu} = 2\pi T)$.

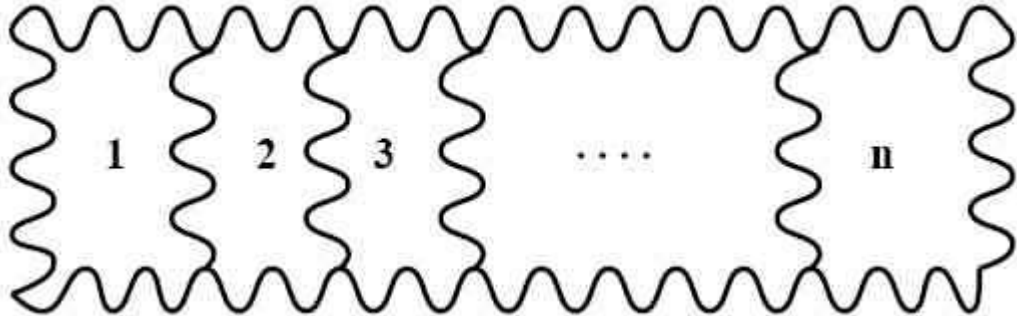
Figure 5 shows the outcome of evaluating the perturbative result (4.5) for the thermodynamic pressure at $N_f = 0$ to order α_s , $\alpha_s^{3/2}$, α_s^2 , and $\alpha_s^{5/2}$, respectively, with a choice of $\bar{\mu} = 2\pi T$. Apparently, there is no convergence for $\alpha_s \gtrsim 0.05$ which in QCD corresponds to $T \lesssim 10^5 T_c$, where T_c is the deconfinement phase transition

perturbatively

there seems to be a complete loss of predictive power at any temperature of interest

The albatross about **wQGP** = weakly coupled QGP at $T \gg T_c$ At 4 and higher loops (slide 5)

(The Linde disease of pQCD)

$$P_{n \text{ loop}}(T) =$$


n -loop diagrams can be estimated as (μ is an IR cutoff):

$$g^{2(n-1)} \left(T \int d^3k \right)^n \frac{k^{2(n-1)}}{(k^2 + \mu^2)^{3(n-1)}}, \quad (1.7)$$

which is of the order $g^6 T^4 \ln(T/\mu)$ if $n = 4$ and of the order $g^6 T^4 (g^2 T/\mu)^{n-4}$ if $n > 4$. (The various factors in eq. (1.7) arise, respectively, from the $2(n-1)$ three-gluon vertices, the n loop integrations, and the $3(n-1)$ propagators.) According to this equation, if $\mu \sim g^2 T$, all the diagrams with $n \geq 4$ loops contribute to the same order, namely to $\mathcal{O}(g^6)$. In other words, the correction of $\mathcal{O}(g^6)$ to the pressure cannot be computed in perturbation theory.

**Unscreened magnetic interactions lead to long range divergences
 Because QCD has bare $m=0$ and no perturbative magnetic monopoles**

Electric interactions are ok and screened by (chromo)charged quarks and glue

$$P^{n \geq 6} = \alpha^3 \left(c_6 \ln \frac{1}{\alpha} + C_\infty \right) T^4$$

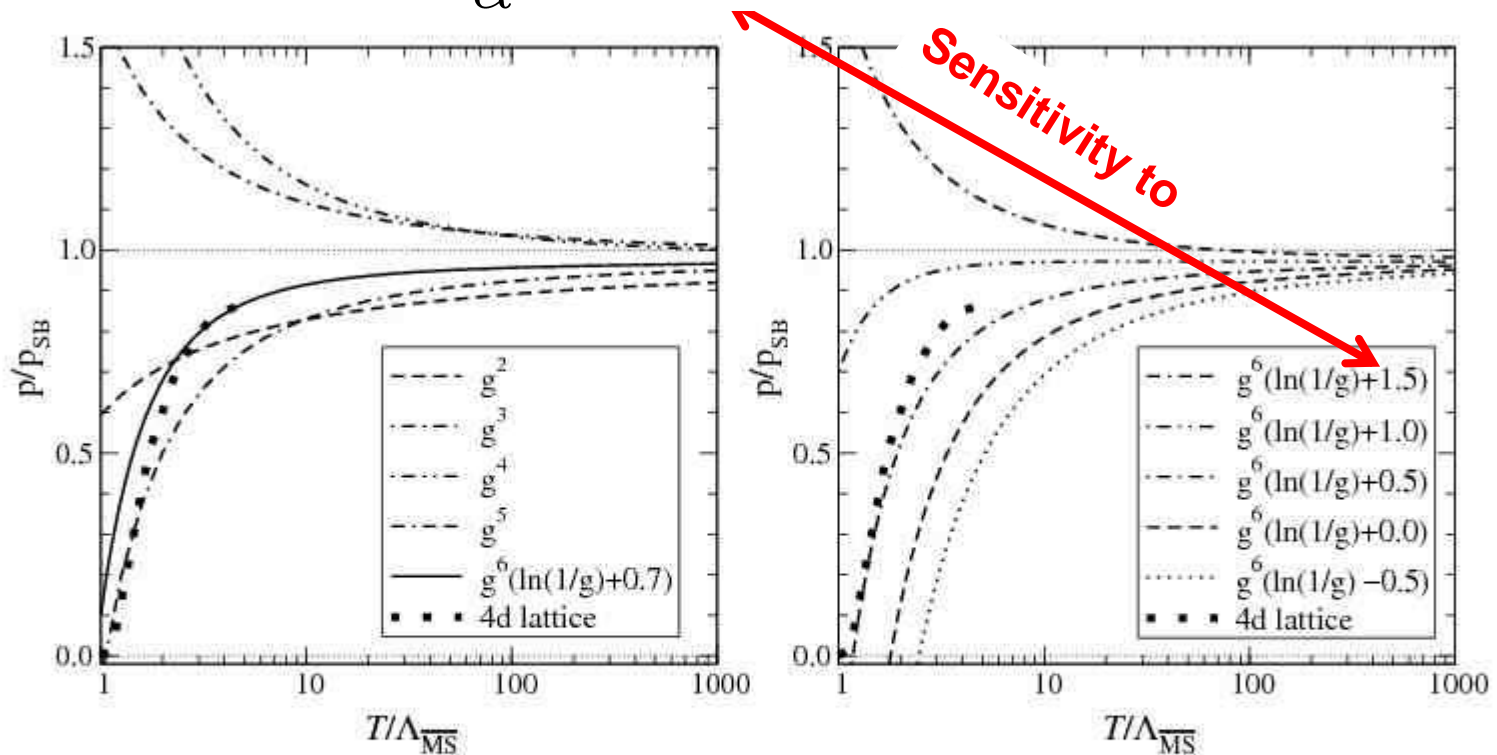


Figure 13: Left panel: Pressure (divided by T^4) as a function of temperature for the pure $[SU(3)_c]$ gauge theory. Several perturbative contributions (up to a given power in g) are shown, as well as lattice QCD data from Ref. [78]. Right panel: Sensitivity of the pressure to the value of the constant δ in the term $\sim g^6$. From Ref. [100].

The Quark gluon plasma in equilibrium.

[Dirk H. Rischke](#) , Prog.Part.Nucl.Phys.52:197-296,2004.

e-Print Archive: [nucl-th/0305030](#)

Try Interpretation of lattice QCD thermo data via Effective Quasiparticle models

SU(3) Gluon EOS with free quasi-gluons + B(T) bag
Fix degrees of freedom (d=16)

$$P(T) = \frac{d}{(2\pi)^3} \int d^3 p \frac{p^2}{3\sqrt{p^2 + M(T)^2}} \left[\exp \frac{\sqrt{p^2 + M(T)^2}}{T} - 1 \right]^{-1} - B(T)$$

$$\varepsilon(T) = \frac{d}{(2\pi)^3} \int d^3 p \sqrt{p^2 + M(T)^2} \left[\exp \frac{\sqrt{p^2 + M(T)^2}}{T} - 1 \right]^{-1} + B(T)$$

$$\epsilon(T), P(T) \leftrightarrow M(T), B(T)$$

Thermo Consistency Requires

Mass + Interaction

$$B(T) = B(T^*) - \sum_{i=g,q,\bar{q}} \frac{D_i}{(2\pi)^3} \int_{T^*}^T dT' M_i(T') \frac{dM_i(T')}{dT'} \int_0^\infty \frac{d^3 k}{\omega_i} f_i(k) . \quad (23)$$

Again T^* is an integration constant. If this expression is satisfied by the functions $M_i(T)$ and $B(T)$, we have a thermodynamically correct quasi-particle description containing effective gluons and quarks with effective thermal masses.

Massive gluons and quarks and the equation of state obtained from SU(3) lattice QCD.

[Peter Levai](#), [Ulrich W. Heinz](#) , Phys.Rev.C57:1879-1890,1998, e-Print: [hep-ph/9710463](#)

**Perturbative QCD expansion similar to an effective mass expansion
For weakly interacting massive Bosons and Fermions**

Ideal Massive Boson Gas Pressure

$$P_B(T, M) = d_B \left\{ \frac{\pi^2}{90} T^4 - \frac{M^2 T^2}{24} + \frac{M^3 T}{12\pi} + \frac{M^4}{64\pi^2} \ln \left(\frac{M^2}{CT^2} \right) + \dots \right\}$$

Ideal Massive Fermion Gas Pressure

$$P_F(T, M) = d_F \frac{7}{8} \left\{ \frac{\pi^2}{90} T^4 - \frac{M^2 T^2}{42} + \frac{M^4}{56\pi^2} \ln \left(\frac{16M^2}{CT^2} \right) + \dots \right\}$$

If one assumes $M = c g(T) T$

$$P(T) = P_{SB}(T) \{ 1 + a_2 g^2(T) + a_3 g^3(T) + a_4 g^4(T) \text{Log } g^2(T) + \dots \}$$

Example Fits with assumed

$$M_i(T) = c_i g(T)T$$

Fig.9. SU(3), $N_f=0,2,4$ --- $M_g(T)$, $M_q(T)$

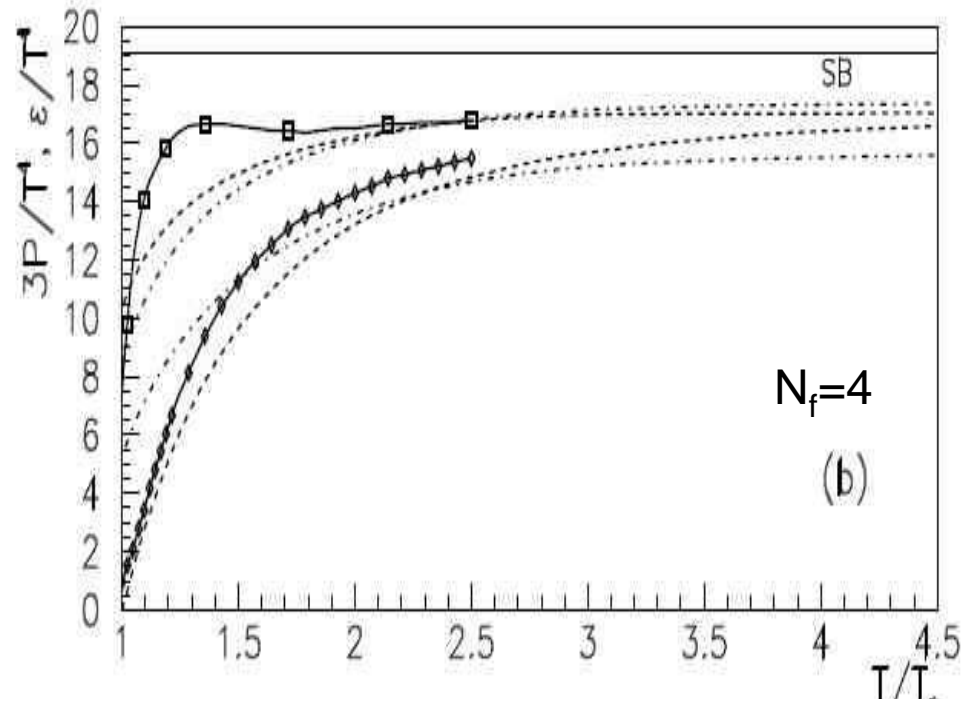
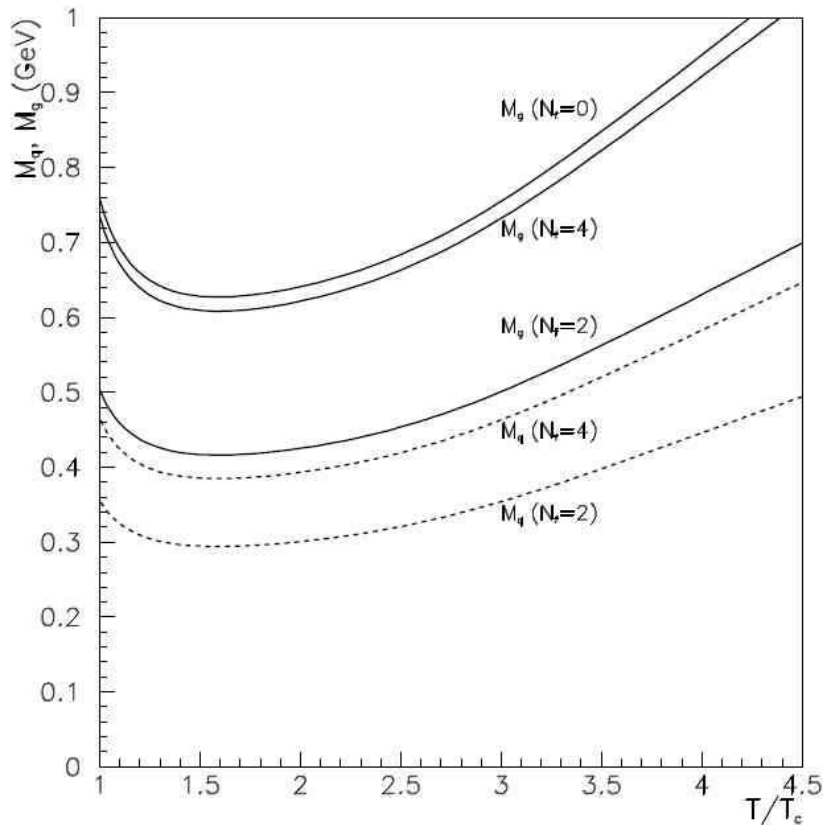
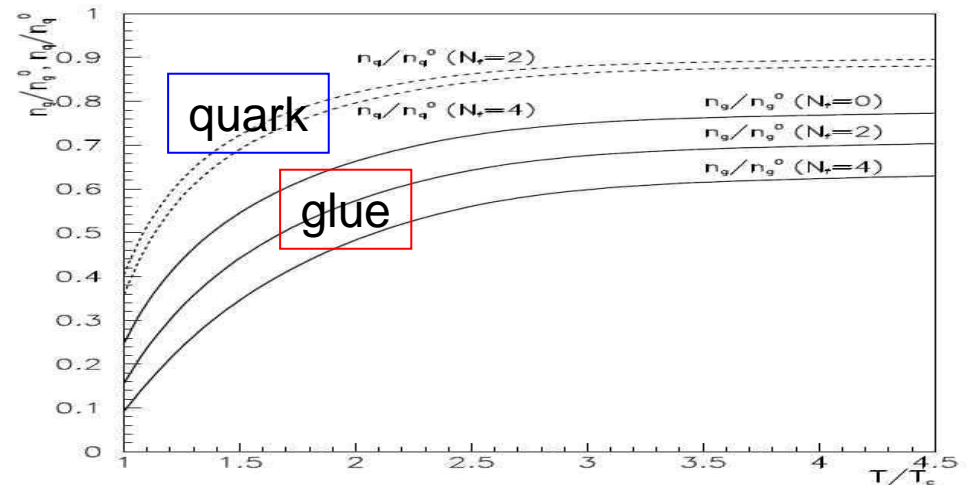


Fig.10. SU(3), $N_f=0,2,4$ --- n_g/n_g^0 , n_q/n_q^0



Massive gluons and quarks and the equation of state obtained from SU(3) lattice QCD.

[Peter Levai](#), [Ulrich W. Heinz](#), Phys.Rev.C57:1879-1890,1998, e-Print: [hep-ph/9710463](#)

Variational HTL Perturbation Theory to Two Loops

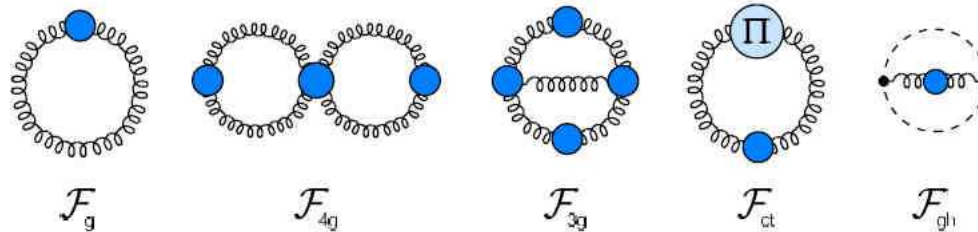


FIG. 2. Diagrams contributing through NLO in HTLpt. Shaded circles indicate dressed HTL propagators and vertices.

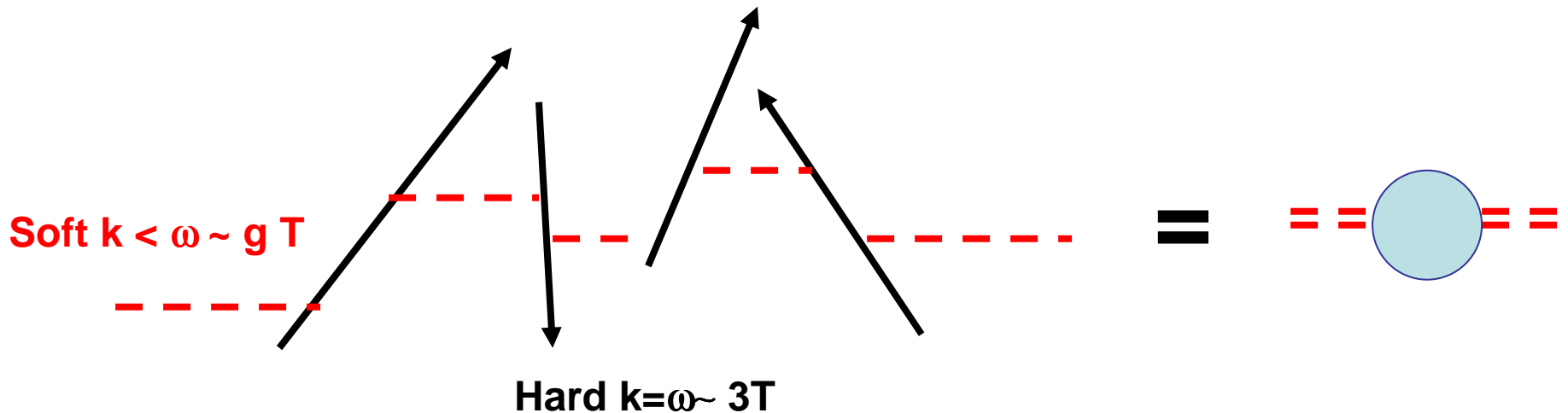
We calculate the pressure for pure-gluon QCD at high temperature to two-loop order using hard-thermal-loop (HTL) perturbation theory. At this order, all the ultraviolet divergences can be absorbed into renormalizations of the vacuum energy density and the HTL mass parameter. We determine the HTL mass parameter by a variational prescription. The resulting predictions for the pressure fail to agree with results from lattice gauge theory at temperatures for which they are available.

ABPS02 = J.O.Andersen, E.Braaten, E.Petitgirard and M.Strickland
Phys.Rev.D66:085016,2002. ;hep-ph/0205085

Hard Thermal Loop idea:

Typical momentum at temp T is $\omega=k \sim 3T$ for $m=0$ partons

Soft gluon field normal modes with $\omega \sim g T$
are Collective “Plasmon” many body excitations



$$\Delta_0^{-1}(\omega, k) = \omega^2 - k^2 + i\epsilon \quad \text{Polarization Self energy } \Pi(\omega, k)$$

$$= \text{Dressed Propagator} = \text{---} + \text{---} \Pi \text{---} = \omega^2 - k^2 - \Pi(\omega, k)$$

$$\text{Debye mass}^2 \approx m_D^2 = g^2 T^2$$

$$\mathcal{F}_{\text{ideal}} = (N_c^2 - 1) \left(-\frac{\pi^2}{45} T^4 \right)$$

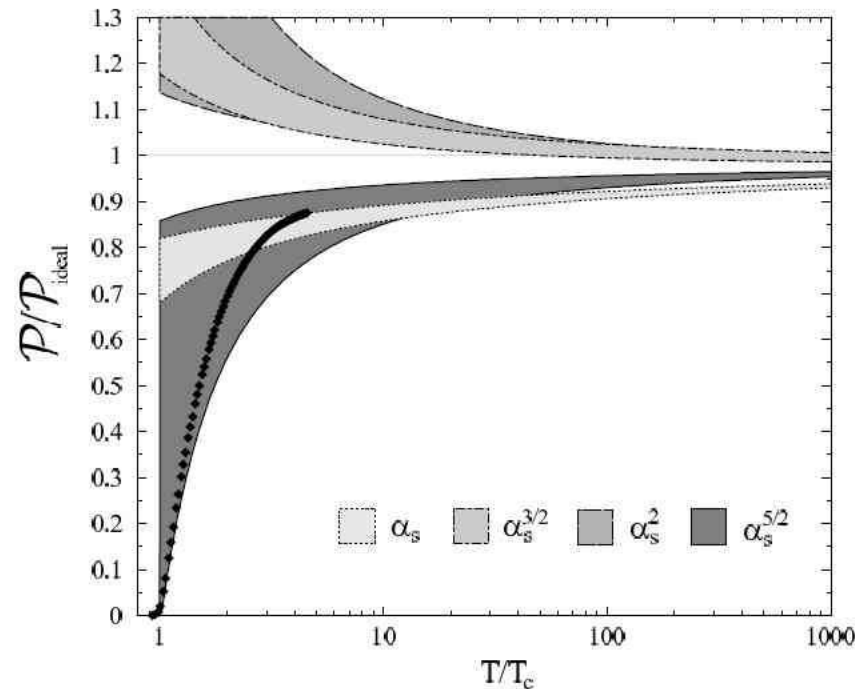
Stefan-Boltzmann

$$\Omega_{\text{LO}} = \mathcal{F}_{\text{ideal}} \left\{ 1 - \frac{15}{2} \hat{m}_D^2 + 30 \hat{m}_D^3 + \frac{45}{4} \left(\log \frac{\hat{\mu}}{2} - \frac{7}{2} + \gamma + \frac{\pi^2}{3} \right) \hat{m}_D^4 \right\}$$

$$\hat{m}_D = \frac{m_D}{2\pi T}$$

$$\hat{\mu} = \frac{\mu}{2\pi T}$$

1-loop HTL



**Failure of
Strict $m=0$
pQCD**

**Motivates
Variational
Ansatz**

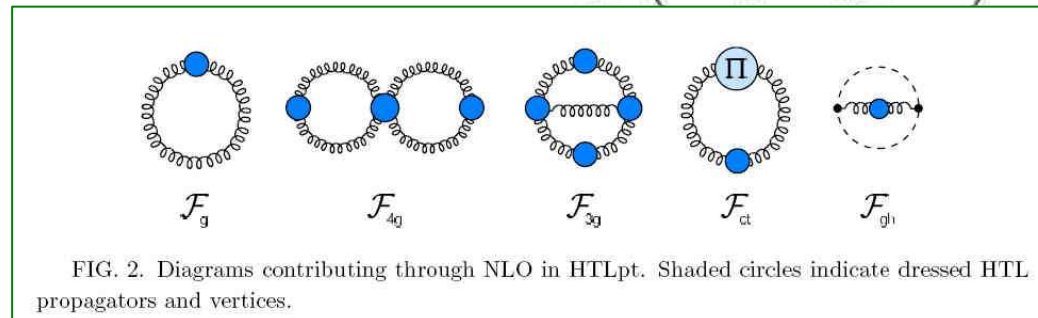
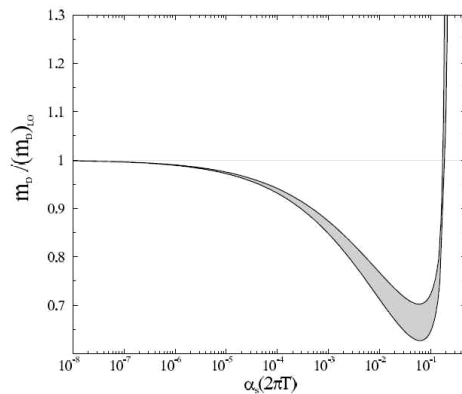
FIG. 1. The free energy for pure-gluon QCD as a function of T/T_c . The weak-coupling expansions through orders α_s , $\alpha_s^{3/2}$, α_s^2 , and $\alpha_s^{5/2}$ are shown as bands that correspond to varying the renormalization scale μ by a factor of two. The diamonds are the lattice result from Boyd et al. [5].

Variationally improved 2-loop HTL, including all terms through ABPS02 order g^5 , is

$$\Omega_{\text{NLO}} = \mathcal{F}_{\text{ideal}} \left\{ 1 - 15\hat{m}_D^3 - \frac{45}{4} \left(\log \frac{\hat{\mu}}{2} - \frac{7}{2} + \gamma + \frac{\pi^2}{3} \right) \hat{m}_D^4 \right. \\ \left. + \frac{N_c \alpha_s}{3\pi} \left[-\frac{15}{4} + 45\hat{m}_D - \frac{165}{4} \left(\log \frac{\hat{\mu}}{2} - \frac{36}{11} \log \hat{m}_D - 2.001 \right) \hat{m}_D^2 \right. \right. \\ \left. \left. + \frac{495}{2} \left(\log \frac{\hat{\mu}}{2} + \frac{5}{22} + \gamma \right) \hat{m}_D^3 \right] \right\}. \quad (62)$$

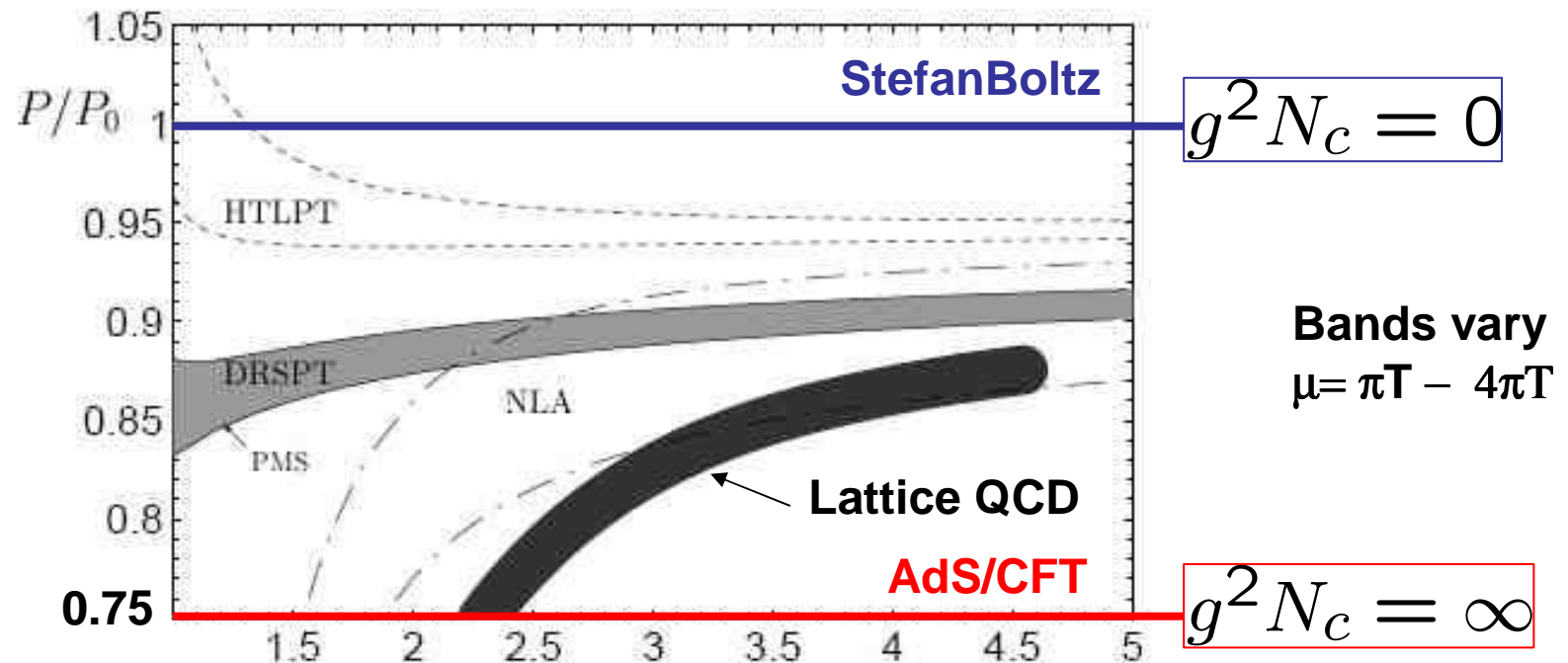
The **Assumed** gap equation which determines m_D is obtained by differentiating (62) with respect to m_D and setting this derivative equal to zero yielding:

$$\hat{m}_D^2 \left[1 + \left(\log \frac{\hat{\mu}}{2} - \frac{7}{2} + \gamma + \frac{\pi^2}{3} \right) \hat{m}_D \right] = \frac{N_c \alpha_s}{3\pi} \left[1 - \frac{11}{6} \left(\log \frac{\hat{\mu}}{2} - \frac{36}{11} \log \hat{m}_D - 3.637 \right) \hat{m}_D \right. \\ \left. + \frac{33}{2} \left(\log \frac{\hat{\mu}}{2} + \frac{5}{22} + \gamma \right) \hat{m}_D^2 \right]. \quad (63)$$



Variationally “improved” extended HTL Gluon Thermodynamics

Blaizot, Iancu, Rebhan
hep-ph/0303045



HTLPT: NLO (2-Loop) Variational Hard Thermal Loop (Braaten et al 02)
 NLA: Variational soft, Effective hard mode Next to Leading Approximate model
 DRSPT: Variationally improved “screened” perturbation theory in
 dimensionally reduced effective theory
 Black Band: Lattice QCD

“Evidently, our main result is that the convergence behavior of successive approximations to the pressure is dramatically improved by abandoning strict perturbation theory in the soft sector. “

“The quantitative predictions for the pressure in 2-loop HTLpt are disappointing. In the range $2T_c < T < 20T_c$, the pressure is predicted to be nearly constant with a value of about 95% of that of an ideal gas of gluons.”

“A possible conclusion is that HTLpt at two-loop order is simply not a useful approximation for thermal QCD. Another possibility is that the problem lies not with HTLpt but with the use of the $m_D/T \sim g$ expansion to approximate the scalar sum-integrals.”

Perhaps the Quasiparticle picture in QCD is incorrect and inconsistent due to strong nonperturbative 2 body correlations

Nuclear physics lesson:

5. THE BRUECKNER THEORY

The nucleon-nucleon potential very probably has a steep repulsive core at small distances. (This will certainly ensure saturation but a proper theory is needed to obtain an energy minimum at the observed nuclear density.) For this v it is clearly impossible to choose V by the Hartree-Fock method, as the matrix elements of

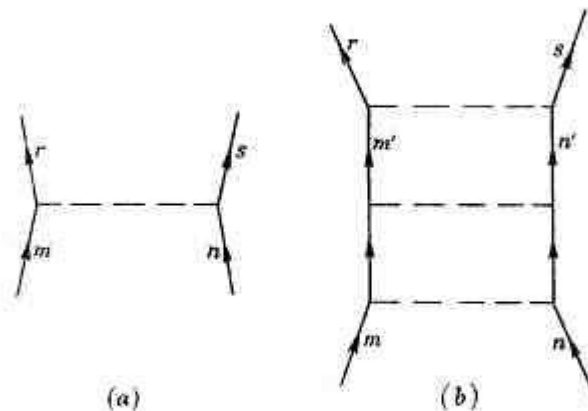


FIGURE 12

Ladder sum
Vs
Quasiparticle
Bubble sum

v will have a large contribution from the core. The Brueckner theory replaces v by a reaction matrix t calculated from a two-body equation of the type

$$t = v + v \frac{1}{E_0 - H_0} t. \quad (5.1)$$

The idea is to derive V from t instead of from v . Since H_0 contains V , V occurs in the

The hot non-perturbative gluon plasma is an almost ideal colored liquid

A. Peshier and W. Cassing : hep-ph/0502138

Entropy Density in Dynamic Quasi Particle Model:

$$n(\omega) = (\exp(\omega/T) - 1)^{-1}$$

$$s^{dqp} = -d_g \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n}{\partial T} (\text{Im} \ln(-\Delta^{-1}) + \text{Im} \Pi \text{Re} \Delta)$$

Instead of HTL, Assume an Ansatz for Dressed Gluon Propagator

$$1/\Delta(\omega, k; T) = \omega^2 - k^2 - M^2(T) + 2i\omega\gamma(T)$$

$$M^2 = \frac{N_c}{6} g^2 T^2$$

, as in HTL, BUT with Width

$$\gamma = \frac{3}{4\pi} \frac{M^2}{T^2} T \ln \frac{c}{(M/T)^2}$$

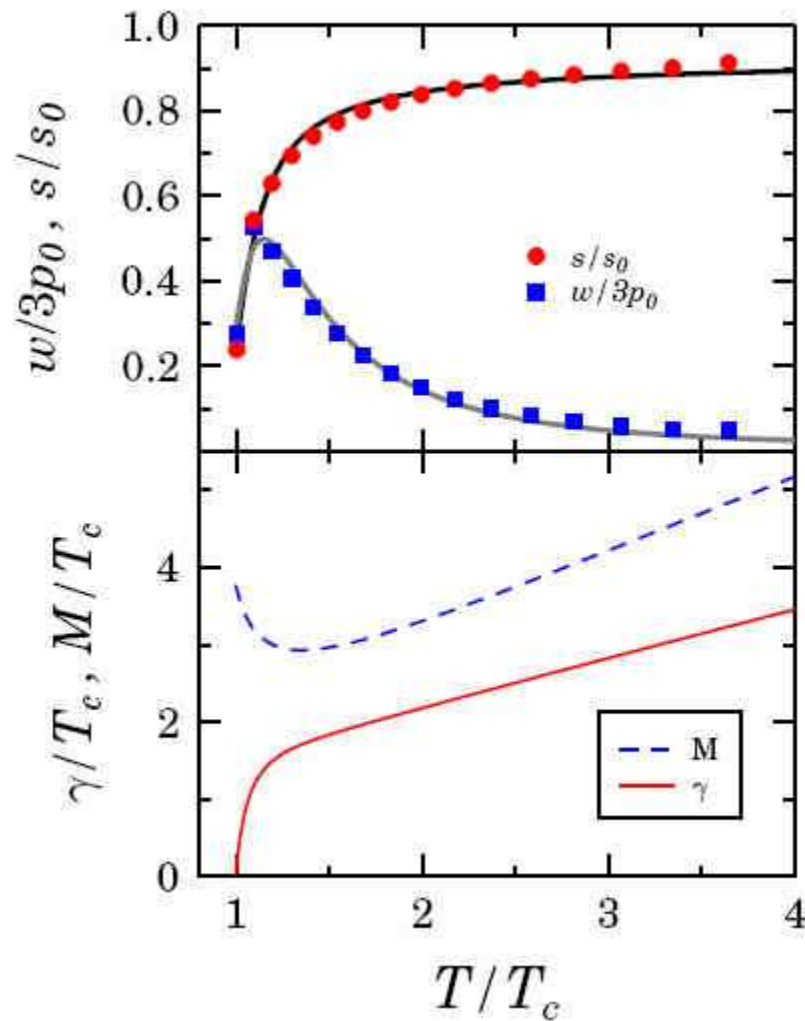
Novel model assumptions :

$$\gamma \sim M$$

and

$$g^2(T) = \frac{48\pi^2}{11N_c \ln(\lambda(T - T_s)/T_c)^2},$$

which permits an enhancement near T_c



The entropy $s(T)$ and the interaction measure $w = e-3p$, in units of the Stefan-Boltzmann limits s_0 and p_0 , from this fitted dynamical quasiparticle model in comparison to lattice calculations. The lower part shows the adjusted mass M and width .

$$1/\Delta(\omega, k; T) = \omega^2 - k^2 - M^2(T) - i\omega\gamma(T)$$

Collision rate in dynamic quasiparticle model

$$dN_{coll}/dV dt = \gamma N_+$$

$$\frac{dN_{coll}}{dV dt} = \tilde{\text{Tr}}_{P_1} \tilde{\text{Tr}}_{P_2} 2\sqrt{\lambda(s, P_1^2, P_2^2)} \sigma_{tot}(P_1, P_2) = \langle \sigma \rangle \tilde{\text{Tr}}_{P_1} \tilde{\text{Tr}}_{P_2} 2\sqrt{\lambda(s, P_1^2, P_2^2)} =: \langle \sigma \rangle I_2, \quad (6)$$

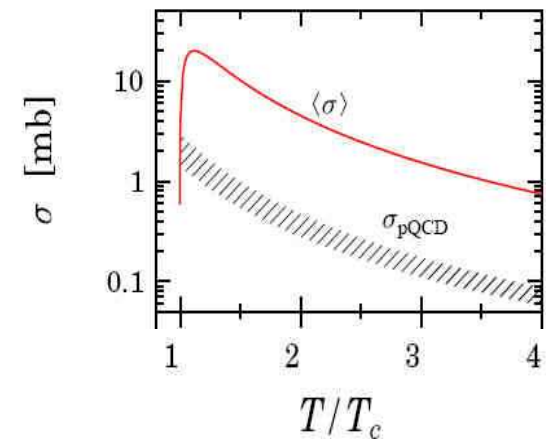
where $\lambda(x, y, z) = (x - y - z)^2 - 4yz$ and $s = (P_1 + P_2)^2$.

$$\tilde{\text{Tr}}_{P \dots} = d_g \int \frac{d\omega}{2\pi} \frac{d^3 p}{(2\pi)^3} 2\omega \rho(\omega) \Theta(\omega) \Theta(P^2) n(\omega) \dots$$

$N_+ = \tilde{\text{Tr}} 1$ = the effective quasi-particle density

$\langle \sigma \rangle = \gamma N_+ / I_2$ = effective total gg cross section

**Large assumed width => Large collision rate
=> Small viscosity**



Very different from “non-interacting” Quasiparticle models

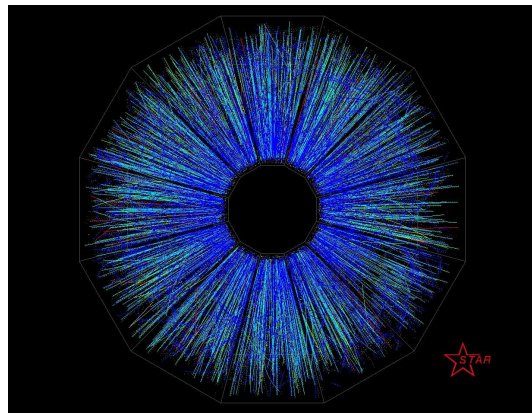
More consistent with Collective flow observables!

Effective mass models are useful for phenomenology

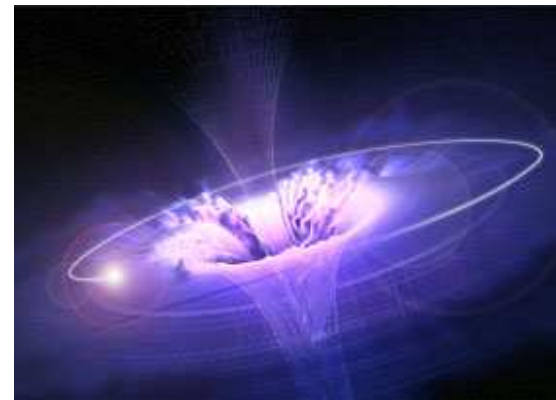
But they do not “explain” why lattice QCD is so close to ideal Stefan-Boltzmann while perturbative wQGP fails so Badly!

QCD thermodynamics is obviously much harder than predicted in 1975!

In last few years radical new ideas are being explored:



**?
=**



Is the **sQGP dual to a 4D Hologram of a 5D Anti de Sitter space Black Hole???**

Look for soluable field theory analogs of the very hard QCD

More Symmetry => More constraints => Solutions are easier

QM examples: SU(N) Harmonic Oscillator
O(4) Hydrogen Atom

In Field Theory it seems that **SO(2,3)** conformal
Super-Symmetric Yang Mills may be exactly soluable
in the super strong coupling limit

(Maldecena, Klebanov, Witten)

$$N_c \rightarrow \infty \text{ and } g^2 N_c \rightarrow \infty$$

Maldecena *Conjectured*:

In this limit, strong coupled quantum SYM in 4D
is dual to classical weak gravity in the 5D curved space time: AdS₅

Conformal SO(2,4) group in 4D ~ Isometry SO(2,4) group of AdS in 5D

The coming year will see a number of interesting developments as the Large Hadron Collider (LHC) goes online. The enormous amount of data generated by the LHC will force us to refine our methods—and explore new ones—for extracting and interpreting information from high energy collisions. This work should lead to new insights into the masses of elementary particles and the consequences of various models for particle physics and cosmology.

Also of interest is the recent application of string theory to the physics being done at the Relativistic Heavy Ion Collider (RHIC), where string theory permits some calculations that would otherwise be intractable. The idea at RHIC is to better understand the strong force that binds together the elements of a nucleon, and 2007 may see the theoretical advances of string theory inform the experimental results from RHIC.

—Lisa Randall, Harvard University

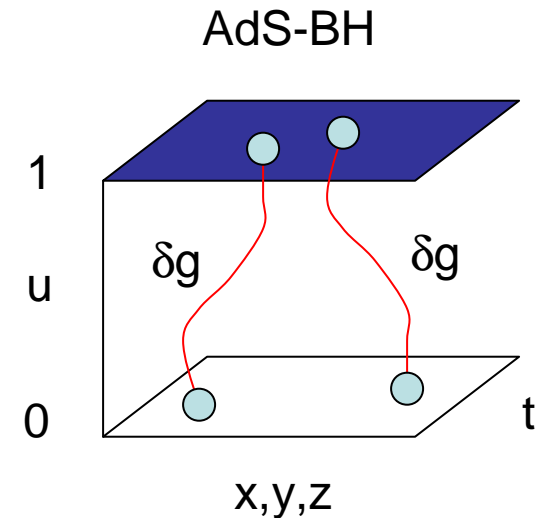
Of course, some may disagree...
...but in the end the “right” approach
will be validated by both
qualitative concepts,
and quantitative predictions



New Yorker, Jan. 8, 2007

**IF the Maldacena conjecture is true
(but no math proof yet),**

then



thermal $\mathcal{N} = 4$ $SU(N_c)$ SYM in the limit of infinite N_c and infinite 't Hooft coupling $g_{YM}^2 N_c$. An equilibrium thermal state of the theory at temperature T is described by a dual five-dimensional AdS-Schwarzschild black hole (more precisely, by a black hole with a translationally invariant horizon) whose metric is given by

$$ds_5^2 = \frac{(\pi T R)^2}{u} (-f(u) dt^2 + dx^2 + dy^2 + dz^2) + \frac{R^2}{4u^2 f(u)} du^2, \quad (3)$$

where $f(u) = 1 - u^2$, T is the Hawking temperature, R is the AdS radius. In Eq. (3), the horizon corresponds to $u = 1$, the spatial infinity to $u = 0$. One may think of the four-dimensional gauge theory as of a theory defined on the “boundary” at $u \rightarrow 0$ of the spacetime (3) with the standard Minkowski coordinates t, x, y, z .

SU(N_c) Gauge Theories with $\mathcal{N}=0,1,2,4$ SUSY

Super-symmetry

$$\beta = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} + \dots$$

$$\beta_0 = \frac{N_c}{3} (11_{\text{glue}} - 2 N_{1/2}^A - N_0^A) - \frac{1}{3} (2 N_{1/2}^F + N_0^F)$$

Number of Adjoint Gluons $N_g = \mathcal{N} \times 2_s \times (N_c^2 - 1)$

For

$$N_c = N_{1/2}^F = 3$$

	$N_{1/2}^A$	N_0^A	$N_{1/2}^F$	N_0^F	β_0	n_{BE}	n_{FD}
QCD: $\mathcal{N} = 0$	0	0	N_{flav}	0	$\frac{11}{3} N_c - \frac{2}{3} N_{\text{flav}}$	16	36
$\mathcal{N} = 1$	1	0	N_{flav}	N_{flav}	$3 N_c - 1 N_{\text{flav}}$	34	34
$\mathcal{N} = 2$	2	1	$2 N_{\text{flav}}$	$2 N_{\text{flav}}$	$2 N_c - 2 N_{\text{flav}}$	68	68
SO(2,3): $\mathcal{N} = 4$	4	3	0	0	0	64	64

Isometry group of AdS₅

$$\frac{\sigma}{T^3} = \frac{4P}{T^4} = \frac{4}{3} \frac{\varepsilon}{T^4} = \frac{2\pi^2}{45} (n_{\text{BE}} + \frac{7}{8} n_{\text{FD}})$$

g=0 SB Equation of State

Weakly coupled Super-Symmetric Yang-Mills Thermodynamics: hep-ph/0611393

TABLE I: Number of adjoint scalar (n_s) and fermionic (n_f) degrees of freedom for different \mathcal{N} , and the resulting common asymptotic thermal mass for all excitations. Also given are the effective numbers of freedom as measured by $g_* \equiv \mathcal{S}_0/(2\pi^2 T^3/45)$; the coefficients of the order- λ term in \mathcal{S} , $f_2 = \mathcal{S}_2/\lambda\mathcal{S}_0$; and the values $\lambda = \lambda_*$ where $\mathcal{S}_3 = |\mathcal{S}_2|$.

\mathcal{N}	n_s	n_f	$m_\infty^2/\lambda T^2$	g_*/N_g	$-\pi^2 f_2$	λ_*
0	0	0	1/6	2	5/16	1.85
1	0	2	1/4	15/4	3/8	2.78
2	2	4	1/2	15/2	3/4	1.91
4	6	8	1	15	3/2	1.14

$$N_g \equiv N_c^2 - 1$$

$$\lambda \equiv g^2 N_c \rightarrow 0$$

Perturbative
Expansion of
Entropy

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_2 + \mathcal{S}_3 + \dots = \mathcal{S}_0(1 - c_2\lambda + c_3\lambda^{3/2} + \dots)$$

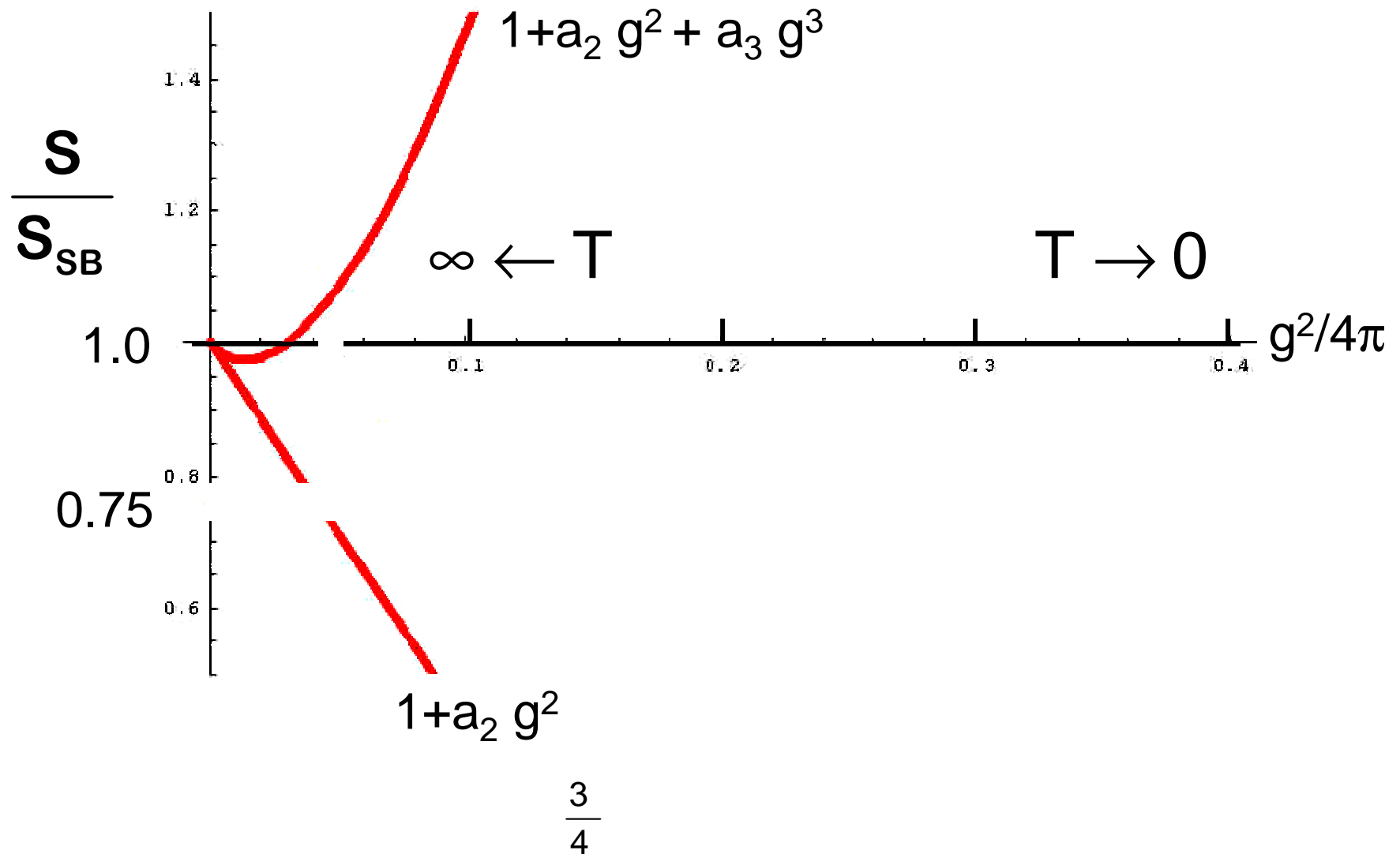
wSYM

$$\mathcal{S}_2 = -T \left\{ \sum_b \frac{m_{\infty(b)}^2}{12} + \sum_f \frac{m_{\infty(f)}^2}{24} \right\}$$

$$\begin{aligned} m_{\infty(s)}^2 = m_{\infty(g)}^2 &= \frac{2 + n_s + n_f/2}{12} \lambda T^2 \\ m_{\infty(f)}^2 &= \frac{2 + n_s}{8} \lambda T^2, \end{aligned}$$

$$\frac{\mathcal{S}_3}{N_g} = \frac{1}{3\pi} \left\{ m_D^3 + n_s m_{\infty(s)}^3 \right\} = \frac{2\sqrt{2} + n_s}{3\pi} m_\infty^3$$

Weak coupling expansion in $\text{AdS}_5 / \text{SYM}_4$ CFT
Is as poorly converging as in wQGP



$$\frac{3}{4}$$

S. S. Gubser, I. R. Klebanov and A. A. Tseytlin, “Coupling constant dependence in the thermodynamics of $N = 4$ supersymmetric Yang-Mills theory,” Nucl. Phys. B **534**, 202 (1998) [arXiv:hep-th/9805156].

A Conjecture from String Theory AdS_5 / SuperYM

A.Buchel, J.T.Liu and A.O.Starinets :hep-th/0406264.

Using the AdS/CFT conjecture, one is able to predict the behavior of the entropy of $\mathcal{N} = 4$ SYM in the regime of strong coupling [14], [15]. In the large N_c limit, the entropy is given by

$$S = \frac{2\pi^2}{3} N_c^2 V_3 T^3 f(g_{YM}^2 N_c), \quad (1.3)$$

where the function $f(g_{YM}^2 N_c)$ interpolates (presumably smoothly) between 1 at zero coupling and 3/4 at infinite coupling. The strong coupling expansion for f was obtained by Gubser, Klebanov and Tseytlin [15]

$$f(g_{YM}^2 N_c) = \frac{3}{4} + \frac{45}{32} \zeta(3) (2g_{YM}^2 N_c)^{-3/2} + \dots \quad (1.4)$$

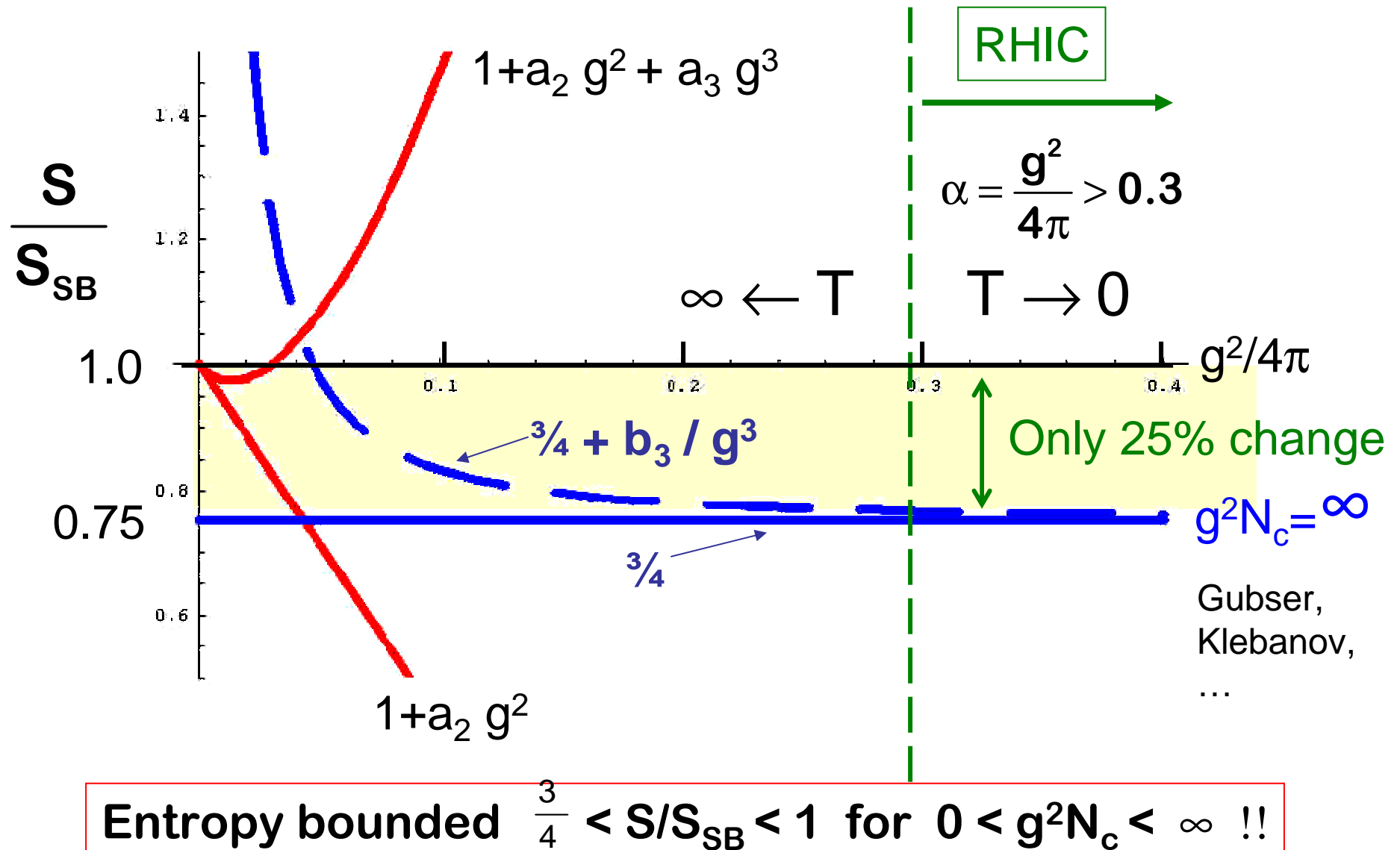
At weak coupling, f can be found by using finite-temperature perturbation theory [16]

$$f(g_{YM}^2 N_c) = 1 - \frac{3}{2\pi^2} g_{YM}^2 N_c + \frac{3 + \sqrt{2}}{\pi^3} (g_{YM}^2 N_c)^{3/2} + \dots \quad (1.5)$$

$$0.75 < S/S_0 < 1.0 \text{ for } 0 < g^2 N_c < \infty$$

Weak vs Strong coupling expansion in $\text{AdS}_5 / \text{SYM}_4$ CFT

A Physics Lesson from String Theory



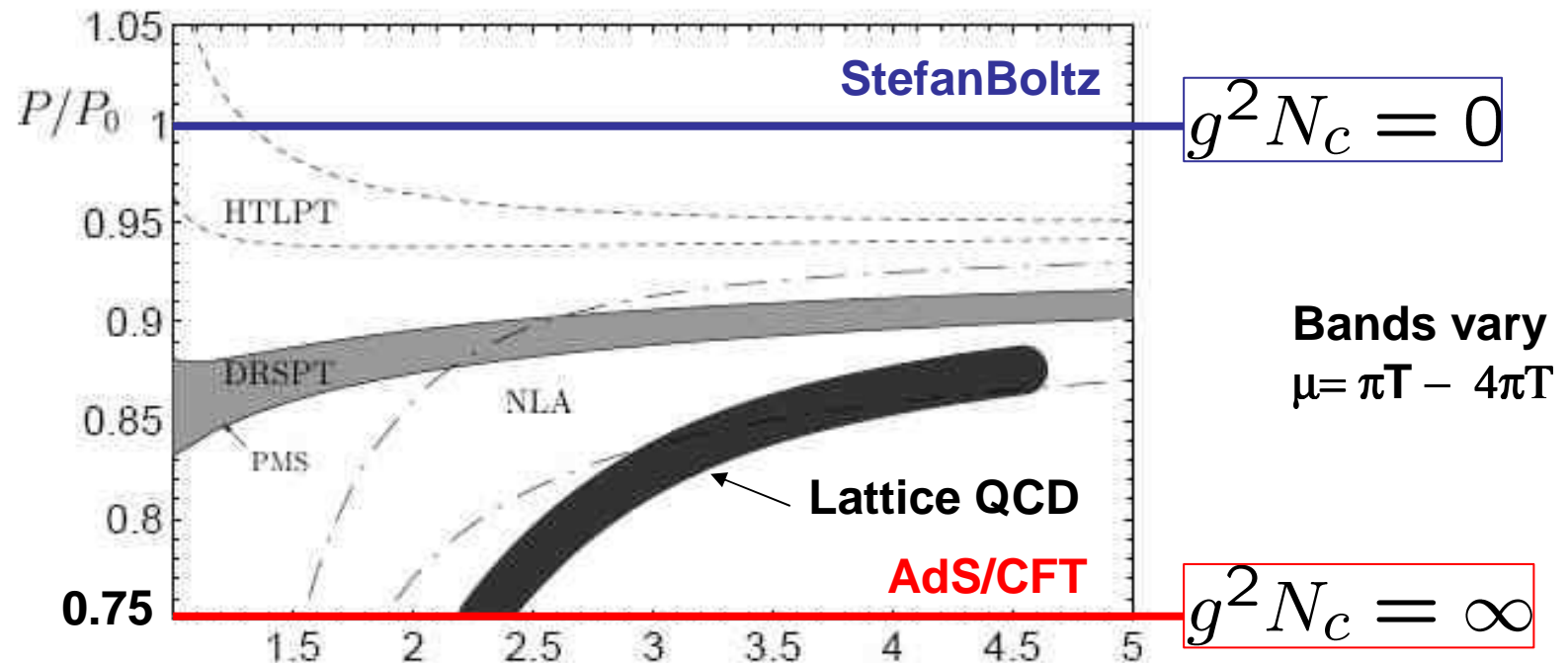
- First great success of AdS/CFT is that it explains how QCD entropy density could

deviate from Stefan Boltzman by only $\sim 20\%$!

- But is this the right explanation?

Variationally “improved” extended HTL Gluon Thermodynamics

Blaizot, Iancu, Rebhan
hep-ph/0303045



HTLPT: NLO (2-Loop) Variational Hard Thermal Loop (Braaten et al 02)
 NLA: Variational soft, Effective hard mode Next to Leading Approximate model
 DRSPT: Variationally improved “screened” perturbation theory in dimensionally reduced effective theory
 Black Band: Lattice QCD

“Evidently, our main result is that the convergence behavior of successive approximations to the pressure is dramatically improved by abandoning strict perturbation theory in the soft sector. “

A very recent snag under active debate

Hard-thermal-loop entropy of supersymmetric Yang-Mills theories.

[J.-P. Blaizot](#), [E. Iancu](#), [U. Kraemmer](#), [A. Rebhan](#) e-Print: [hep-ph/0611393](#)

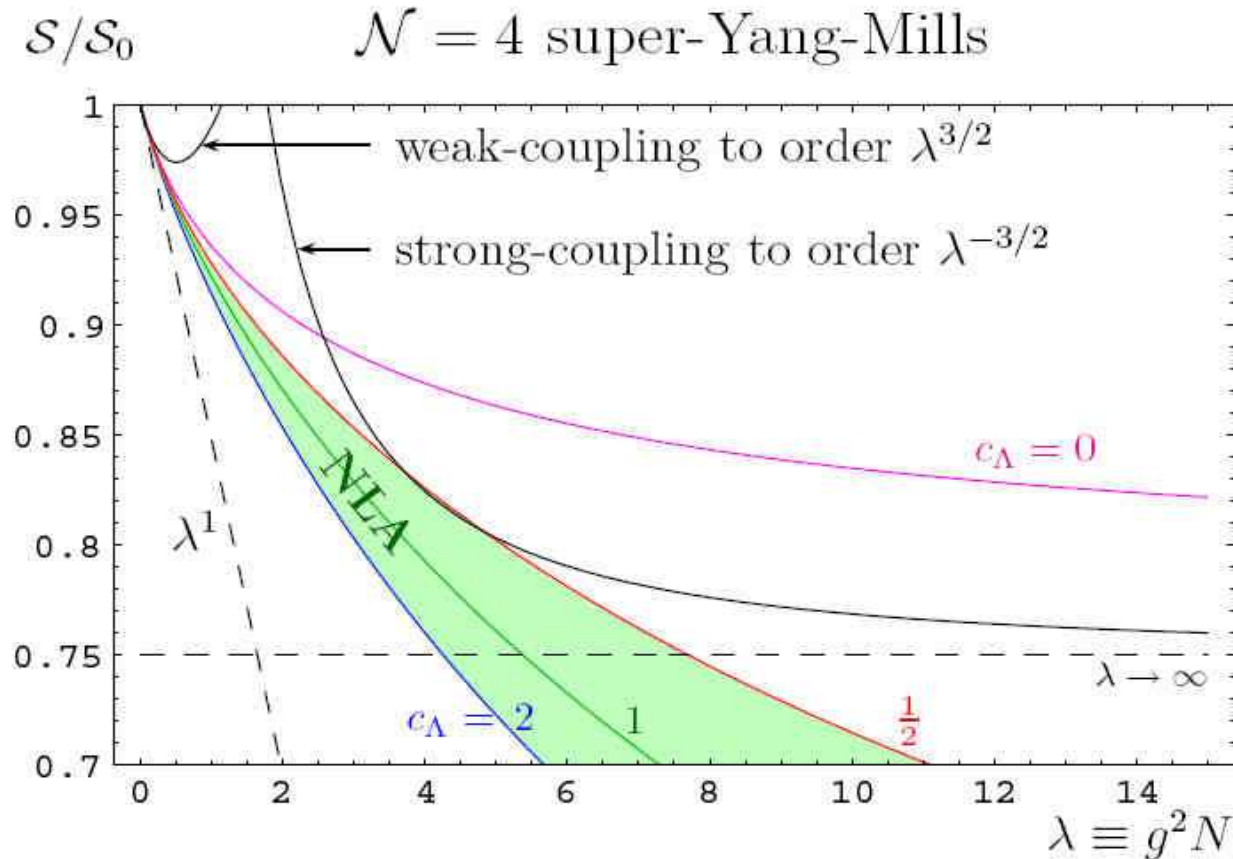
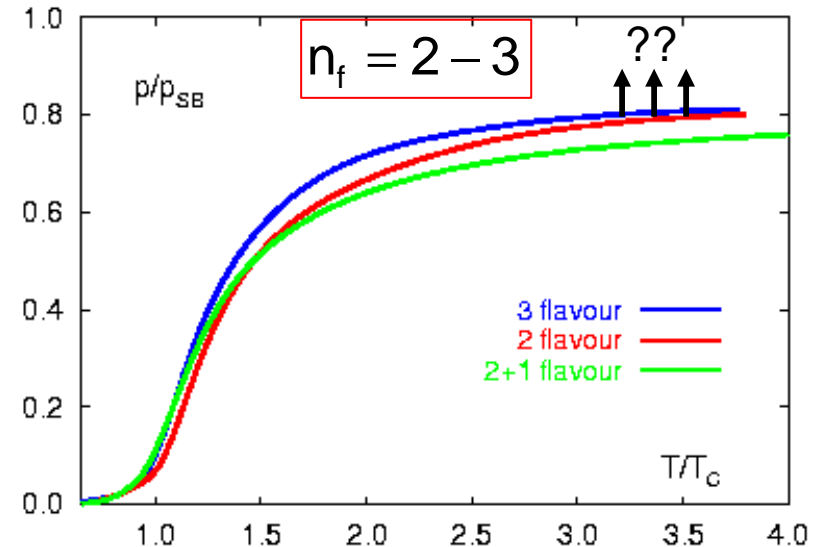
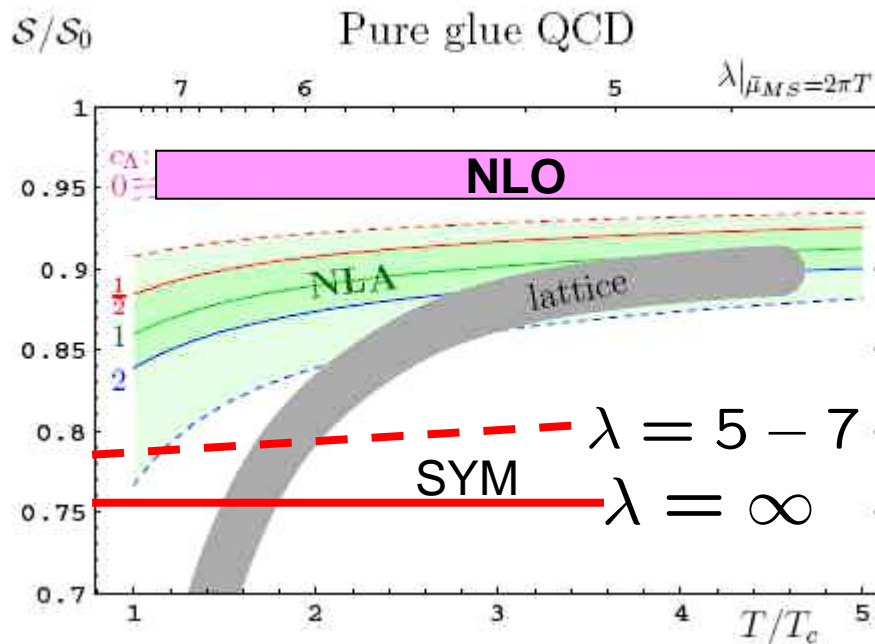


FIG. 2: Weak and strong coupling results for the entropy density of $\mathcal{N} = 4$ SYM theory together with the NLA results obtained in analogy to QCD (cp. Fig. [1](#)), but as a function of λ , which here is a free parameter.

Lattice QCD: F. Karsch et al



“The lattice results for QCD that are frequently referred to are usually results with quarks which have not been continuum extrapolated and so typically give somewhat smaller numbers. Reliable continuum extrapolations have so far been done only for purely gluonic QCD.”

Hard-thermal-loop entropy of supersymmetric Yang-Mills theories.

[J.-P. Blaizot](#), [E. Iancu](#), [U. Kraemmer](#), [A. Rebhan](#) e-Print: [hep-ph/0611393](#)

So, What the Hell is a sQGP?



**Hell if I know. Stay tuned . No one yet knows.
This is frontier research, Man!
You are invited to join the active research and
furious debate to find the right answer.**