

# IEOR E4706: FE: Discrete-Time Asset Pricing (Fall 2004)

Columbia University

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Assignment 2: Due Tuesday October 5th

## Question 1 (Luenberger 10.1)

The current price of gold is \$412 per ounce. The storage cost is \$2 per ounce per year, payable quarterly in advance. Assuming a constant interest rate of 9% compounded quarterly, what is the theoretical forward price of gold for delivery in 9 months.

## Question 2 (Luenberger 10.7)

A certain 10-year bond is currently selling for \$920. A friend of yours owns a forward contract on this bond that has delivery date in 1 year and a delivery price of \$940. The bond pays coupons of \$80 every 6 months, with one due 6 months from now and another just before maturity of the forward. The current interest rates for 6 months and 1 year (compounded semiannually) are 7% and 8%, respectively (annual rates compounded every 6 months). What is the current value of the forward contract?

## Question 3 (Luenberger 10.9)

Mr A. Gaylord manages a pension fund and believes that his stock selection ability is excellent. However, he is worried because the market could go down. He considers entering an equity swap where each quarter  $i$ , up to quarter  $M$ , he pays counterparty  $B$  the previous quarter's total rate of return  $r_i$  on the S&P 500 index times some notional principal and receives payments at a fixed rate  $r$  on the same principal. The total rate of return includes dividends. Specifically,  $1 + r_i = (S_i + d_i)/S_{i-1}$ , where  $S_i$  and  $d_i$  are the values of the index at  $i$  and the dividends received from  $i - 1$  to  $i$ , respectively. Derive the value of such a swap by the following steps:

(a) Let  $V_{i-1}(S_i + d_i)$  denote the value at time  $i - 1$  of receiving  $S_i + d_i$  at time  $i$ . Argue that  $V_{i-1}(S_i + d_i) = S_{i-1}$  and find  $V_{i-1}(r_i)$ .

(b) Find  $V_0(r_i)$ .

(c) Find  $\sum_{i=1}^M V_0(r_i)$ .

(d) Find the value of the swap.

## Question 4 (Luenberger 10.13)

Farmer D. Jones has a crop of grapefruit juice that will be ready for harvest and sale as 150,000 pounds of grapefruit juice in 3 months. Jones is worried about possible price changes, so he is considering hedging. There is no futures contract for grapefruit juice, but there is a futures contract for orange juice. His son, Gavin, recently studied minimum-variance hedging and suggests it as a possible approach. Currently the spot prices are \$1.20 per pound for

orange juice and \$1.50 per pound for grapefruit juice. The standard deviation of the prices of orange juice and grapefruit juice is about 20% per year, and the correlation coefficient between them is about .7. What is the minimum variance hedge for farmer Jones, and how effective is this hedge as compared to no hedge?