

POLITICAL SCIENCE W4210 (FALL 2005)

RESEARCH TOPICS IN GAME THEORY

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Time: Mondays, 4:10pm-6:00pm
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Research topics in game theory will cover the study of dynamic games, games of incomplete information, and evolutionary games, with applications in the fields of voting, bargaining, lobbying and violent conflict. Results from the study of social choice theory and mechanism design will also be treated. The course will concentrate on mathematical techniques for constructing and solving games. Students will be required to develop a topic relating political science and game theory and to write a formal research paper. Prerequisite: W4209 or instructor's permission.

Course Overview

Political Science W4210 is a continuation of W4209. It aims to teach the skills you need to start producing your own formal models for research in political science.

- In the first section of the course we introduce the tools you need to construct a formal model and prove results. We consider the choices that modelers need to make and the set of options that they have and we review approaches to constructing models and proving results.
- In the final section the course will take on more of a seminar format, engaging in close reading of models that use the techniques we have seen to study political problems.

Requirements

The readings are typically light in terms of page numbers but are compact and heavy in notation. As one of the aims of the course is to develop skills not just in reading but in developing models, you will be expected to work through the proofs of all propositions and theorems covered in the course. Notes on close readings of these texts follow below. In addition:

1. You will be required to write an original paper presenting a model or theorem. This paper is your key output from this course, ideally it should contribute directly to the writing of your dissertation. The paper should motivate a problem, develop a model and prove ensuing propositions, and identify testable predictions resulting from the model. This research paper will account for 55% of the final grade. You may be asked to present parts of your model in class for discussion by the group. The paper is due on 14 December.
2. There will be problem sets and exercises to complete throughout the first part of the course; these are intended to evaluate your understanding of the material and to allow for deeper exploration of models studied, and, especially, to practice model construction and proof writing. These account for 15% of the course grade and typically have to be handed in the week after they are assigned. *Late problem sets will not be accepted.*
3. In one of the weeks of the second section of the course you will be required to review one of the central models / theorems studied in the course. For this presentation you will be

- expected to (i) give an overview of the question under study (ii) give a brief presentation of the proof, (iii) evaluate the model's assumptions—are all assumptions necessary? are all assumptions reasonable? (iv) discuss the solution concept employed (v) discuss the generality of field of application of the results (vi) suggest ways in which the results could be pushed further. In some weeks these presentations may be constructed in somewhat of a debate format where rival papers are discussed by different students. This presentation will account for 10% of your grade.
4. You will be required to participate in weekly sections where problem sets will be reviewed, and class and research material will be discussed. The final 20% of the grade will be based on participation in these sections.
 5. All submitted writing, for your research paper or your problem sets, should be typed up on a word processor capable of handling the mathematics and symbols. It is strongly recommended, if you do not already know how, that you learn to use either Scientific Word/ Workplace (<http://www.mackichan.com/index.html?products/sw.html~mainFrame>) or LaTeX (<http://www.latex-project.org> <http://www.maths.tcd.ie/~dwilkins/LaTeXPrimer>) during the course of the term and use these tools to write your papers.

Topics

Part I: *Tools for Constructing and Solving Games*

Week 1	[12 SEPTEMBER] INTRODUCTION: STUDIES OF COLLECTIVE ACTION: FROM SOCIAL CHOICE TO NON-COOPERATIVE GAME THEORY
<p>THEOREMS AND CONCEPTS: Social Choice, Cooperative and Non-cooperative approaches to modeling public goods problems: Arrow's Impossibility Theorem, the Impossibility of a Paretian Liberal, the Coase Theorem, Olson, Hardin and the tragedy of the commons.</p>	
<p>Readings</p> <ul style="list-style-type: none"> • Class Notes: 1 & 2 • Coase, Ronald. 1960. "The Problem of Social Cost." <u>Journal of Law and Economics</u> (http://www.sfu.ca/~allen/CoaseJLE1960.pdf) • Geanakoplos, John, 2001. Three Brief Proofs of Arrow's Impossibility Theorem http://cowles.econ.yale.edu/P/cd/d11a/d1123RRR.pdf • Hardin, Garrett. 1968. "The Tragedy of the Commons." <i>Science</i> 162 (13 December) http://www.sciencemag.org/cgi/reprint/162/3859/1243.pdf 1243-1248 	
<p>Further reading:</p> <ul style="list-style-type: none"> • Aivazian, V.A. and Jeffrey L. Callen. 1981 "The Coase Theorem and the Empty Core." <u>Journal of Law and Economics</u> v.24: 175 -181. See also Coase's response, same journal. <i>Not on-line</i>. • Sen, Amartya. 1970. "The Impossibility of a Paretian Liberal." <u>The Journal of Political Economy</u>, 78:1, pp. 152-57. http://www.jstor.org/view/00223808/di950932/95p0121q/0 	

Week 2 [19 SEPTEMBER] HOW TO PROVE IT: STRATEGIES OF PROOF

THEOREMS AND CONCEPTS: Direct Method of Proof, Proof by Contradiction, by Induction, Truth Tables, WLOG, Existence, Uniqueness, Monotonicity, Weierstrass Theorem, Intermediate Value Theorem, Fixed Point Theorems: (Brouwer, Kakutani, Borzák-Ulam, Gale-Nikaido, Hairy Ball). Proof of Nash's theorem.

Required Readings

- Class Notes: 3 & 4
- Velleman, Daniel J. 1994. How to Prove It: A Structured Approach. Cambridge: UP. Ch 1-3.
- How to Write Mathematics (from the webpages of the Mathematics Society of Trent) http://xaravve.trentu.ca/mascot/handbook/SEC_write.pdf

Recommended Readings:

- Osborne and Rubinstein, Section 2.4.
- The rest of Velleman
- For an older political science article that uses a fixed point theorem read Herbert A. Simon's short 1954 piece: "[Bandwagon and Underdog Effects and the Possibility of Election Predictions](http://links.jstor.org/sici?sici=0033-362X%28195423%2918%3A3%3C245%3ABAUEAT%3E2.0.CO%3B2-Y)" Public Opinion Quarterly, 18:3. pp. 245-253. <http://links.jstor.org/sici?sici=0033-362X%28195423%2918%3A3%3C245%3ABAUEAT%3E2.0.CO%3B2-Y>
- For more on the mathematical results see the appendices in Mas-Colell, Whinston, and Green or in Rasmusen's Games and Information, or in topology texts such as Berge's, Topological Spaces.

Week 3 [26 SEPTEMBER] WHAT TO PROVE: DEFINING GAMES, REPRESENTING PREFERENCES

THEOREMS AND CONCEPTS: Defining Games Formally. Generality of Propositions and Strength of Assumptions, Choices (Discrete v. Continuous Action Spaces, Discrete v. Continuous Time, Atomic v. Continuous Distributions, Finite v. Infinite Horizons), Genericity and Robustness. Choices for representing preferences: Representing attitudes to risk, single and multidimensional problems, intertemporal utility. The Expected utility theorem.

Required Reading

- Class Notes 5, 6 & 7 (include readings on proofs)
- Varian, Hal [How to Build an Economic Model in Your Spare Time](http://www.sims.berkeley.edu/%7Ehal/Papers/how.pdf) <http://www.sims.berkeley.edu/%7Ehal/Papers/how.pdf>
- Starmer, Chris. 2000. "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk" Journal of Economic Literature Vol. XXXVIII 332-382 http://www.nottingham.ac.uk/~lezcs/pdf_files/STARMER_JEL.PDF
- /Davies, Philip et al. "The Creation of New Mathematics: An Application of the Lakatos Heuristic," Ch 44 in Rasmusen, Readings (Call Number: QA269 .R42 2001).

Further Reading

- Harry Roberts and Roman Weil, 1970. "Starting Research Early," University of Chicago, Graduate School of Business, http://pacioli.bus.indiana.edu/erasmuse/GI_reader/05c.roberts.htm
- Recommended to help your writing: MIT's guidelines: [Writing a Math Phase Two Paper](http://www.mit.edu/afs/athena.mit.edu/course/other/mathp2/www/piil.html), <http://www.mit.edu/afs/athena.mit.edu/course/other/mathp2/www/piil.html> .
- Of historical interest: John Von Neumann and Oscar Morgenstern, 1990. Theory of Games and Economic Behavior, Princeton University Press, Chapter 3: "The Notion of Utility."

Week 4 [3 OCTOBER] REPRESENTING INFORMATION

THEOREMS AND CONCEPTS: Using Bayes' Rule, Common Knowledge, Can you agree to disagree? No trade theorems, Common Knowledge and Asymmetric Information, Rationality and Backwards Induction

Required Readings:

- Class Notes: 8
- John Geanakoplos Common Knowledge *The Journal of Economic Perspectives* Vol. 6, No. 4 (Autumn, 1992), pp. 53-82 Stable URL: <http://links.jstor.org/sici?sici=0895-3309%28199223%296%3A4%3C53%3ACK%3E2.0.CO%3B2-4>
- Roger Myerson, "Harsanyi's Games with Incomplete Information", *Management Science*, <http://home.uchicago.edu/~rmyerson/research/harsinfo.pdf>

Recommended Readings:

- John C. Harsanyi "Games with incomplete information played by Bayesian players," *Management Science* 14 (1967-1968), 159-182, 320-334, 486-502.
- Adam Brandenburger ["The Power of Paradox: Some Recent Developments in Interactive Epistemology."](http://pages.stern.nyu.edu/%7Efabranden/teachingmaterials/paradox120402.pdf) <http://pages.stern.nyu.edu/%7Efabranden/teachingmaterials/paradox120402.pdf>
- Aumann, Robert and Adam Brandenburger 1995. "Epistemic Conditions for Nash Equilibrium", *Econometrica*, 63: 1161-1180.

Week 5 [10 OCTOBER] SOLUTION CONCEPTS FOR NORMAL FORM GAMES AND EVOLUTIONARY GAMES

THEOREMS AND CONCEPTS: Elimination of Dominated Strategies, Rationalizability, Nash Equilibrium, Resistance, Risk Dominance, Viscosity, Strong Equilibrium, Coalition-Proof Equilibrium, Correlated Equilibrium. Evolutionarily Stable Strategies, Weak Evolutionarily Stable Strategies, Neutrally Stable Strategies, Limit Evolutionarily Stable Strategies, Evolutionarily Stable Sets, Proper Equilibrium, Asymptotically Stable Equilibria, Stochastically Stable Equilibrium.

Required Readings

- Class Notes: 9, 10
- Osborne, Martin J, and Ariel Rubinstein. 1994. *A Course in Game Theory*, MIT University Press, Chapters 2, 3, 4, and 13.

Recommended Readings:

- Samuelson, Larry. 1998. *Evolutionary Games and Equilibrium Selection* Cambridge: MIT Press. Chapters 2, 3 and 4.
- H. Peyton Young. 1998. *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*. Princeton: Princeton University Press.
- Brian Skyrms, 1996. *Evolution of the Social Contract*. Cambridge: Cambridge University Press.

Week 6 [17 OCTOBER] DISCUSSION OF MODEL IDEAS FOR FINAL PAPER [1 PAGE MEMO TO BE CIRCULATED IN ADVANCE]

Week 7 [24 OCTOBER] SOLUTION CONCEPTS AND TOOLS FOR EXTENSIVE FORM GAMES I

CONCEPTS AND THEOREMS: SPNE, Fuzzy SPNE, Refinements, Zermelo's Theorem. The One Stage Deviation Principle, The Bellman Equation, Existence and Uniqueness of Equilibrium in Stahl-Rubinstein Game, Folk-theoretic results.

Required Readings:

- Class Notes 11 & 12
- Muthoo, Abhinay. 1999. [Bargaining Theory with Applications](#), Cambridge: Cambridge University Press. 3.1-.2
- Osborne and Rubinstein, Sections 8.1-5.
- Ferejohn, John. 1986. "Incumbent Performance and Electoral Control" *Public Choice* 50: 5-26 or Chapter 1 in Perrson and Tabellini. 1995. *Monetary and Fiscal Policy 2: Politics*, Cambridge, MIT University Press.

Recommended Readings:

- For a more general presentation of the Rubinstein model, see Osborne and Rubinstein, Sections 7.2-3 or the rest of Muthoo Chapter 3
- For a richer but tougher model than Ferejohn's try: Jeffrey S. Banks and Rangarajan Sundaram. "Adverse Selection and Moral Hazard in a Repeated Elections Model," in *Political Economy: Institutions, Information, Competition, and Representation*, (W. Barnett, et al, Eds.), Cambridge University Press, Cambridge and New York, 1993.
- A recent application of folk theoretic ideas to the study of ethnic politics is found in Fearon, James D. and David D. Laitin. 1996. "Explaining Interethnic Cooperation," *APSR*, 90:715-735. <http://links.jstor.org/sici?sici=0003-0554%28199612%2990%3A4%3C715%3AEIC%3E2.0.CO%3B2-8>

Week 8 [31 OCTOBER] SOLUTION CONCEPTS AND TOOLS FOR EXTENSIVE FORM GAMES II (WITH MORE UNCERTAINTY)

CONCEPTS AND THEOREMS: Identifying Weak Perfect Bayesian Equilibria, Using equilibrium refinements, Sequential equilibrium, Trembling Hand Perfection, Forward Induction, The Intuitive Criterion, Signaling and Screening, The Single-Crossing Property.

Required Readings:

- Class Notes 13
- Nalebuff, Barry (1991), "Rational Deterrence in an Imperfect World" *World Politics*, 43:3, pp. 313-335.
- Mas-Colell, Andreu, Michael Whinston, and Jerry Green, 1995. *Microeconomic Theory*. Oxford, Oxford University Press. See BUSINESS: [HB172 .M6247 1995](#). Chapter 13

Further Reading

- Osborne and Rubinstein, Chapter 12.
- To see some of the refinements "in action" see Banks, Jeffrey, Colin Camerer, and David Porter, David., 1996. "[An Experimental Analysis of Nash Refinements in Signaling Games](#)," *Games and Economic Behavior*. 6: 1-31.

! DUE! : * HAND IN WRITTEN MODEL OUTLINE *****

**Week 9 [7 NOVEMBER] UNIVERSITY HOLIDAY
[OPTIONAL: SOLUTION CONCEPTS FOR COOPERATIVE GAMES]**

CONCEPTS AND THEOREMS: THE NEGOTIATION SET, THE NASH BARGAINING SOLUTION, THE CORE, THE NUCLEOLIS, THE SHAPLEY VALUE, PLOTT'S THEOREM, THE UNCOVERED SET. MIXING COOPERATIVE AND NON-COOPERATIVE GAME THEORY.

Required Readings:

- Class Notes 14
- Mas-Colell, Andreu, Michael Whinston, and Jerry Green, 1995. *Microeconomic Theory*. Oxford, Oxford University Press. See BUSINESS: [HB172 .M6247 1995](#). Chapter 13
- Diermeier, Daniel and Keith Krehbiel, 2003. "Institutionalism as a Methodology" *Journal of Theoretical Politics* 15: 2. <http://ejournals.ebsco.com/Article.asp?ContributionID=4476343>
- Austen-Smith, David and Jeffrey S. Banks. 1998. "Social Choice Theory, Game Theory and Positive Political Theory" in N.W. Polsby [Eds.] *Annual Review of Political Science*, Palo Alto: Annual Reviews. <http://arjournals.annualreviews.org/doi/pdf/10.1146/annurev.polisci.1.1.259>

Further Reading

- Aumann, Robert. 1998. "On the State of the Art in Game Theory," *Games and Economic Behavior*. 24: 181-210. <http://sv5.vwl.tuwien.ac.at/literatur/GEB/Vol24/0612a.pdf>

Week 10 [14 NOVEMBER] TURNING GAME THEORY ON ITS HEAD: MECHANISM DESIGN, AUCTION THEORY

THEOREMS AND CONCEPTS: The Revelation Principle, The Gibbard-Satterthwaite Theorem, the Revenue Equivalence Theorem. The Myerson-Satterthwaite Theorem.

Required Reading

- Class Notes 15
- Osborne and Rubinstein, Chapter 10.
- Klemperer, Paul. 2000. *Auction Theory: A Guide to the Literature* in *The Economic Theory of Auctions*, Klemperer (ed.); on-line: <http://www.nuff.ox.ac.uk/users/klemperer/Survey.pdf>

Further Reading

- Reny, 2000. "Arrow's Theorem and the Gibbard Satterthwaite Theorem: A Unified Approach," Working Paper. <http://www.src.uchicago.edu/users/preny/arrow-gibbard-satterthwaite.pdf>
- Myerson, Roger. B., and Satterthwaite, M. A. 1983. "Efficient mechanisms for bilateral trading." *Journal of Economic Theory* 28:265--281. (*Not on-line*)

Part II: *Game Theory Applied to Politics*

Week 11 [21 NOVEMBER] NEW WORK IN CONFLICT AND IR

Required Readings:

- Ethan Buna de Mesquita. 2005. "The Quality of Terror" *American Journal of Political Science*. 49 (3)
- Robert Powell. 2004. The Inefficient Use of Power: Costly Conflict with Complete Information. *American Political Science Review*. 98 (2)
- Schultz, K. The Politics of Risking Peace: Do Hawks or Doves Deliver the Olive Branch? *International Organization*

Week 12 [28 OCTOBER] APPLICATIONS II: MEET THE AUTHOR I

Required Reading:

- Michael Laver 2005 "Policy and the Dynamics of Political Competition." *American Political Science Review* 99 (2).

Week 13 [5 DECEMBER] APPLICATIONS III: THEORY AND DATA

Required Reading

- Sanford C. Gordon and Catherine Hafer 2005 "Flexing Muscle: Corporate Political Expenditures as Signals to the Bureaucracy" *American Political Science Review* 99 (2)
- Goeree, Jacob and Charles Holt. 2005. "An Explanation of Anomalous Behavior in Models of Political Participation" *American Political Science Review* 99 (2).
- Cameron, Charles, Jeffrey Segal and Donald Songer. 2000. "Strategic Auditing in a Political hierarchy: An Informational model of the Supreme Court's Certorari Decisions." *American Political Science Review*. 95 (1)

Week 14 [12 DECEMBER] APPLICATIONS IV: MEET THE AUTHOR II

Required Reading: TBA