HUNGARIAN ACADEMY OF SCIENCES
WIGNER RESEARCH CENTRE FOR PHYSICS

Is Perfect Fluidity of the sQGP *necessary* in light of recent BES & D+Au & p+Au data from RHIC and LHC ?

Gyulassy Miklós
Wigner Research Center KFKI
(Columbia University)

Collaborators: Levai P, Vitev I, Biro T

Abstract:
Recent low pT<2 GeV azimuthal correlation data from the beam energy scan (BES) and D+Au at RHIC/BNL and the especially the surprising low pT azimuthal v_n(pT) in p+Pb at LHC challenge long held assumptions about the necessity of perfect fluidity (minimal viscosity to entropy ~ 1/4pi) to account for azimuthal asymmetric "flow" patterns in A+A.

Perfect fluidity is certainly sufficient to fit all A+A and even p+A, data,
but is it really necessary, i.e., is it a unique property of sQGP?

I discuss **basic** pQCD interference phenomena from beam jet color antenna arrays that may help unravel current BES+DA+pA vs AA puzzles without requiring perfect fluid hydrodynamic or CGC Glasma diagrams , but only LO Feynmann diagrams.

Slide 3-4: The Case for Perfect Fluidity and CGC/Glasma *Sufficiency* for Flow Harmonics in Non-central A+A as of QM12

Slides 5-14 : Trouble for Fluidity since 2012

a) Beam Energy *independence*
b) p(D)+A = A+A v2 and v3 Beam size *independence*
c) large rapidity-even v1 Dipole

Slides 15-27 : Is Perfect Fluidity *Necessary ??*
Are vn in B+A mostly Nonabelian Wave interference?

(see also QM talks by Venugopalan , Dusling, Schenke... & poster of Biro, Schram)

Could rapidity-even v odd Dipole,Triangular, etc harmonics be just fingerprints of basic pQCD beam jet anisotropic bremsstrahlung?

Work in Progress, all order in opacity pQCD beam jets vn Bremsstrahlung in P+A via HIJING replacing ARIADNE with VGB

Non-Abelian Bremsstralung and Azimuthal Asymmetries in High Energy p+A

M. Gyulassy,[1,2,*] P. Levai,[1] I. Vitev,[3] and T. Biro[1]

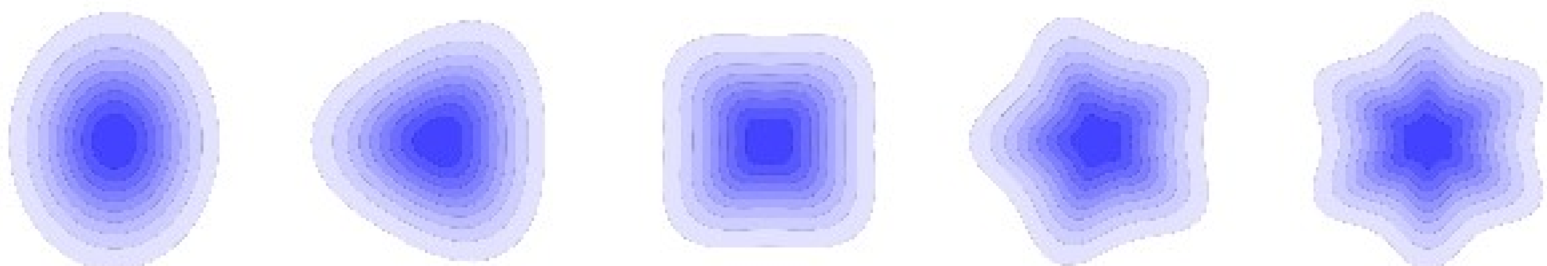
[1] *MTA WIGNER Research Centre for Physics, RMI, Budapest, Hungary*
[2] *Department of Physics, Columbia University, New York, NY 10027, USA*
[3] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
(Dated: May 13, 2014 v11)

slide of B. Schenke QM12

Higher harmonic flow **Fourier starts at n=1**

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_{n=2}(2v_n \cos[n(\phi - \psi_n)]\right)$$

When including fluctuations, all moments appear:



$n=2$ $n=3$ $n=4$ $n=5$ $n=6$

also v_1 and $n > 6$

Compute $v_n = \langle\cos[n(\phi - \psi_n)]\rangle$
with the event-plane angle $\psi_n = \frac{1}{n}\arctan\frac{\langle\sin(n\phi)\rangle}{\langle\cos(n\phi)\rangle}$

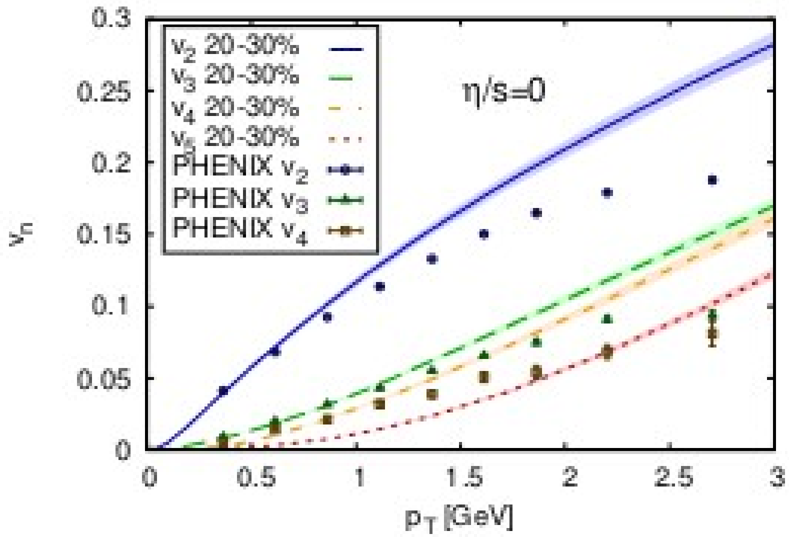
Part 1: case for perfect fluidity with CGC IS

Using higher harmonics to determine η/s

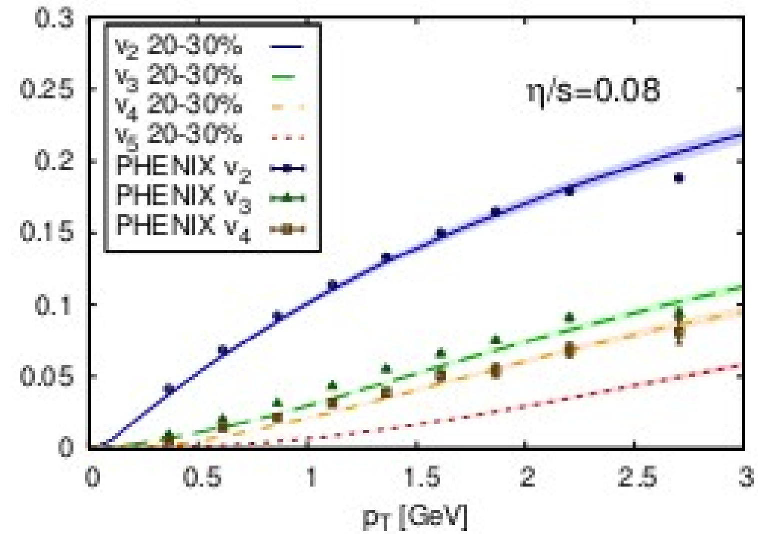
Viscocity/entropy
Of perfect fluid QGP

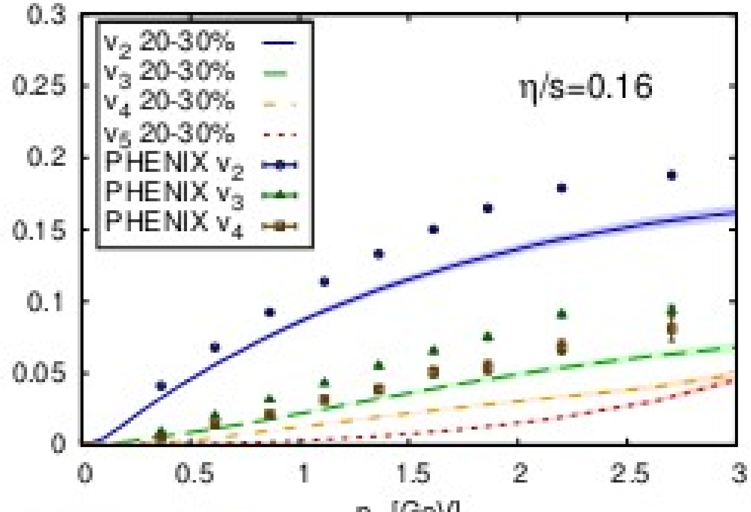
B. Schenke, S. Jeon, C. Gale, arXiv:1109.6289

Data is from event-plane method. Calculations are $\sqrt{\langle v_n^2 \rangle}$.



MC-Glauber initial conditions

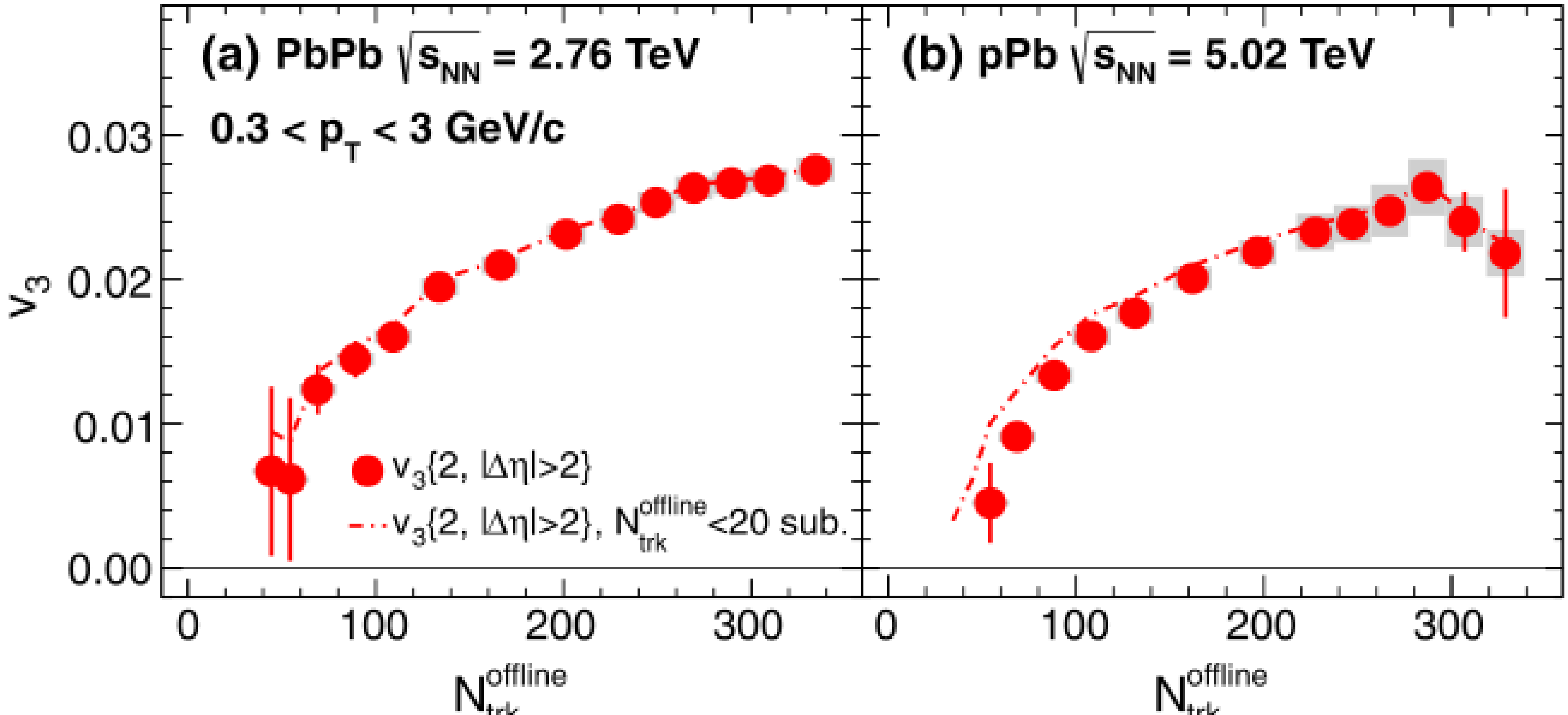


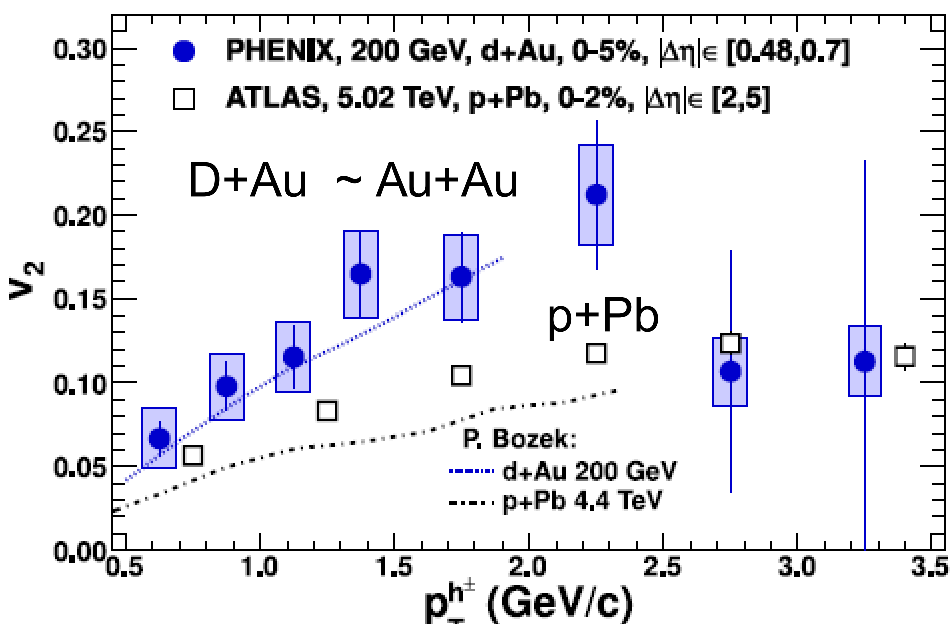


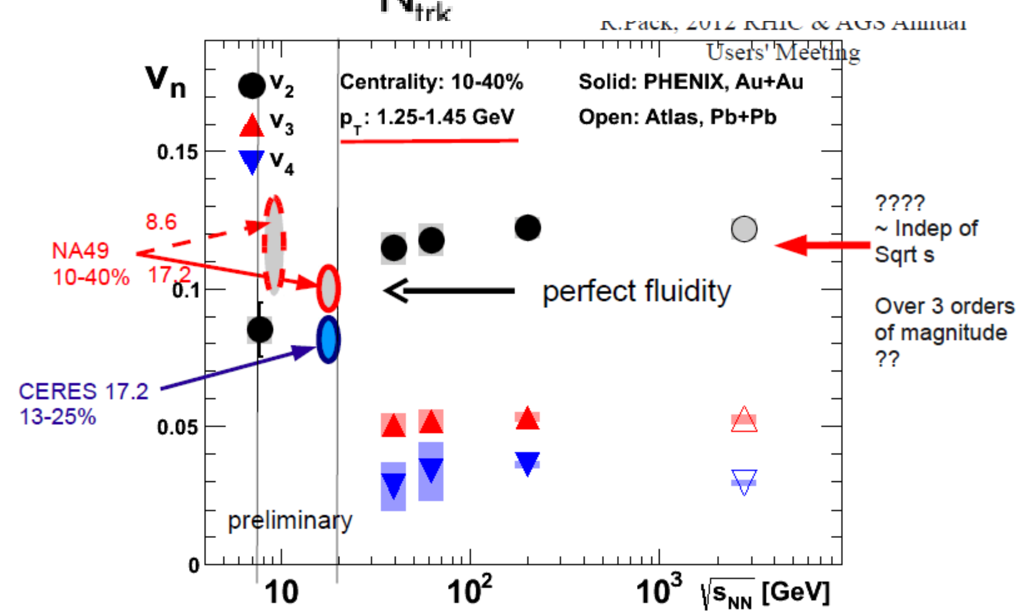
This is promising.
Need systematic study of all v_n as function of initial conditions, granularity, η/s, ...

Experimental data: PHENIX, arXiv:1105.3928

QM12 p(D) + A & BES surprises: v3(p+Pb) =v3(Pb+Pb) ; v2(D+Au)=v2(Au+Au) ; Ecm independ





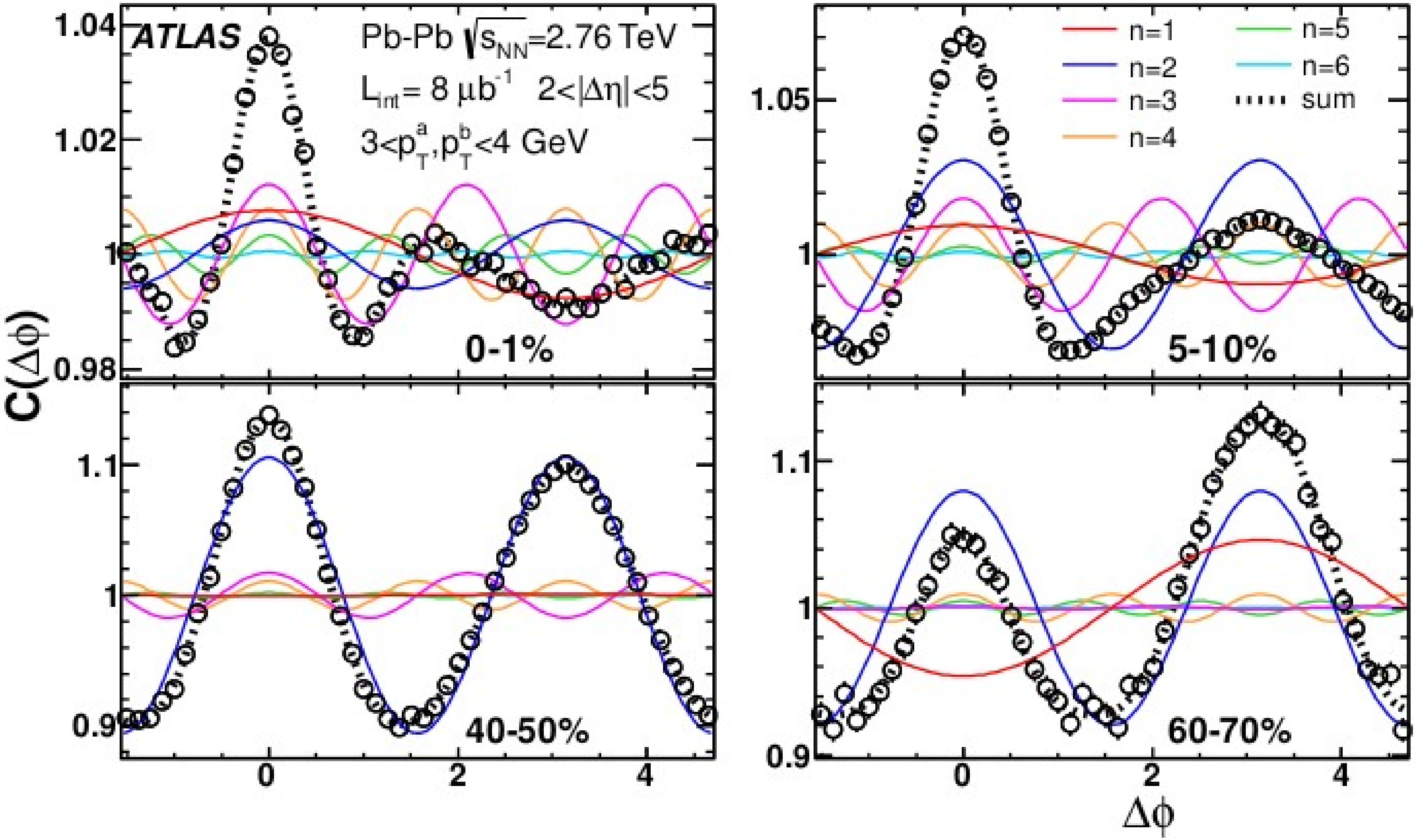


BES vn independent os Ecm

Another surprise : v1

[ATLAS Collaboration], PRC86, 014907 (2012)

PQCD Color Scintillations or Lumpy CGC perfect hydro response ? or complex combo of many effects?

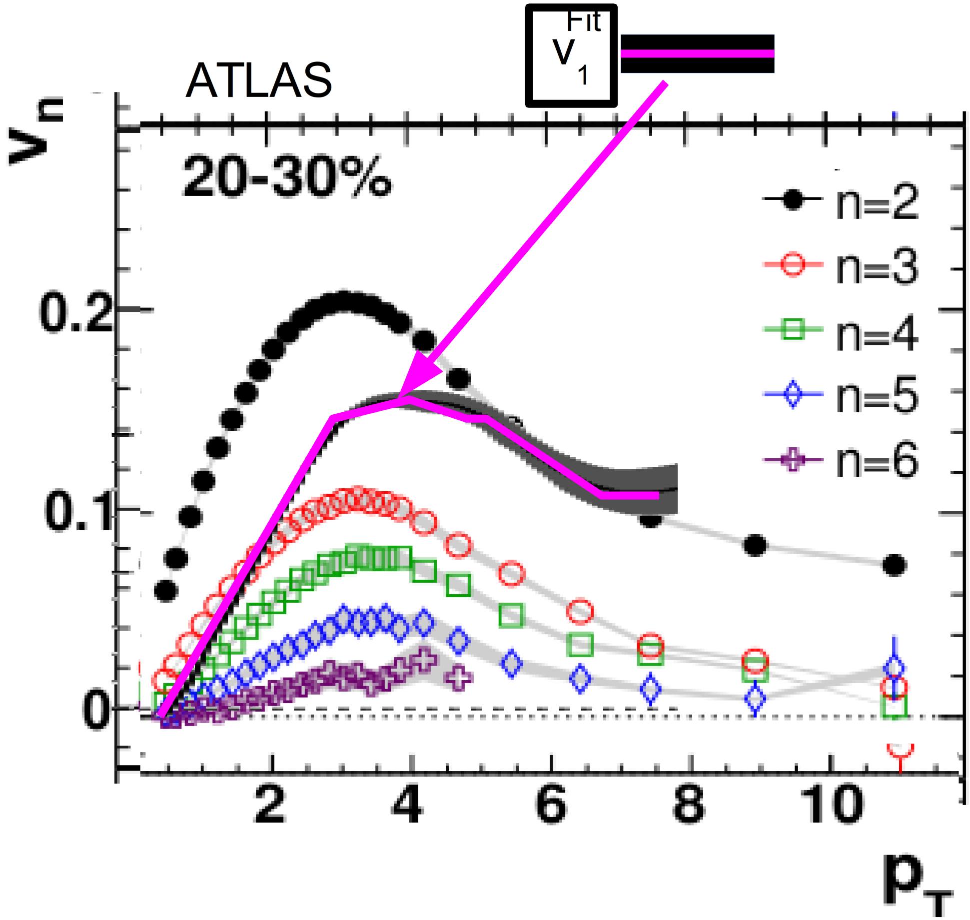


The magnitude of v1,1=<Cos[ϕ1 - ϕ2]> <u>Red</u> is large for p1~p2~ 3-4GeV

After subtracting dijets and mom.consv a new rapidity even Dipole!

$$\chi^2 = \sum_{a,b} \frac{\left(v_{1,1}(p_T^a, p_T^b) - [\boxed{v_1^{\rm Fit}}(p_T^a)\boxed{v_1^{\rm Fit}}(p_T^b) - cp_T^a p_T^b]\right)^2}{\left(\sigma_{a,b}^{\rm stat}\right)^2 + \left(\sigma_{a,b}^{\rm sys,p2p}\right)^2},$$

Rapidity-even, positive collective dipole <Cos[φ1 -φ1]> ~ - (negative mom.conserv. term)



Is v1 an artifact of fit? See *

A hydro response ? See Schenke

or is it a fingerprint of basic

pQCD Color Scintillation?

*Beware v1 is notoriously tricky, see L. Csernai, H. Stoecker, JPG 2014

Below we show rapidity-even ATLAS v1 Ridge component natural from pQCD Bremsstrahlung

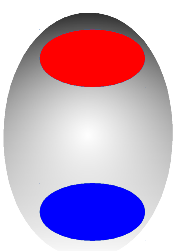
Fuctuating Dipole can also appear

Higher harmonic flow

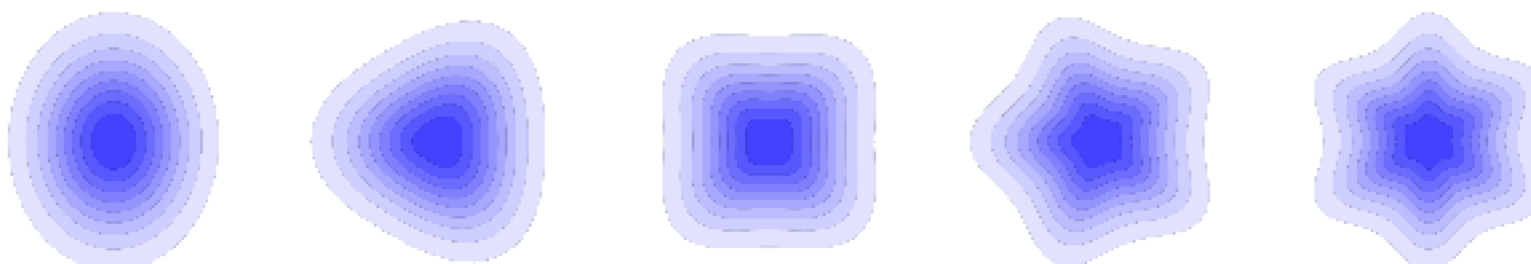
Fourier starts at n=1

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_{n=1} (2v_n \cos[n(\phi - \psi_n)]\right)$$

When including fluctuations, all moments appear:



n=1



$n = 2$ $n = 3$ $n = 4$ $n = 5$ $n = 6$

also v_1 and $n > 6$

Compute $v_n = \langle \cos[n(\phi - \psi_n)] \rangle$
with the event-plane angle $\psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle}$

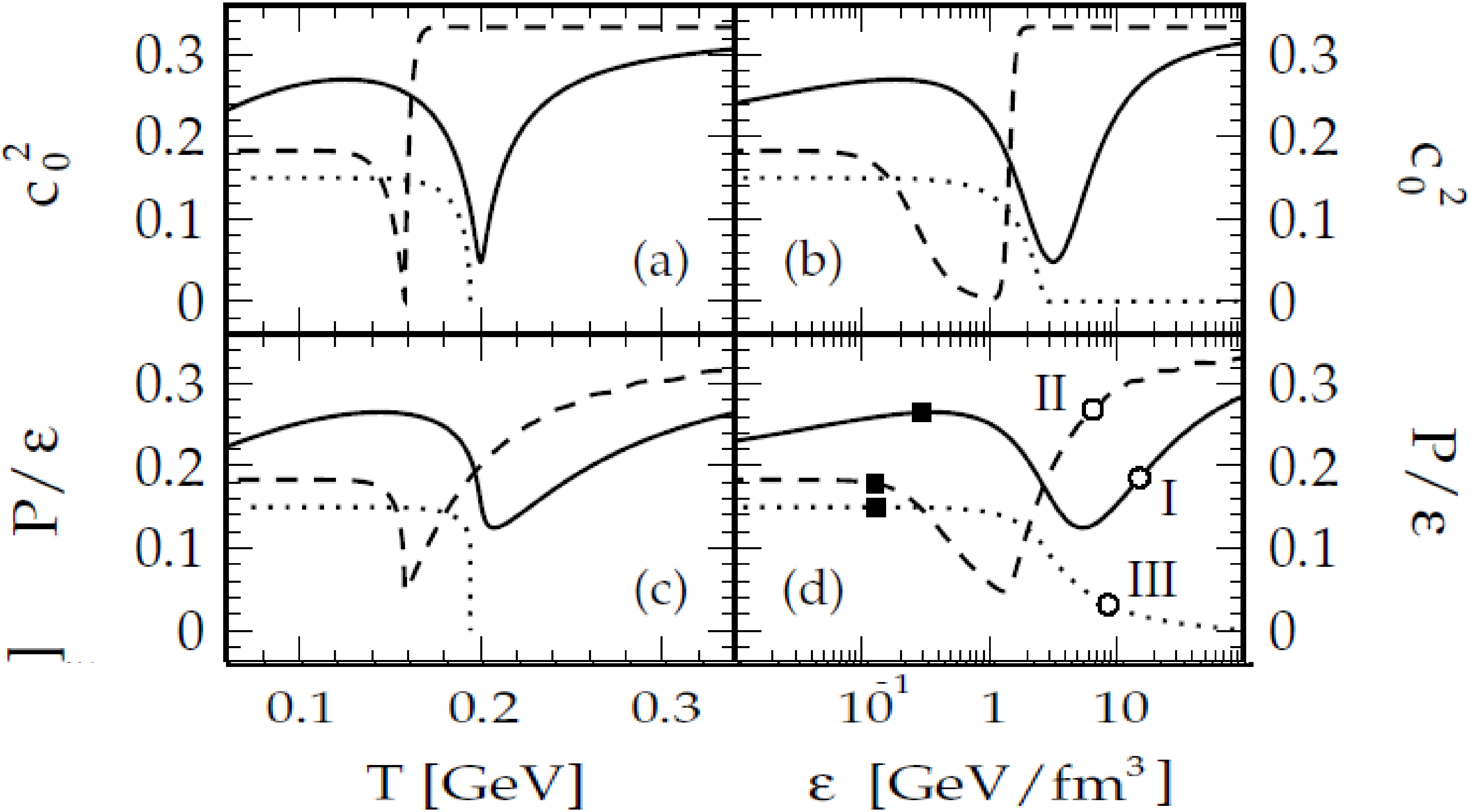
[! Danger ! Hydro can fit anything]

158-GeV/A Pb + Pb

B.R. Schlei, D. Strottman , N. Xu

Nucl-th/9801045; 9710047, 9706037

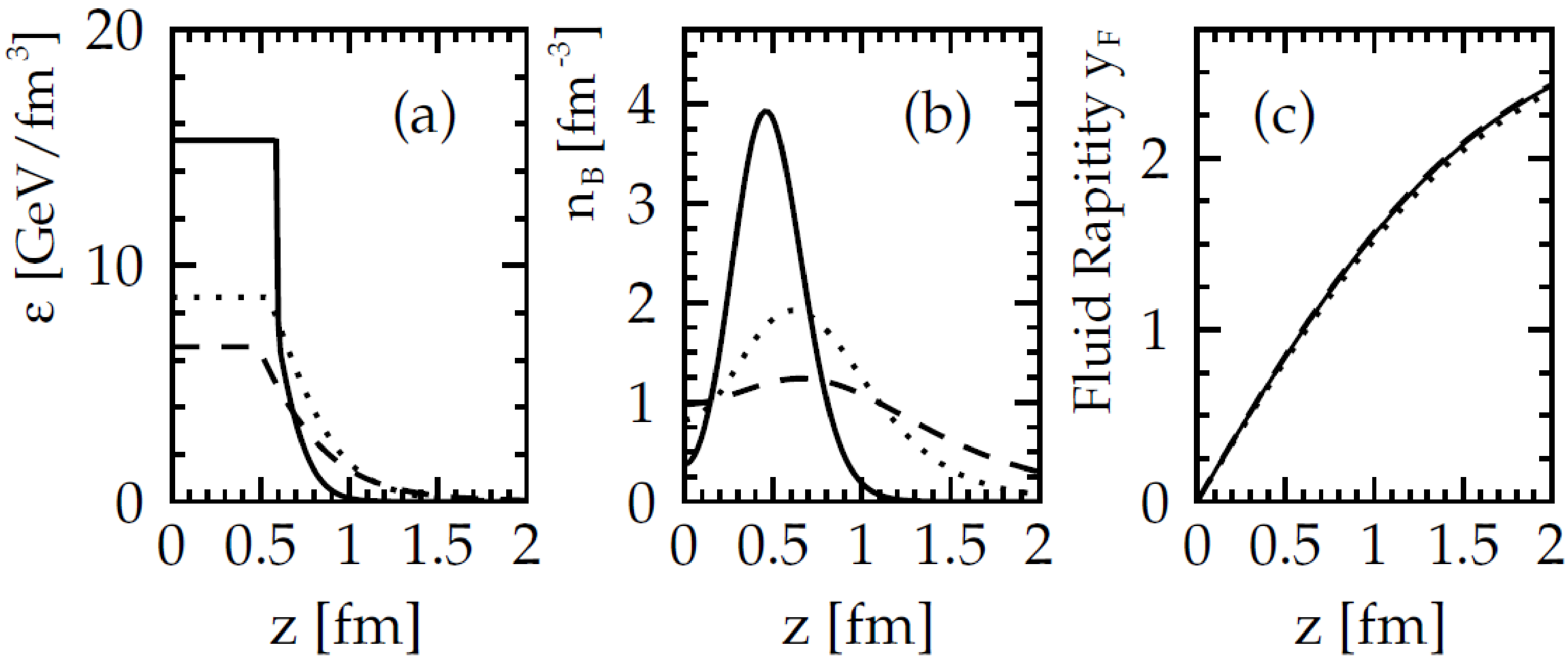
Ideal Landau 3D Hydro with different EOS I (solid) , II (dashed), III(dotted)



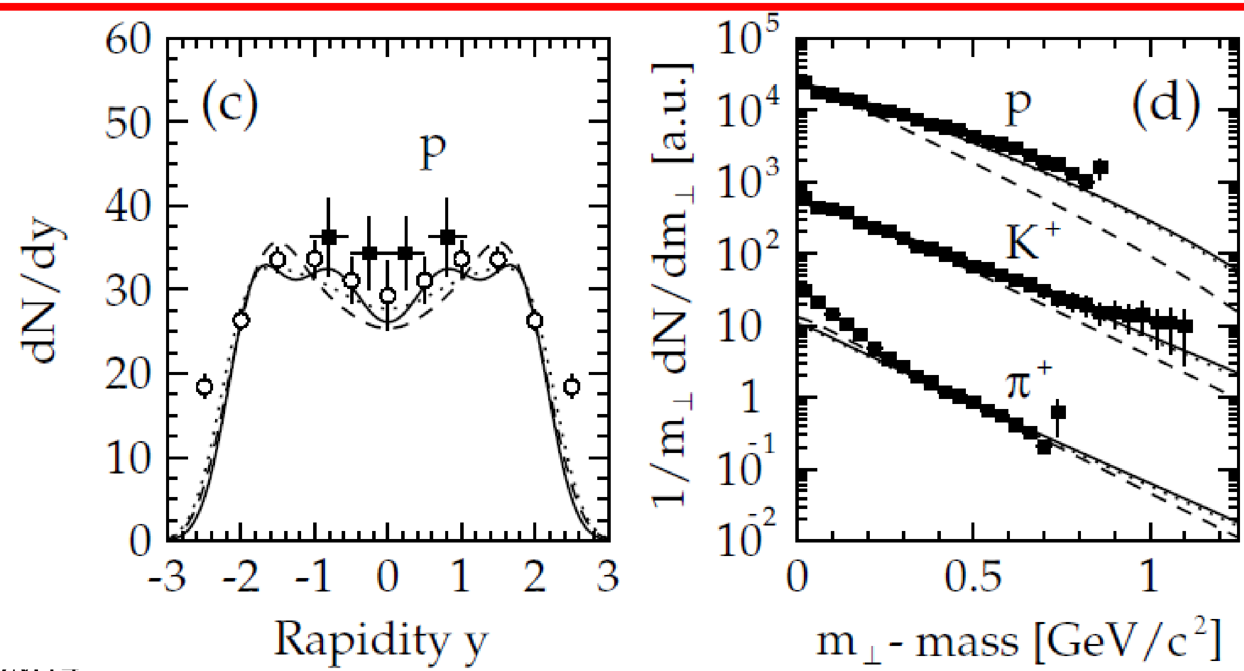
NA44 and NA49 data could be post-dicted with Ideal relativistic hydrodynamics with ***ANY Equation of State***

Given freedom to adjust **Initial** and **Freeze-out** Conditions

Three different hydro evolutions that could fit SPS data



Adjusted initial/final conditions *sufficient* for even ideal hydro to fit the data



Mathematician's delight can be Physicist's Nightmare

K. Weierstrass (1885) Theorem:

“Über die analytische Darstellbarkeit sogenannter willkürlicher Functionen einer reellen Veränderlichen'

Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin, 1885 (II).

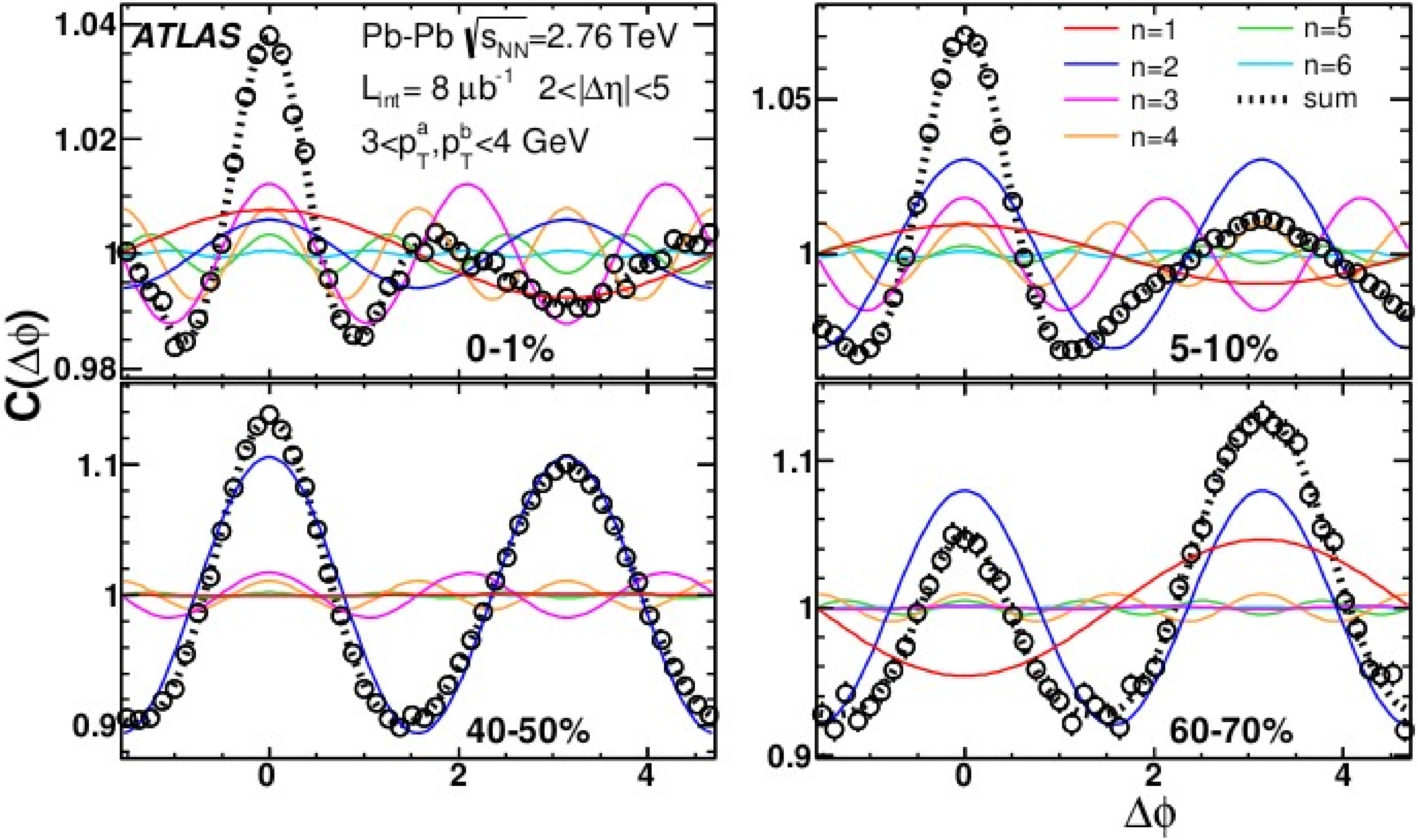
If f is a continuous real-valued function defined on the set $[a,b] \times [c,d]$ and $\varepsilon > 0$, then **there exists a polynomial function** in two variables such that
$| f(x,y) - p(x,y) | < \varepsilon$
for all x in [a,b] and y in [c,d].

Corrolary 1: Fourier transforms exists

Corrolary 2: ideal hydro can fit anything

[ATLAS Collaboration], PRC86, 014907 (2012)

pQCD Color Scintillations or Lumpy CGC+perfect hydro response ? or complex combo of many effects?



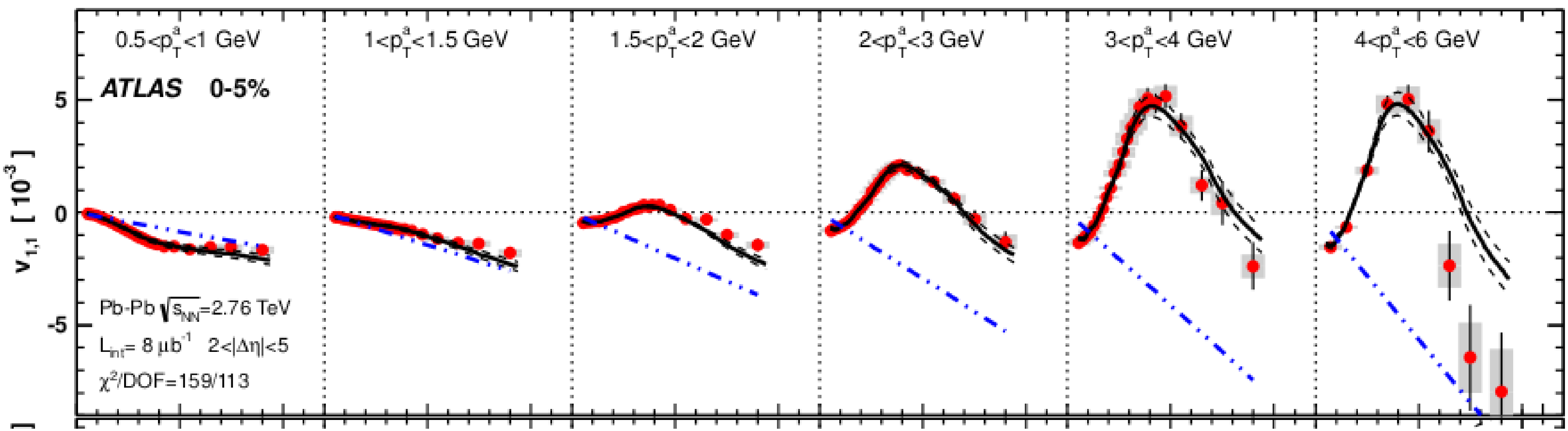
The magnitude of v1,1=<Cos[ϕ1 -ϕ1]> Red is large for p1~p2~ 3-3GeV

General problem: Have we lost calibration control of vn Barometer to verify lattice EOS?

The magnitude of v1,1=<Cos[ϕ1 -ϕ1]>   Red is large for p1~p2~ 3GeV

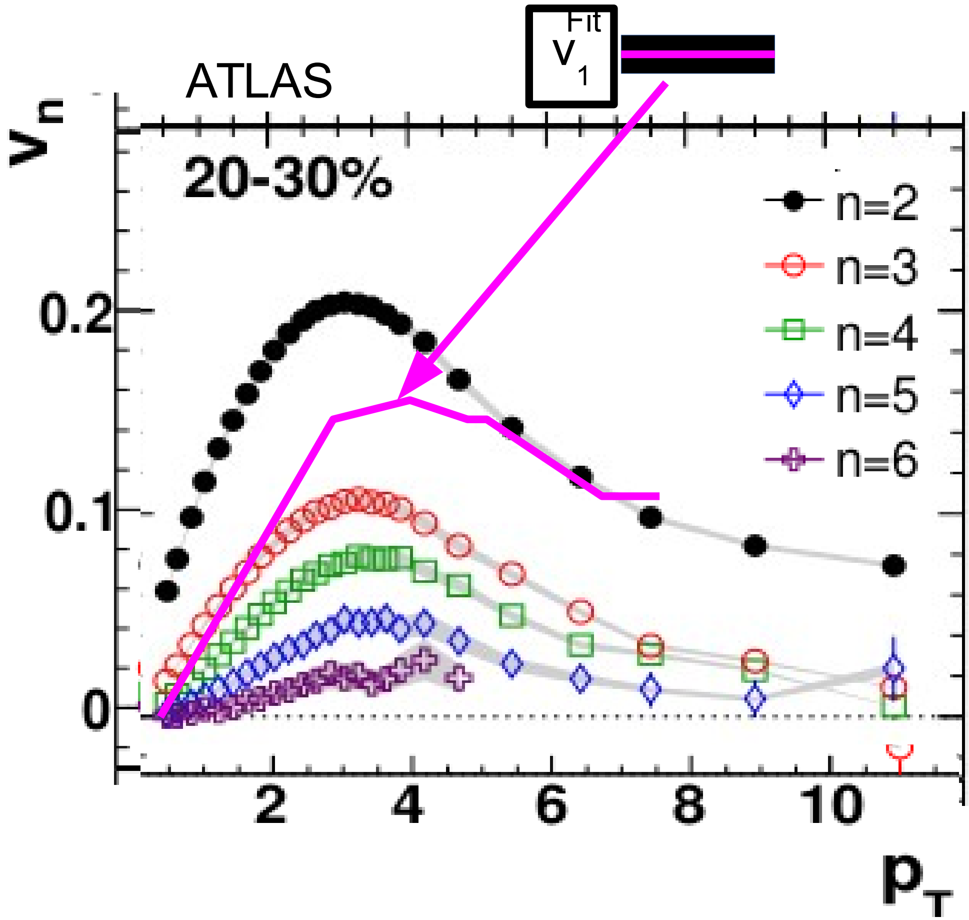
below). The rapidity-even v_1 signal is believed to arise from the dipole asymmetry of the nuclear overlap due to fluctuations in the initial geometry [11, 68]. This spatial asymmetry results in a dipole anisotropy of the pressure gradient, which drives the v_1. If the rapidity-even v_1 depends weakly on η [68], similar to the higher-order v_n, its contribution to $v_{1,1}$ should be positive and vary weakly with $\Delta\eta$. In this case, Eq. 19 can be simplified to:

$$v_{1,1}(p_{\mathrm{T}}^{\mathrm{a}}, p_{\mathrm{T}}^{\mathrm{b}}) \approx v_1(p_{\mathrm{T}}^{\mathrm{a}}) v_1(p_{\mathrm{T}}^{\mathrm{b}}) - \frac{p_{\mathrm{T}}^{\mathrm{a}} p_{\mathrm{T}}^{\mathrm{b}}}{M \langle p_{\mathrm{T}}^2 \rangle} . \quad (20)$$



$$\chi^2 = \sum_{\mathrm{a,b}} \frac{\left(v_{1,1}(p_{\mathrm{T}}^{\mathrm{a}}, p_{\mathrm{T}}^{\mathrm{b}}) - [v_1^{\mathrm{Fit}}(p_{\mathrm{T}}^{\mathrm{a}}) v_1^{\mathrm{Fit}}(p_{\mathrm{T}}^{\mathrm{b}}) - c p_{\mathrm{T}}^{\mathrm{a}} p_{\mathrm{T}}^{\mathrm{b}}]\right)^2}{\left(\sigma_{\mathrm{a,b}}^{\mathrm{stat}}\right)^2 + \left(\sigma_{\mathrm{a,b}}^{\mathrm{sys,p2p}}\right)^2},$$

Rapidity-even, positive collective dipole <Cos[ϕ1 -ϕ1]> ~ - (negative mom.conserv. term)



Is v1 an artifact of fit? See *

A hydro response ? See Schenke

or is it a fingerprint of basic

pQCD Color Scintillation?

*Beware v1 is notoriously tricky, see L. Csernai, H. Stoecker, JPG 2014

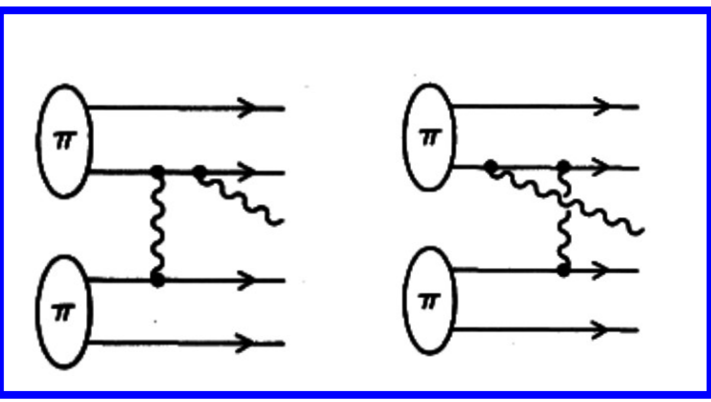
Here we take rapidity-even ATLAS v1 Ridge component as physics beyond naïve v2,v3 flow

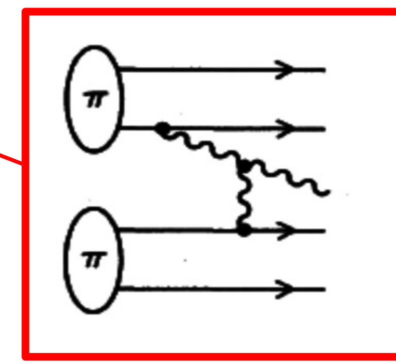
II. FIRST ORDER IN OPACITY (GB) BREMSSTRAHLUNG AND AZIMUTHAL ASYMMETRIES v_n

The above puzzles with BES and $D + Au$ at RHIC and with $p + Pb$ at LHC and models proposed so far motivate us to consider simpler more basic perturbative QCD sources of azimuthal asymmetries.The well known non-abelian bremstralung Gunion-Bertsch (GB) formula[29] for the soft gluon radiation single inclusive distribution is

Color Dipole
Form factor

$$\frac{dN_g^{(1)}}{d\eta d^2\mathbf{k} d^2\mathbf{q}} \equiv f(\eta,\mathbf{k},\mathbf{q}) = \frac{C_R\alpha_s}{\pi^2 k^2}\frac{\mu^2}{(\mathbf{q}^2+\mu^2)}\frac{P_\eta}{(\mathbf{k}-\mathbf{q})^2+\mu^2} \equiv \frac{F\cdot P}{A_{kq}-\cos(\phi-\psi)}$$



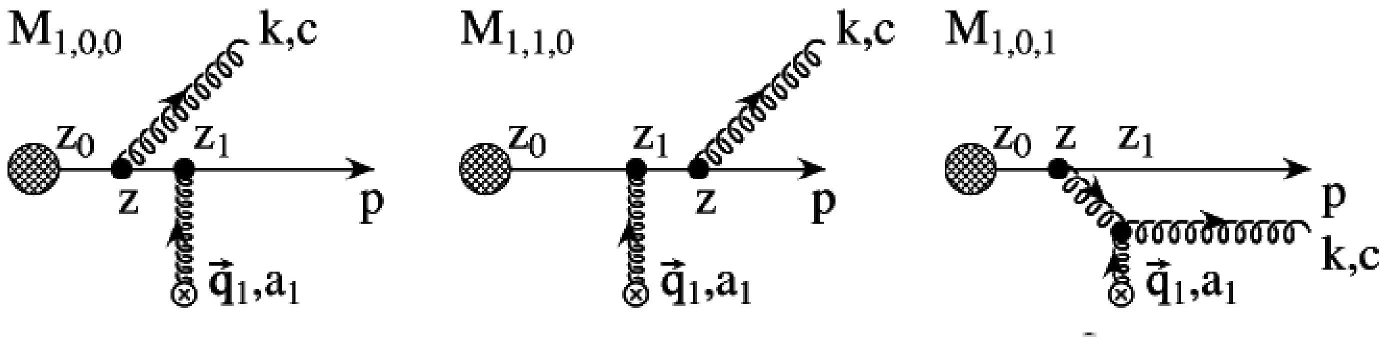


Gluon Bremsstrahlung peaks in transverse direction near net momentum transfer $\vec{Q}$ = (Q, Psi) that also defined reaction Event Plane (EP)

Basic Non-Abelian feature: uniform *rapidity-even* distributed (unlike QED)

Of course also peaks in beam direction 1/k^2 (as in QED)

Basic lowest order pQCD Bremsstrahlung



$$M_{1,0,1} = J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0,\mathbf{q}_1)e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times 2ig_s \frac{\epsilon\cdot(\mathbf{k}-\mathbf{q}_1)}{(\mathbf{k}-\mathbf{q}_1)^2} e^{i(\omega_0-\omega_1)z_1}\left(e^{i\omega_1 z_1} - e^{i\omega_1 z_0}\right)[c,a_1]T_{a_1}$$

$$M_{1,0,0} + M_{1,1,0} = J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0,\mathbf{q}_1)\, e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times (-2ig_s) \frac{\epsilon\cdot\mathbf{k}}{\mathbf{k}^2} e^{i\omega_0 z}(ca_1T_{a_1} - a_1 c\, T_{a1})$$

GLV, Nucl.Phys. B594 (2001) 371

M. Djordjevic, M. Gyulassy / Nuclear Physics A 733 (2004) 265–298

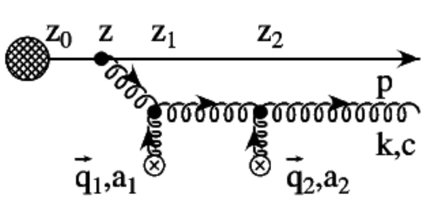
$$\frac{dE_{\text{ind}}^{(1)}}{dx} = \frac{C_R\alpha_S}{\pi}\frac{L}{\lambda}E\int\frac{d\mathbf{k}^2}{\mathbf{k}^2+m_g^2+M^2x^2}\int\frac{d^2\mathbf{q}_1}{\pi}\frac{\mu^2}{(\mathbf{q}_1^2+\mu^2)^2}$$
$$\times 2\frac{\mathbf{k}\cdot\mathbf{q}_1(\mathbf{k}-\mathbf{q}_1)^2+(m_g^2+M^2x^2)\mathbf{q}_1\cdot(\mathbf{q}_1-\mathbf{k})}{(\frac{4Ex}{L})^2+((\mathbf{k}-\mathbf{q}_1)^2+M^2x^2+m_g^2)^2},$$

LPM
Formation
Coherence

Triple gluon vertex
Radiation Dipole focuser
Into reaction plane

"Dead Cone"
Massive quarks
HTL gluons

A higher order opacity amplitude



...

$L/\lambda=1$ $\quad$ $L/\lambda=2$ $\quad$ $L/\lambda=M$

$k - q1 \longrightarrow k - (q1 + q2) \longrightarrow k - (q1 + \ldots + qM)$

e-b-e In Plane focusing gets stronger with opacity

ϕ is the azimuthal angle of $\mathbf{k}$ and ψ is the azimuthal angle of $\mathbf{q}$ and abreviations

$$A \equiv A_{kq} \equiv (k^2 + q^2 + \mu^2)/(2k\,q) \geq 1$$

$$F \equiv F_{kq} \equiv \frac{C_R \alpha_s}{\pi^2 k^2} \frac{\mu^2}{(\mathbf{q}^2 + \mu^2)} \frac{1}{2kq} P_\eta$$

Kinematic rapidity envelope

$$P_\eta \equiv (1 - e^{Y_T - \eta})^{n_f} (1 - e^{\eta - Y_P})^{n_f} \quad ,$$

$$v_n(k,q,\psi) f_0(k,q) \equiv \int \frac{d\phi}{2\pi} \cos(n\phi) f(\eta,k,\phi,q,\psi)$$

$$= F \int \frac{d\phi}{2\pi} \frac{\cos(n\phi)}{A - \cos(\phi - \psi)}$$

$$= \cos(n\psi)\, F \int \frac{d\phi}{2\pi} \frac{\cos(n\phi)}{A - \cos(\phi)} \quad .$$

$f_0 \equiv \int d\phi f = \int d\phi\; d^7N/d\eta dk^2 d\phi dq^2 d\psi$ is the ϕ integrated single gluon inclusive

$$dN/d\eta dk^2 = F_{kq} P_\eta / (A_{kq}^2 - 1)^{1/2}$$

A single GB color antennas has analytic vn:

$$A_{kq} \equiv (k^2+q^2+\mu^2)/(2k\,q) \geq 1$$

$$v_1^{GB}(k,q,\psi) = \cos[\psi](A_{kq} - \sqrt{A_{kq}^2 - 1})$$

$$\lim_{\mu\to 0} v_1^{GB}(k,q,0) = (k/q)\,\theta(q-k)$$

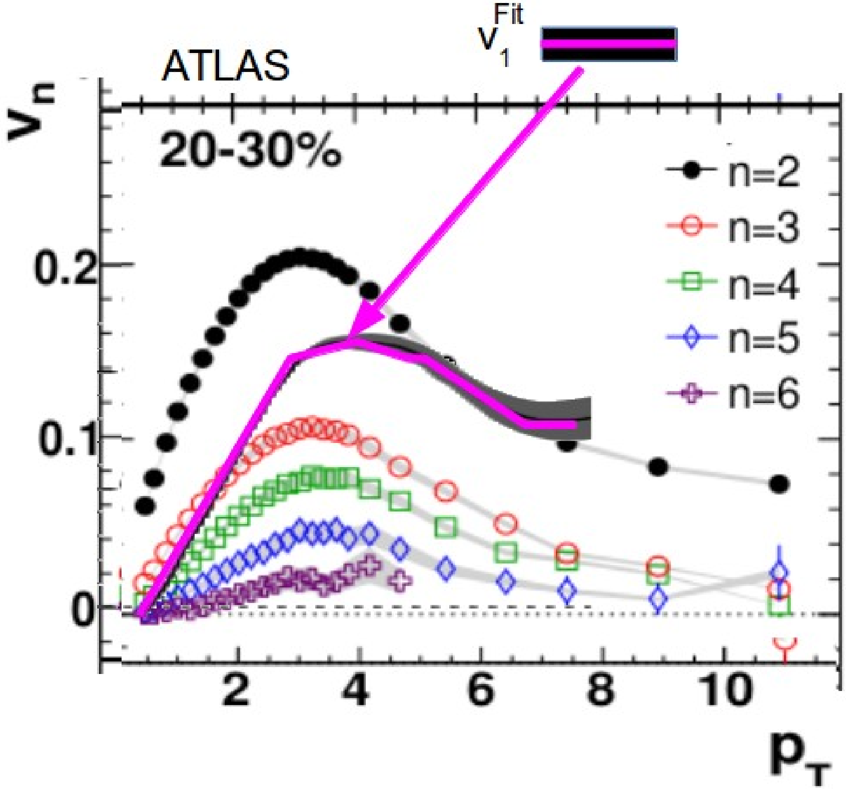
$$v_n^{GB}(k,q,\psi) = \cos[n\psi]\,(v_1^{GB}(k,q,0))^n$$

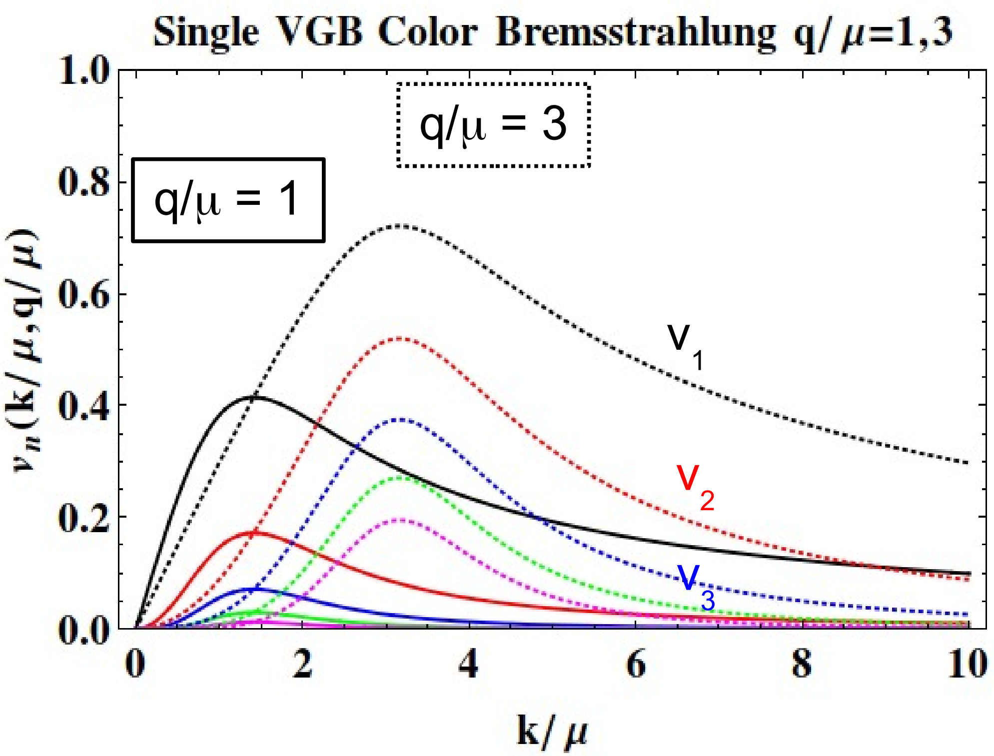
$$\lim_{\mu\to 0} v_n^{GB}(k,q,0) = (k/q)^n\,\theta(q-k)\ .$$

1/ Dipole size

$\boxed{\text{Perfect } v_n^{1/n} = v_1 \text{ Scaling}}$

Two particle vn from ATLAS

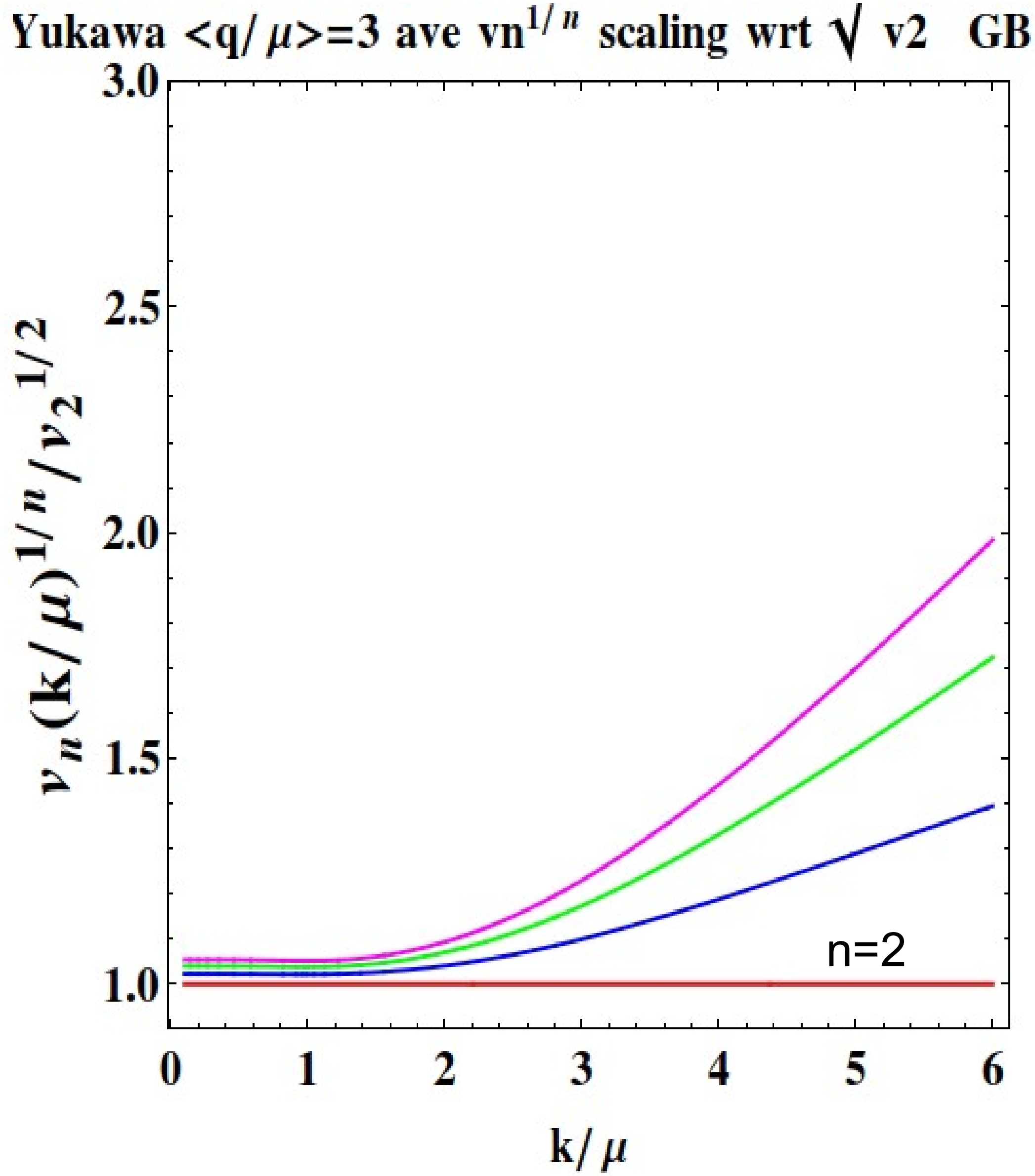


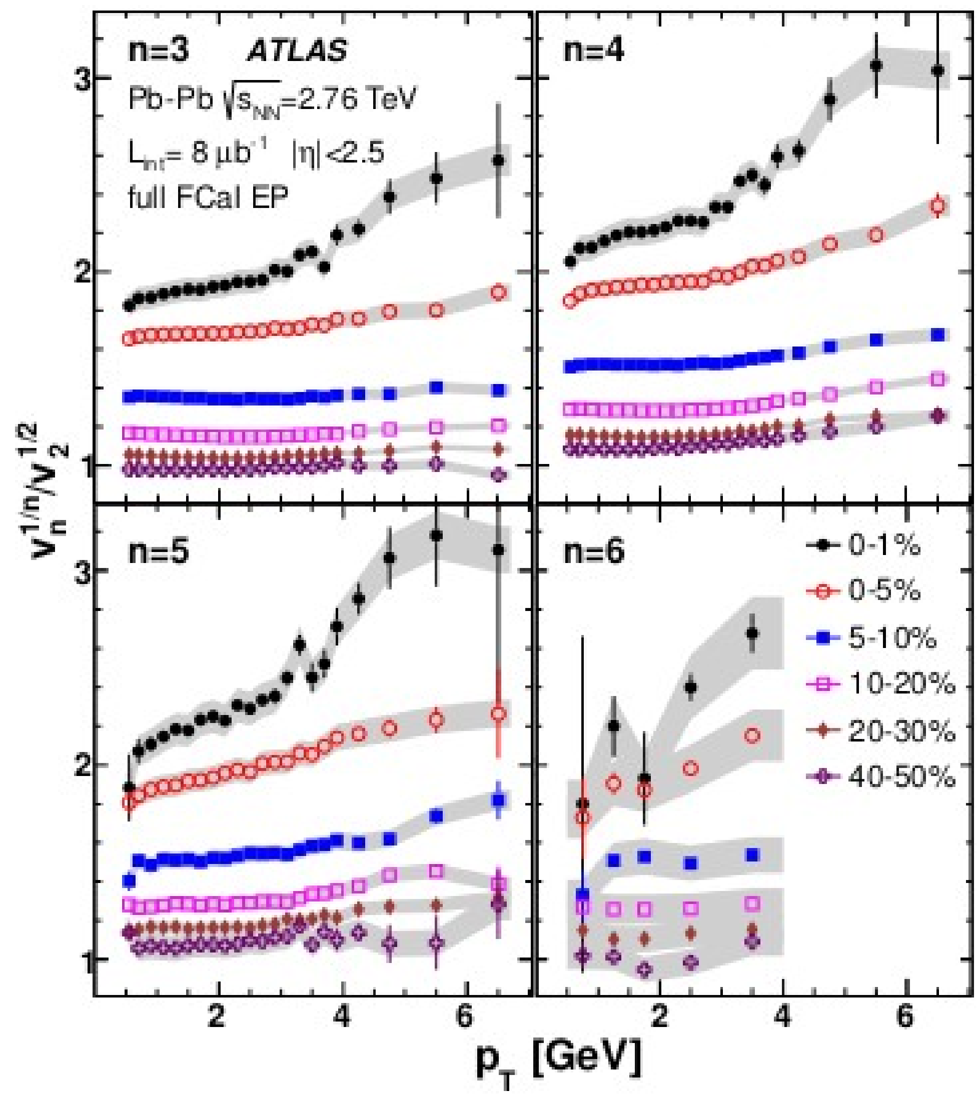


Fixed q GB pQCD Bremsstrahlung harmonics scale perfectly via 1/n power law

$$[v_n^{GB}(k,q,0)]^{1/n} \;=\; [v_m^{GB}(k,q,0)]^{1/m}$$

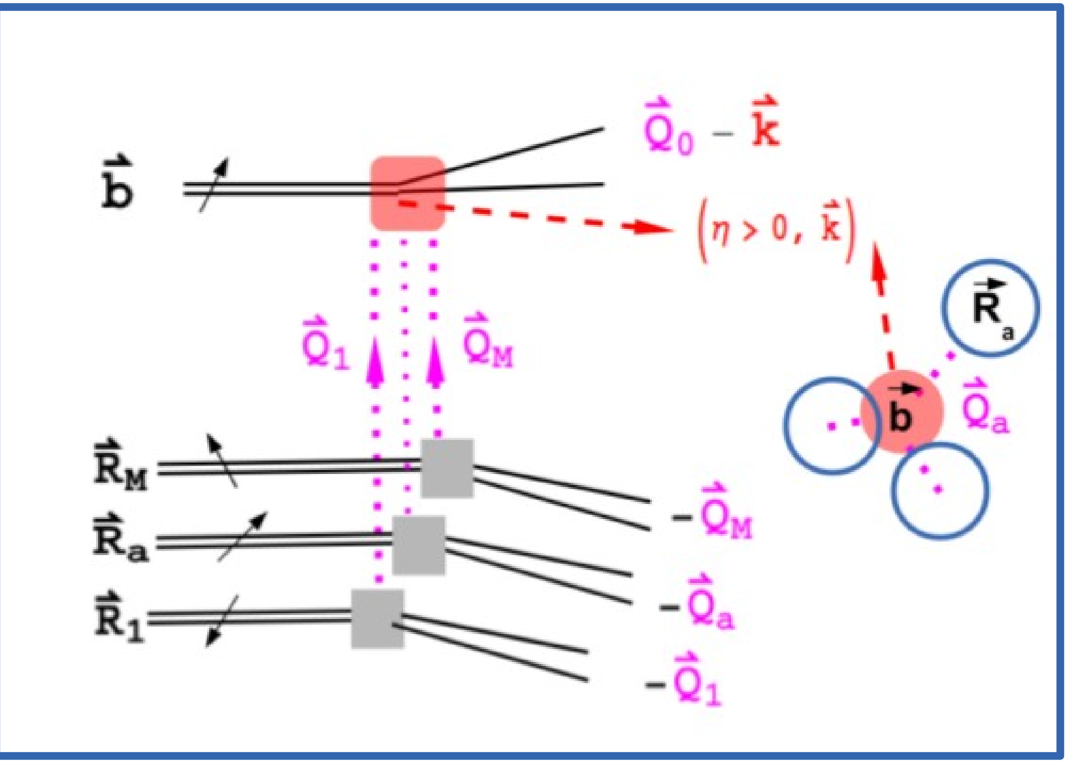
For Yukawa averaged <q/mu>=√M, GB 1/n scaling hold for k< √M and breaks down for k>√M



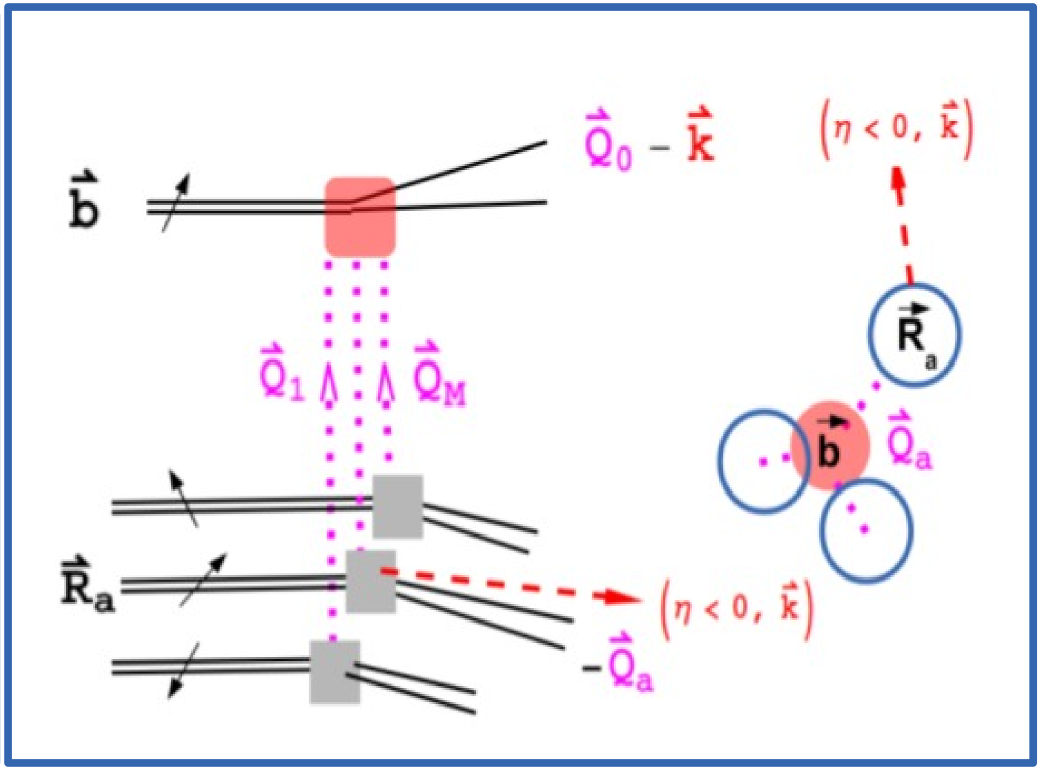


Combined projectile and target participants soft recoil Bemsstrahlung

Projectile Beam Jet Brems



Participant Recoil Target Beam Jets Brems



Target dipoles act
Coherently if transverse
separation cannot be resolved

$$R_{ij} \lesssim d(k) = \frac{c}{k}$$

1< M < N coherent target clusters defined by Resolution scale 1/k

If $i \in I_a$ and $j \in I_a$ as well as $j \in I_b$, then j is added to I_a if its $\langle d_{ij} \rangle_{i \in I_a} < \langle d_{ij} \rangle_{i \in I_b}$

Vitev all order in opacity multiple scattering generalization of GB Brems

$$dN^{VGB}_{coh}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d^2\mathbf{Q}\, P^{el}_n(\mathbf{Q})\, dN^{GB}(\mathbf{k},\mathbf{Q})$$

$$P^{el}_n(\mathbf{Q}) \;=\; \exp[-\chi]\frac{\chi^n}{n!} \int \left\{ \prod_{j=1}^{n} \frac{d^2\mathbf{q}_j}{\sigma_{el}} \frac{d\sigma_{el}}{d^2\mathbf{q}_j} \right\} \times \delta^2(\mathbf{Q} - (\mathbf{q}_1 + \cdots + \mathbf{q}_n))$$

Cumulative momentum transfer from n coherent scatterings

At n=N th order in opacity with M coherent target clusters that can resolved by k
Projectile plus Target bremsstrahlung sums to

$$\begin{aligned} dN^{M,N} \;&=\; dN^{N}_{P}(\eta,\mathbf{k}_1;\mathbf{Q}_P) + dN^{M,N}_{T}(\eta,\mathbf{k}_1;\{\mathbf{Q}_a\}) \\ &=\; \sum_{a=0}^{M} \frac{B_{1a}}{(\mathbf{k}_1+\mathbf{Q}_a)^2+\mu_a^2}\;, \end{aligned}$$

$$\boxed{\begin{aligned} B_{ia} &\equiv F_{k_i,Q_a}\, P_a(\eta_i) \\ \mathbf{Q}_0 &\equiv -\mathbf{Q}_P = -\textstyle\sum_a \mathbf{Q}_a \end{aligned}}$$

2 glue Brems in independent emission approx

$$dN^{N,M}_2(\mathbf{k}_1,\mathbf{k}_2) \;=\; \sum_{a=0}^{M}\sum_{b=0}^{M} \frac{B_{1a}}{A_{1a}-\cos(\phi_1+\psi_a)} \frac{B_{2b}}{A_{2b}-\cos(\phi_2+\psi_b)}$$

Two gluon relative Cos(n (ϕ1 – ϕ2)) analytic azimuthal harmonics VGA color antennas

$$f_n^{N,M}(k_1,k_2) \;\equiv\; \int_{-\pi}^{\pi} d\Phi \int_{-\pi}^{\pi} d\Delta\phi\,\cos(n\Delta\phi)\,dN_2^{N,M}(k_1,\Phi+\Delta\phi/2,k_2,\Phi-\Delta\phi/2)$$

$$=\sum_{a,b=0}^{M} B_{1a}B_{2b}\int_{-\pi}^{\pi} d\Phi' \frac{1}{A_{1a}-\cos(\Phi')}\int_{-\pi}^{\pi} d\Delta\phi \frac{\cos(n\Delta\phi)}{A_{2b}-\cos((\Phi'+\psi_b-\psi_a)-\Delta\phi)}$$

$$=\sum_{a,b=0}^{M} B_{1a}B_{2b}\; f_{0,1,a}\; f_{0,2,b}\boxed{(v_1^{GB}(k_1,Q_a)v_1^{GB}(k_2,Q_b))^n\;\cos(n(\psi_b-\psi_a))}$$

$$f_{n,1,a}=\int_{-\pi}^{\pi} d\Phi \frac{\cos(n\Phi)}{A_{1a}-\cos(\Phi)} = (v_1^{GB}(k_1,Q_a))^n f_{0,1,a} = \frac{\left(A_{k_1,Q_a}-\sqrt{A^2_{k_1,Q_a}-1}\right)^n}{\sqrt{A^2_{k_1,Q_a}-1)}}$$

$$\boxed{v_n^{M,N}\{2\}[k_1,k_2] \equiv= \langle\cos(n(\phi_1-\phi_2))\rangle_{k_1,k_2} = \frac{\langle f_n^{M,N}(k_1,k_2)\rangle}{\langle f_0^{M,N}(k_1,k_2)\rangle}}$$

$$\langle\cdots\rangle = \int\left\{\prod_{a=0}^{M} d\mathbf{Q}_a\right\}\delta(\sum_{a=0}^{M}\mathbf{Q}_a)\sum_{m_1,\cdots,m_M}\delta(N-\sum_{a=1}^{M} m_a)\;p^{M,N}_{\{m_j\}}\;P^{el}_{m_1}(\mathbf{Q}_1)\cdots P^{el}_{m_1}(\mathbf{Q}_M)$$

A. Special Z_n color antenna arrays

From Eq,(36,42) it is clear that a particularly simple special case arises if $M = n-1$ target clusters have similar number of members $m_a = N/M = N/(n-1)$ and transfer similar $Q_a^2 = N/M\mu^2$ to the projectile but only at special angles , $\{\psi_a\} = 2\pi a/n$, corresponding to the discrete unitary group of $n = M+1$ roots of unity,

$$Z_n = \{z_{a,n} = e^{i2\pi a/n} | a = 0, \cdots, n-1; \sum_{a=0}^{n-1} z_{a,n} = 0\} \quad . \tag{43}$$

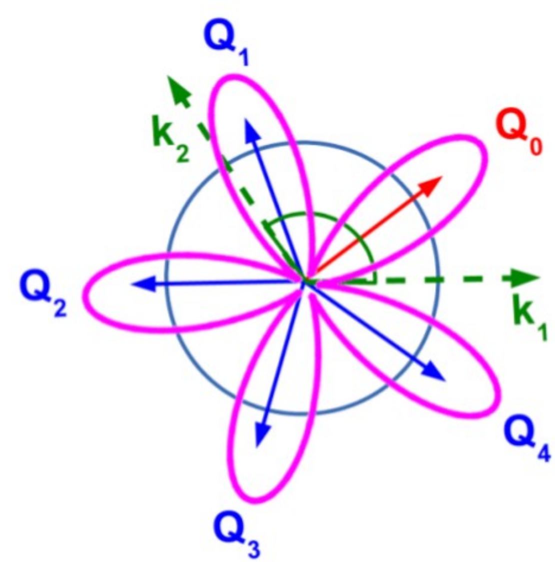
For this special geometry of projectile and target color dipole antennas the double sum over a and b decouple because

$$\cos(n(\psi_a - \psi_b)) = \cos(2\pi(a-b)) = 1 \quad , \tag{44}$$

$$v_n^{M,N}\{2\}(k_1,k_2) \xrightarrow{Z_n} \delta_{n,M+1} v_{M+1}^{GB}(k_1,Q_0) v_{M+1}^{GB}(k_2,Q_0)$$

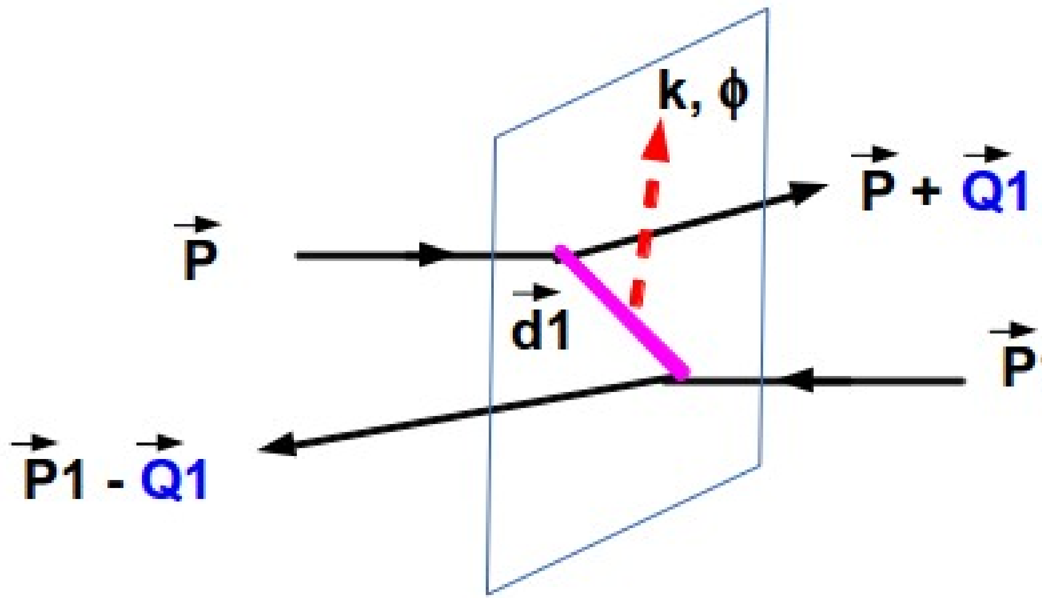
$$\frac{v_n^{M,N}\{2\}(k_1,k_2)}{v_{M+1}^{GB}(k_2,Q_0)} \xrightarrow{Z_n} \delta_{n,M+1} \left(v_1^{GB}(k_1,Q_0)\right)^{M+1}$$

Zn color dipole antennas produce perfect pitch
Unique multipole vn azimuthal harmonics

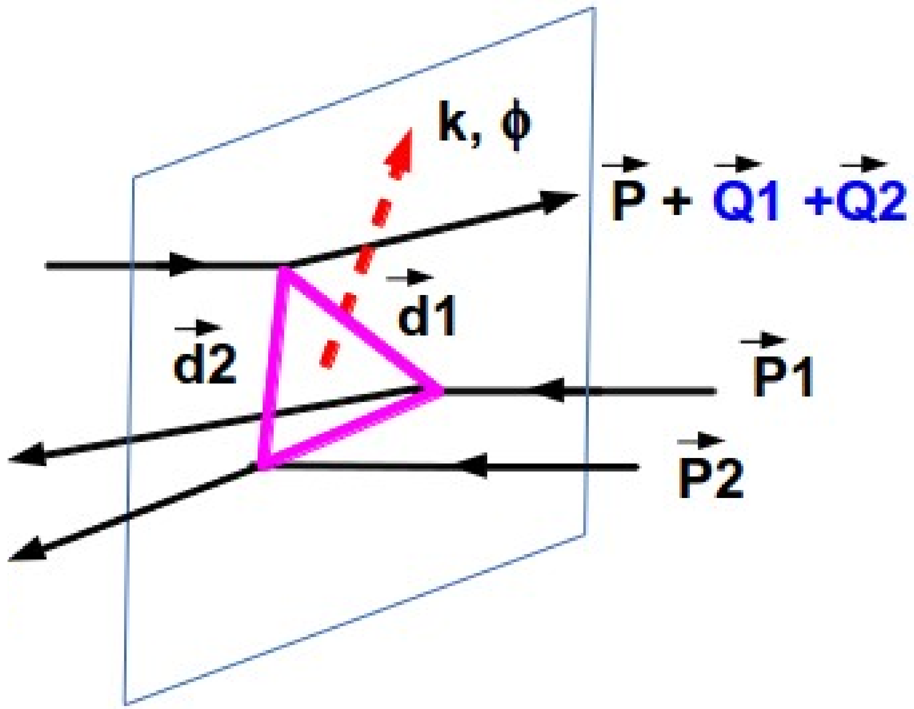


Z_5 "star fish" antenna array with $M = 4$ target clusters recoiling off projectile with $\mathbf{Q}_0 = -\sum_{a=1}^{M} \mathbf{Q}_a$

Classical Color Field Produced by Interfering 3 dipole currents



Two BG dipole antenna array



Three BG dipole antenna array

Produce only n=2,4,6, ...

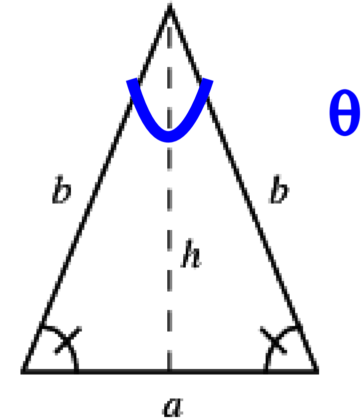
Produce all n=1, 2, 3, 4, ...

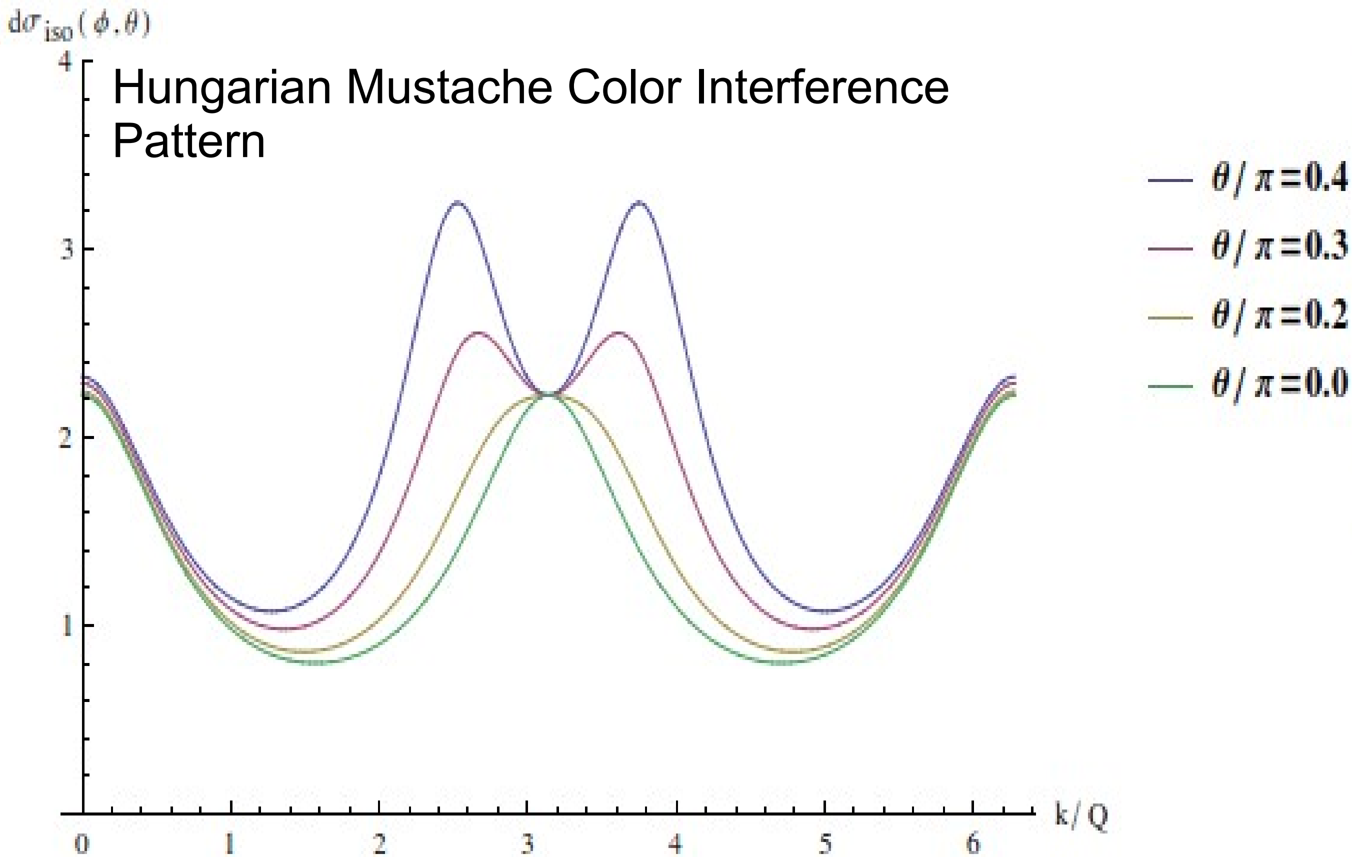
Example: Color Scintillating Arrays can easily produce large v2, v3 Interferences

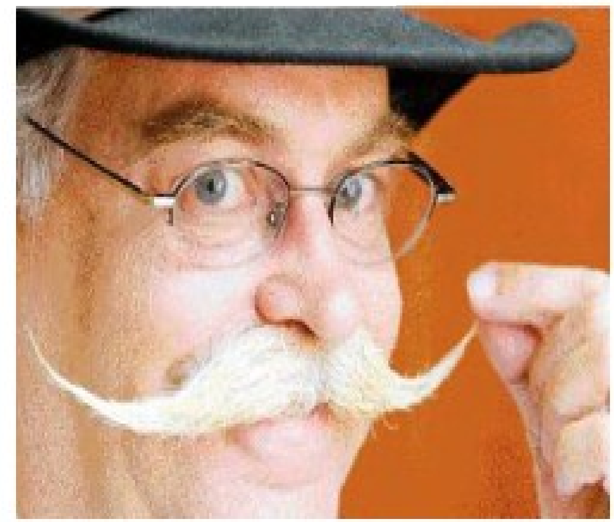
Isoscelese Triangle GB antenna (Magyar Bajusz) array

$$\frac{d\sigma_{\text{iso}}}{d\eta\, dk\, d\phi}[\{\vec{Q}_a\}] = \frac{C_A\,\alpha}{k^2}\ \sum_{a=1}^{3} \frac{Q_a{}^2}{k^2 + Q_a{}^2 - 2\,k\,Q_a\,\mathrm{Cos}[\phi - \Theta_a]}$$

$$v_n(k/Q_1, \theta) = \int d\phi\ \mathrm{Cos}[n\ \phi]\ d\sigma_{\text{iso}}\ /\ \left(\int d\phi\ d\sigma_{\text{iso}}\right)$$







MG, P. Levai, I. Vitev, and T. Biro work in progress

B. Random recoil color antenna arrays

Another simple limit is when the recoil azimuthal angles ψ_a are in random $[0, 2\pi]$ and the $\mathbf{Q}_a$ are distributed with a Gaussian of same width squared $\langle Q_a^2 \rangle = Q_T^2 = (N/M)\mu^2$ for $a \in [1, \cdots, M]$ In this antenna array t, the projectile $\mathbf{Q}_0$ is also Gaussian distributed with zero mean but with an enhanced second moment,

$$\langle Q_0^2 \rangle = M Q_T^2 = N\mu^2 \quad . \tag{48}$$

Unlike for perfect n^{th} harmonic antenna arrays with Eq,(49), in the random Gaussian distributed case

$$\cos(n(\psi_a - \psi_b)) = \delta_{a,b} \quad , \tag{49}$$

$$\boxed{f_n^{N,M}(k,k) \overset{Gauss}{\rightarrow} \int d^2\mathbf{Q} \left\{ \frac{\exp[-Q^2/(2N\mu^2)]}{2\pi N\mu^2} + M \frac{\exp[-Q^2/(2(N/M)\mu^2)]}{2\pi (N/M)\mu^2} \right\} \{B_{kQ} f_{0,k,Q} v_n^{GB}(k,Q)\}^2}$$

$$\sqrt{f_n^{N,M}(k,k)} \approx \left(\frac{C_R \alpha_s \mu^2}{\pi^2 k^2}\right) \left\{ \frac{P_P(\eta)}{(N+1)\mu^2} \frac{(v_1^{GB}(k, \sqrt{N}\mu))^n}{((k^2 + (N+1)\mu^2)^2 - 4Nk^2\mu^2)^{1/2}} + \frac{M(\eta)}{(N/M(\eta)+1)\mu^2} \frac{(v_1^{GB}(k, \sqrt{N/M(\eta)}\mu))^n}{((k^2 + (N/M(\eta)+1)\mu^2)^2 - 4(N/M(\eta))k^2\mu^2)^{1/2}} \right\}$$

Mixture of two scale GB harmonics with $Q^2 \sim N\,\mu^2$ from projectile beam jet
And M $Q^2 \sim (N/M)\,\mu^2$ recoil target beam jet clusters that depends of eta

**C: Work in progress implement correlated VGA brems into HIJING Monte Carlo generator
Replace phi ave ARIADNE in JETSET with VGA recoil correlated gluon Bremsstrahlung**

Slide 3-4: The Case for Perfect Fluidity and CGC/Glasma *Sufficiency*
for Flow Harmonics in Non-central A+A as of QM12

Slides 5-14 : Trouble for Fluidity since 2012

a) Beam Energy *independence*
b) p(D)+A = A+A v2 and v3 Beam size *independence*
c) large rapidity-even v1 Dipole

Slides 15-27 : Is Perfect Fluidity *Necessary ??*
Are vn in B+A mostly Nonabelian Wave interference?

(see also QM talks by Venugopalan , Dusling, Schenke... & poster of Biro, Schram)

Could rapidity-even v odd Dipole,Triangular, etc harmonics be just fingerprints of basic pQCD beam jet anisotropic bremsstrahlung?

(My current working hypothesis is Yes, and that it will increase eta/s fits closer to 1985 pQCD)

Work in Progress: all order in opacity pQCD beam jets vn
Bremsstrahlung in P+A via HIJING replacing ARIADNE with VGB

Non-Abelian Bremsstralung and Azimuthal Asymmetries in High Energy p+A

M. Gyulassy,[1,2,*] P. Levai,[1] I. Vitev,[3] and T. Biro[1]

[1] *MTA WIGNER Research Centre for Physics, RMI, Budapest, Hungary*
[2] *Department of Physics, Columbia University, New York, NY 10027, USA*
[3] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

(Dated: May 13, 2014 v11)