Handling and Manhandling Civilians: Derivation of Hypotheses (August 2006)

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1 Environment

Consider the following strategic environment:

Players. Assume that in each period, civilian i, living in some region, inelastically generates per period income y, using one unit of labor. What the civilian produces can be consumed or transferred, but not saved or invested. Further, assume that production for each civilian takes place subject to some base consumption requirement in the previous period $\underline{c}< y$.

Control over the region is divided between m combatant groups. We consider two ways in which groups may encounter civilians.

- We first consider **random encounters**. In this case, let the probability that at each point in time any given group encounters a civilian be proportionate to its relative size in the region, p_j.

- The second protocol we consider involves **sequenced encounters**. In this case in each period the civilian encounters each of the groups in some predetermined order.[1]

The latter protocol may be appropriate if groups hold fixed positions on routes used by traders or producers. The former may be more appropriate if fighters roam more broadly. A key difference between the two is that in the latter case, with sequenced encounters, for one group to refrain from abuses, all other must also refrain—that is, refraining implies tacit collusion. With random matching a group may refrain even though

[1] Other protocols might include random matching in each period with possibly multiple encounters in each period or matching with all groups but in a random order.

others do not, in the hopes of re-engaging with the civilian before more abusive groups encounter the civilian.

Strategy Sets. The strategy sets available to combatants are simple. If any combatant, j, encounters a civilian holding income y', he may choose to forcefully extract any amount y_j in the range $[0, y']$. We call an action by a combatant, j, as *abusive* if $y_j > y' - \underline{c}$. Hence an action is abusive if it is of a scale sufficient to prevent survival.

Preferences. Per period utility to a combatant group j is given by the amount of resources extracted by the combatant plus a bonus, $k_{i,j}$ (possibly negative), that occurs in the event that j's behavior toward the civilian is abusive. We assume that $k_{i,j} \geq -\underline{c}$: that is, we focus on cases where abuse is tempting in the sense that the short-run benefits to a group of taking *all* of a civilian's assets, given by $(y + k_{i,j})$, outweigh the short-run benefits of extracting a non-abusive amount $(y - \underline{c})$. Hence: $v_j(y_j) = \{y_j \text{ if } y' - y_j \geq \underline{c},\ \ y_j + k_{i,j} \text{ otherwise}\}$. Both combatants and civilians have additive discounted utility with a common time-invariant discount factor, δ, and infinite horizons.

2 Random Encounters

If civilians are matched randomly with one group in each period, then the condition under which group j will *refrain* from abuse (and extract exactly $y - \underline{c}$) in an equilibrium in which the total share of groups desisting from abuse is given by z is:

$$(y - \underline{c}) + \delta p_j (y - \underline{c}) + \delta^2 z p_j (y - \underline{c}) + \delta^3 z^2 p_j (y - \underline{c}) + ... \geq (y + k_{i,j})$$

The first term on the left hand sides represents the maximum return in the present period from desisting; the second represents the discounted maximum expected return in the second period, which obtains conditional upon non-abusive behavior in the first period and the likelihood that the group encounters the civilian again in the second period; the third term is the twice discounted reward from non abusive behavior which obtains only if no abusive group encountered the civilian in the previous period (z) and group j encounters the group in the third period. This condition can be more compactly written as:

$$y \geq \underline{c} + \frac{(\underline{c} + k_{i,j})(1 - \delta z)}{\delta p_j} \tag{1}$$

Note that the condition implies that groups are more likely to desist if they value the future highly and if the requirements for subsistence are low. In addition, condition (1) can be used to provide foundations for each of our hypotheses.

- [Hypothesis 1] A given group is more likely to desist from abuse the larger is y
- [Hypotheses 2 and 3] A given group is more likely to desist from abuse the smaller (or more negative) is $k_{i,j}$
- [Hypothesis 4] The relationship between p_j and condition (*) is more complex. *Ceteris paribus*, groups are more likely to desist when other groups also desist (when z is large), but this in turn depends on similar conditions being satisfied for other desisting groups; compounding the effects of y and $k_{i,j}$. Although z is not independent of p_j (z depends on the relative size of other groups and the strategies they employ), two implications on relative size follow immediately from condition (1). First, we can derive a *necessary* condition for an equilibrium to obtain in which a given group j desists from abuse. Group j will desist, in an equilibrium in which all other groups also desist (that is, if $z = 1$) iff $y \geq \underline{c} + \frac{(\underline{c}+k_{i,j})(1-\delta)}{\delta p_j}$ or equivalently $p_j \geq \frac{(\underline{c}+k_{i,j})(1-\delta)}{\delta(y-\underline{c})}$. For $p_j < \frac{(\underline{c}+k_{i,j})(1-\delta)}{\delta(y-\underline{c})}$, no stationary equilibrium obtains in which j desists from abuse in each period. Second, we can derive a *sufficient* condition for non-abusive equilibrium behavior by j: in particular, group j will refrain from abuse even if all other groups act in an abusive manner (that is, if $z = p_j$) iff: $p_j \geq \frac{\underline{c}+k_{i,j}}{\delta(y+k_{i,j})}$. If this condition holds, then condition (1) is satisfied independent of the actions of the other players. This argument then provides our foundation for Hypothesis 4. With higher values of p_j, both the necessary and the sufficient condition for non-abusive behavior are more easily fulfilled.[2]
- [Hypothesis 5 − 8] Insofar as the within-group collective action problem can be represented as a collective action problem between

[2] Again we emphasize that within the range $[\frac{(\underline{c}+k_{i,j})(1-\delta)}{\delta(y-\underline{c})}, \frac{\underline{c}+k_{i,j}}{\delta(y+k_{i,j})}]$ whether or not a player acts abusively depends on the actions of other players. For some parameter values both abusive and non-abusive equilibria may obtain. In some circumstances within this range, a fall in p_j for one group, j, if coupled with a rise in p_h for another group, h, may result in a shift from an equilibrium in which j acts abusively to one in which it acts non-abusively. As an example, consider a case with only two combatant groups, j and h in which $k_{i,j} = k_{i,h} = 0$, $\delta = .8$, $\underline{c} = 1$ and $y = 2$. If $p_j = p_h = .5$, an equilibrium obtains in which neither group abstains from abuse (another exists in which both abstain). With a decline in p_j to .25 however (and a corresponding rise in p_h to .75), h always desists from abuse in equilibrium and j's best response is also to desist.

multiple individuals or sub-factions within a group, we see that Hypotheses 5-8 follow from the same logic as that underpinning Hypothesis 4. Consider any partitioning of group j, with size p_j, into subgroups of size q_1 and q_2, with $q_1 + q_2 = p_j$ and for whom $k_{i,1} = k_{i,2} = k_{i,j}$. A non-cohesive group of size p_j that is formed of subgroups of size q_1 and q_2 can always act non-abusively if $y \geq \underline{c} + \frac{(\underline{c}+k_{i,j})(1-\delta z)}{\delta \min(q_1,q_2)}$; a cohesive group, however, can act non-abusively if $y \geq \underline{c} + \frac{(\underline{c}+k_{i,j})(1-\delta z)}{\delta p_j}$. Since $\underline{c} + \frac{(\underline{c}+k_{i,j})(1-\delta z)}{\delta p_j} \leq \underline{c} + \frac{(\underline{c}+k_{i,j})(1-\delta z)}{\delta \min(q_1,q_2)}$ we have that the conditions for non-abusive behavior by fragmented groups are more severe than the conditions for cohesive groups. More generally, if Condition (1) is satisfied for all subgroups then it is also satisfied for the aggregate group j; the converse is not true however. Condition (1) may be satisfied for group j but not for each subgroup.

3 Ordered Encounters

Consider next a situation in which multiple groups visit a civilian in each period in a fixed order. In this case the condition for desisting from abuse depends on the order of the visitation, with earlier groups generally requiring higher payouts then later groups, to prevent them from taking more early on. In this case, desisting from abuse by one group can be sustained in an equilibrium only if *all* groups desist from abuse.

The condition for the first group to desist from abuse (condtional on all others also desisting) is that:

$$\frac{y_1^*}{1-\delta} \geq y + k_{i,1}$$
$$\leftrightarrow$$
$$\delta y - k_{i,1}(1-\delta) \geq y - y_1^*$$

For the second group, the condition is that:

$$\frac{y_2^*}{1-\delta} \geq y - y_1^* + k_{i,1}$$
$$\leftrightarrow$$
$$y - y_1^* \geq \frac{y - y_1^* - y_2^* + k_{i,2}(1-\delta)}{\delta}$$

Combining the conditions for the first two groups yields:

$$\delta y - k_{i,1}(1-\delta) \geq \frac{y - y_1^* - y_2^* + k_{i,2}(1-\delta)}{\delta}$$
$$\leftrightarrow$$
$$\delta^2 y - (1-\delta)(k_{i,1}\delta + k_{i,2}) \geq y - y_1^* - y_2^*$$

Continuing in this manner, the condition for groups 1 to m together imply:

$$\delta^m y - (1-\delta)\sum_{j=1}^{m} \delta^{m-j} k_{i,j} \geq y - \sum_{j=1}^{m} y_j^*$$

The condition for the *final* player in the sequence not to engage in abuse when extracting y_j^*, conditional upon all others not having engaged in abuse is that:

$$y - \sum_{j=1}^{m} y_j^* \geq \underline{c}$$

Hence a necessary condition for a non-abusive equilibrium to exist is that:

$$\delta^m y - (1-\delta)\sum_{j=1}^{m} \delta^{m-j} k_{i,j} \geq \underline{c}$$

Or:

$$y \geq \frac{1}{\delta^m}\left[\underline{c} + (1-\delta)\sum_{j=1}^{m} \delta^{m-j} k_{i,j}\right] \tag{2}$$

For sufficiently tight distributions of the k_j parameters (in particular, if for some $k_{min} \geq -\underline{c}$ we have $k_{i,j} \in [k_{min}, \frac{k_{min}+\delta^{m-1}y}{1-\delta^{m-1}}]$, for all j) then this is also a sufficient condition. For the derivation of hypotheses we focus on such cases. In this case, for all j, let:

$$y_j^* = (1-\delta)[k_{i,j} + \delta^{j-1} y - (1-\delta)\sum_{h=1}^{j-1} \delta^{j-h-1} k_{i,h}]$$

In this case y_j^* is positive, for each j, and the sum of the extracted amounts is:

$$\sum_{s=1}^{j} y_s^* = \sum_{s=1}^{j} (1-\delta)[k_{i,s} + \delta^{s-1} y - (1-\delta)\sum_{h=1}^{s-1} \delta^{s-h-1} k_{i,h}]$$
$$= [1-\delta^j] y + (1-\delta)\sum_{s=1}^{j} \delta^{j-s} k_{i,s}$$

To verify that each player is satisfied (or more precisely, indifferent) accepting y_j^* in perpetuity, note that the value of refraining from abuse is:

$$\frac{y_j^*}{(1-\delta)} = k_{i,j} + \delta^{j-1}y - (1-\delta)\sum_{h=1}^{j-1}\delta^{j-h-1}k_{i,h}$$

The value to deviating and taking the maximum possible is:

$$\begin{aligned} y - \sum_{s=1}^{j-1} y_s^* + k_{i,j} &= y - [1-\delta^{j-1}]y + (1-\delta)\sum_{s=1}^{j-1}\delta^{j-s-1}k_{i,s} + k_{i,j} \\ &= k_{i,j} + \delta^{j-1}y - (1-\delta)\sum_{s=1}^{j-1}\delta^{j-s-1}k_{i,s} \end{aligned}$$

Hence with Condition 2 satisifed this (and possibly other) on-abusive equilibrium can be sustained. Again we can derive each of our hypotheses from Condition 2. In particular:

- Condition 2 is most easily satisfied for large y (Hypothesis 1)
- Condition 2 is most easily satisfied for low $k_{i,j}$ (Hypothesis 2-3)
- Condition 2 is also, though less obviously, more easily satisfied for lower m (Hypothesis 4) To see this, let y^* denote the minimum income level needed to support cooperation with some collection of m players:

$$y^* = \frac{1}{\delta^m}\left[\underline{c} + (1-\delta)\sum_{j=1}^{m}\delta^{m-j}k_{i,j}\right]$$

Let y^{**} denote the corresponding conditions that would obtain for these same players (with the same indices) but in the absence of arbitrary player h:

$$\begin{aligned} y^{**} &= \frac{1}{\delta^{m-1}}\left[\underline{c} + (1-\delta)\sum_{j=1}^{h-1}\delta^{m-j-1}k_{i,j} + (1-\delta)\sum_{j=h+1}^{m}\delta^{m-j}k_{i,j}\right] \\ &= \frac{1}{\delta^m}\left[\delta\underline{c} + (1-\delta)\sum_{j=1}^{h-1}\delta^{m-j}k_{i,j} + (1-\delta)\sum_{j=h+1}^{m}\delta^{m-j+1}k_{i,j}\right] \end{aligned}$$

Defining $\Delta = y^* - y^{**}$ we then have:

$$\Delta = \frac{1-\delta}{\delta^m}[\underline{c} + \delta^{m-h}k_{i,h} + \sum_{j=h+1}^{m}(\delta^{m-j} - \delta^{m-j+1})k_{i,j}]$$

To demonstrate that Δ is non-negative, note that since $(\delta^{m-j} - \delta^{m-j+1}) > 0$, Δ is increasing in each $k_{i,j}$.The lowest value that Δ can take is when $k_{i,j} = -\underline{c}$ for all j and hence $\Delta = (1-\delta)[\underline{c} - \delta^{m-h}\underline{c} - \sum_{j=h+1}^{m}(\delta^{m-j} - \delta^{m-j+1})\underline{c}] = 0$. With Δ increasing in $k_{i,j}$, Δ is positive if $k_{i,j} > -\underline{c}$ for some j. Hence, the more groups that coexist the more difficult it is to sustain non-abusive equilibria. Unlike market situations, competition among the consumers for the producer's surplus is bad for the producer.

- Finally, as in the case with random ordering, Hypotheses 5-8 are based on a logic similar to that underpinning Hypothesis 4. Benefits to cohesion accrue when cooperative arrangements cannot be maintained through repeat play with competing extractors, but failure to achieve a cooperative arrangement is Pareto inefficient. With internally divided factions in which subfactions (or individuals) act unilaterally, n_j each group j, non-abusive behavior requires that a version of Condition 2 holds in which the number of factions, m, is replaced by the effective number of units acting independently, $m^* = \sum_{j=1}^{m} n_j$. Since $m^* > m$ whenever $n_j > 1$ for some group, we have immediately that, *ceteris paribus*, a rise in cohesion of units is associated with an expected decline in the level of abuse for any given group.