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(Quark Matter 2018)

Precision Dijet Acoplanarity Tomography of the Chromo Structure of Perfect QCD Fluids

M. Gyulassy^{a,b,c,d}, P. Levai^b, J. Liao^{e,d}, S. Shi^e, F. Yuan^a, X.N. Wang^{a,d}

^a*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*

^b*MTA Wigner Research Centre for Physics, 1525 Budapest, Hungary*

^c*Pupin Lab MS-5202, Department of Physics, Columbia University, New York, NY 10027, USA*

^d*Institute of Particle Physics, Central China Normal University, Wuhan, China*

^e*Physics Department and Center for Exploration of Energy and Matter, Indiana University,
2401 N Milo B. Sampson Lane, Bloomington, IN 47408, USA*

This is an update from my 2017 lecture at CCNU: see link

<http://www.columbia.edu/~mg150/Talks/2017/MGyulassy-Lec2-CCNU-101817.pdf>

Key references on which this work was built

A. H. Mueller, B. Wu, B. W. Xiao and F. Yuan,
Probing Transverse Momentum Broadening in Heavy Ion Collisions
Phys. Lett. B 763, 208 (2016)

Probing Transverse Momentum Broadening via Dihadron and Hadron-jet Angular
Correlations in Relativistic Heavy-ion Collisions
Phys. Rev. D 95, 034007 (2017)

L. Chen, G. Y. Qin, S. Y. Wei, B. W. Xiao and H. Z. Zhang,
Probing Transverse Momentum Broadening via Dihadron and Hadron-jet Angular
Correlations in Relativistic Heavy-ion Collisions
Phys. Lett. B 773, 672 (2017) [arXiv:1607.01932 [hep-ph]]

ALICE Collaboration: Measurement of jet quenching with semi-inclusive
hadron-jet distributions in central Pb-Pb collisions at $\sqrt{s_{NN}} = \sqrt{2.76}$ TeV, JHEP1509 (2015)

STAR Collaboration: Measurements of jet quenching with semi-inclusive hadron+jet
distributions in Au+Au collisions at $\sqrt{s_{NN}} = \sqrt{200}$ GeV, Phys.Rev. C96 (2017)

Outline

Section 1: Introduction

Section 2: Some details of the calculation

Section 3: Numerical examples and conclusions

My interest in acoplanarity was motivated by a Peter Jacob question after my INT 2017 talk on

**Consistency of Perfect Fluidity and Jet Quenching
in semi-Quark-Gluon-Monopole-Plasmas (sQGMP)**

Jiechen Xu , J.Liao, MG, Chin.Phys.Lett. 32 (2015) and JHEP 1602 (2016) 169
Shuzhe Shi, J.Xu, J.Liao, MG, Nucl.Phys. A967 (2017) 648
Shuzhe Shi, J.Liao, MG: Chin.Phys. C 42 (2018) 104104,

Global χ^2 RHIC and LHC Data Constraints on Soft-Hard Transport Properties of sQGMP
[via CIBJET= EbE-VISHNU+CUJET3.1 , arXiv:1808.05461 [hep-ph]]

Peter Jacob’s question (my paraphrase) :

Can **future high precision** dijet acoplanarity measurements help to falsify **sQGMP** or **wQGP** or **AdS-BH** models of the color structure of QCD perfect fluids?

Or is acoplanarity limited to the extraction of only one BDMS medium saturation scale, **Qs**, as is already determined by jet and dijet nuclear modification ratio data RAA(pT) and IAA ??

$$Q_s^2(a) \equiv \left\langle q_\perp^2 \frac{L}{\lambda} \right\rangle_a \equiv \int dt \sum_b \hat{q}_{ab}(x(t), t) \equiv \sum_b \int dt d^2 q_\perp q_\perp^2 \Gamma_{ab}(q_\perp, t)$$

Can acoplanarity **distribution shapes** help to extract information on the color d.o.f in near perfect QCD fluids and their microscopic differential scattering rates, Γ_{ab} , near $T \sim T_c$?

$$\Gamma_{ab}(q_\perp, T) = \rho_b(T) d^2 \sigma_{ab}(T) / d^2 q_\perp$$

Does any Γ_{ab} exhibit critical opalescence near T_c that could account for ~ perfect fluidity?

Jet Transport Coefficients = q_T^2 moment of $\sum_b \Gamma_{ab}(q_\perp, T)$ in CIBJET semiQGMP

$$\hat{q}_F(E, T) = \int_0^{6ET} dq_\perp^2 \frac{2\pi}{(\mathbf{q}_\perp^2 + f_E^2 \mu^2(z))(\mathbf{q}_\perp^2 + f_M^2 \mu^2(z))} \rho(T)$$

$$\begin{aligned} q(q+g) \\ qm \end{aligned} \quad \left\{ [C_{qq}f_q + C_{qg}f_g] \cdot [\alpha_s^2(\mathbf{q}_\perp^2)] \cdot [f_E^2 \mathbf{q}_\perp^2 + f_E^2 f_M^2 \mu^2(z)] + \right. \\ \left. [C_{qm}(1 - f_q - f_g)] \cdot [1] \cdot [f_M^2 \mathbf{q}_\perp^2 + f_E^2 f_M^2 \mu^2(z)] \right\}, \quad (14)$$

$$\hat{q}_g(E, T) = \int_0^{6ET} dq_\perp^2 \frac{2\pi}{(\mathbf{q}_\perp^2 + f_E^2 \mu^2(z))(\mathbf{q}_\perp^2 + f_M^2 \mu^2(z))} \rho(T)$$

$$\begin{aligned} g(q+g) \\ gm \end{aligned} \quad \left\{ [C_{gq}f_q + C_{gg}f_g] \cdot [\alpha_s^2(\mathbf{q}_\perp^2)] \cdot [f_E^2 \mathbf{q}_\perp^2 + f_E^2 f_M^2 \mu^2(z)] + \right. \\ \left. [C_{gm}(1 - f_q - f_g)] \cdot [1] \cdot [f_M^2 \mathbf{q}_\perp^2 + f_E^2 f_M^2 \mu^2(z)] \right\}, \quad (15)$$

Note Γ_{qm} & Γ_{gm} => Critical Opalescence near Tc because $\alpha_E \alpha_M = 1 \gg \alpha_E^2$ Dirac

Can acoplanarity distribution shapes test the existence of such novel color dynamics in
 \approx Perfect QCD fluids near Tc and constrain the multicomponent differential scattering rates?

$$\Gamma_{ab}(q_\perp, T) = \rho_b(T) d^2 \sigma_{ab}(T) / d^2 q_\perp$$

Note that CUJET dE/dL is **not** proportional to $\hat{q}L$ but given by a generalized DGLV formula

(See eq2.23 J.Xu, J.Liao, MG, JHEP 02 (2016) 169)

Dijet acoplanarity is an A+B observable among many needed to help falsify models of A+A dynamics and probe deeper into the unknown the color structure of QCD perfect fluids produced at RHIC and LHC

dijet initial
orientation
 ψ_2^{hard}

Event plane
 ψ_2^{soft}

Most sensitive
to $\hat{q}(E, T)$

Soft $p_T < 2$
Hard $p_T > 10$
Correlations

Raa & V2
@ varied centrality

(Q_1, ϕ_1)

Di-jet Acoplanarity $I_{AA}(Q_1, Q_2, \phi_1 - \phi_2)$

PASS!

**Heavy
& Light**

**200GeV
& 2.76TeV
& 5.02TeV**

(Q_2, ϕ_2)

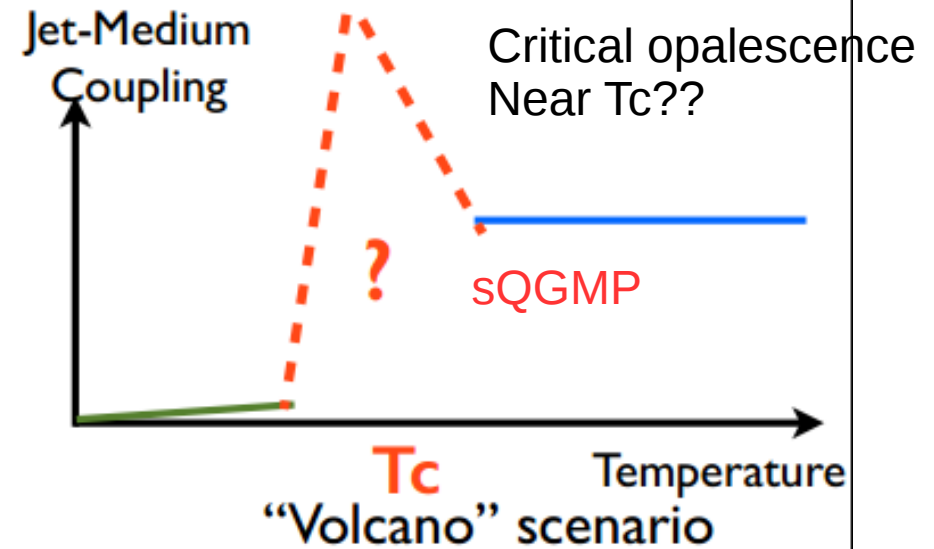
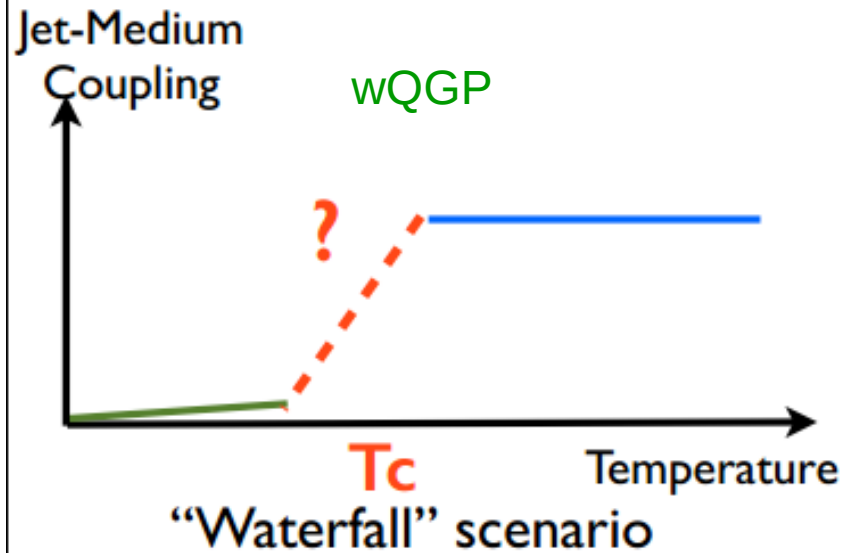
The challenge is to
Reduce the Volume of
Dynamical A+A models
Via multiple independent
Soft-Hard exp constraints

Monopole component near T_c
could account for near perfect fluidity

$$\frac{d\sigma_{EM}}{dq_{\perp}^2} \sim \frac{\alpha_E \alpha_M}{q_{\perp}^4} \sim \frac{1}{\alpha_E^2} \frac{d\sigma_{EE}}{dq_{\perp}^2} \gg \frac{d\sigma_{EE}}{dq_{\perp}^2}$$

Ed Shuryak
Jinfeng Liao

From “Transparency” to Opaqueness



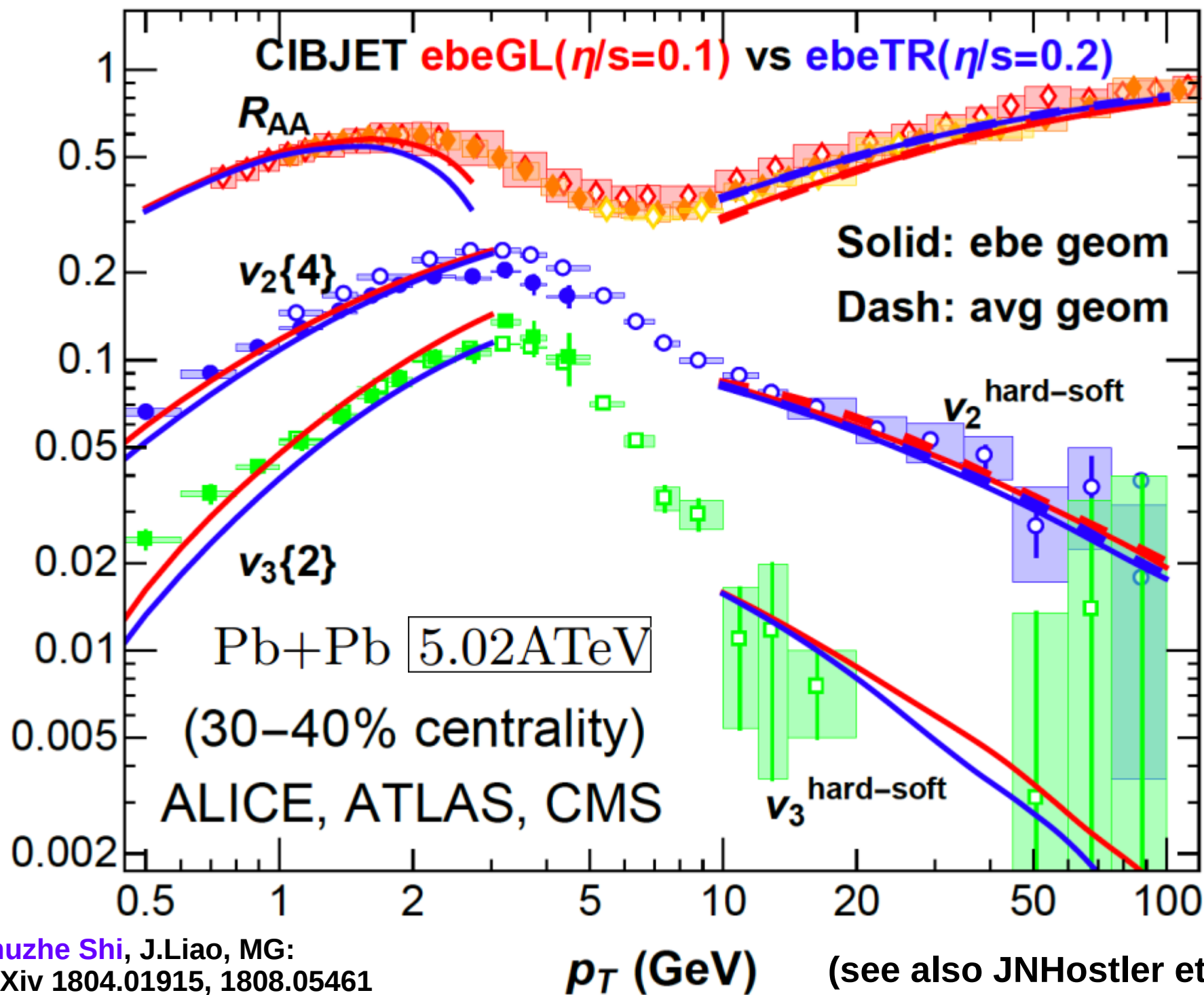
**The temperature dependence of jet-medium coupling
has profound consequences!**

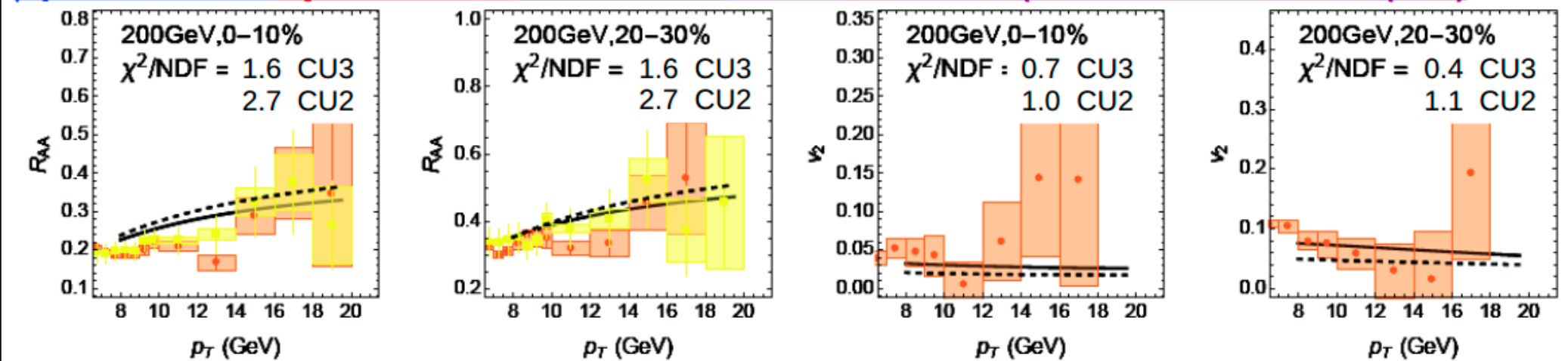
J.Liao 2015

CIBJET was developed by A. Buzzatti, J.Xu, Shuzhe Shi, Jinfeng Liao, MG

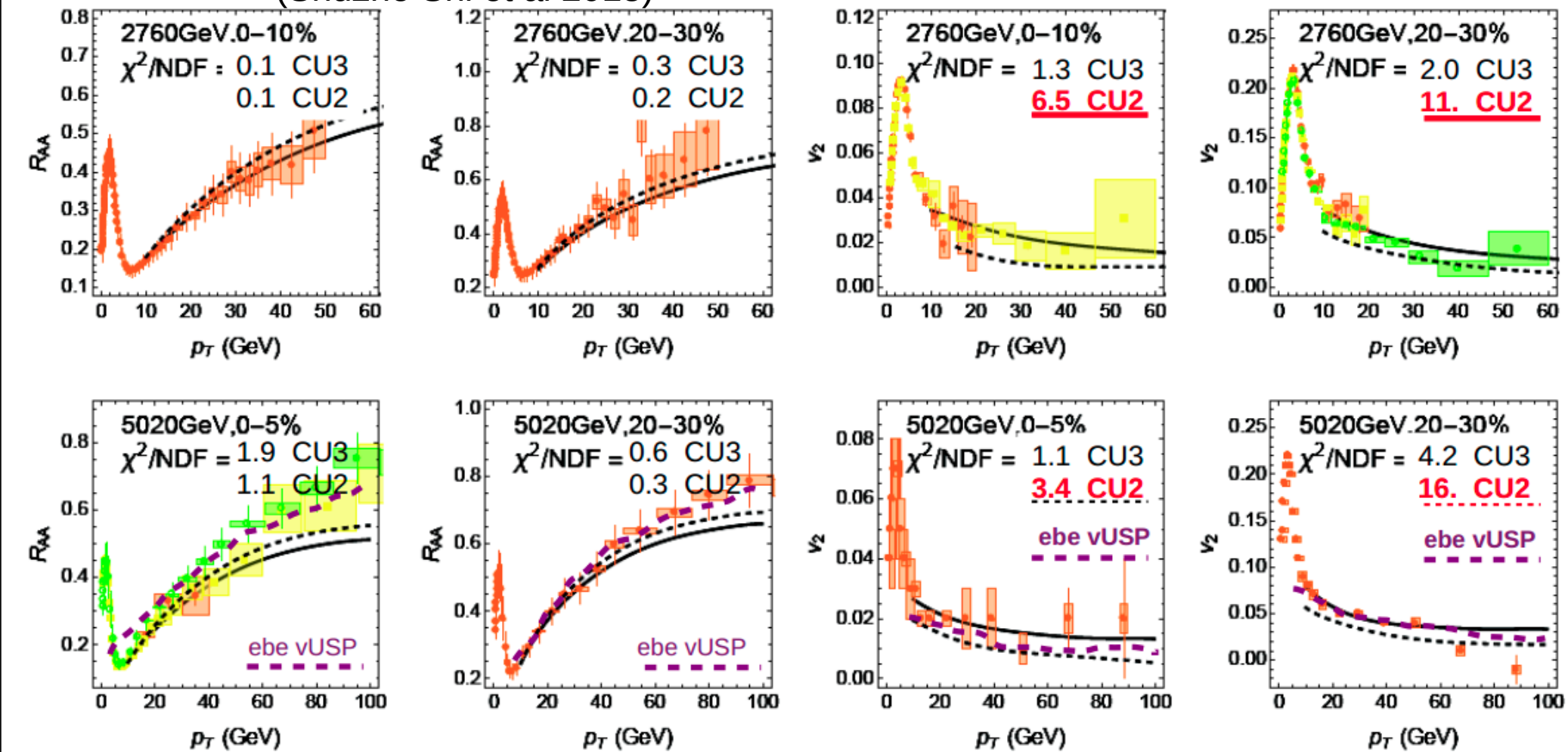
MG 10.15.18 CCNU to test quantitatively this idea with SPS, RHIC and LHC RAA, v2, v3 data

Quantitative Test of “Volcano scenario” with CIBJET sQGMP





(Shuzhe Shi et al 2018)



Global RHIC+LHC1+LHC2 RAA+v2 $\chi^2(\alpha_c, c_m)$ fit contours

sQGMP=(Suppressed $\chi_T^L = c_q L + c_g L^2$ elec semi-Q+G) + (Emergen(1 - χ_T^L) mag.monopoles)

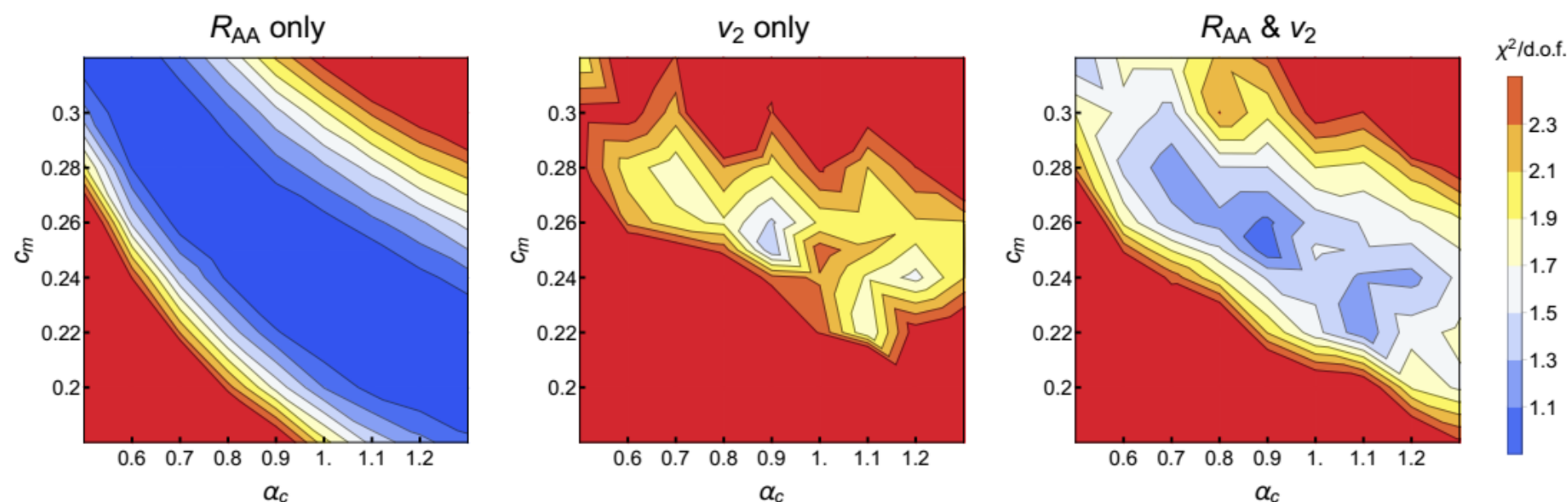


FIG. 1: (color online) $\chi^2/\text{d.o.f.}$ comparing χ_T^L -scheme CUJET3 results with RHIC and LHC data. Left: $\chi^2/\text{d.o.f.}$ for R_{AA} only. Middle: $\chi^2/\text{d.o.f.}$ for v_2 only. Right: $\chi^2/\text{d.o.f.}$ including both R_{AA} and v_2

With CIBJET = **ebe** IC+VISHNU+CUJET3.1 framework

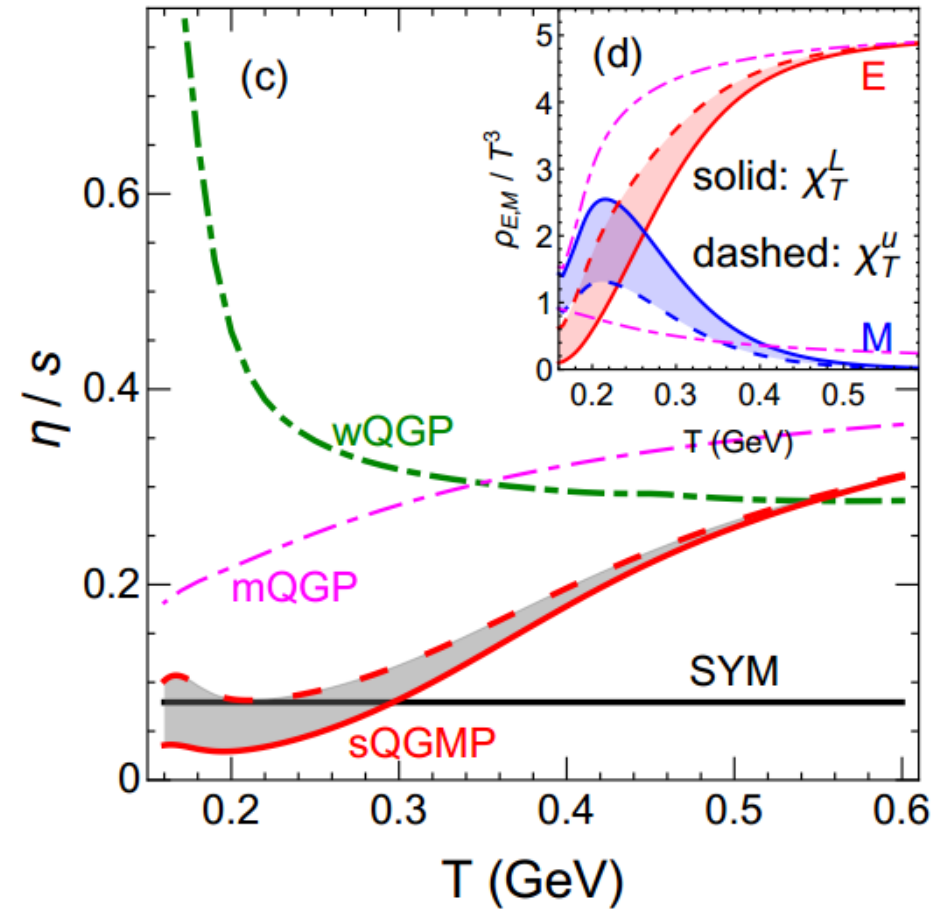
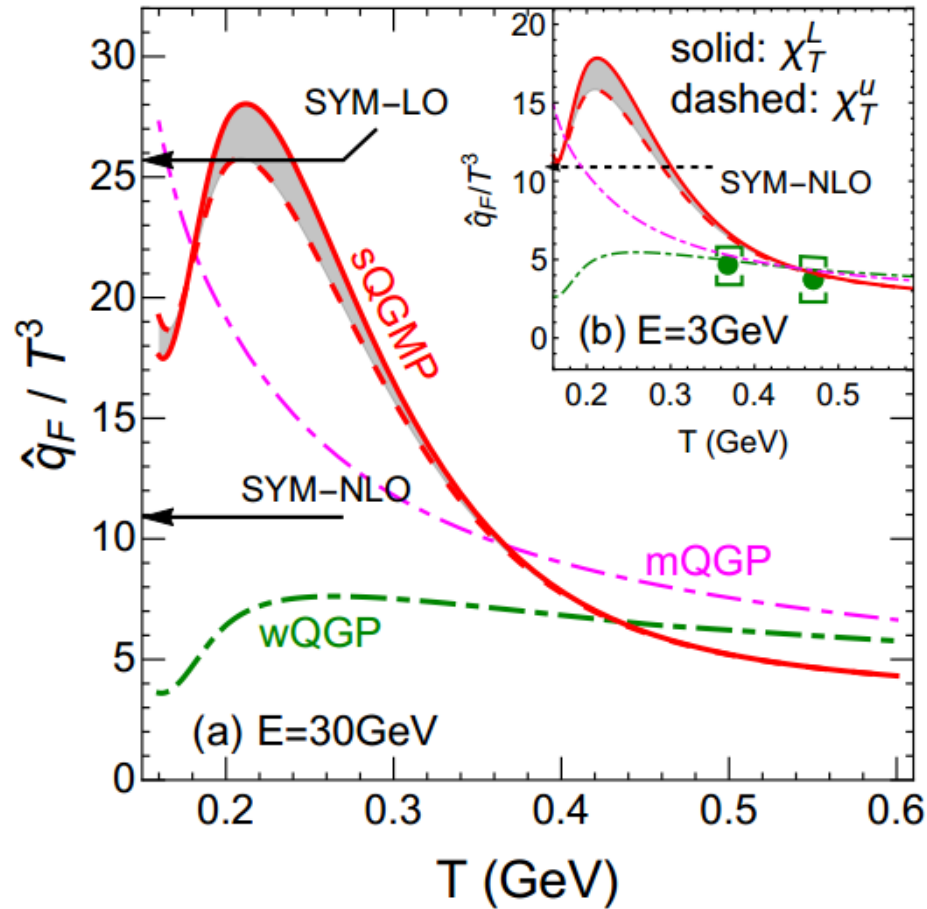
Shuzhe Shi found that ebe only makes $\sim 10\%$ changes to hard v_2 relative using event ave geom.

There is tension between CIBJET and vUSPhydro SHEE framework interpretations, but

MG 10.15.18 CCNU CIBJET qhat has the advantage that it bridges jet quenching with perfect fluidity₁₀

Quantitative extraction of $\hat{q}_F(E, T)$ jet transport field and $\eta/s(T)$ via CIBJET

The q+g suppressed semi-QGP components of **sQGMP** require large monopole density near T_c to compensate the loss of color electric dof and still fit the lattice Eq of State: P/T or S(T)



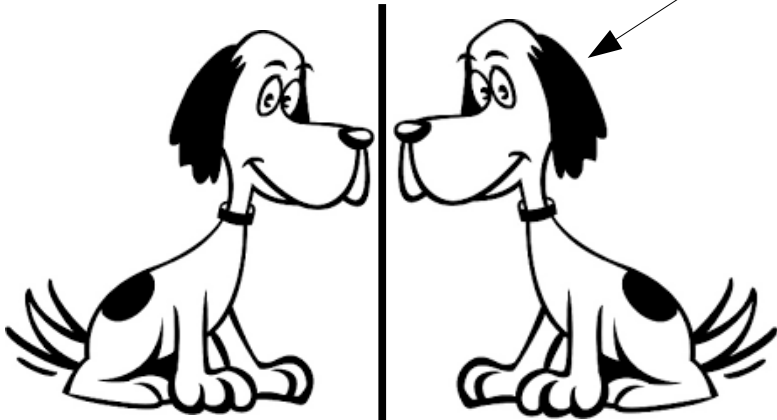
Lattice constrained sQGMP color composition model accounts not only for global RHIC&LHC RAA, v_2 , v_3 data but uniquely accounts for bulk perfect fluidity due to Near unitary bound q+m and g+m scattering rate near T_c !

Can we learn **more** about the QCD perfect fluid color structure $Q_{sat}^2 = \langle \hat{q} L \rangle = \langle \chi \mu^2 \rangle$ than just its average BDMS second moment jet transport parameter?

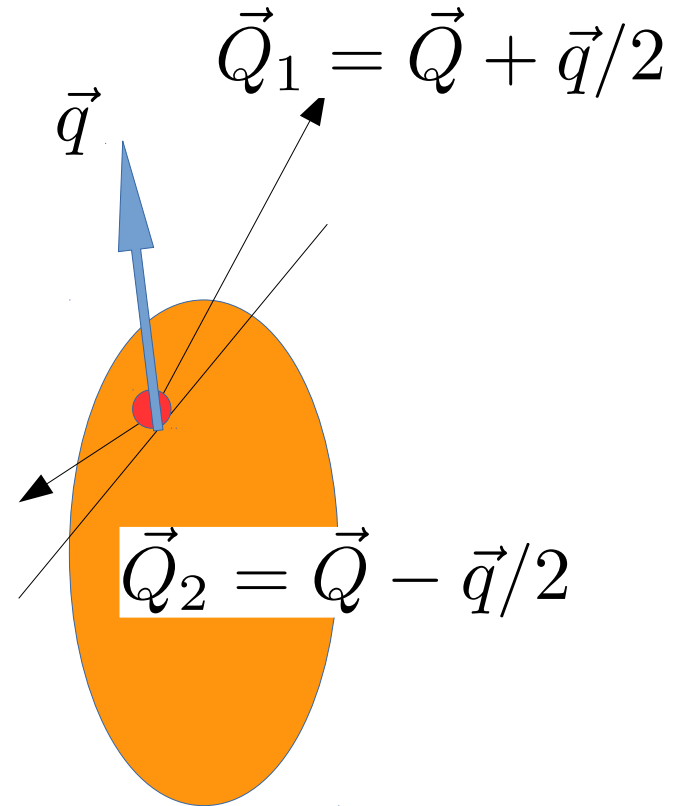
Can we determine the **opacity** $\chi = L/\lambda(T)$ and the **screening scale** $\mu(T)$ separately Using future precise data on the Landau and Rutherford multiple scattering tails of the acoplanarity distribution ?

Sudakov \otimes BDMS Gaussian “Ears”

$$\exp[-\delta\phi^2 / (Q_s/Q)^2]$$



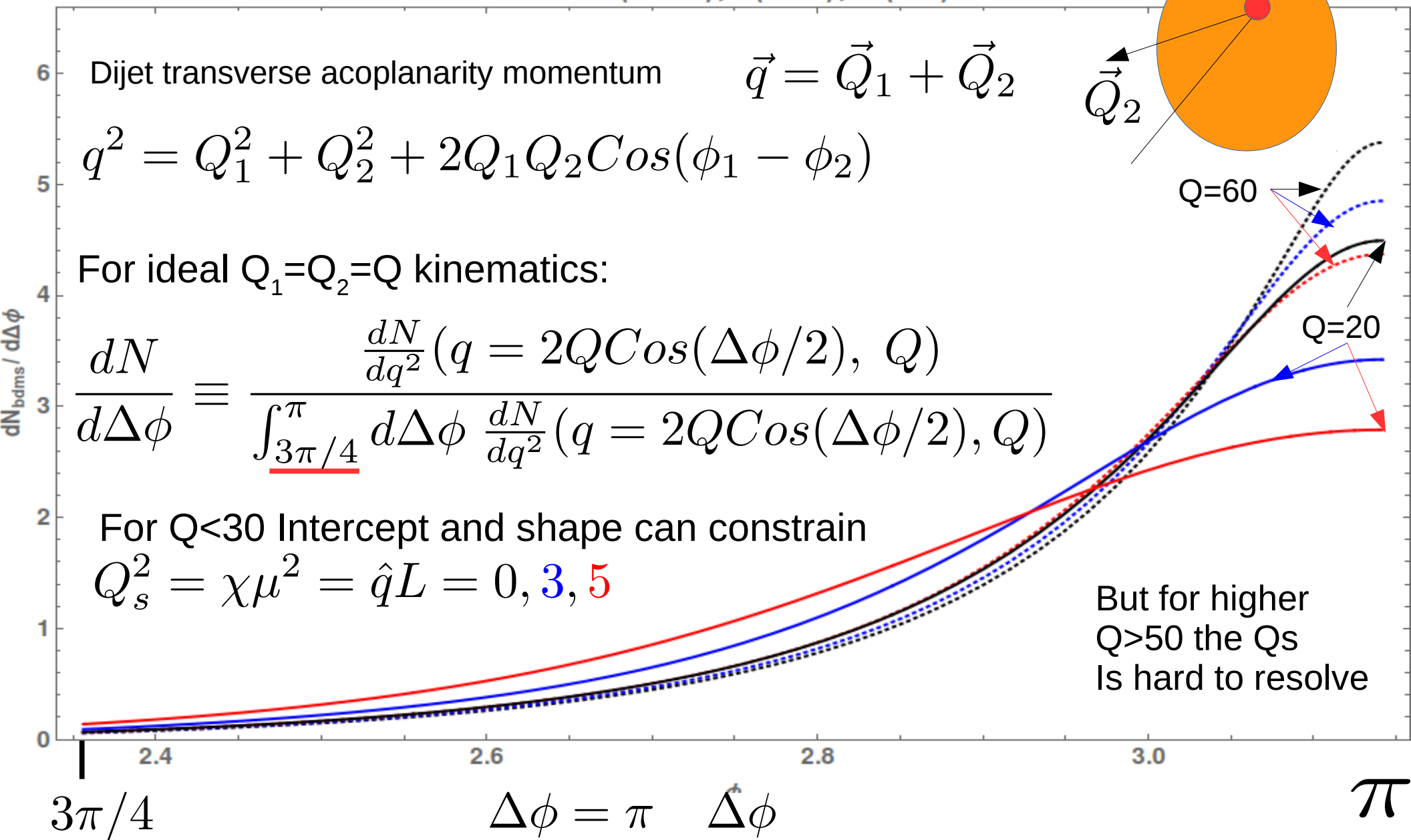
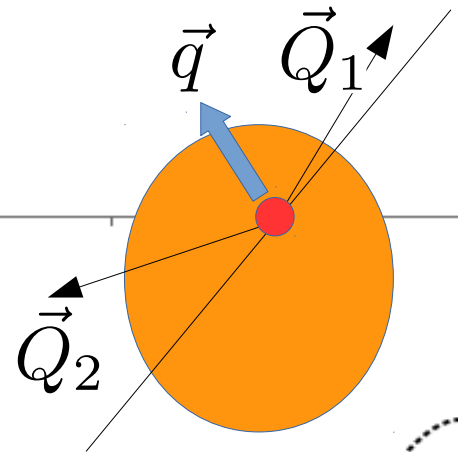
Sudakov \otimes Rutherford “Tails”



$$\sim \frac{\alpha}{\delta\phi^2} + \frac{(Q_s/Q)^2}{\delta\phi^4}$$

$$\Delta\phi = \phi_1 - \phi_2 = \pi - \delta\phi$$

h+Jet Acoplanarity $dN_{\text{bdms}}/d\Delta\phi$ vs $\Delta\phi$
for Vac+BDMS $\alpha=0.09$ for $Q=20$ (solid), 60 (dots)
 $Q_s = 0$ (black), 3 (blue), 5 (red)



distribution in the Sudakov resummation formalism as follows

$$\frac{d\sigma}{d\Delta\phi} = \sum_{a,b,c,d} \int p_{\perp\gamma} dp_{\perp\gamma} \int p_{\perp J} dp_{\perp J} \int dy_{\gamma} \int dy_J \int db \\ \times x_a f_a(x_a, \mu_b) x_b f_b(x_b, \mu_b) \frac{1}{\pi} \frac{d\sigma_{ab\rightarrow cd}}{d\hat{t}} b J_0(|\vec{q}_{\perp}|b) e^{-S(Q,b)}, \quad (1)$$

where J_0 is the Bessel function of the first kind, q_{\perp} is the transverse momentum imbalance between the photon and the jet $\vec{q}_{\perp} \equiv \vec{p}_{\perp\gamma} + \vec{p}_{\perp J}$, which takes into account both initial and final transverse momentum kicks from vacuum Sudakov radiations and medium gluon radiations. Here we define $x_{a,b} = \max(p_{\perp\gamma}, p_{\perp J})(e^{\pm y_{\gamma}} + e^{\pm y_J})/\sqrt{s_{NN}}$ as

The vacuum Sudakov factor $S_{pp}(Q, b)$ is defined as

$$S_{pp}(Q, b) = S_P(Q, b) + S_{NP}(Q, b) \quad (2)$$

where the perturbative S_P Sudakov factor depends on the incoming parton flavour and outgoing jet cone size. The perturbative Sudakov factors can be written as [35–37]

pQCD Vacuum Shower
$$S_P(Q, b) = \sum_{q,g} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B + D \ln \frac{1}{R^2} \right] \quad (3)$$

At the next-to-leading-log (NLL) accuracy, the coefficients can be expressed as $A = A_1 \frac{\alpha_s}{2\pi} + A_2 (\frac{\alpha_s}{2\pi})^2$, $B = B_1 \frac{\alpha_s}{2\pi}$ and $D = D_1 \frac{\alpha_s}{2\pi}$, with the value of individual terms given by the following table, where both A and B terms are summed over the corresponding incoming parton flavours.

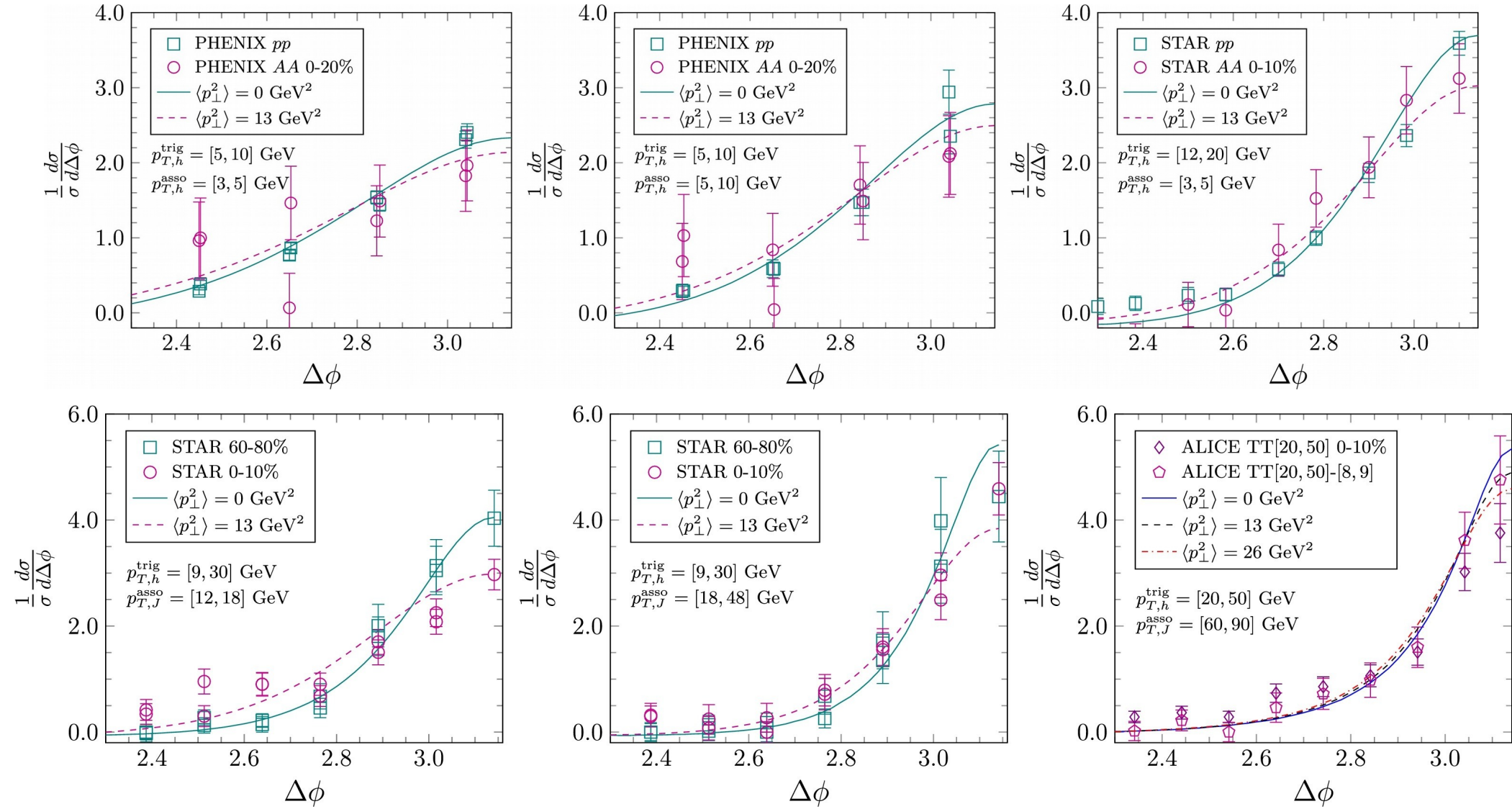
	A_1	A_2	B_1	D_1
quark	C_F	$K \cdot C_F$	$-\frac{3}{2}C_F$	C_F
gluon	C_A	$K \cdot C_A$	$-2\beta C_A$	C_A

Here C_A and C_F are the gluon and quark Casimir factor, respectively. $\beta = \frac{11}{12} - \frac{N_f}{18}$, and $K = (\frac{67}{18} - \frac{\pi^2}{6})C_A - \frac{10}{9}N_f T_R$. $R^2 = \Delta\eta^2 + \Delta\phi^2$ represents the jet cone-size, which is set to match the experimental setup. The implementation of the non-perturbative Sudakov factor $S_{NP}(Q, b)$ follows the prescription given in Refs [61, 62]. In the Sudakov resummation formalism, following the usual b^* prescription, the factorization scale is set to be $\mu_b \equiv \frac{c_0}{b_{\perp}} \sqrt{1 + b_{\perp}^2/b_{max}^2}$,

Jet-hadron acoplanarity azimuthal distribution from Chen,Qin,Xiao,Zhang PLB773, 2017

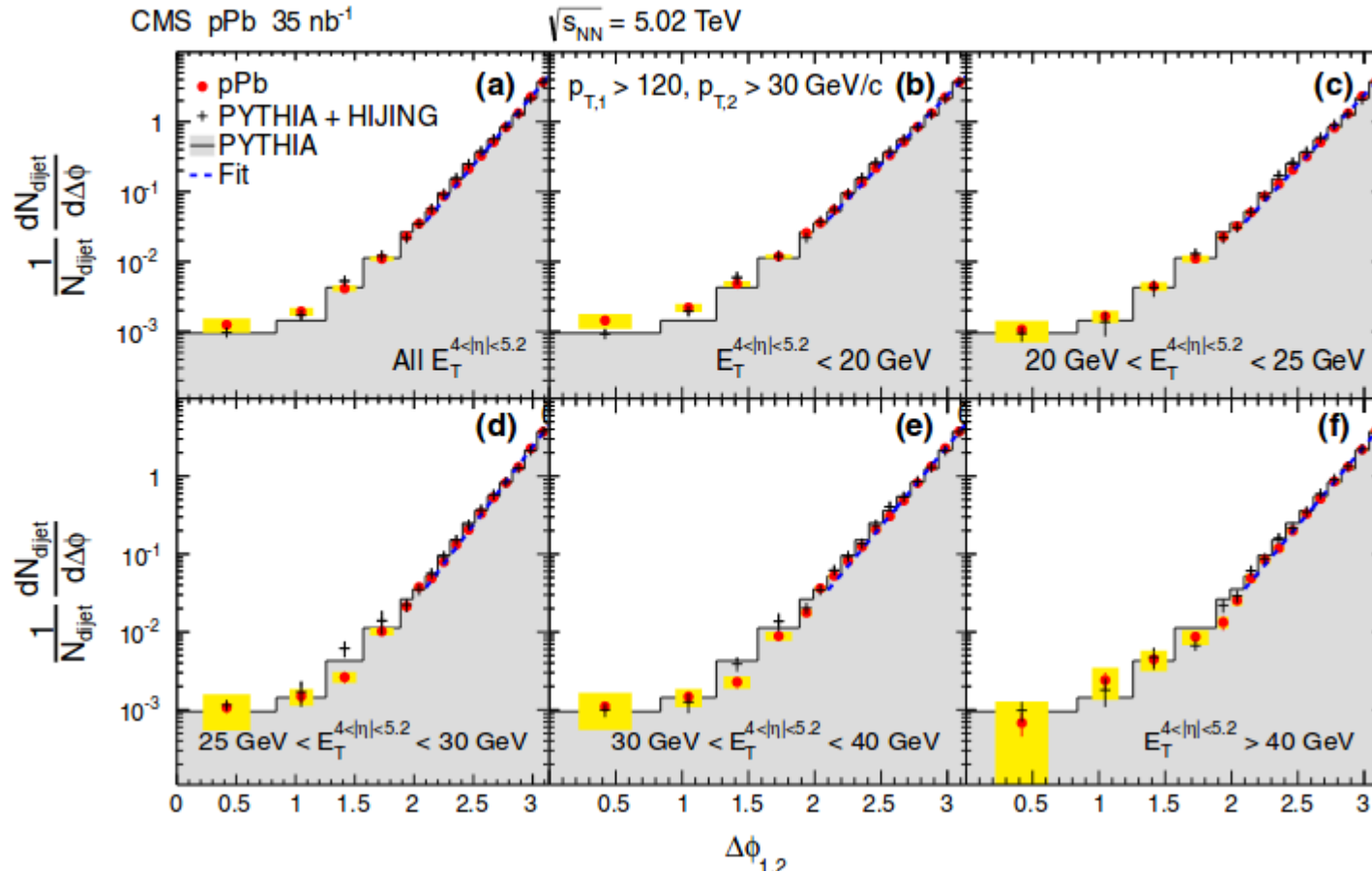
A+A Vacuum Sudakov+ BDMS(Qs) model compared to RHIC and LHC data

Current Status of A+A dijet acoplanarity at RHIC and LHC



[Current exp precision does not constrain Qs or qhat better than RAA(pT) & v2(pT) already do. Much higher precision future data needed in order to test color dof $n_a(T)$ and $d\sigma_{ab}/dq^2$ with acopl

CMS Studies of dijet transverse momentum imbalance and acoplanarity distributions in pPb collisions at 5.02 TeV have achieved great precision



Very high precision has (after 30 years) been reached at LHC in pp and pA to quantify ***vacuum*** induced Sudakov acoplanarity due to jet gluon showers. Thus pQCD Sudakov (A, B and D) factors can now be tuned to higher accuracy. Small deviations from Sudakov distribution due to jet-medium multiple collision interactions can thus help to discriminate between competing models of the color structure of QCD perfect fluids in A+A reactions

Multiple jets and γ -jet correlation in high-energy heavy-ion collisions

Luo, Cao, He, Wang CCNU
PLB782 (2018) , 1803.06785 [

High $p_T \sim 100$ GeV makes small angle
Deviations from π nearly independent
Of medium effect and are dominated
by Vacuum Sudakov radiation effects.

At large angles < 2 there is a predicted
suppression of gamma-jet correlations
due to multiple induced medium response

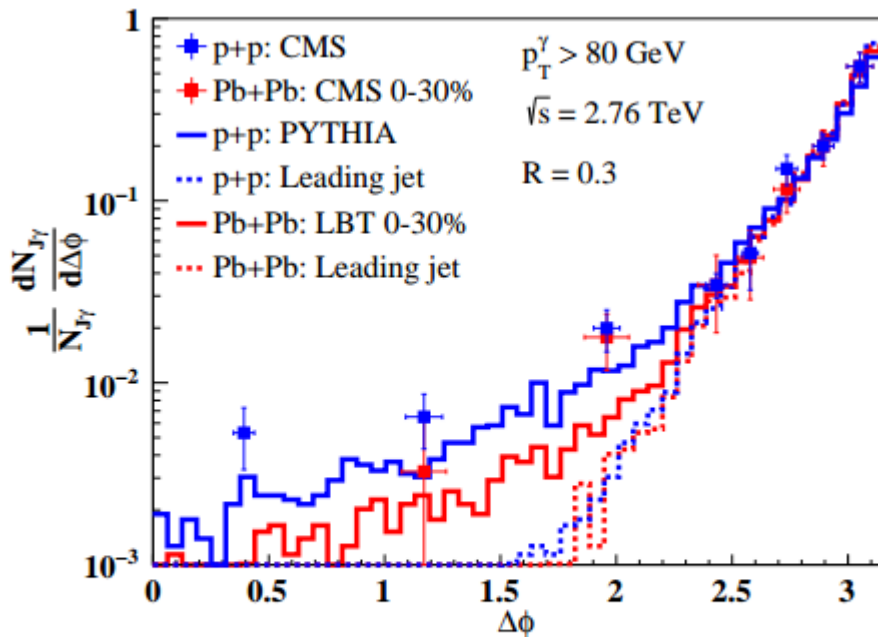


FIG. 6: (Color online) Angular distribution of γ -jet in central (0-30%) Pb+Pb (red) and p+p collisions (blue) at $\sqrt{s} = 2.76$

Exp should focus on the “sweet window”

$$2.4 < \Delta\phi < \pi$$

To reduce distortion due to quenching of gluon
showers and medium recoil contributions

“Dominance of the Sudakov form factor in γ -jet correlation from soft gluon radiation in large p_T hard processes pose a challenge for using γ -jet azimuthal correlation to study medium properties via large angle parton-medium interaction.”

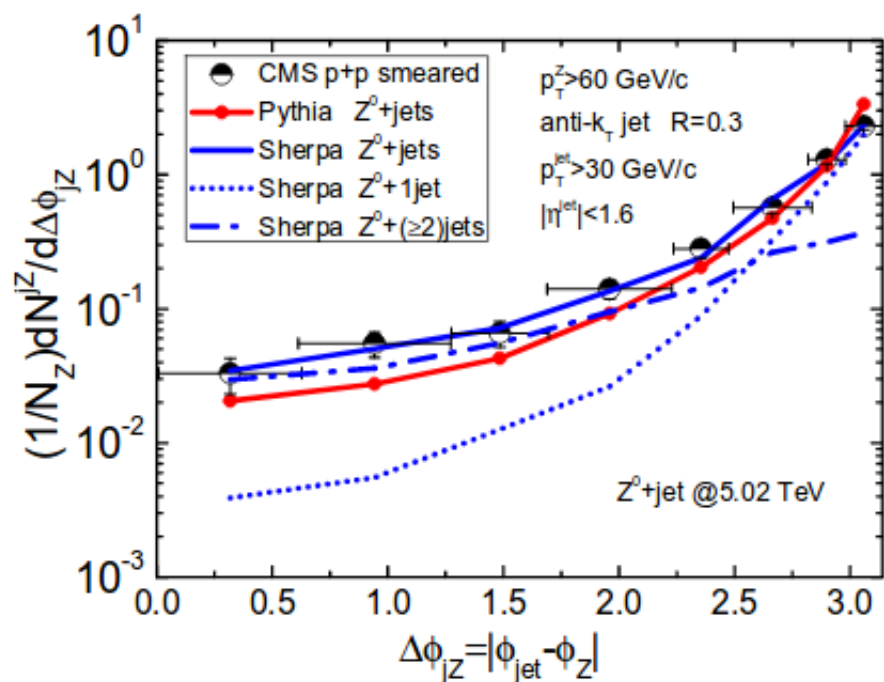


FIG. 1. Comparison between the azimuthal angle correlation $\Delta\phi_{jZ}$ of Z^0 +jet by CMS data [28] and theoretical simulations of SHERPA (Blue) and Pythia (Red) in p+p collisions at $\sqrt{s} = 5.02$ TeV. The dotted (the dash-dotted) line shows the contribution from $Z^0 + 1\text{jet}$ ($Z^0 + (\geq 2)\text{jets}$).

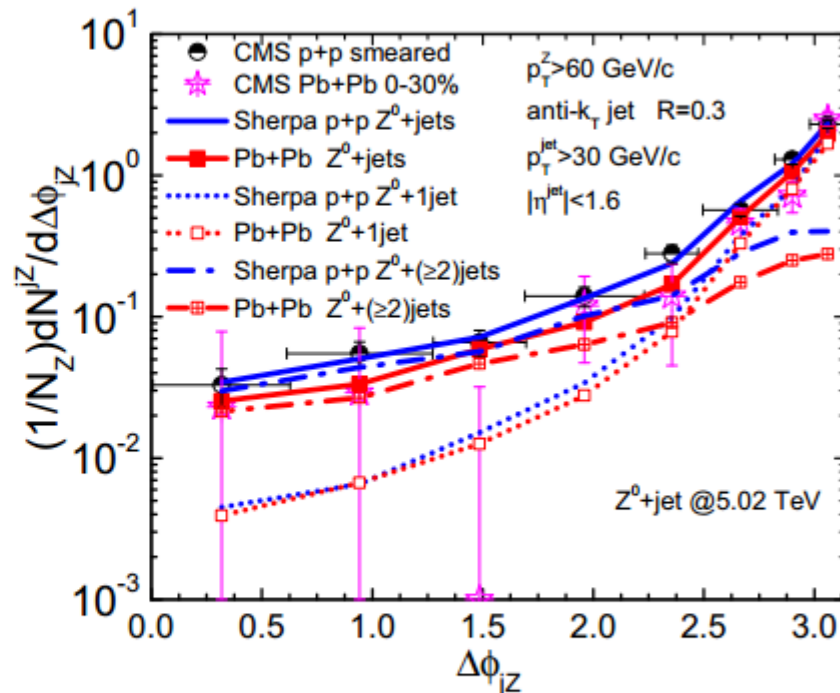


FIG. 5. Numerical results of the azimuthal angle correlation in $\Delta\phi_{jZ}$ in p+p (blue) and Pb+Pb (red) collisions at $\sqrt{s} = 5.02$ TeV as compared to CMS data [28]. The dotted (dash-dotted) lines show the contributions from $Z^0 + 1\text{jet}$ ($Z^0 + (\geq 2)\text{jets}$).

decorrelation of the Z^0 +jet in azimuthal angle from $Z^0 + 1\text{jet}$ processes in this region is dominated by soft and collinear radiation, the resummation of which can be described by a Sudakov form factor. The transverse momentum broadening of this leading jet due to jet-medium interaction is negligible to that caused by soft and collinear radiation ;

future experimental data with much better statistics are needed to observe this suppression of the small-angle Z^0 +jet correlation unambiguously.

Section 2: Some details of the calculation

D.Appel 1986

J.P.Blaizot, L.McLerran(1986); M. Greco,(1985); V. Sudakov (1956)

Acoplanarity in
p+p is due to
Gluon radiation
from dijet antenna

In the parton model there are no bremsstrahlung effects, so we have simply $dP/dK_\eta = \delta(K_\eta)$. With perturbative QCD, multiple gluon emission from the hard scattering can be resummed in perturbation theory,¹⁴ and for the one-dimensional normal momentum density has the form

$$\frac{1}{\sigma_0(p,p_T)} \frac{1}{p_T} \frac{d\sigma}{d\phi} = \frac{dP}{dK_\eta} = \frac{1}{\pi} \int_0^\infty db \cos(K_\eta b) \exp[\tilde{B}(b)] .$$

In Double leading log
Sudakov approx

$$\tilde{B}(b) = - \int_{(b_0/b)^2}^{Q^2} \frac{dq^2}{q^2} \left[\ln \left[\frac{Q^2}{q^2} \right] A'(\alpha_s(q)) + B'(\alpha_s(q)) \right]$$

Acoplanarity in A+A arises from convolution of Sudakov and Jet-medium multiple scattering probabilities

$$F(\ell_T) \propto L \rho d\sigma/d^2\ell_T \quad \text{Depends on the medium}$$

$$\frac{dP}{dK_\eta} = \sum_{n=0}^\infty \sum_{m=0}^\infty \left[\frac{1}{n!} \prod_{i=1}^n \int d^2k_{Ti} B(\mathbf{k}_{Ti}) \frac{1}{m!} \prod_{j=1}^m \int d^2l_{Tj} F(l_{Tj}) \delta \left[K_\eta - \sum_{i=1}^n (\mathbf{k}_{Ti})_\eta - \sum_{j=1}^m (l_{Tj})_\eta \right] \right]$$

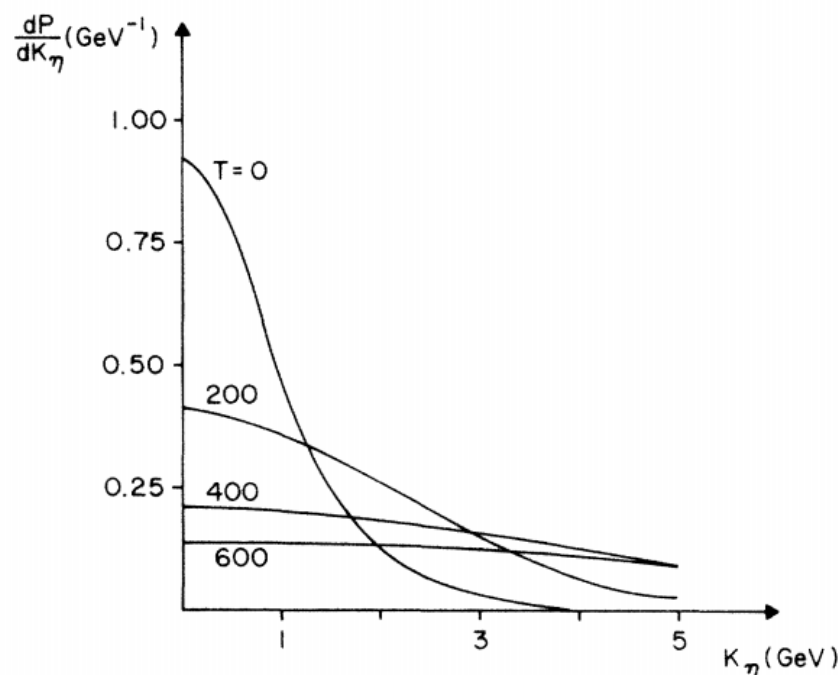
$$\int_{-\infty}^{+\infty} dK_\eta \exp(iK_\eta b) \frac{dP}{dK_\eta} = \exp[\tilde{B}(b) + \tilde{F}(b)]$$

For $F(l_T)$, the probability density for scattering elastically off the plasma constituents with transverse-momentum transfer l_T , we propose the following form:

$$F(l_T) = \sum_x n_x R \frac{d^2 \sigma_x}{d^2 l_T}, \quad (11)$$

where x runs over the different particle types comprising the plasma ($x = g, q_i, \bar{q}_i$), with n_x their number density. This equation essentially relates the plasma mean free path to the available distance for scattering (R) for each particular l_T .

Stefan-Boltzmann wQGP model estimates



$$F(l_T) = 9aRT^3 \left[1 + \frac{N_F}{4} \right] \frac{\alpha_s^2(l_T)}{l_T^4}$$

Cut off soft divergence below pQCD Debye mass

$$\ell_{\perp} \sim gT$$

“Based on this, one is encouraged to conjecture that someday jet behavior could be used as an effective thermometer of a QCD plasma.”

Confirmed by J.P.Blaizot, L.McLerran(1986)
In more realistic detail

Logarithmic approximations, quark form factors, and quantum chromodynamics

S. D. Ellis, N. Fleishon, and W. J. Stirling

1394

S. D. ELLIS, N. FLEISHON

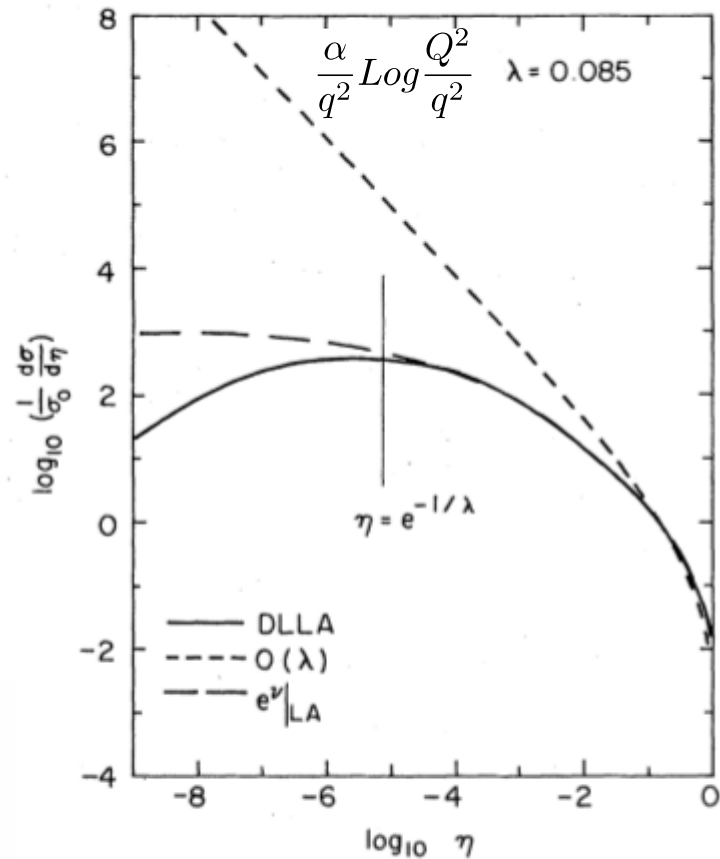


FIG. 4. Theoretical approximations to the cross section defined in the text. The long-dashed line is the soft logarithmic approximation [LA, (1), (2), (3)]. The solid line is the DLLA Eq. (2.12). The dashed line is the corresponding one-gluon contribution.

It is convenient to return to the general notation of the Introduction and define $\eta = Q_T^2/s$ and $\lambda = \alpha_s C_F/\pi$. Thus Eq. (2.11) can be written as

$$\begin{aligned} \frac{1}{\sigma_0} \frac{d\sigma}{d\eta} \Big|_{\text{DLLA}} &= \frac{\lambda}{\eta} \ln \frac{1}{\eta} \exp\left(-\frac{\lambda}{2} \ln^2 \eta\right) \theta(1-\eta) \\ &= \frac{d}{d\eta} F_{\text{DLLA}}(\eta) \theta(1-\eta) \end{aligned} \quad (2.12)$$

with $F_{\text{DLLA}}(\eta)$ identified from Eq. (1.1).

with $C_F = 4/3$, $T_R = 1/2$ and $N = 3$. It is instructive to see how the logarithms in b -space generate logarithms in q_T -space. For illustration, we take only the leading coefficient $A^{(1)} = 2C_F$ to be non-zero in $e^{S(b, Q^2)}$, and assume a fixed coupling α_s . This corresponds to

$$\frac{d\sigma}{dq_T^2} = \frac{\sigma_0}{2} \int_0^\infty b db J_0(q_T b) \exp\left[-\frac{\alpha_s C_F}{2\pi} \ln^2\left(\frac{Q^2 b^2}{b_0^2}\right)\right]. \quad (6)$$

The expressions are made more compact by defining new variables $\eta = q_T^2/Q^2$, $z = b^2 Q^2$, $\lambda = \alpha_s C_F/\pi$, $z_0 = 4 \exp(-2\gamma_E) = b_0^2$. Then

$$\frac{1}{\sigma_0} \frac{d\sigma}{d\eta} = \frac{1}{4} \int_0^\infty dz J_0(\sqrt{z\eta}) e^{-\frac{\lambda}{2} \ln^2(z/z_0)} \quad (7)$$

and we encounter the same expression as in [6], which describes the emission of soft and collinear gluons with transverse momentum conservation taken into account. The result

The conclusion is then that the *subleading logarithms* which arise from a correct treatment of transverse-momentum conservation can play a major role in filling in the zero at $\eta=0$ and obscuring the maximum which was present near $\ln 1/\eta \sim 1/\lambda$ in the DLLA. It is informative to di-

$$\frac{dN_{BDMS}}{dq_T^2} = \frac{e^{-q_T^2/Q_s^2}}{Q_s^2} \quad \text{versus} \quad \frac{dN_{GLV}}{dq_T^2} = f(q_T, \mu, \chi)$$

Moliere Gaussian (Qs) vs Elastic scattering opacity series $(\mu, \chi = L/\lambda, Q_s^2 = \chi\mu^2\zeta)$

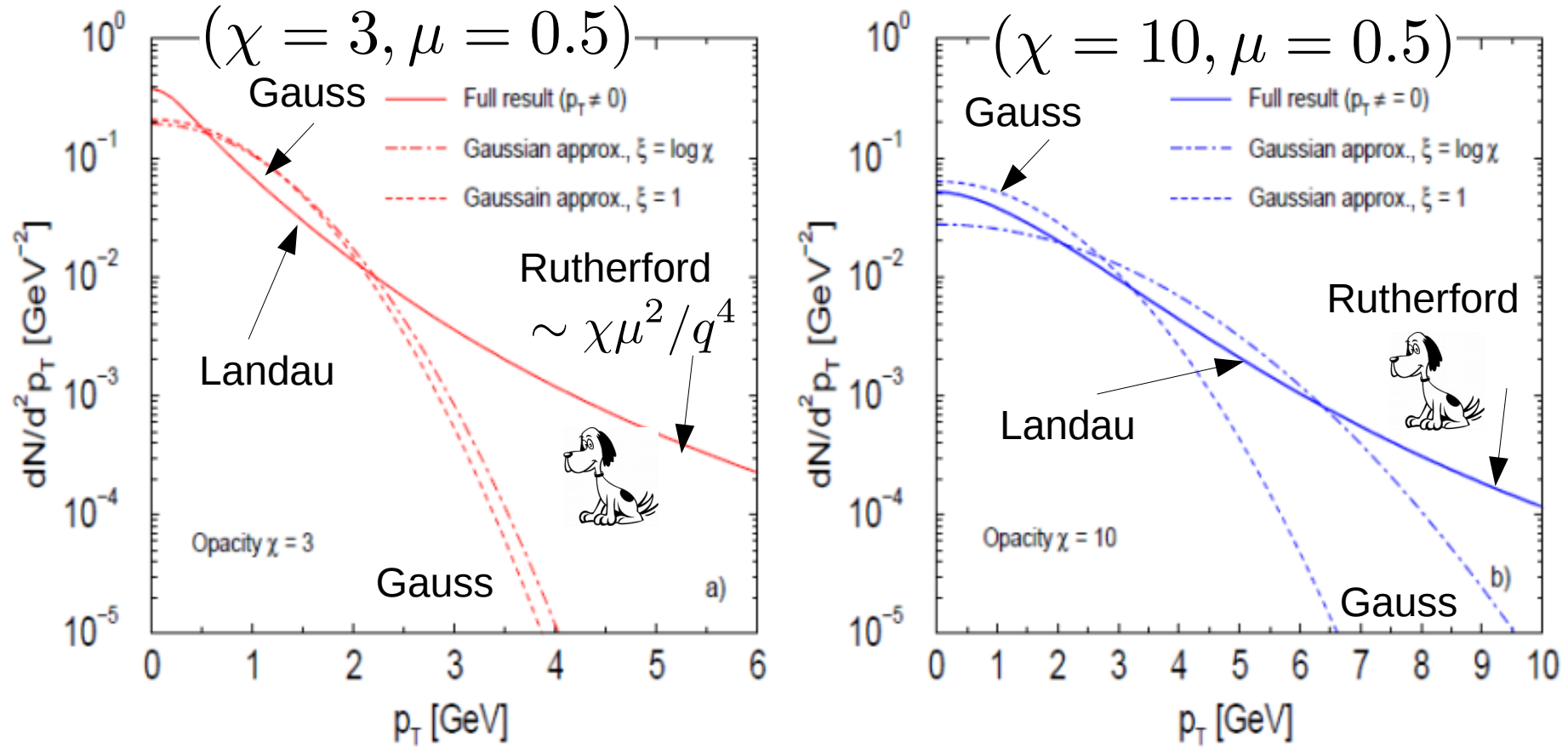


FIG 3. The final jet p_T distribution is shown versus p_T for two different opacities $\chi = 3$ (Fig. 3a) and $\chi = 10$ (Fig. 3b). We compare the full result (without the delta function contribution at $p_T \sim 0$) to the Moliere Gaussian approximation with $\xi = 1$ and $\xi = \log \chi$. In this example we use $\mu^2 = 0.25$ GeV 2 .

Medium Induced Acoplanarity Distribution shapes due to multiple collisions depend on at least two parameters (μ, χ)

e.g Yukawa $\mu \approx gT$ screened parton elastic scattering

$$\frac{d\tilde{\sigma}_{el}}{d^2\mathbf{q}}(\mathbf{b}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{q}\cdot\mathbf{b}} \frac{1}{\pi} \frac{\mu^2}{(\mathbf{q}^2 + \mu^2)^2} = \frac{\mu b}{4\pi^2} K_1(\mu b)$$

Mult.coll. opacity χ^n series can be summed in b-space

$$dN(\mathbf{p}) = e^{-\sigma_{el}T(\mathbf{b}_0)} \int d^2\mathbf{b} e^{i\mathbf{p}\cdot\mathbf{b}} e^{\tilde{\sigma}_{el}(\mathbf{b})T(\mathbf{b}_0)} dN^{(0)}(\mathbf{b})$$

$$\chi = \langle L/\lambda \rangle = \int dt \int d^2\mathbf{q} \{d\sigma_{el}(t)/d^2\mathbf{q}\} \rho(t, \mathbf{b}_0) \sim O(\alpha T L)$$

$$dN(\mathbf{p} \gg \chi\mu^2\xi) \approx (\chi\mu^2\xi)/\mathbf{p}^4 \quad \text{Rutherford tail}$$

In large $\chi \gg 1$ lim, distrib. approaches Moliere form

$$dN(\mathbf{p}) = \int d^2\mathbf{b} e^{i\mathbf{p}\cdot\mathbf{b}} \frac{1}{(2\pi)^2} \frac{e^{-\chi\mu^2\xi b^2/2}}{\chi\mu^2\xi} = \frac{1}{2\pi} \frac{e^{-p^2/2\chi\mu^2\xi}}{\chi\mu^2\xi}$$

In BDMS approx this Gaussian form depends on only one "saturation scale"

$$Q_s^2 = \chi\mu^2\xi = \int dt \hat{q}(t)$$

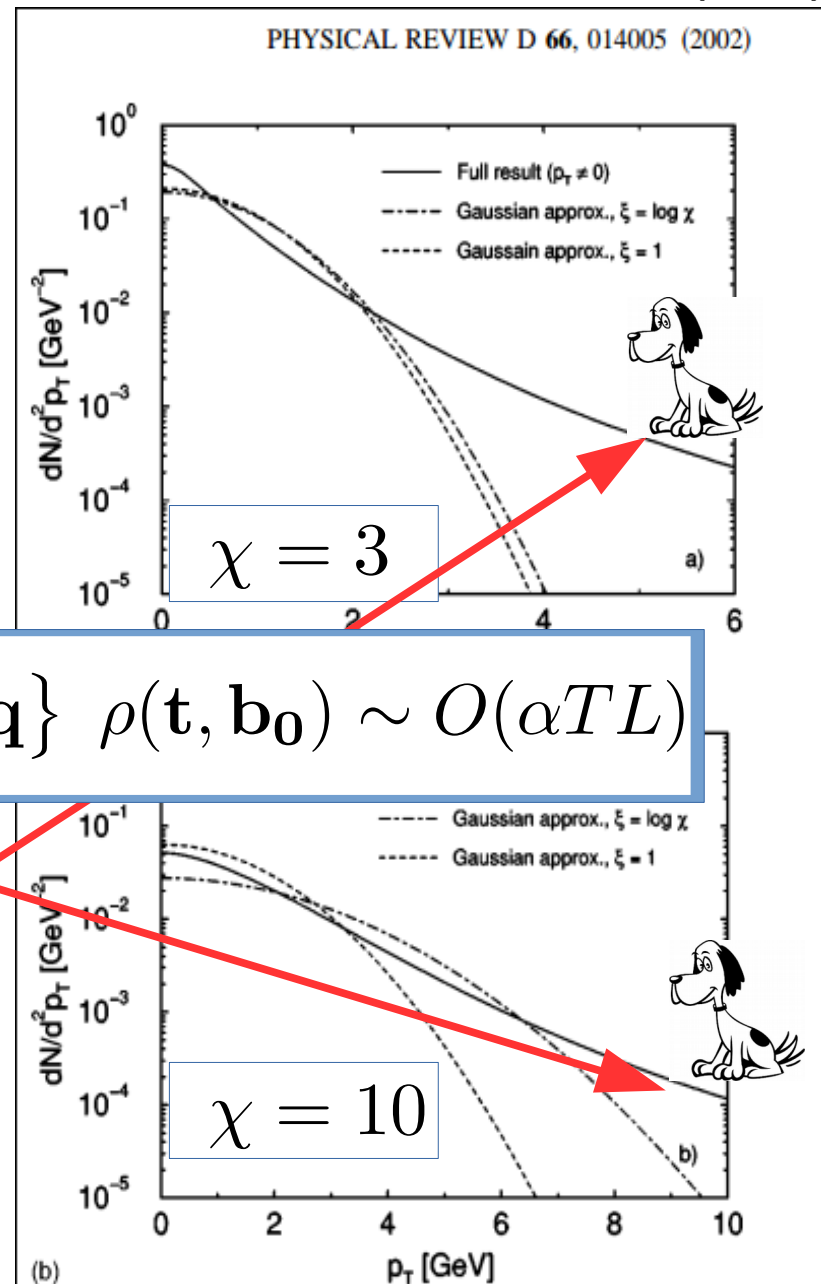


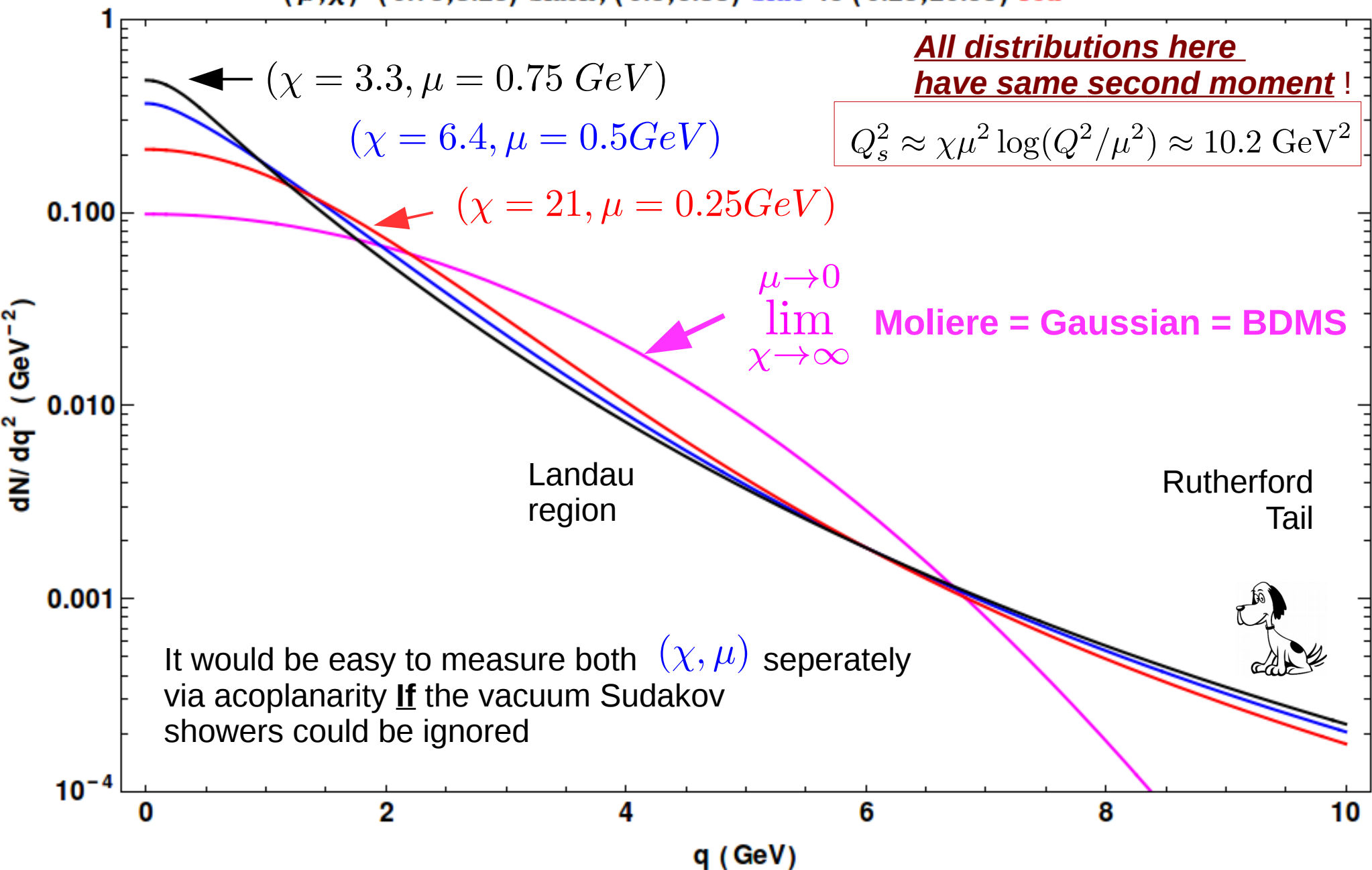
FIG. 3. The final parton p_T distribution is shown versus p_T for two different opacities $\chi=3$ (a) and $\chi=10$ (b). We compare the full result (without the delta function contribution at $p_T \sim 0$) to the Moliere Gaussian approximation with $\xi=1$ and $\xi=\log \chi$. In this example we use $\mu^2=0.25$ GeV 2 .

Section 3: Numerical examples and conclusions

Summary of medium pT broadening distribution

$Q=20$ dN/dq^2 Gauss(Q_s) (magenta) vs GLV(μ, χ) shapes for fixed $Q_s^2 = 10.19$

$(\mu, \chi) = (0.75, 3.25)$ black, $(0.5, 6.38)$ blue vs $(0.25, 20.99)$ red



Summary 2:

Vacuum Sudakov dominates over medium induced dijet acoplanarity as Mueller et al and Chen et al emphasized

Percent level precision would be needed to resolve BDMS Qs into (χ, μ)

$Q=20$ dN/dq^2 Gauss(Q_s^2) blue, GLV(μ, χ) orange, Yukawa(μ) black

$\mu=0.5$ GeV ; $\chi=1,2,4,8$; $Q_s^2/\chi=1.59575$ GeV²

Can future exp resolve the high q non-Gaussian power-law like Landau and Rutherford tails of the jet-medium multiple collisions hiding below the dominant vacuum

Sudakov $\sim \alpha/q^2$

Sudakov \otimes BDMS

GLV $\chi=8$

$\sim \chi\mu^2/q^4$

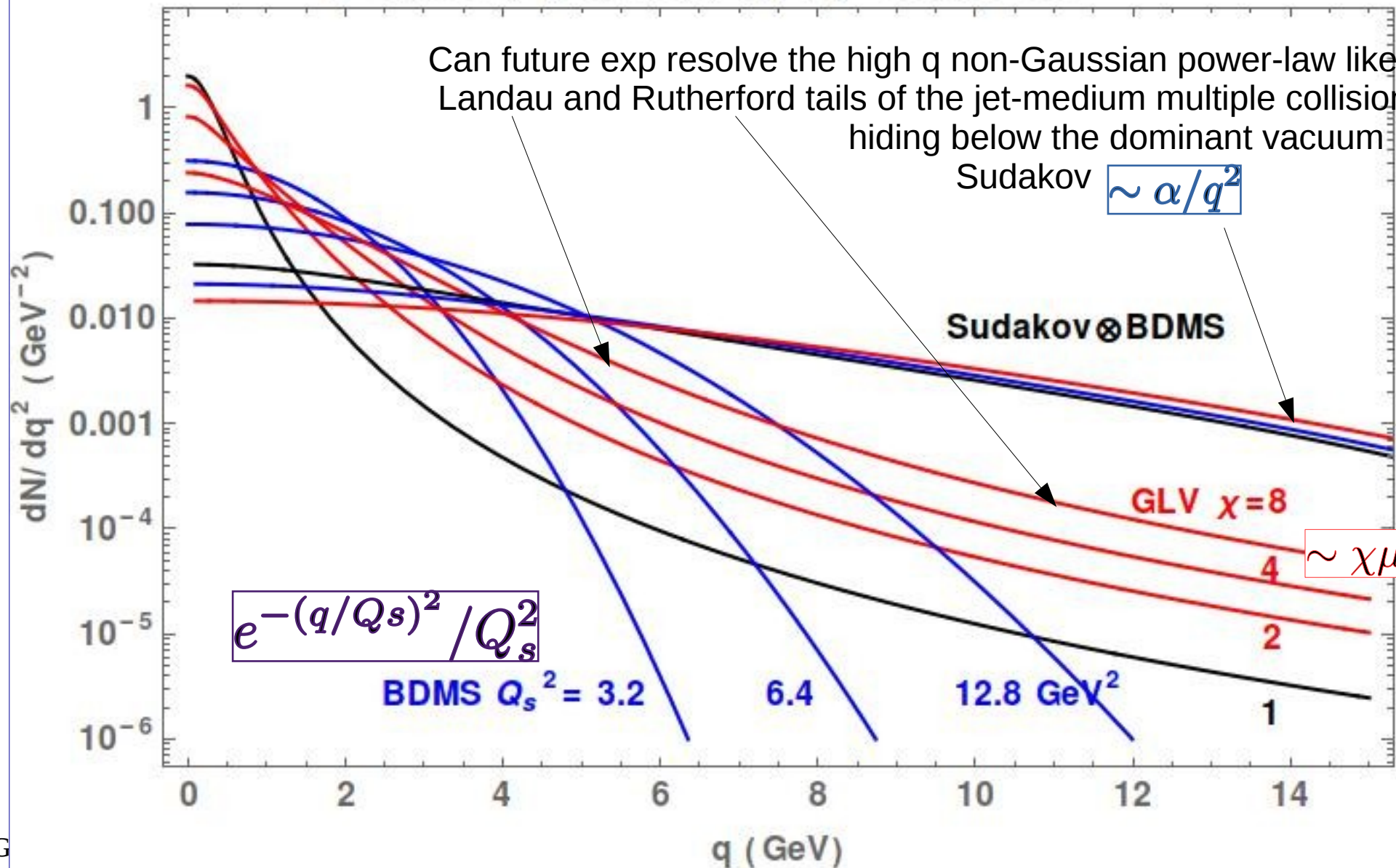
$e^{-(q/Q_s)^2}/Q_s^2$

BDMS $Q_s^2 = 3.2$

6.4

12.8 GeV²

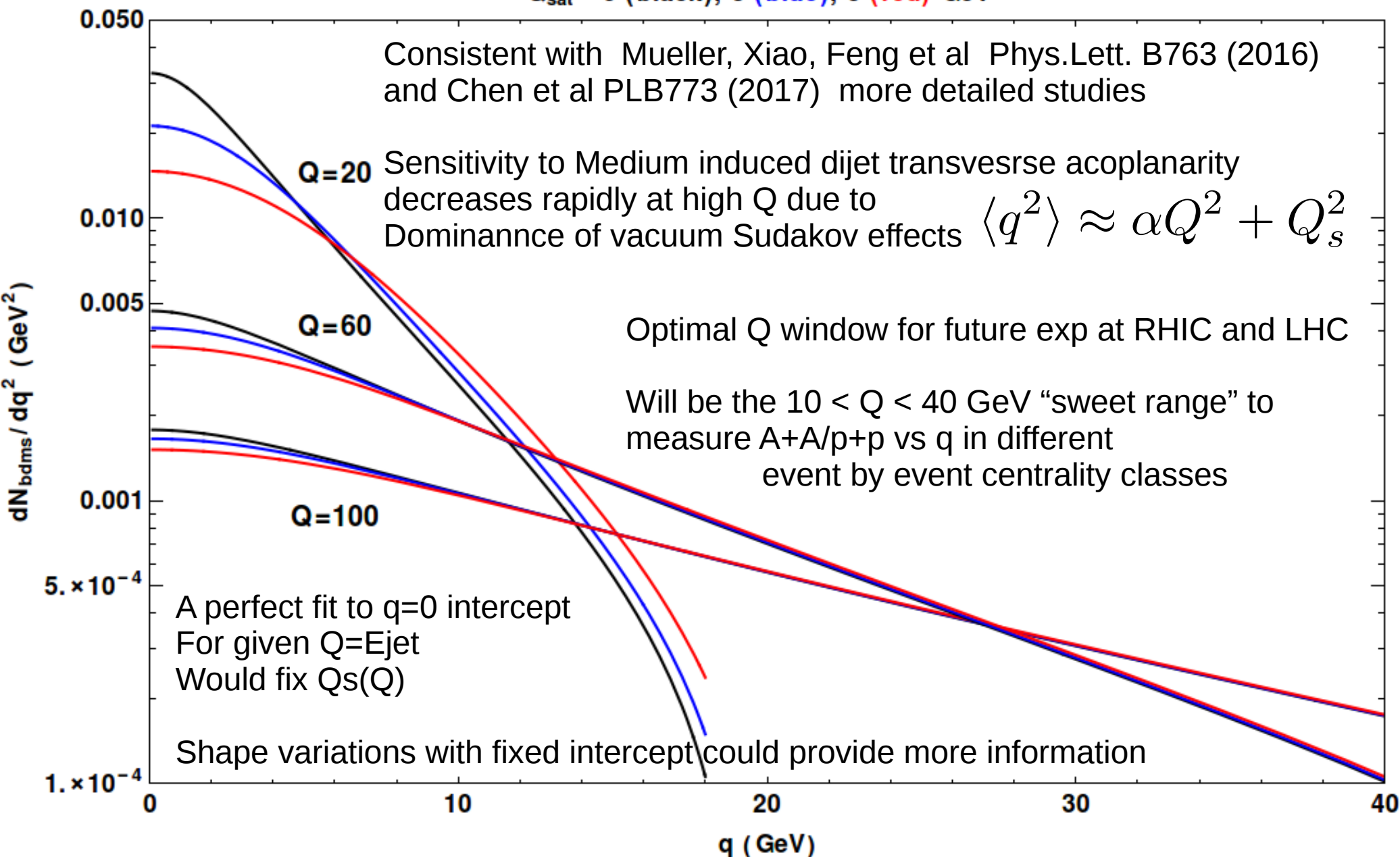
1



One parameter, Q_s , BDMS medium convoluted with Sudakov dijet transverse distributions

Hadron-Jet Vac \otimes BDMS dN_{bdms}/dq^2 vs q for $Q = 20, 60, 100$ GeV

$Q_{\text{sat}} = 0$ (black), 3 (blue), 5 (red) GeV



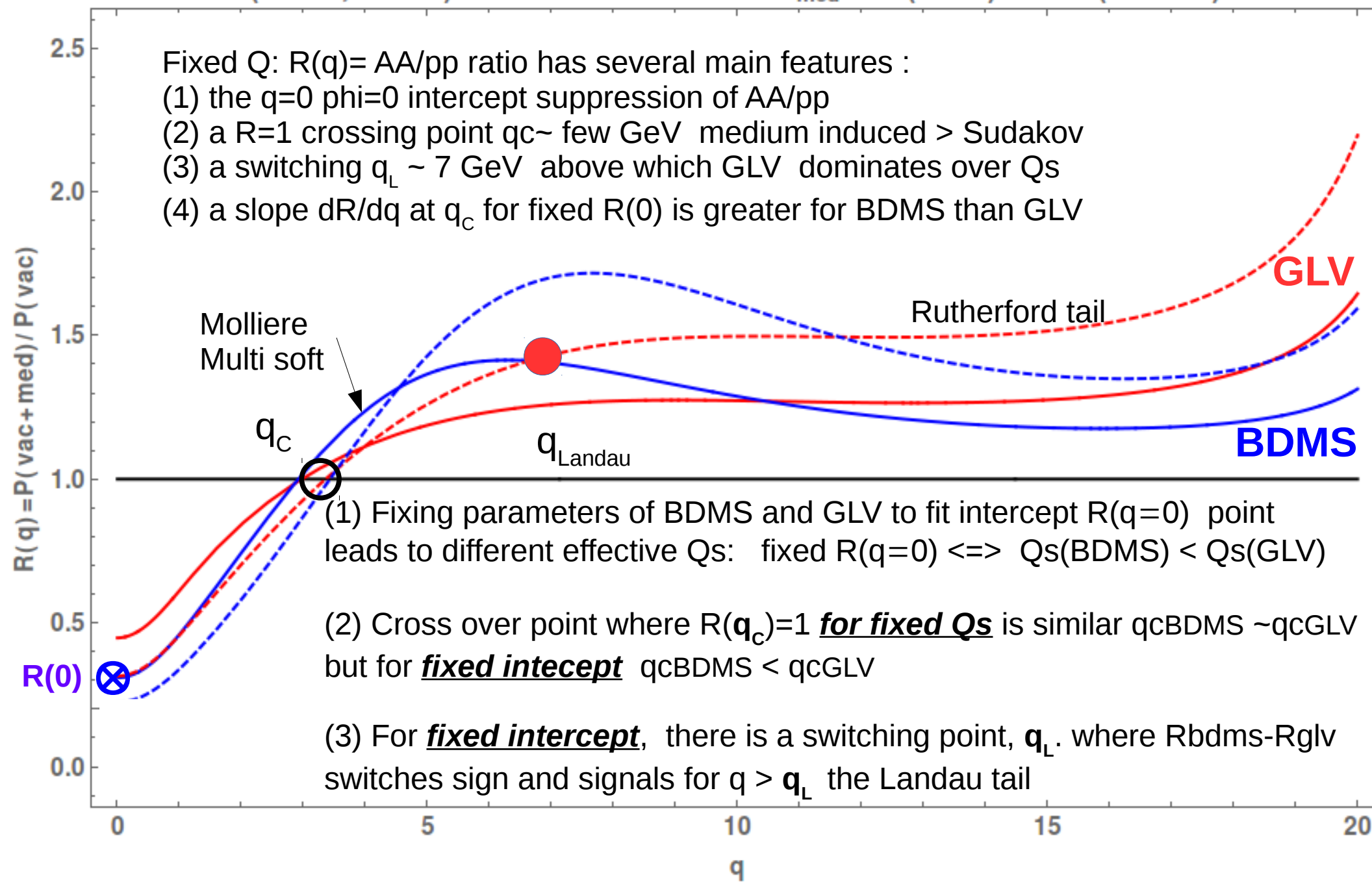
This example is Exaggerated with

Ratio of Probabilities for dijet transverse momentum q
Relative to Vacuum for BDMS(blue) and GLV(Red)

($Q=20$, $\alpha=0.3$) with medium induced $Q_{\text{med}}^2=9$ (solid) vs 16 (dashed)

Fixed Q : $R(q) = A A / p p$ ratio has several main features :

- (1) the $q=0$ $\phi=0$ intercept suppression of $A A / p p$
- (2) a $R=1$ crossing point $q_c \sim$ few GeV medium induced $>$ Sudakov
- (3) a switching $q_L \sim 7$ GeV above which GLV dominates over Q_s
- (4) a slope dR/dq at q_c for fixed $R(0)$ is greater for BDMS than GLV



This example is Exaggerated with

Ratio $R(\Delta\phi)$ of Jet Medium to Vacuum Acoplanarity

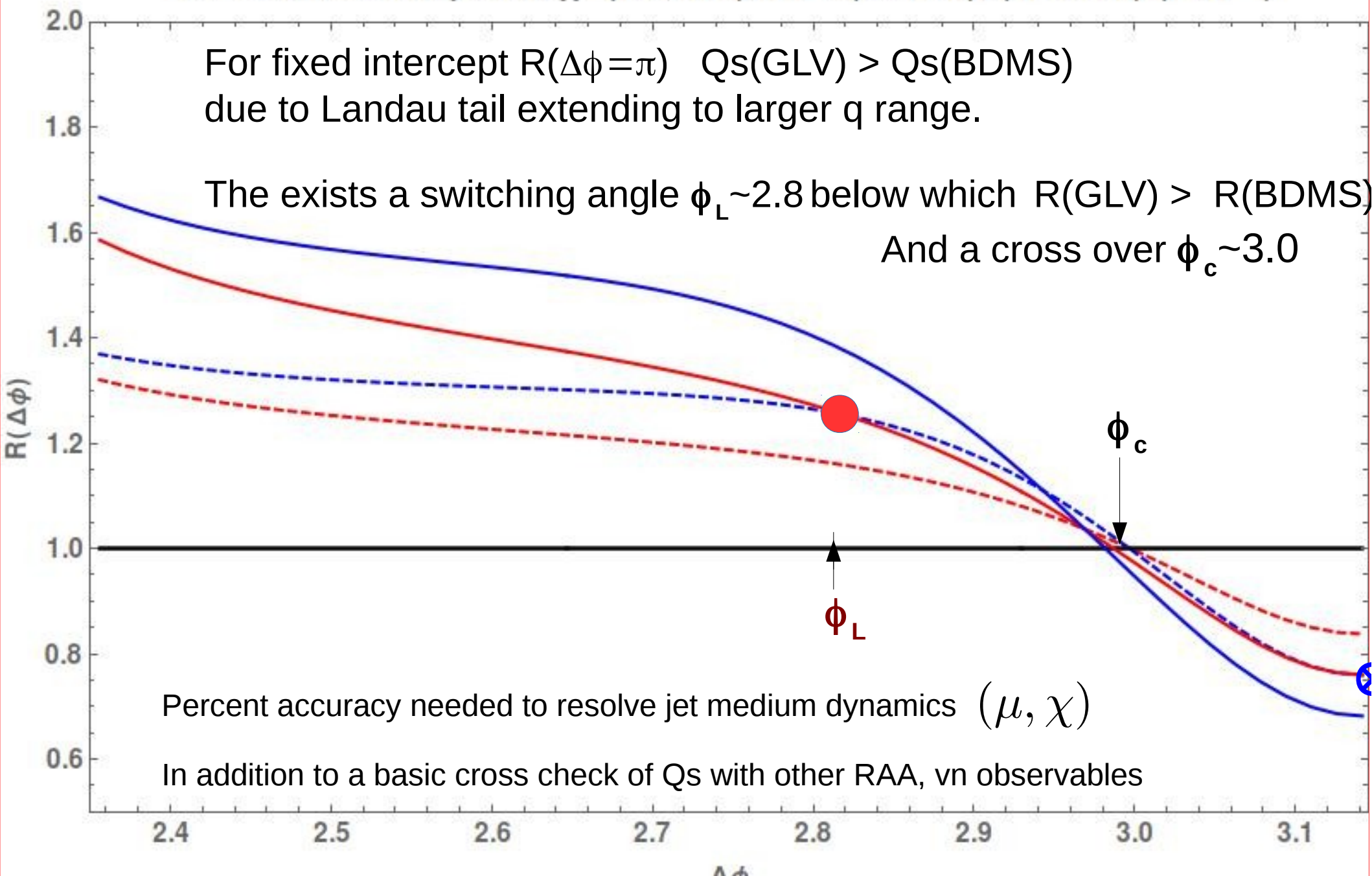
(Vac+GLV)/Vac (red) vs (Vac+BDMS)/Vac (blue)

for $Q=20$, $\alpha=0.3$, $\mu=0.5$, $\chi=(5.7, 10.2)$, $Q_s^2=(9 \text{ dash}), (16 \text{ solid}) \text{ (GeV}^2\text{)}$

For fixed intercept $R(\Delta\phi=\pi)$ $Q_s(\text{GLV}) > Q_s(\text{BDMS})$
due to Landau tail extending to larger q range.

There exists a switching angle $\phi_L \sim 2.8$ below which $R(\text{GLV}) > R(\text{BDMS})$

And a cross over $\phi_c \sim 3.0$

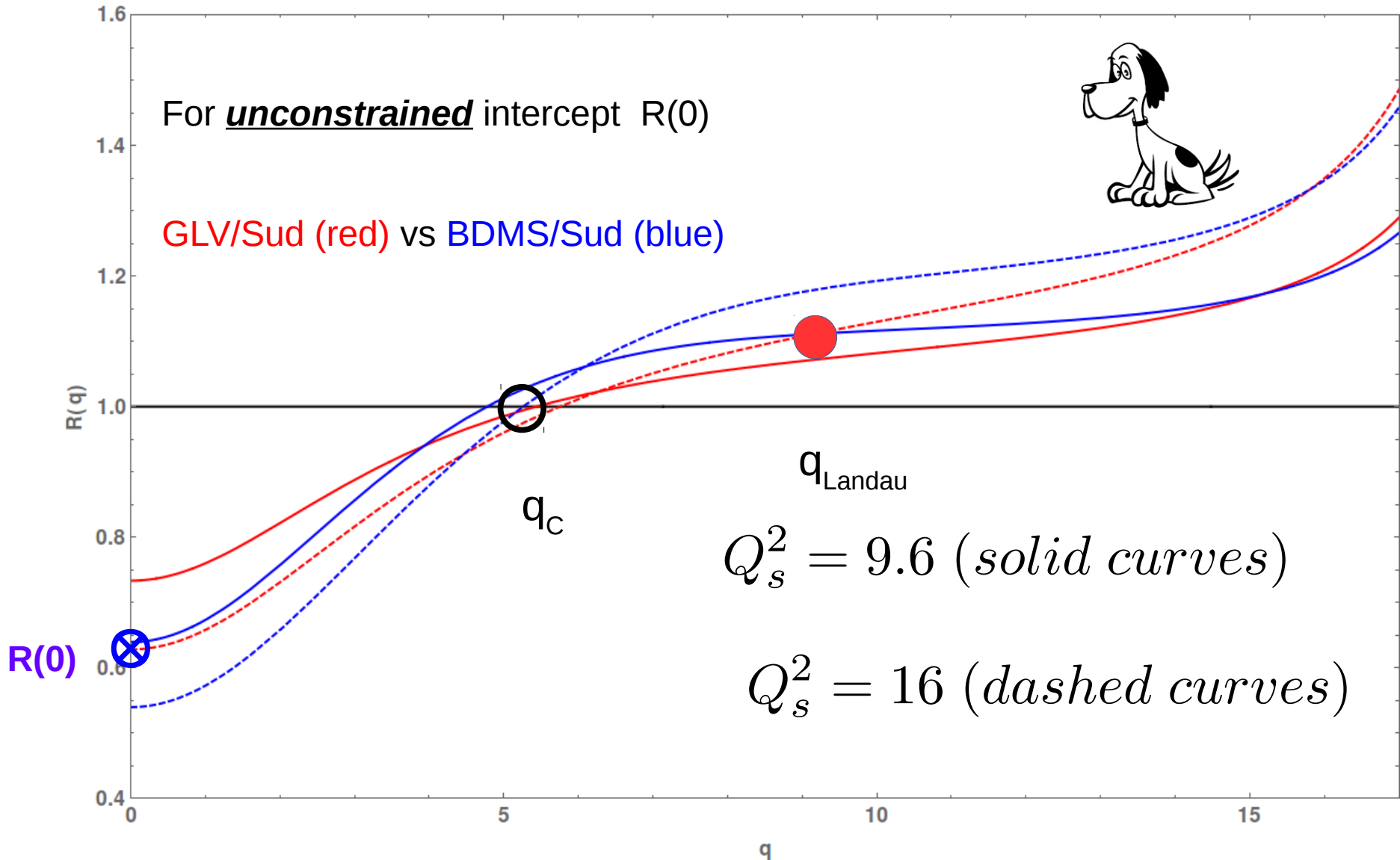


For more realistic Sudakov fits to p+p set $\alpha \approx 0.09$

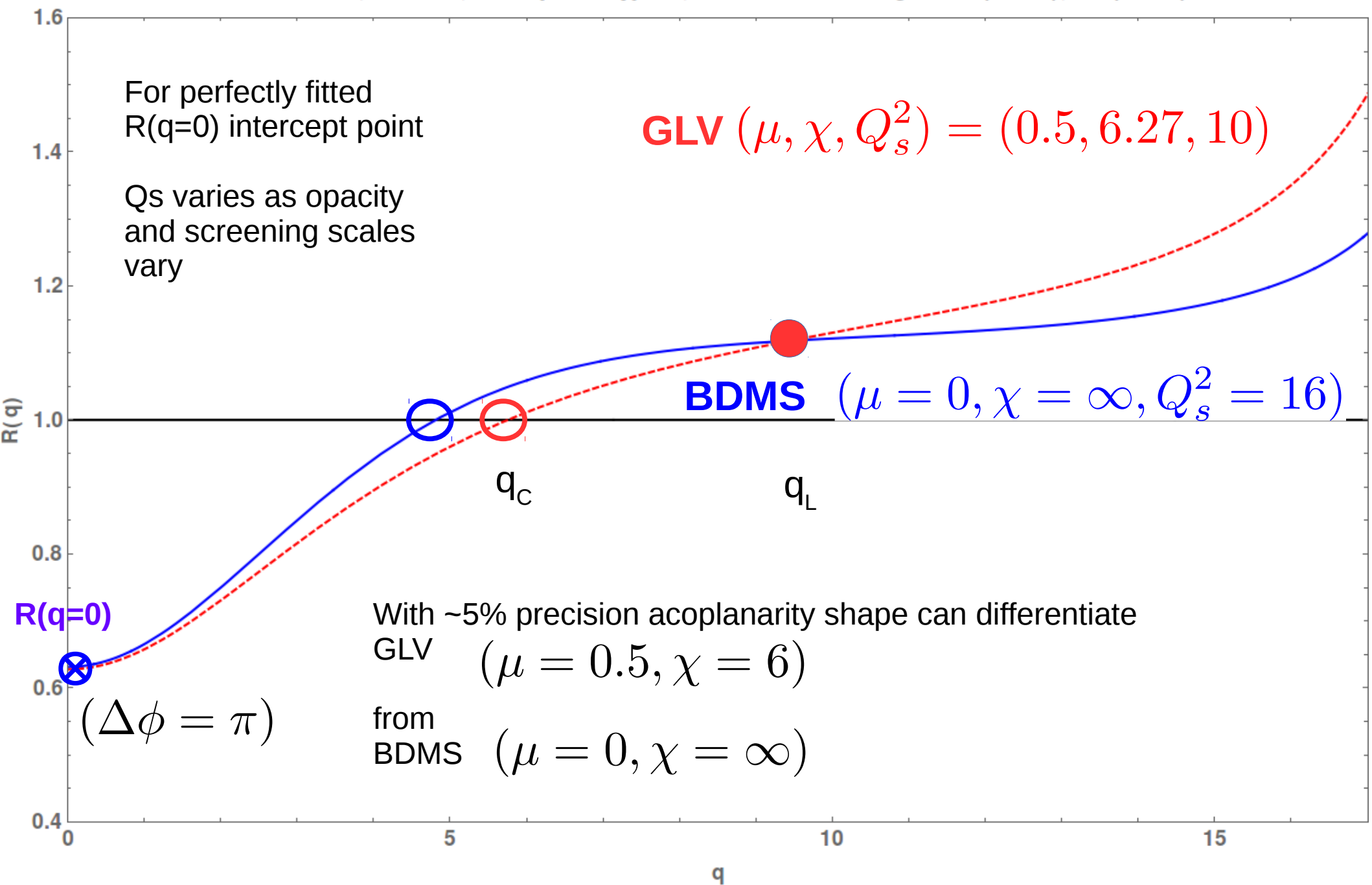
Requires much higher precision to resolve GLV finite (χ, μ) from BDMS(Q_s)

Ratio $dN(\text{Vac}+\text{GLV})/dN(\text{Vac})$ (red) vs $dN(\text{Vac}+\text{BDMS})/dN(\text{Vac})$ (blue) vs q

for $Q=20$, $\alpha=0.09$, GLV $\mu=0.5$ $\chi=6, 10 \iff$ BDMS $Q_s^2=9.57449$ (solid), 15.9575 (dash)



Ratio $dN(\text{Vac}+\text{GLV})/dN(\text{Vac})$ (red) vs $dN(\text{Vac}+\text{BDMS})/dN(\text{Vac})$ (blue) vs q
 for $Q=20$, $\alpha=0.09$, **GLV** $\mu=0.5$ $\chi=6, 10 \Leftrightarrow$ **BDMS** $Q_s^2=10$. (solid), 16. (dash)



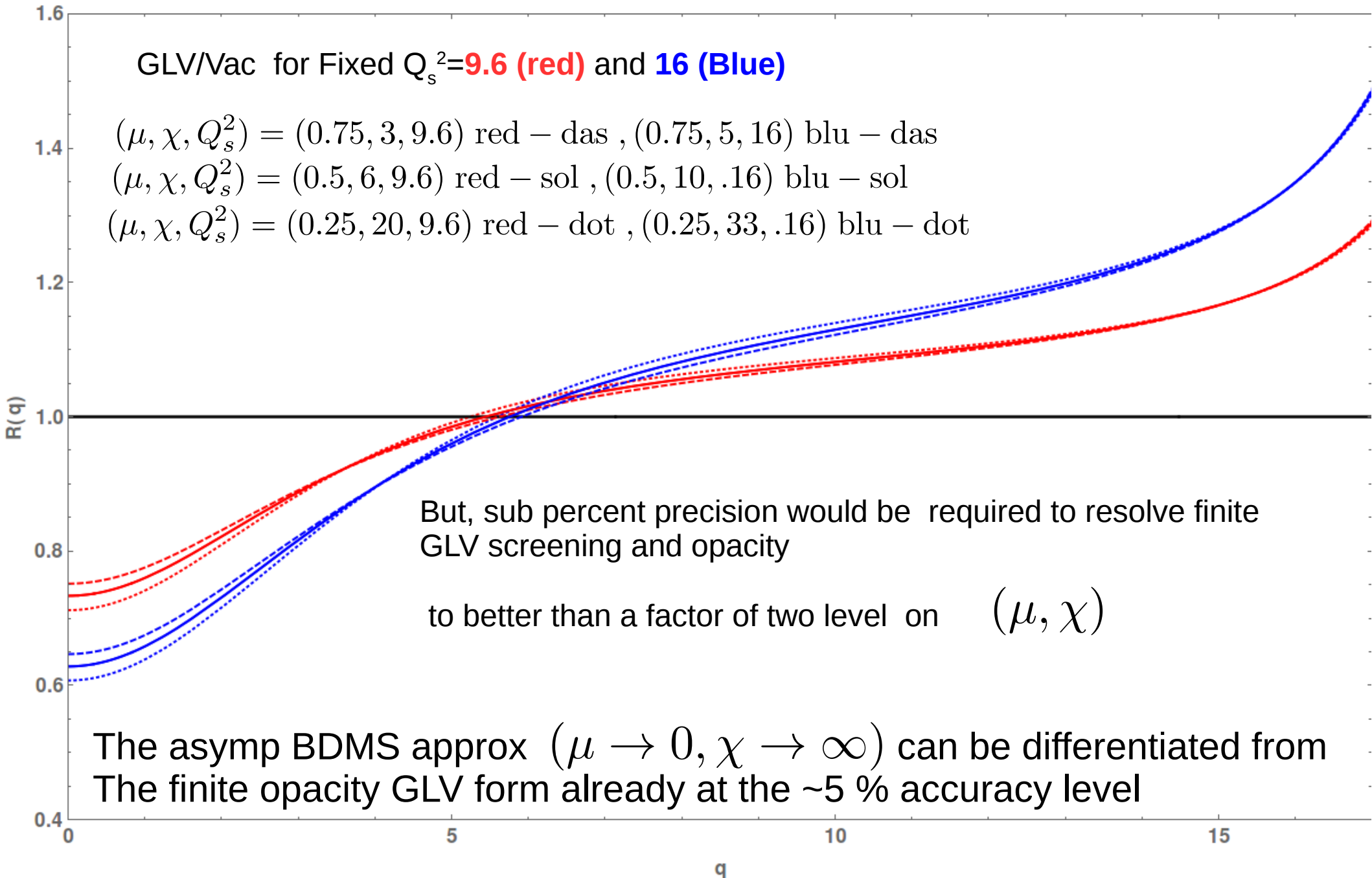
Ratio $dN(\text{Vac}+\text{GLV})/dN(\text{Vac})$ $Q_s^2=9.6$ (red), 16 (blue) vs q
 for $Q=20$, $\alpha=0.09$, $(\mu, \chi)=(0.5, 6\&10)$ sol, $(0.75, 3.1\&5.1)$ dash, $(0.25, 20\&33)$ dot

GLV/Vac for Fixed $Q_s^2=9.6$ (red) and 16 (Blue)

$(\mu, \chi, Q_s^2) = (0.75, 3, 9.6)$ red – das , $(0.75, 5, 16)$ blu – das

$(\mu, \chi, Q_s^2) = (0.5, 6, 9.6)$ red – sol , $(0.5, 10, .16)$ blu – sol

$(\mu, \chi, Q_s^2) = (0.25, 20, 9.6)$ red – dot , $(0.25, 33, .16)$ blu – dot



Final remarks:

Is the extra experimental and theoretical effort needed to try to extract dynamical information such as $\Gamma_{ab}(q_\perp, T) = \rho_b(T) d^2 \sigma_{ab}(T) / d^2 q_\perp$ from the *very tiny* medium modifications of azimuthal acoplanarity observables worth it?

Yes, because we need more ways to falsify competing microscopic dynamical mechanisms such as critical opalescence in sQGMP or non-conformal holography to gain more insight into the novel chromo dynamics responsible for the observed perfect fluidity of the bulk in A+A and the intricate hard jet and dijet quenching patterns correlated So strongly with the soft perfect fluid flow observable

Appendix: extra slides and links to longer lectures

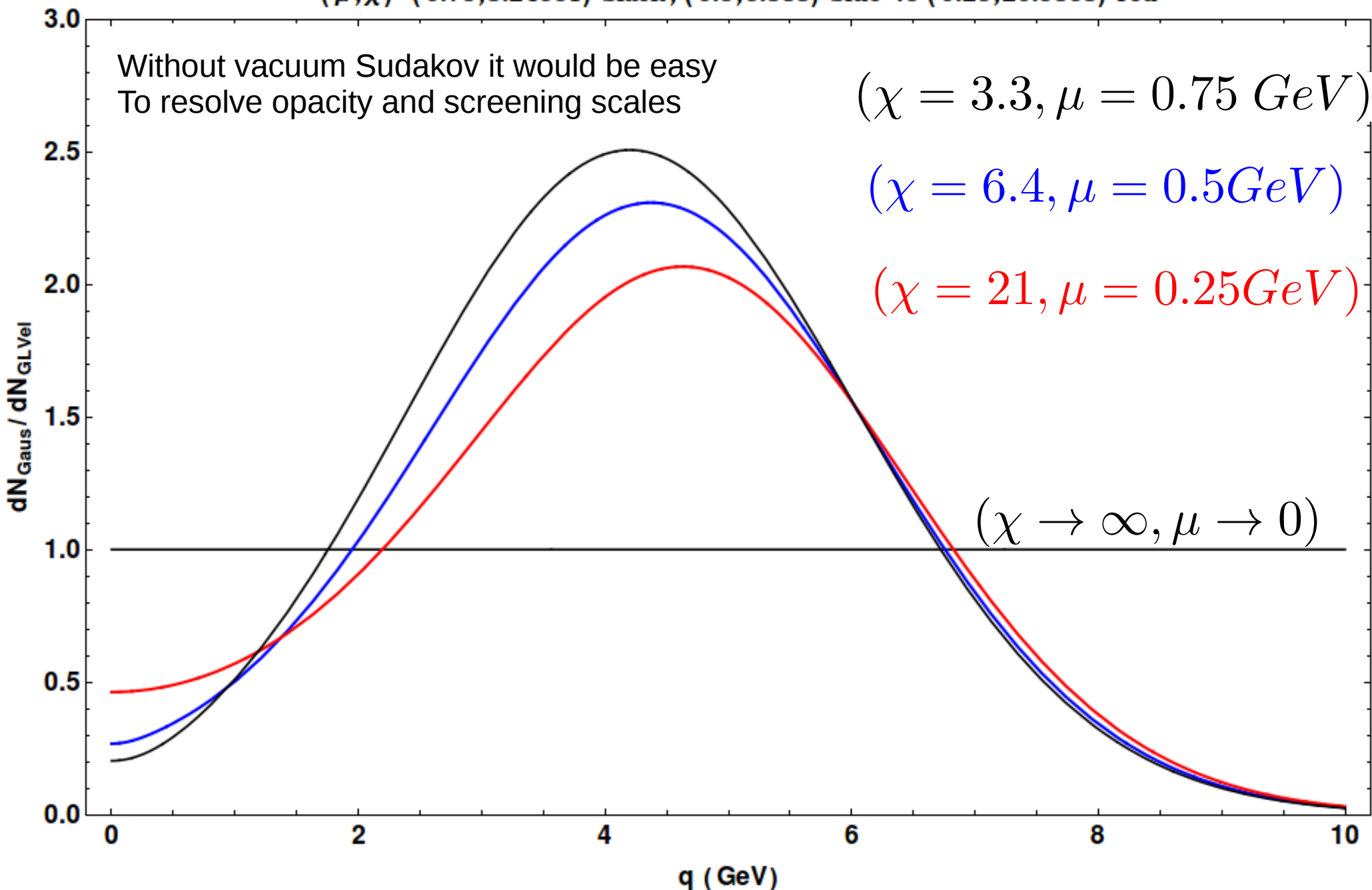
<http://www.columbia.edu/~mg150/Talks/2017/MGyulassy-Lec1-CCNU-101817.pdf>

<http://www.columbia.edu/~mg150/Talks/2017/MGyulassy-Lec2-CCNU-101817.pdf>

Conclusion 1:

$Q=20$ Ratio $dN_{\text{Gaus}}(Q_s)/dN_{\text{GLVel}}(\mu, \chi)$ vs q for $Q_s^2 = 10.1857$

$(\mu, \chi) = (0.75, 3.24661)$ black, $(0.5, 6.383)$ blue vs $(0.25, 20.9863)$ red



Hybrid: Pythia+ N=4 SYM holography model with added Gaussian transverse momentum
Distributed with BDMS Gaussian approximation controlled by a parameter K

$$Q_s^2 \equiv \langle \hat{q} L \rangle \equiv K \langle T^3 L \rangle$$

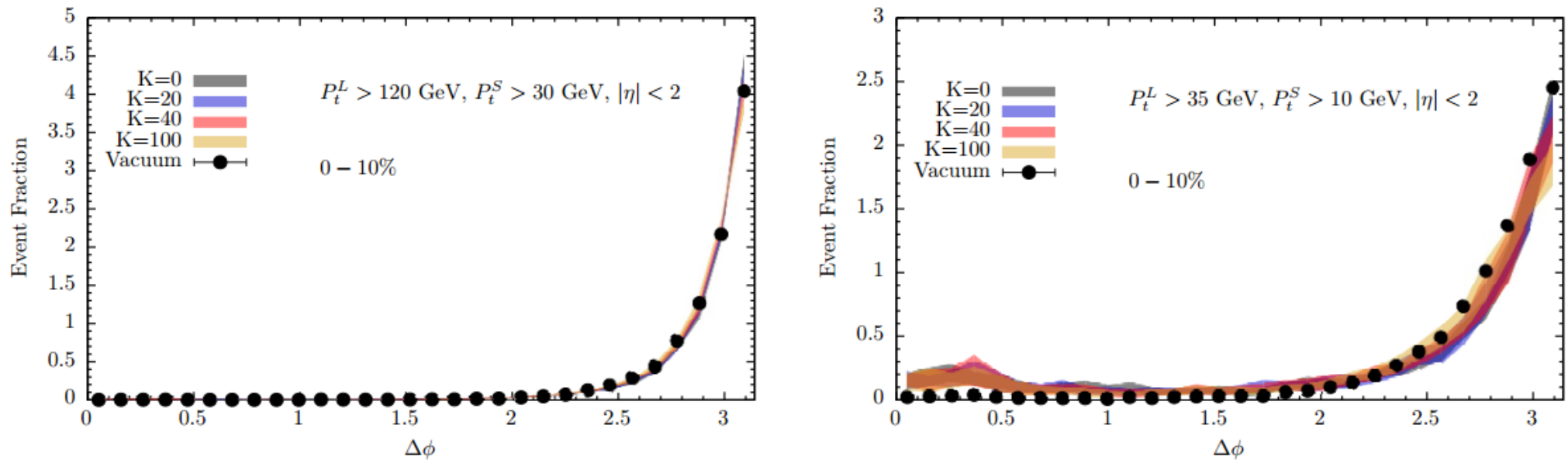


Figure 3. Dijet acoplanarity distribution for high-energy (left) and low-energy (right) dijets in LHC heavy ion collisions with $\sqrt{s} = 2.76$ ATeV for two different values of the broadening parameter K . For comparison, the black dots show the acoplanarity in proton-proton collisions as simulated by PYTHIA.

the effects of medium broadening on the acoplanarity distribution are small

For $E \sim 30$ GeV strong coupling broadening could be tested in the future to falsify

holographic or perturbative or other hybrid model combinations