IEOR E4706: Financial Eng: Discrete-Time Asset Pricing

Columbia University
Instructor: Martin Haugh
Midterm Examination October 26th 2004
Total Marks: 100
Time: 2 hours

Notes that are on either side of a single letter size sheet of paper may be used during the exam.

Question 1 (30 marks)
(a) Consider two 3-year bonds: one has an 8% coupon and sells for 102 while the other has a 6% coupon and sells for 97. Find the price of a 3-year zero-coupon bond. (5 marks)

(b) A 5-year 10% coupon bond with semi-annual coupon payments and face value \$100 is currently trading at par, i.e., \$100. The next coupon payment is due in exactly six months. What is the yield to maturity of this bond? (5 marks)

(c) Suppose the CAPM holds and the expected return on a particular risky asset is significantly less than the risk-free rate of interest. True or False: such an asset should *rarely* be held in an agent's optimal portfolio? Give a reason for your answer. (5 marks)

(d) The current price of a forward contract for delivery of copper in 6 months is 84 cents per pound and the current price of copper in the spot market is 82 cents per pound. Assuming that there are no storage costs and that 6-month T-bills are currently selling for \$950, do you think there is an arbitrage opportunity? If so, explain briefly how you would take advantage of it. (5 marks)

(e) You have an obligation to pay \$5m in 7 years time. You wish to immunize this obligation against changes in the yield-to-maturity by constructing a portfolio that has zero duration. The portfolio is to consist of the obligation and positions in two bonds, A and B. Bond A is a zero-coupon bond that matures in 5 years time. Bond B is a coupon bond with coupon rate 10% per annum (a coupon has just been paid) and it matures in 8 years time. As an individual investor, you are not allowed to take short positions in any bonds. Under what circumstances will it be possible for you to immunize the obligation? (You are not required to perform any calculations.) (5 marks)

(f) Suppose we are in a one period world where $t = 0$ at the beginning of the period, and $t = 1$ at the end. There are three possible states at $t = 1$ and there are two securities available for trading at $t = 0$. The payoffs of these securities at $t = 1$ are $(1, 2, 1)$ and $(3, -2, 4)$, respectively, so for example, the payoff of the second security at $t = 1$ is -2 if the second state occurs. In this world, what, if any, elementary securities are available? (5 marks)

Question 2 (20 marks)
Consider a Treasury bond with face value \$10,000, an annual coupon rate of 7% (paid semi-annually), and several years to maturity. Currently the bond sells for \$9,100. The current interest rates for 6 months and 1 year, (compounded semiannually) are 5% and 6% respectively. You may assume that a coupon has just been paid and that short selling is allowed.

(a) What is the forward price for delivery of this bond in 1 year? (10 marks)

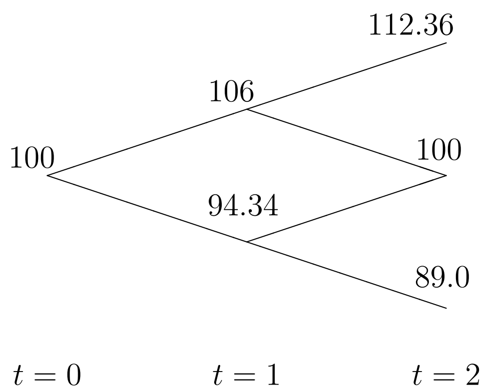
(b) Suppose the market quotes a forward price of \$9,100. Is there an arbitrage opportunity and if so, explain how you would take advantage of it? (10 marks)

Question 3 (25 marks)
An airline must purchase 100,000 barrels of aviation fuel in 3 months time but it is concerned about its exposure to adverse price movements. As a result it decides to hedge this exposure by trading in an oil futures contract that expires in exactly three months. An oil futures contract is for 1,000 barrels and the current spot price of oil is \$28 per barrel. The current spot price of aviation fuel is \$30 per barrel and the correlation coefficient between aviation fuel and oil price fluctuations is approximately .8. The standard deviation of oil prices is about 6% per quarter, (i.e. the standard deviation of the 3-month spot price of oil is $.06 \times 28$ dollars), while that of aviation fuel is about 5%.

(a) Find the minimum-variance hedging position. (15 marks)

(b) Do you think an *equal-and-opposite* hedge would require a position in a larger number of contracts or smaller? Explain your answer. (10 marks)

Question 4 (25 marks)
Consider the binomial lattice below that describes the evolution of a non-dividend paying stock in a 2 period world. The initial price of the stock is \$100 and in each period the stock price either increases by a factor of $u = 1.06$ or decreases by a factor $d = 1/u$. You may assume that there is a risk-free asset which pays a total return of $R = 1.005$ per period.



(a) Explain precisely why there are no arbitrage opportunities in this financial market. (9 marks)

(b) Is this financial market complete? Explain your answer. (8 marks)

(c) Consider a contingent claim that at date $t = 2$ pays \$10 if the date $t = 2$ stock price is greater than \$95, and \$0 otherwise. Find the replicating portfolio for this contingent claim. (8 marks)