Set Parameters

$d := .9$ $x1 := 20$ $x2 := 15$ $y1 := 10$ $y2 := 15$ $k := 500$

Define Utility Functions

$u1(P1,P2,z1,z2,Y1,Y2) := k - (P1 - x1)^2 - (P2 - x2)^2$ $u2(P1,P2,z1,z2,X1,X2) := k - (P1 - y1)^2 - (P2 - y2)^2$ $u3(P1,P2,z1,z2) := k - (P1 - z1)^2 - (P2 - z2)^2$

Sample policy: $\begin{pmatrix} P1 \\ P2 \end{pmatrix} := \begin{pmatrix} 10 \\ 5 \end{pmatrix}$

Define best reply functions starting values: $z1 := 20$ $z2 := 20$

Given $u3(P1,P2,z1,z2) > d \cdot u3(Y1,Y2,z1,z2)$ $u2(P1,P2,z1,z2,Y1,Y2) > d \cdot u2(Y1,Y2,z1,z2,Y1,Y2)$ $f1(z1,z2,Y1,Y2) := \text{Maximize}(u1,P1,P2)$

Given $u3(P1,P2,z1,z2) > d \cdot u3(X1,X2,z1,z2)$ $u1(P1,P2,z1,z2,X1,X2) > d \cdot u1(X1,X2,z1,z2,X1,X2)$ $f2(z1,z2,X1,X2) := \text{Maximize}(u2,P1,P2)$

Example of repeated application of maximization to convergence:

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1,z2,Y1,Y2)$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1,z2,X1,X2)$ $\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 20 \\ 15 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.933 \\ 15.25 \end{pmatrix}$

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1,z2,Y1,Y2)$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1,z2,X1,X2)$ $\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.603 \\ 14.999 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.576 \\ 15.249 \end{pmatrix}$

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1,z2,Y1,Y2)$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1,z2,X1,X2)$ $\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.485 \\ 15 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.542 \\ 15.25 \end{pmatrix}$

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1,z2,Y1,Y2)$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1,z2,X1,X2)$ $\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.475 \\ 15 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.538 \\ 15.25 \end{pmatrix}$

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1,z2,Y1,Y2)$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1,z2,X1,X2)$ $\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.474 \\ 15 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.538 \\ 15.25 \end{pmatrix}$

$\begin{pmatrix} X1 \\ X2 \end{pmatrix} := f1(z1,z2,Y1,Y2)$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} := f2(z1,z2,X1,X2)$ $\begin{pmatrix} X1 \\ X2 \end{pmatrix} = \begin{pmatrix} 17.474 \\ 15 \end{pmatrix}$ $\begin{pmatrix} Y1 \\ Y2 \end{pmatrix} = \begin{pmatrix} 12.538 \\ 15.25 \end{pmatrix}$

Program to iterate this procedure (ex post check for convergence):

$$
\begin{aligned}
&\text{GUESS}(z1, z2, \text{startX1}, \text{startX2}, \text{startY1}, \text{startY2}, \text{tries}) := \\
&\quad X \leftarrow f1(z1, z2, \text{startY1}, \text{startY2}) \\
&\quad X1 \leftarrow X_0 \\
&\quad X2 \leftarrow X_1 \\
&\quad Y \leftarrow f2(z1, z2, \text{startX1}, \text{startX2}) \\
&\quad Y1 \leftarrow Y_0 \\
&\quad Y2 \leftarrow Y_1 \\
&\quad \text{for } j \in 1 .. \text{tries} \\
&\qquad X \leftarrow f1(z1, z2, Y1, Y2) \\
&\qquad X1 \leftarrow X_0 \\
&\qquad X2 \leftarrow X_1 \\
&\qquad Y \leftarrow f2(z1, z2, X1, X2) \\
&\qquad Y1 \leftarrow Y_0 \\
&\qquad Y2 \leftarrow Y_1 \\
&\quad OUT_0 \leftarrow X1 \\
&\quad OUT_1 \leftarrow X2 \\
&\quad OUT_2 \leftarrow Y1 \\
&\quad OUT_3 \leftarrow Y2 \\
&\quad OUT
\end{aligned}
$$

Example with 10 and 15 iterations

$$
\text{GUESS}(z1, z2, 0, 0, 0, 0, 10) = \begin{pmatrix} 17.473 \\ 15 \\ 12.538 \\ 15.25 \end{pmatrix} \qquad \text{GUESS}(z1, z2, 0, 0, 0, 0, 15) = \begin{pmatrix} 17.473 \\ 15 \\ 12.538 \\ 15.25 \end{pmatrix}
$$

Utilities at optimum

```
U1(Xgrid, Ygrid, tries) := | START ← GUESS(0,0,0,0,0,0,tries·2)
                           | X1t ← START_0
                           | X2t ← START_1
                           | Y1t ← START_2
                           | Y2t ← START_3
                           | for i ∈ 0 .. Xgrid
                           |   for j ∈ 0 .. Ygrid
                           |     | GO ← GUESS(i, j, X1t, X2t, Y1t, Y2t, tries)
                           |     | X1t ← GO_0
                           |     | X2t ← GO_1
                           |     | Y1t ← GO_2
                           |     | Y2t ← GO_3
                           |     | OUTPUT_(i,j) ← u1(X1t, X2t, 0, 0, 0, 0)
                           | OUTPUT
```

```
U2(Xgrid, Ygrid, tries) := | START ← GUESS(0,0,0,0,0,0,tries·2)
                           | X1t ← START_0
                           | X2t ← START_1
                           | Y1t ← START_2
                           | Y2t ← START_3
                           | for i ∈ 0 .. Xgrid
                           |   for j ∈ 0 .. Ygrid
                           |     | GO ← GUESS(i, j, X1t, X2t, Y1t, Y2t, tries)
                           |     | X1t ← GO_0
                           |     | X2t ← GO_1
                           |     | Y1t ← GO_2
                           |     | Y2t ← GO_3
                           |     | OUTPUT2_(i,j) ← u2(X1t, X2t, 0, 0, 0, 0)
                           | OUTPUT2
```

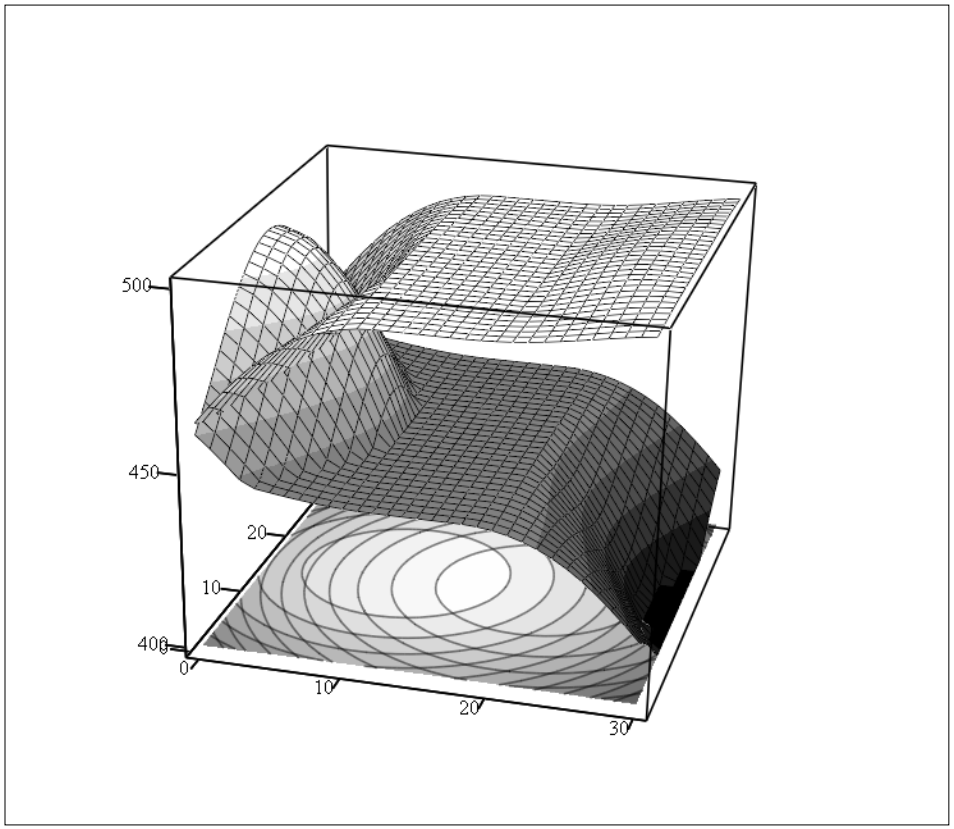
Set grid size $gr := 30$

Estimate utilities on grid: $V1 := U1(gr, gr, 10)$ $V2 := U2(gr, gr, 10)$

Set graph indices $i := 0 .. gr$ $j := 0 .. gr$

Utility graphs for contour plots $Fl1_{i,j} := u1(i, j, 0, 0, 0, 0)^1$ $Fl2_{i,j} := u2(i, j, 0, 0, 0, 0)^1$

Figure 1



V1, V2, Fl1, Fl2