Question 1
(a) Let $F(x) := 1 - (\kappa/(\kappa + x))^\alpha$ for $\alpha > 0$, $\kappa > 0$ and $x \geq 0$ denote the CDF of the Pareto distribution. By considering the normalizing sequences $c_n = \kappa n^{1/\alpha}$ and $d_n = \kappa n^{1/\alpha} - \kappa$, show that $F \in MDA(H_\xi)$. What is the value of $\xi$?

(b) Confirm your result of part (a) by applying a theorem from the lecture notes regarding the Fréchet MDA. In particular, you should specify the function, $L(\cdot)$, that is slowly varying at infinity.

Question 2
Let $Z \sim H_{\xi,\mu,\sigma}$. Show that $W := \left(1 + \xi \frac{Z - \mu}{\sigma}\right)^{-1/\xi}$, $\xi \neq 0$ has an exponential distribution with mean 1. Explain how this might be used to check the GEV model’s goodness of fit given data $Z_1, \ldots, Z_n$.

Question 3
Use the threshold exceedance method to estimate $ES_{99}$ using the Danish fire data. Compute a 95% confidence interval for your estimate by assuming the maximum likelihood estimates of $\xi$ and $\beta$ have a bivariate normal distribution and then using Monte-Carlo simulation. Compare your confidence interval with the empirical estimate of $ES_{99}$. (This is a very approximate way to construct confidence intervals. There are better ways based on re-parametrization or possibly bootstrapping methods.)

Hint: You can load the data in R by installing the `evir` package and then typing `data(danish)` at the R prompt. Further useful commands include `out <- gpd(danish,x)` where $x$ is the threshold level that you need to specify, and `riskmeasures(out, c(0.99))`. 