Outsourcing, Vertical Integration, and Cost Reduction*

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Abstract

We study a buyer's incentives to source internally or externally in a stylized model of procurement. In stage one, all suppliers invest in cost reductions. In stage two, the suppliers compete in prices. In stage three, the buyer selects a supplier or abandons the project. Vertical integration gives the buyer the option to source internally, which is advantageous for the buyer as it avoids a markup payment, but disadvantageous insofar as this option discourages investments by independent suppliers. Just as suggested by Williamson's puzzle of selective intervention, the integrated firm can do exactly the same as the two stand alone entities, and can sometimes do better. But this ability to do better has detrimental incentive effects for the behavior of non-integrated suppliers. For a model with exponential distributions of costs and valuations and quadratic investment costs, we derive conditions under which these detrimental effects outweigh the advantageous effects of vertical integration.

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1 Introduction

Pressures of a global economy forced a dramatic transformation of American manufacturing at the end of the twentieth century, away from vertical integration and toward outsourcing. The emergent customer-supplier relationships shifted and blurred the boundaries of firms, as original equipment manufacturers increasingly relied on independent suppliers for both the production and the design of specialized parts. In his case study of durable metal manufacturing, Whitford (2005) describes colorfully the new organizational form as "contested collaboration," whereby a customer-supplier pair cooperate gracefully on cost-reducing design innovations, but contest awkwardly over price. As another case in point, consider Pepsi's problem of procuring potatoes to satisfy the requirements of its Chinese potato chip business as described by a Harvard Business School case (Lu, Tao and Loo, 2008). When Pepsi introduced Lay's potato chips in China in 1997, it needed domestic potatoes with particular characteristics. The raw potatoes had to be large, round, low in sugar and water, and unbruised. Since Chinese agriculture initially was not well adapted to the task, Pepsi both integrated into farming potatoes itself and developed a network of independent farmers to meet its requirements. As Pepsi's potato chip business in China grew, so did its need for potatoes. The case raises several interesting questions. Why did Pepsi outsource when it had the ability to produce internally? As its potato chip business grew, how did Pepsi choose between internal sourcing and outsourcing to meet incremental procurement requirements? Did Pepsi have reason to consider divesting its potato farming assets?

The Pepsi potato chip problem illustrates a general scenario in which such questions are pertinent. Imagine a "customer" who is looking to commercialize a new product or improve (or to expand distribution of) an existing one in a downstream market for which the design of a specialized input process potentially has significant cost consequences. The customer has access to a group of qualified suppliers with different ideas and capabilities who invest in product and process design to prepare proposals for supplying the input. The customer selects the most attractive supply source or abandons the project if it is not commercially viable. A vertically integrated customer has the option to source internally if that is more cost effective.

Vertical integration has a tradeoff in this setup. On the one hand, there are efficiency and rent-seeking advantages from avoiding a markup when the input is sourced internally. Markup avoidance increases efficiency because the project is pursued whenever its value exceeds the cost of internal sourcing, and also shifts rents away from lower cost independent suppliers by distorting the sourcing decision. On the other hand, vertical integration has a disadvantageous "discouragement effect" on the investment incentives of the independent suppliers. Because the procurement process is tilted in favor of internal sourcing, independent suppliers are less inclined to make cost-reducing investments in preparation of proposals. It is costly for the integrated firm can compensate by increasing its own *ex ante* investment, and, if the net investment discouragement effect outweighs the markup avoidance advantages, then the customer has reason to divest its internal division as a way to commit to a level playing field. This is the tradeoff we study.

We examine the interplay between outsourcing and vertical integration and the implications for cost reduction in a stylized model of procurement. The model uses exponential distributions to parameterize cost and demand uncertainty, and a linear marginal cost of investment to parameterize opportunities for cost reduction. A key feature of the model is that the customer has a limited ability to commit to the procurement process. We formalize this by modelling procurement as a three stage game. In stage one, potential suppliers invest in cost reduction. These investments shift independent probability distributions over costs. In stage two, the independent suppliers observe costs and quote prices. In stage three, the customer either selects a source or abandons the project.

As the input has a specialized design and a complex production process, the buyer is not able to make any prior commitments about the sourcing decisions at stage three, except the commitment to be vertically integrated or not. Non-integration in our model is equivalent to a commitment to a first-price auction with an unobservable reserve price equal to the buyer's realized value for the project, and vertical integration can be understood as a commitment to an even lower unobservable reserve price due to the additional option of sourcing internally when the realized cost of the integrated supplier is below the low-price bid of the independent suppliers.

The model abstracts from agency problems inside the firm and instead focuses on how vertical integrations alters sourcing and investment incentives. The research issue is to determine how market structure matters for the efficiency and profitability of partial vertical integration compared to complete outsourcing. While the model is stylized and the parametric assumptions restrictive, the model nevertheless is quite rich and the parameters have relevant economic interpretations.

Our analysis identifies a tradeoff between complete outsourcing and partial vertical integration, and demonstrates that under particular market conditions vertical divestiture of upstream assets is a profitable strategy. Under conditions such that independent suppliers invest symmetrically, vertical integration improves the investment incentives of the integrated supplier but diminishes the incentives of independent suppliers. The reason for discouragement of investment by independent firms is that the integrated firm is biased in favor of its own supply division because it avoids paying a markup on cost by sourcing internally. If the investment discouragement effect of vertical integration is sufficiently detrimental, then the integrated firm benefits from divesting its supply division in order to encourage independent firms to invest more in cost reduction when preparing proposals.

The efficient pattern of investments depends on supply conditions. Within the confines of our exponential-quadratic model, if cost heterogeneity is sufficiently large relative to opportunities for cost reduction, then symmetric investments by suppliers is socially optimal. This result is intuitive sensible. When there are many symmetric suppliers and the cost variance is high, it is likely to be costly to concentrate investment on a particular supplier, because the favored supplier is likely have a significantly higher cost draw than the others. The magnitude of the symmetric investment will also depend on demand conditions.

The profitability of vertical divesture depends on both demand and supply conditions. Under conditions such that symmetric investment is socially efficient, vertical divestiture is a profitable organizational strategy only when cost heterogeneity is not too great and there are sufficiently many suppliers, and when the expected value of the project or demand volatility are sufficiently great. These results also are intuitively sensible. First, if supplier investments are important for cost reduction, then the investment discouragement effect of vertical integration can be detrimental. Second, since a highly valuable project is rarely abandoned under non-integration, vertical integration can have only a very limited market expansion benefit in this case. Third, less demand uncertainty reduces markups by making demand more elastic, and thus diminishes the rent-seeking incentive for vertical integration. Fourth, more upstream competition squeezes markups, and thus also diminishes the rent-seeking incentive for vertical integration. Fifth, if cost heterogeneity is too great, then the resulting high markups make irresistible the rent-seeking incentive for vertical integration, even though the resulting sourcing distortions raise investment costs.

The relatively recent trend toward outsourcing gives renewed salience to the puzzle of selective interventions posed by (Williamson, 1985). Why can't a merged firm do everything the two separate firms can do, and do strictly better by intervening selectively? Most recent theories of vertical integration frame the problem in bilateral terms, focusing on how agency problems inside an integrated firm compare with contracting problems across separate firms. As Cremer (2010) explains, the key to these theories is that the "principal does not quit the stage" after vertical integration, meaning that contracts between the owner (principal) and managers unavoidably are incomplete. Thus, fearing expropriation by an owner who is unable to commit to fair treatment, an employee-manager has low-powered incentives to invest in the relationship.

The current theories are most compelling for evaluating incentive tradeoffs surrounding the vertical acquisition of an owner-managed firm. As observed by Williamson (1985), however, the explanation for vertical integration is more elusive when a separation of ownership and control prevails and diminishes incentives both upstream and downstream irrespective of the identity of the owner Our approach is to view the procurement problem in multilateral terms by embedding Williamson's puzzle in a broader market context while abstracting from agency problems inside the firm. Like in most contemporary theories, the principal does not quit the stage in our theory either. However, the problem with vertical integration in our model is not an inability of the owner to make commitments to managers, but rather an inability of the vertically integrated firm to make credible commitments to independent firms on whom it wants to rely for expertise.

If the vertically integrated firm simply replicated the way it procured before integrating, the profit of the integrated entity would just be equal to the joint profit of the two independent firms. However, just like Williamson (1985) argued, it can do strictly better than that because it can now avoid paying the markup for procuring from outside suppliers whenever the cost of production of the integrated supplier is below the lowest bid of the outside suppliers. In this sense, the vertically integrated firm's flexibility to change its behavior after integration is to its benefit. This seems to contrast sharply with the existing literature, where the vertically integrated firm's inability to commit may render integration unprofitable (Cremer 2010). But it raises the question why vertical integration would not always be profitable in our model. Essentially, the answer is that, because the integrated firm procures differently, the incentives for the outside suppliers to invest in cost-reduction decrease. Depending on parameters, this effect can be so strong that it dominates all the benefits from integration. Therefore, it is exactly the ability of the vertically integrated firm to do better than it does without integration that ultimately may hinder it from so doing because this ability changes the investment behavior of the outside suppliers, which is outside the control of the integrated firm.

That vertical market structure matters for relationship-specific investments is well known. Williamson (1985) argues that asset specificity, incompete contracts, and opportunism conspire to undermine efficient investments. Grossman and Hart (1986) and Hart and Moore (1990) echo the sentiment by modeling how asset specificity and incomplete contracting causes a holdup problem that diminishes the investment incentive of the party lacking control rights over productive assets. Riordan (1990) argued in a different vein, but still consistent with Cremer's interpretation of contemporary theories, that the changed information structure of a vertically integrated creates a holdup problem because the owner cannot commit to incentives for the employee-manager. Bolton and Whinston (1993) added that vertical integration may cause investment distortions motivated by the pursuit of a bargaining advantage.

The basic technological assumptions in our model extend those in Riordan (1990) to a multilateral setting. A supplier makes a non-contractible investment that determines a cost realization prior to price determination. The realized cost is the private information of the supplier under non-integration, but is observed by the owner-customer under vertical integration. However, there are two differences. First, our model abstracts from the internal hold-up problem by assuming that the integrated firm is able to control the investment of its upstream division. Second, by explicitly placing the vertical integration problem in a multilateral setting, we are able to model how the investment incentives of independent suppliers change with vertical divestiture. This extension is key to identifying a new tradeoff between markup avoidance and investment incentives, and to articulating conditions under which vertical divestiture becomes an attractive organizational strategy.

Our emphasis on multilateral supply relationships and our argument that vertical is motivated partly by rent-seeking is reminiscent of Bolton and Whinston (1993). Bolton and Whinston (1993) considers how forward integration enables an upstream supplier to extract rents from independent downstream customers who make relationship-specific investments, whereas our model turns the incentives around to consider how backward integration reduces the rents of upstream suppliers who make relationship specific investments in cost reduction. While the direction of vertical integration is mainly a matter of interpretation, there are other important differences between the models. First, the models make different assumptions about information and the procurement mechanism. The Bolton-Whinston (BW) model assumes efficient bargaining process under complete information to allocate scarce supplies. Vertical integration creates an "outside option" of the bargaining process that for given investments only influences the division of rents. In contrast, our model features incomplete information about cost reduction and realized value in a competitive procurement process, and for given investments vertical integration impacts the sourcing decision as well as the division of rents. Second, the logic of the investment distortions arising from vertical integration is different as a result of the difference in procurement mechanisms. In the BW model, the integrated downstream firm overinvests to create a more powerful outside option when bargaining with independent customers, but the *ex post* allocation decision is efficient conditional on investments. In other words, *ex post* allocation is distorted relative to the first best only because of the *ex ante* investment distortions. In our model, investments are efficient conditional on the allocation decisions, and the causality of distortions is reversed. The rent-seeking advantage of vertical integration leads to *ex post* sourcing distortions, which in turn create *ex ante* investment distortions relative to the first best.

Furthermore, our conclusions differ from those of Bolton and Whinston (1993). Bolton and Whinston (1993) argue there are strong bilateral incentives for partial vertical integration precisely when non-integration is the more socially efficient market structure. The reason for this conclusion is that, as long as the outside option of internal sourcing is binding, the investment disincentives of the independent firm do not matter for the profits of the integrated firm. In contrast, we demonstrate that vertical integration can be privately disadvantageous when the symmetric outcomes of non-integration are attractive from a social efficiency perspective, and the reason is that the investment disincentives of the independent sector very much matter for the profits of the integrated firm. Thus our theory of the private incentives for outsourcing versus partial vertical integration is novel and quite different from the one proposed by Bolton and Whinston (1993), even though the theories share an emphasis on multilateral procurement relationships.

Vertical integration in our model effectively establishes a preferred supplier, who serves to limit the market power of non-integrated suppliers as in Burguet and Perry (2009). The integrated firm avoids giving away rents by allocating production to its upstream division whenever its cost is below the low bid. These allocation distortions from a preferred suppler are similar to those analyzed by Burguet and Perry (2009). Our model goes further by analyzing the consequences for investment in cost reduction. As result of endogenous investments, the preferred supplier has a more favorable cost distribution than the independent suppliers in our model, in contrast to the Burguet and Perry model which assumes identical cost distributions. Obviously, endogenous investments are an additional dimension along which to consider the consequences of a preferred supplier. Our model also extends the Burguet-Perry framework to allow for demand uncertainty.

Our model also relates to an older industrial organization literature in which vertical integration is motivated by a downstream firm's incentive to integrate backwards in order to avoid paying above-cost prices to upstream suppliers of inputs (Perry, 1989). In the double markups literature, vertical integration of successive monopolists achieves greater efficiency by reducing the markup to the single monopoly level. In the variable proportions literatures, a non-integrated firm inefficiently substitutes away from a monopoly-provided input at the margin, and vertical integration of the downstream customer with monopoly supplier corrects the resulting variable proportions distortions. In our model, the alternative suppliers can be viewed as substitute inputs for producing the final good, but, because symmetric suppliers have the same degree of market power under non-integration in our model, there is no distortion in the input choice. Nevertheless, the downstream firm still has an incentive to integrate backward to avoid paying above cost for the input when it procures internally, the result of which is a sourcing distortion. In this sense, our theory turns the logic of the variable proportions literature on its head. At the same time, a lower input price expands the market by increasing the probability that the downstream project succeeds similarly to the double markup literature. The resulting ambiguity from these two effects for economic efficiency is reminiscent of the social welfare ambiguity in the variable proportions literature.

The rest of the paper is organized as follows. Section 2 lays out our model of vertical market structure, demand, and technology. Section 3 examines social efficiency from the perspective of a benevolent social planner. Section 4 characterizes equilibrium sourcing and investments for a non-integrated market structure, and Section 5 does the same for a vertically-integrated market structure. Section 6 identifies conditions under which vertical divesture is an attractive organizational strategy, and Section 7 concludes. Proofs are in the Appendix.

2 Market structure model

2.1 Overview

There is a downstream firm, the customer, who demands a fixed requirement of a specialized input for a project. The returns from the project are uncertain, depending, for example, on realized demand conditions in a downstream market in which the customer is launching a differentiated product.

There are n downstream firms, the suppliers, capable of providing possibly different versions of the required input. Each of the suppliers makes a non-contractible investment in designing the input by exerting effort before making a proposal. *Ex ante*, that is, prior to the investment in effort, a supplier's cost of producing the input is uncertain. *Ex post*, that is after the investment, the supplier privately observes the realized cost. More effort shifts the supplier's cost distribution downward in the sense of first-order stochastic dominance, and, more specifically, shifts mean cost downward.

There are two possible modes of vertical market organization. The customer either is independent of the n suppliers, which we refer to as "non-integration", or is under common ownership with one of the suppliers, which is referred to as "integration". Restricting attention to limited partial integration serves to focus the analysis on vertical rather than horizontal market structure.[1]

2.2 Demand

The customer has value v for the input, drawn from a probability distribution $F(v)$ with support on $[\alpha, \infty)$. The mean of the distribution might be interpreted to indicate the expected profitability of the downstream market, and the variance might be interpreted as indicating uncertainty about product differentiation.

The special case of inelastic demand corresponds to the limit of a sequence of distributions such that $F(v) \to 0$ for all $v < \infty$. For example, the inelastic case occurs as a limit of exponential distributions as the rate parameter goes to zero (see below). Inelastic demand is a leading case in our analysis of incentives for vertical divestiture. The inelastic case captures in the extreme the idea that the likely value of the downstream good is very large relative to the likely cost of the input. This might be so for a highly valuable and differentiated downstream product.

2.3 Costs

Supplier i makes a non-negative investment x_i, and draws a production cost c_i from a probability distribution $G(c_i; x_i)$ with support on the interval $[\underline{c}(x_i), \infty)$. If investment only shifts the mean of the distribution, then

$$G(c_i; x_i) = G(c_i + x_i; 0) \equiv G_0(c_i + x_i)$$

and

$$\underline{c}(x_i) = \underline{c}(0) - x_i \equiv \beta - x_i$$

Thus, mean-shifting investment is the same as in the Laffont and Tirole (1993) model of procurement. In contrast to the typical Laffont-Tirole model, however, the realized cost is the private information of the supplier.

[1] A more ambitious analysis could also consider horizontal consolidations that bring addition suppliers under common ownership, but a thorough analysis along such lines needs a richer model of downstream market structure to consider adequately the antitrust issues. If the upstream industry were diversified horizontally into supplying other downstream firms in the same market or into other markets entirely, then a horizontal consolidation of the industry would attract antitrust scrutiny.

The distribution of the minimum cost with n suppliers with a vector of investments $\mathbf{x} = (x_1, ...x_n)$ is

$$L(c; \mathbf{x}) = 1 - \prod_{i=1}^{n} [1 - G(c; x_i)]$$

with support $[\underline{c}(\mathbf{x}), \infty)$ with $\underline{c}(\mathbf{x}) = \min\{\underline{c}(x_1), ...\underline{c}(x_n)\}$. If the investments are symmetric, then

$$L(c; \mathbf{x}) = 1 - [1 - G_0(c + x)]^n \equiv L_0(c + x)$$

The investment cost function is $\Psi(x_i)$ is increasing and convex.

2.4 Procurement

The customer solicits bids from the suppliers in a reverse auction. There is no reserve price because the precise input specifications are non-contractible *ex ante*, and the buyer cannot commit to reject a profitable offer. Each supplier simultaneously makes an *ex ante* effort choice x_i, privately observes its *ex post* cost c_i.

Under non-integration, each supplier bids a price p_i. The bids are simultaneous. The customer selects the low-bid supplier if $\min\{p_1, ...p_n\} \leq v$, and otherwise cancels the project.

Under integration, the first supplier is owned by the customer. The remaining $n - 1$ independent suppliers simultaneously each bid a price p_i. The customer sources internally if $c_1 \leq \min\{v, p_2, ...p_n\}$, purchases from the low-bid independent supplier if $\min\{p_1, ...p_n\} \leq \min\{c_1, v\}$, and cancels the project otherwise.

2.5 Exponential-quadratic model

If demand uncertainty is captured by an exponential distribution, then

$$F(v) = 1 - e^{-\lambda(v-\alpha)}$$

with $\lambda > 0$. The mean and standard deviation of exponential demand are respectively $\alpha + \frac{1}{\lambda}$ and $\frac{1}{\lambda}$. The special case of inelastic demand case corresponds to the limiting distribution as $\lambda \to 0$, as the mean and variance both become infinite.

Similarly, if mean-shifting cost uncertainty is exponential, then

$$G_0(c) = 1 - e^{-\mu(c-\beta)}$$

and positive effort by supplier i shifts the cost distribution according to

$$G_0(c_i + x_i) = 1 - e^{-\mu(c_i + x_i - \beta)}$$

If the investment cost function is quadratic, then

$$\Psi(x_i) = \frac{a}{2}x_i^2,$$

with $a > 0$. The parameter a is the slope of the linear marginal cost of effort function, and thus indicates the scope for cost reduction.

3 Social Efficiency

3.1 Planning problem

The social planner chooses investments and allocates production to maximize expected social surplus.

An *ex post* efficient allocation awards production to the low cost supplier if $\min\{c_1, ...c_n\} \leq v$, and cancels the project otherwise. Therefore, the expected surplus from an *ex post* efficient allocation given investments is

$$S(\mathbf{x}) = \int_{\alpha}^{\infty} \int_{\underline{c}(\mathbf{x})}^{\infty} \max\{v - c, 0\} dL(c; \mathbf{x}) dF(v)$$

Ex ante efficient investments maximize the expected net surplus from *ex post* efficient allocations. Thus the planner's problem is to maximize

$$W(\mathbf{x}) = S(\mathbf{x}) - \sum_{i=1}^{n} \Psi(x_i)$$

3.2 Symmetric investments

Proposition 1 *Assume mean-shifting investments. (a) A symmetric solution to the planner's problem satisfies:*

$$\Psi'(x) = \frac{1}{n} \int_{\alpha}^{\infty} L_0(v + x) dF(v)$$

if

$$\alpha \geq \beta - x,$$

and

$$\Psi'(x) = \frac{1}{n} \int_{\beta - x}^{\infty} [1 - F(c)] dL_0(c + x)$$

otherwise. (b) In the exponential-quadratic model, there exists a symmetric solution to the planner's problem if $\mu \leq a$. The symmetric solution satisfies

$$1 - \frac{\lambda}{\lambda + n\mu} e^{n\mu(\beta - \alpha - x)} = nax$$

if $\alpha \geq \beta - x$*, and*

$$\frac{n\mu}{\lambda + n\mu} e^{-\lambda(\beta-\alpha-x)} = nax$$

otherwise.

Part (a) states that the planner equates marginal cost of effort to the probability that the project is not canceled ("expected production"). There are two cases, which only matters for expressing expected production as a simple integral. Part (b) provides a simple sufficient condition for a symmetric solution in the exponential-quadratic case, and provides parametric characterizations of the first-order conditions for optimality in each of the two cases.

The result that symmetric investments are socially optimal when cost heterogeneity is high is intuitive. The variance of the minimum order statistic is high when μ is small. Spreading investments across suppliers is wasteful *ex post*, because only one supplier is selected. Obviously, if the most efficient supplier were known in advance, then it would be optimal to concentrate investment on that supplier. But with cost uncertainty it is a risky bet to concentrate investment, because a neglected supplier might end up with a much better efficiency draw. For this reason, the planner diversifies her bet by investing symmetrically. If instead the cost variance were low, then it would be much less risky to concentrate investment. It is striking in the exponential-quadratic model that this tradeoff does not depend on the number of suppliers.

3.3 Inelastic demand

In the limiting case of inelastic demand ($\lambda \to 0$), the project is never canceled. Integrating by parts, the solution to the planner's problem in part (a) can be written alternatively as

$$n\Psi'(x) = \int_{\alpha}^{\infty} [1 - F(c)] dL_0(v + x) + L_0(\alpha + x)$$

Therefore, inelastic demand corresponds to $n\Psi'(x) = 1$, and in the quadratic case $x = \frac{1}{an}$. In the inelastic-exponential-quadratic (IEQ) case, it can be shown that symmetric investments are social optimal if and only if $\mu \leq a$.

3.4 Asymmetric investments

TBD: When is it optimal for the planner to concentrate investment on a single supplier?

4 Outsourcing

4.1 Procurement game

The timing of the procurement under nonintegration is as follows:

1. Suppliers simultaneously choose investments x_i and observe costs c_i.
2. Suppliers simultaneously submit bids p_i.
3. The customer either selects the low-bid supplier, or cancels the project.

The payoff of the buyer is $v - p_{\hat{\imath}}$, the payoff of a supplier is $p_{\hat{\imath}} - c_{\hat{\imath}} - \Psi(x_{\hat{\imath}})$ if selected, and $-\Psi(x_i)$ otherwise. This is an extensive form game in which suppliers choose investments in the first stage, and submit bids in the second stage. The appropriate equilibrium concept is subgame perfection. Since the investments are unobserved, the normal formal of the game has firms simultaneously choosing an investment and bidding strategy. We focus on symmetric equilibria, by which we mean equilibria in which all firms choose the same investment level x at the first stage, so that all firms draw their costs independently from the same distribution $G_0(c+x)$ and accordingly employ the same bidding function $b(c)$ at the second stage.

4.2 Bidding

The structure of equilibrium bidding given symmetric investments x is well understood from Burguet and Perry (2009). Consider the bidding incentives of a representative firm with cost realization c when rival bidders use an invertible bid strategy $b(c)$. A representative bidder chooses p to maximize $(p-c)[1-F(p)][1-G(b^{-1}(p);x)]^{n-1}$. Therefore, a symmetric equilibrium bidding strategy $b(c)$ is such that

$$c = \arg\max_{z} \left\{ [b(z) - c] \, [1 - F(b(z))] \, [1 - G(c;x)]^{n-1} \right\}.$$

For the exponential case, we obtain the following closed form solution:

Lemma 2 *Letting $\hat{c} := \alpha - \frac{1}{\lambda + (n-1)\mu}$, the symmetric equilibrium bidding function given symmetric investments x is*

$$b(c) = c + \frac{1}{\lambda + (n-1)\mu} + \left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu} \right] \left[1 - e^{-(n-1)\mu \max\{\hat{c} - c, 0\}} \right].$$

Observe that for $c \geq \hat{c}$, $b(c)$ has a constant markup $\frac{1}{\lambda+(n-1)\mu}$, that is, $b(c) = c + \frac{1}{\lambda+(n-1)\mu}$. Notice also that although the optimal bid is independent of the level of investment x, the expected profit at the optimal bid is decreasing in x.

The bidding function in the lemma only applies to the support of the symmetric equilibrium cost distribution. To characterize the equilibrium investment, however, it is necessary (at least implicitly) to consider unilateral deviations from equilibrium investment. A unilateral decrease of investment obviously is unproblematic, because the support of the deviant's cost distribution remains in the equilibrium support, and a unilateral increase of investment, say from x to x', it is straightford to deal with. If the deviant gets a cost realization $c \in [\beta - x', -x)$ low enough to completely ignore rivals, then it would bid the monopoly price $c + \frac{1}{\lambda} \leq b(\beta - x)$. Otherwise, constrained by possible competition from a very low cost rival, it would just match the minimum equilibrium bid $b(\beta - x)$.

4.3 Symmetric equilibrium

4.3.1 Inefficient cancelation

Attention is focused on the case in which at a symmetric equilibrium there is a positive probability of cancelation for all possible cost realization. This means that the lower bound of the bid distribution exceeds the lower bound of the value distribution, i.e. $b(\beta - x) \geq \alpha$. The purpose of imposing this restriction is that it enables us to work with a constant markup bid function.

4.3.2 Profits

Assuming $b(\beta - x) \geq \alpha$ in a symmetric equilibrium in which each supplier invests x, the expected profit of the customer is

$$\Pi(x) = n \int_{b(\beta-x)}^{\infty} \int_{\beta-x}^{b^{-1}(v)} [v - b(c)][1 - G_0(c+x)]^{n-1} dG_0(c+x) dF(v)$$

and the expected profit of a representative supplier is

$$\pi(x) = \int_{b(\beta-x)}^{\infty} \int_{\beta-x}^{b^{-1}(v)} [b(c) - c][1 - G_0(c+x)]^{n-1} dG_0(c+x) dF(v) - \Psi(x)$$

4.3.3 Investment

Each supplier chooses an investment to maximize profit given the investments of its rivals. This gives rise to an equilibrium first-order condition for symmetric investments.

Proposition 3 *Assume mean-shifting investments. (a) In symmetric equilibrium the investment of each firm solves*

$$\Psi'(x) = \int_{\beta-x}^{\infty} [1 - F(b(c))] [1 - G_0(c+x)]^{n-1} dG_0(c+x)$$

assuming

$$b(\beta - x) > \alpha$$

at the solution. (b) In the exponential-quadratic model, a symmetric equilibrium satisfying $b(\beta - x) > \alpha$ *exists if*

$$\frac{1}{\lambda + (n-1)\mu} + \beta - \alpha \geq \frac{\mu}{a[\lambda + n\mu]}.$$

The symmetric bid function is

$$b(c) = c + \frac{1}{\lambda + (n-1)\mu}$$

and the parameter restriction reduces to

$$\frac{1}{\lambda + (n-1)\mu} + \beta - \alpha \geq x.$$

If the parameter restriction holds, then symmetric investment solves

$$x = \frac{1}{a}\frac{\mu}{\lambda + n\mu}e^{-\lambda\left[\frac{1}{\lambda+(n-1)\mu}+\beta-\alpha-x\right]}$$

The equilibrium first-order condition for symmetric investment in part (a) equates marginal cost to each supplier's share of the probability of production. The condition departs from the first-order condition that solves the planner's problem because the bid markup compromises the viability of the project. As a result of the production distortion, symmetric equilibrium investment is below the socially efficient level. Part (b) details the parameter restriction for the exponential-quadratic model, and provides a parametric characterization of the investment level for the exponential-quadratic model. The parameter restriction puts an upper bound on $\alpha - \beta$. The bound is illustrated in Figure 1 for different values of μ and n (setting $a = 1$ and $\mu = 0.85$).

4.3.4 Inelastic-exponential-quadratic (IEQ) case

As $\lambda \to 0$ the existence condition for the exponential-quadratic model converges to

$$\beta - \alpha \geq \frac{1}{an}\left[\mu - a\frac{n}{(n-1)}\right]$$

The right-hand-side is negative whenever symmetric investments solve the planners problems, so that the condition says $\beta - \alpha$ cannot be too negative. This condition assures a symmetric equilibrium satisfying $b(\beta - x) \geq \alpha$. More generally, as $\lambda \to 0$, no restriction on $\beta - \alpha$ is necessary for the existence of a symmetric equilibrium. A symmetric equilibrium with $x \to \frac{1}{n}$ exists in the limit if and only if $\mu \leq \frac{n}{a(n-1)}$. In the limit, a symmetric non-integration equilibrium is equal to the solution to the planners problem. Thus in the IEQ case, a symmetric non-integration equilibrium is efficient whenever symmetric investments solve the planner's problem.

The equilibrium expected procurement cost to the buyer under nonintegration equals the expected low bid. Given symmetric investment levels x, the formula is

$$\begin{aligned} PC_N &= \int_{\beta-x}^{\infty} b(c)dL_0(c+x) = \mu n \int_{\beta-x}^{\infty} ce^{-\mu n(c+x-\beta)}dc + \frac{1}{\mu(n-1)} \\ &= \beta - x + \frac{1}{\mu n} + \frac{1}{\mu(n-1)}. \end{aligned}$$

Evaluated at the equilibrium value $x = \frac{1}{an}$, we thus get

$$PC_N = \beta - \frac{1}{an} + \frac{1}{\mu}\frac{2n-1}{n(n-1)},$$

and the expected profit of a representative supplier is

$$\Pi_N = \frac{1}{\mu n(n-1)} - \frac{1}{2}\frac{1}{an^2}.$$

5 Vertical integration

5.1 Procurement game

Under vertical integration, the customer owns supplier 1. Vertical integration effectively establishes a preferred supplier, who serves to limit the market power of non-integrated suppliers as in Burguet and Perry (2009). The procurement game is the same as under non-integration, except the customer has the option of producing internally. The timing is a follows.

1. Suppliers simultaneously choose investments x_i and observe costs c_i.
2. Independent suppliers simultaneously submit bids p_i.
3. The customer accepts the low bid, produces internally, or cancels the project.

If $p_{\hat{\imath}} = \min\{p_2, ...p_n\}$ is the low bid, then the payoff of the buyer is $v - p_{\hat{\imath}}$ with outsourcing, $v - c_1$ with internal sourcing, and 0 with project cancellation. A supplier receives $p_{\hat{\imath}} - c_{\hat{\imath}} - \Psi(x_{\hat{\imath}})$ if selected, and $-\Psi(x_i)$ otherwise. We focus on subgame perfection equilibria in which independent supplier invest and bid symmetrically.

5.2 Bidding

Given investments x_1 by the integrated supplier and symmetric investments x_2 by all other non-integrated firms, the symmetric equilibrium bidding function $b(c)$ is such that

$$c = \arg\max_z \left\{ [b(z) - c]\,[1 - F(b(z))][1 - G(b(z); x_1)]\,[1 - G(z; x_2)]^{n-2} \right\}.$$

For the exponential case, bidding by the independent sector for vertical integration is the same as under integration if the integrated firm has a greater investment.

Lemma 4 *With exponential distributions, the symmetric equilibrium bidding function given investments x_1 and x_2 with $x_1 \geq x_2$ is*

$$b(c) = c + \frac{1}{\lambda + (n-1)\mu}$$

if $x_2 \leq \beta - \alpha + \frac{1}{\lambda+(n-1)\mu}$. *For* $x_2 > \beta - \alpha + \frac{1}{\lambda+(n-1)\mu}$, *it is*

$$b(c) = c + \frac{1}{\lambda + (n-1)\mu} + \left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right]\left[1 - e^{-(n-1)\mu \max\{\hat{c}-c,0\}}\right].$$

where $\hat{c} := \alpha - \frac{1}{\lambda+(n-1)\mu}$.

Our analysis focuses on the equilibrium case in which the integrated supplier invests x_1 and the $n-1$ independent suppliers symmetrically invest x_2, and $b(\beta - x_2) \geq \max\{\alpha, \beta - x_1\}$. This allows us to work with the same constant markup bid function as before.

5.3 Distribution of procurement costs

The probability that procurement costs are less or equal to $b(c)$ for $c \in [\beta - x_2, \infty)$ is

$$P(c; x_1, x_2) = 1 - [1 - G_0(b(c) + x_1)][1 - G_0(b(c) + x_2)]^{n-1}$$

5.4 Profits

Assuming $b(\beta - x_2) \geq \alpha \geq \beta - x_1$ in equilibrium, the expected profit of the integrated firm is[2]

$$\begin{aligned}
\hat{\Pi}(x_1, x_2) &= \int_{b(\beta-x_2)}^{\infty} \int_{\beta-x_2}^{b^{-1}(v)} [v - b(c)] dP(c; x_1, x_2) dF(v) \\
&+ \int_{b(\beta-x_2)}^{\infty} \int_{\beta-x_1}^{b(\beta-x_2)} (v - c) dG_0(c + x_2) dF(v) \\
&+ \int_{\alpha}^{b(\beta-x_2)} \int_{\beta-x_1}^{v} (v - c) dG_0(c + x_2) dF(v) \\
&-\Psi(x_1)
\end{aligned}$$

and the expected profit of a representative independent supplier is

$$\hat{\pi}(x_1, x_2) = \int_{b(\beta-x_2)}^{\infty} \int_{\beta-x_2}^{b^{-1}(v)} [b(c)-c][1-G_0(b(c)+x_1)][1-G_0(c+x_2)]^{n-2} dG_0(c+x_2) dF(v) - \Psi(x_2)$$

5.5 Investment incentives

Proposition 5 *Assume mean-shifting investments. (a) In equilibrium, if independent suppliers invest symmetrically, then the investment of the integrated*

[2] An equivalent expression is

$$\begin{aligned}
\hat{\Pi}(x_1, x_2) &= (n-1) \int_{b(\beta-x_2)}^{\infty} \int_{\beta-x_2}^{b^{-1}(v)} [v - b(c)][1 - G_0(b(c) + x_1)][1 - G_0(b(c) + x_2)]^{n-2} dG_0(c + x_2) dF(v) \\
&+ \int_{\alpha}^{\infty} \int_{\beta-x_1}^{v} (v - c) dG_0(c + x_1) dF(v) \\
&- \int_{b(\beta-x_2)}^{\infty} \int_{b(\beta-x_2)}^{v} (v - c)\{1 - [1 - G_0(b^{-1}(c) + x_2)]^{n-1}\} dG_0(c + x_1) dF(v) \\
&-\Psi(x_1)
\end{aligned}$$

and independent suppliers solve

$$\Psi'(x_1) = \int_{\beta-x_2}^{\infty} [1-F(b(c))]\,[1-G_0(c+x_2)]^{n-1}\, dG_0(b(c)+x_1) + \int_{\alpha}^{b(\beta-x_2)} [1-F(v)]\, dG_0(c+x_1) + G_0(\alpha+x_1)$$

$$\Psi'(x_2) = \int_{\beta-x_2}^{\infty} [1-F(b(c))]\,[1-G_0(b(c)+x_1)]\,[1-G_0(c+x_2)]^{n-2}\, dG_0(c+x_2)$$

assuming

$$b(\beta-x_2) \geq \alpha \geq \beta - x_1$$

at the solution. (b) In the exponential-quadratic model, if (i)

$$\frac{1}{\lambda+\mu} > \frac{1}{a}\frac{\mu}{\lambda+n\mu}e^{-\frac{\lambda+\mu}{\lambda+(n-1)\mu}}$$

(ii)

$$\frac{1}{a}\frac{\mu}{\lambda+n\mu}e^{-\frac{\lambda+\mu}{\lambda+(n-1)\mu}} \geq \beta-\alpha$$

and (iii)

$$\frac{1}{\lambda+\mu} - \frac{1}{\lambda+(n-1)\mu} \geq \beta-\alpha$$

then there exists an equilibrium in which the independent suppliers bid and invest symmetrically and

$$b(\beta-x_2) \geq \alpha \geq \beta - x_1.$$

The equilibrium bid function is

$$b(c) = c + \frac{1}{\lambda+(n-1)\mu}$$

and the equilibrium investments of the integrated supplier and an independent supplier satisfy

$$x_1 = x_2 + \frac{1}{a}\frac{\mu}{\lambda+\mu}e^{-\mu(x_1-x_2)}\left[e^{\mu(\beta-\alpha-x_2)} - e^{-\lambda(\beta-\alpha-x_2)-\frac{\lambda+\mu}{\lambda+(n-1)\mu}}\right]$$

and

$$x_2 = \frac{1}{a}\frac{\mu}{\lambda+n\mu}e^{-\lambda(\beta-\alpha-x_2)-\mu(x_1-x_2)-\frac{\lambda+\mu}{\lambda+(n-1)\mu}}.$$

Observe that condition (i) in part (b) will be satisfied, for example, if $\mu \leq a$.

5.6 IEQ case

As $\lambda \to 0$, the project is never canceled and demand is inelastic. The equilibrium investments satisfy

$$x_1 = \frac{1}{an} + \frac{n-1}{n}\Delta$$

$$x_2 = \frac{1}{an}(1 - a\Delta)$$

and

$$\Delta = \frac{1}{a}\left(1 - e^{-\mu\Delta - \frac{1}{n-1}}\right).$$

The equation for Δ has a unique positive solution as a function of a, μ and n, and therefore (x_1, x_2) has a unique solution as a function of μ and n.

The expected procurement cost of the vertically integrated firm equals the expected price paid to the independent suppliers, plus the expected production cost of self supply, plus the investment cost of the integrated supplier. The simplified formula is

$$PC_I = \beta + \frac{a-\mu}{\mu}x_1 + \frac{a}{2}x_1^2$$

where x_1 is the equilibrium investment the integrated supplier as determined by the solution for Δ.

6 Profit comparisons

6.1 Stability of vertical integration

We refer to an ownership structure as "unstable" if there exists a bilateral acquisition or divesture that makes both parties to the transaction better off. In order to focus on vertical rather than horizontal market structure, we rule out transactions that consolidate the upstream industry. In particular, we focus on whether there is mutual incentive for the vertically integrated firm to divest its upstream division to an independent owner. Furthermore, we restrict attention to environments for which symmetry is a unique equilibrium under non-integration. In these circumstances, vertical integration is unstable if the profit of the vertically integrated firm is less than the sum of profits of the customer and a representative supplier under symmetric non-integration.

6.2 IEQ case

We consider the limiting case in the exponential-quadratic model as $\lambda \to 0$ and $a = 1$. In the limit, the mean and variance of the value of project go to infinity, and at the limit demand for the required input is inelastic. Thus, the limiting case is equivalent to a model in which the project is consummated with probability one, and the customer seeks to minimize expected procurement cost.

Formally, in the IEQ case, vertical integration is unstable if the profit of a representative supplier in a symmetric equilibrium under non-integration exceeds the difference in procurement costs between vertical integration and non-integration. If this is so, then the vertically integrated firm has the incentive to divest its upstream assets to a willing independent supplier and to procure its requirements under non-integration. Substituting the definitions of procurement cost and independent supplier profit for the IEQ case, the condition for unstable vertical integration as $\lambda \to 0$ is equivalent to

$$\begin{aligned}\Phi(\mu, n, a) &\equiv [\frac{a-\mu}{\mu}x(\mu,n,a) + \frac{a}{2}x(\mu,n,a)^2] - [\frac{2n-1}{\mu n(n-1)} - \frac{1}{an}] + [\frac{1}{\mu n(n-1)} - \frac{1}{2}\frac{1}{an^2}] \\ &= \frac{a-\mu}{\mu}x(\mu,n,a) + \frac{a}{2}x(\mu,n,a)^2 - \frac{2}{\mu n} - \frac{1}{2}\frac{1}{an^2} + \frac{1}{an} > 0\end{aligned}$$

with $x = x(\mu, n, a)$ determined by

$$x = \frac{1}{a}\left[\Delta + \frac{1}{n}(1-\Delta)\right]$$

and

$$\Delta = \frac{1}{a}\left[1 - e^{-\mu\Delta - \frac{1}{n-1}}\right].$$

Vertical divesture is profitable in the IEQ case if $\Phi(\mu, n, q) > 0$. Figure 2 graphs $\Phi(\mu, n, 1)$ as function on n for different values of μ. By continuity, the same qualitative conclusions hold for the elastic demand case assuming λ is not too large. This is supported in the numerical analysis that follows below.

Proposition 6 *In the exponential-quadratic model with $a = 1$, λ sufficiently small, and μ close to 1, vertical integration is stable when n is small and vertical integration is unstable when n is sufficiently large, while for μ close to 0 vertical integration is stable for any n.*

TBD: Relax $a = 1$.

6.3 Intuition

To appreciate this result, it is important to understand the powerful advantages of vertical integration in the IEQ case. With mean-shifting investment, inelastic demand and quadratic effort cost, the aggregate investment in effort is the same under non-integration and integration. This follows because the equilibrium marginal costs of effort are equal to market shares which sum to one. Furthermore, since the exponential distribution has a constant hazard rate, the distribution of minimum production cost is more favorable under vertical integration. The support of minimum cost distribution is the union of the supports of the cost distributions of the integrated and independent suppliers, and depends only on aggregate investment on the support of an independent firm. Because the additional investment of the integrated firm shifts its support downward, however, the minimum cost distribution shifts to the left. On top of that advantage of vertical integration, the integrated firm self-sources in some instances, thereby avoiding paying a markup and further reducing its procurement cost compared to non-integration.

From this perspective, the downside to vertical integration might seem more modest. Because the cost of effort is convex, the total effort cost increases as the same total investment is redistributed from independent suppliers to the independent supplier. In other words, even though the vertically integrated firm fully compensates for the investment discouragement of the independent suppliers, it does so at a higher cost.

Notice that a "revealed preference argument" that the customer can do no worse by changing its conduct under vertical integration does not apply to this situation because of the response of the independent suppliers. Even though the integrated firm could keep its investment at the pre-integration level but chooses not to, and the integrated firm could source its requirements the same as under nonintegration but chooses not to, the other firms nevertheless reduce their investments in equilibrium. All we can conclude from revealed preference is that, given that the other firms reduce their investments, the integrated buyer prefers slightly more to less investment, but this does not allow us to conclude that it is better off with integration.

6.4 Curvature of the cost of effort

TBD: We have demonstrated the instability of vertical integration for the quadratic case, which is neutral in the inelastic demand case in the sense that vertical integration redistributes the same total amount of investment as under nonintegration. If the cost function were even more convex, however, then total investment would be less, and the instability of vertical integration even more pronounced.

6.5 Numerical analysis

Figure 3 graphs the customer's profit under vertical integration relative to the profits from divesture and non-integration for various cases of the exponential-quadratic model. The number of firms is $n \in \{5, 10, 15\}$, cost parameters are $\mu \in \{0.35, 0.85\}$, $\beta = 0$, and $a = 1$, and the demand parameters are $\alpha \in [-1, 0]$ and $\lambda \in (0, 1]$. The demand parameter region is well within the bounds for the constraint $b(\beta - x) \geq \alpha$ under non-integration, as provided for in Proposition 2(b) and illustrated in Figure 1. It also can we verified numerically that the parameter constraints of Proposition 3(b) hold in this demand region, so the investment formulas of both Propositions 2(b) and 3(b) are applicable for computing the profit comparison.

Part (a) of the figure sets $\mu \in 0.85$ and $n = 10$ to illustrate a subset of demand parameter region for which vertical integration is the more profitable organization. The profitable vertical integration region is where the profit comparison plane (blue) rises above the zero plane (red). In this region, the difference in customer profits between integration and non-integration exceeds the divested value of the supply division of the integrated firm. Vertical divestiture is the more profitable organizational strategy, however, if λ is sufficiently close to 0, consistent with Proposition 4. It can also be seen that vertical divestiture is profitable for much higher values of λ for negative α near 0, i.e. if $\beta - \alpha$ is not too positive. This could be illustrated even more dramatically for small positive values of α that remain consistent with all the constraints (but is not in the graph for economy of exposition.)

The other parts of the figure illustrate comparative static effects. Part (b) increases cost heterogeneity by reducing μ to 0.35, while Part (c) instead keeps μ at 0.85 but reduces n to 5. For both parameter changes, vertical integration becomes stable over the entire demand parameter region under consideration. As discussed above, a high degree of heterogeneity results in high markups under non-integrtion, which makes the markup avoidance benefit of vertical integration very valuable. The higher markups resulting from fewer competitors similary increases markups similarly and enhances the relative profitability of vertical integration. Finally, Part (d) increases n to 15 which diminishes the markup avoidance benefit of vertical integration.

In summary, these numerical analyses show that the comparative static conclusions for the IEQ case are robust to allowing demand to be elastic.

6.6 Bargaining models

TBD: Discuss simple models of acquisition and divestiture.

7 Conclusion

We study a simple, stylized model of procurement and vertical integration in which relationship-specific investments by suppliers decrease expected production costs and procurement occurs via a first-price auction without a reserve price. For given investments, vertical integration improves the profits of the procurer because it enables it to avoid paying the markup over costs it must pay absent integration. Moreover, if the procurer's demand is elastic, integration increases efficiency and further increases profits, keeping investments fixed, because the markup avoidance also leads to an output expansion. Therefore, just like in Williamson (1985)'s famous puzzle of selective intervention, an integrated firm can do the same as the separate entities do, and sometimes it can do strictly better. This would seemingly lead to the conclusion that vertical integration is inevitably profitable. This prediction is puzzling because it is at odds with the empirical observations, which include the recent trend towards outsourcing.

However, in our model, and we would argue in the real world, vertical integration is not always profitable because it changes the incentives to invest for the suppliers, making optimal investment levels smaller for non-integrated suppliers and larger for the integrated supplier. Thus vertical integration effectively reallocates investment away from independent suppliers and toward the integrated supplier. Such a reallocation raises total investment costs because the marginal cost of investment is increasing. This change-of-equilibrium-investments effect can be so strong that it outweighs the afore mentioned benefits from vertical integration. Not only does vertical integration change the behavior of the integrated entity in the way suggested by Williamson, but, exactly because it does so, it also changes the behavior of the non-integrated firms. Put differently, vertical integration occurs within a competitive environment, and depending on how this environment's behavior is affected by vertical integration, insourcing or outsourcing may be the procurer's preferred organizational structure.

Williamson's puzzle interprets the vertical integration decision in a narrow bilateral context, implicitly holding constant the conduct of outside parties. All that seems to matter for the decision are the incentives of the manager of the supply division and the ability of the integrated firm to adapt to the external environment. Accounting for the investment response of independent suppliers, however, creates a tradeoff between the advantages of markup avoidance on the one hand, and the cost disadvantage of realigned investment incentives on the other. In this multilateral setting, the puzzle vanishes. The tradeoff favors vertical integration in some circumstances, and vertical divestiture and outsourcing in others.

Our procurement model is motivated by the idea that specialized suppliers make non-contractible investments in cost-reducing product and process design, consistent with Whitford (2005)'s description of the type of customer-supplier relationships that emerged in manufacturing at the end of the 20th century. The model predicts that vertical divesture is under certain circumstances an attractive strategy to encourage these investments by independent suppliers. These conditions favoring vertical divestiture include an increase in the number of potential suppliers, greater cost heterogeneity, and greater demand volatility. Each of these conditions contributes to reducing supplier markups, thus weakening the markup avoidance advantages of vertical integration. These predictions help explain the trend away from vertical integration and toward outsourcing in an increasingly uncertain global economy marked by rapid technological change and short product cycles.

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9 Appendix

9.1 Proof of Proposition 1

9.1.1 Planner's problem

The planner chooses $\mathbf{x} = (x_1, ...x_n)$ to maximize $S(\mathbf{x}) - \sum_{i=1}^{n} \Psi(x_i)$. Extending

$$\begin{aligned} S(\mathbf{x}) &= \int_{\alpha}^{\infty} \int_{\underline{c}(\mathbf{x})}^{\max\{v,\underline{c}(\mathbf{x})\}} (v-c) dL(c;\mathbf{x}) dF(v) \\ &= \int_{\alpha}^{\infty} \int_{\underline{c}(\mathbf{x})}^{\max\{v,\underline{c}(\mathbf{x})\}} L(c;\mathbf{x}) dc dF(v) \\ &= \int_{\underline{c}(\mathbf{x})}^{\infty} [1-F(c)] L(c;\mathbf{x}) dc \end{aligned}$$

by integration by parts, where $F(v)$ is defined in the usual manner on an extended support if necessary. The first-order condtions are

$$\Psi'(x_i) = \frac{\partial S(\mathbf{x})}{\partial x_i}$$

With mean-shifting investments

$$\begin{aligned} \frac{\partial S(\mathbf{x})}{\partial x_i} &= \int_{\alpha}^{\infty} \int_{\beta - x_i}^{\max\{v,\beta - x_i\}} \prod_{j=1, j\neq i}^{n} [1 - G_0(c + x_j)] dG(c + x_i) dF(v) \\ &= \int_{\beta - x_i}^{\infty} [1 - F(c)] \prod_{j=1, j\neq i}^{n} [1 - G_0(c + x_j)] dG(c + x_i) \end{aligned}$$

9.1.2 Adding up

Summing the first-order conditions for mean-shifting investments gives

$$\begin{aligned}
\sum_{i=1}^{n} \Psi'(x_i) &= \sum_{i=1}^{n} \frac{\partial S(\mathbf{x})}{\partial x_i} \\
&= \sum_{i=1}^{n} \int_{\beta - x_i}^{\infty} [1 - F(c)] \prod_{j=1, j \neq i}^{n} [1 - G_0(c + x_j)] dG(c + x_i) \\
&= \int_{\underline{c}(\mathbf{x})}^{\infty} [1 - F(c)] \sum_{i=1}^{n} \prod_{j=1, j \neq i}^{n} [1 - G_0(c + x_j)] g_0(c + x_i) dc \\
&= \int_{\underline{c}(\mathbf{x})}^{\infty} [1 - F(c)] dL(c; \mathbf{x}) < 1
\end{aligned}$$

where the $G_0(c + x_j)$ and corresponding densities $g_0(c + x_i)$ are defined on an extended support as necessary.

9.1.3 Symmetric solution

At a symmetric solution, $\mathbf{x} = (x, ...x)$, assuming $\alpha \geq \beta - x$,

$$\begin{aligned}
S(\mathbf{x}) &= \int_{\alpha}^{\infty} \int_{\beta - x}^{v} (v - c) dL_0(c + x) dF(v) \\
&= \int_{\alpha}^{\infty} \int_{\beta - x}^{v} L_0(c + x) dc dF(v)
\end{aligned}$$

by integration by parts. Furthermore,

$$\begin{aligned}
\frac{dS(x, ...x)}{dx} &= \int_{\alpha}^{\infty} \int_{\beta}^{v+x} \frac{dL_0(z)}{dz} dz dF(v) \\
&= \int_{\alpha}^{\infty} L_0(v + x) dF(v)
\end{aligned}$$

by a change of variables.

Similarly, if $\beta - x \geq \alpha$, then

$$\frac{dS(x, ...x)}{dx} = \int_{\beta - x}^{\infty} F(c) dL_0(c + x)$$

This proves part (a).

9.1.4 Exponential-quadratic model

Symmetric solution Without loss of generality $x_1 \geq x_2 .. \geq x_n$, define

$$X_i = \sum_{j=1}^{i} x_j$$

There are three possible cases: either (1) $x_n \geq \beta - \alpha$, or (2) $\beta - \alpha \geq x_1$, or (3) $x_k \geq \beta - \alpha \geq x_{k+1}$ for some $k < n$. In each case, the solution to the first-order conditions of the Planner's Problem must be symmetric if $a \geq \mu$.

Part (1) $x_n \geq \beta - \alpha$

$$\begin{aligned}
\frac{\partial S(\mathbf{x})}{\partial x_i} &= \mu \int_{\alpha}^{\infty} e^{-(\lambda+\mu n)c+\lambda\alpha+\mu n\beta-\mu X_n} dc \\
&+\mu \int_{\beta-x_n}^{\alpha} e^{-\mu n c+\mu n\beta-\mu X_n} dc \\
&+\mu \int_{\beta-x_{n-1}}^{\beta-x_n} e^{-\mu(n-1)c+\mu(n-1)\beta-\mu X_{n-1}} dc \\
&.... \\
&+\mu \int_{\beta-x_i}^{\beta-x_{i+1}} e^{-\mu i c+\mu i\beta-\mu X_i} dc
\end{aligned}$$

Therefore, for $i = 1,,, n-1$

$$\begin{aligned}
\frac{\partial S(\mathbf{x})}{\partial x_i} - \frac{\partial S(\mathbf{x})}{\partial x_{i+1}} &= \mu \int_{\beta-x_i}^{\beta-x_{i+1}} e^{-\mu i c+\mu i\beta-\mu X_i} dc \\
&= \frac{1}{i} e^{-\mu(X_i - i x_i)} [1 - e^{-\mu i(x_i - x_{i+1})}]
\end{aligned}$$

and, consequently, the differenced first-order conditions for the exponential-quadratic model imply

$$a(x_i - x_{i+1}) = \frac{1}{i} e^{-\mu(X_i - i x_i)} [1 - e^{-\mu i(x_i - x_{i+1})}]$$

Given x_i, this equation obviously has a solution at $x_i - x_{i+1} = 0$. Furthermore, since the slope of the LHS in $(x_i - x_{i+1})$ is a and the slope of the RHS is $\mu e^{-\mu(X_i - i x_i) - \mu i(x_i - x_{i+1})}$ and decreasing, if $X_i = i x_i$, then there is no strictly positive solution if $a \geq \mu$. Provided this condition holds, iteration establishes that $x_i = x_1$ is the unique solution to the simplified difference equation for all $i = 1, .., n$ for an arbitrary x_1.

Part (2) $\beta - \alpha \geq x_1$

$$
\begin{aligned}
\frac{\partial S(\mathbf{x})}{\partial x_i} &= \mu \int_{\beta - x_n}^{\infty} e^{-(\lambda+\mu n)c+\lambda\alpha+\mu n\beta-\mu X_n} dc \\
&\quad +\mu \int_{\beta - x_{n-1}}^{\beta - x_n} e^{-[\lambda+\mu(n-1)]c+\lambda\alpha+\mu(n-1)\beta-\mu X_{n-1}} dc \\
&\quad .... \\
&\quad +\mu \int_{\beta - x_i}^{\beta - x_{i+1}} e^{-(\lambda+\mu i)c+\lambda\alpha+\mu i\beta-\mu X_i} dc
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial S(\mathbf{x})}{\partial x_i} - \frac{\partial S(\mathbf{x})}{\partial x_{i+1}} &= \mu \int_{\beta - x_i}^{\beta - x_{i+1}} e^{-(\lambda+\mu i)c+\lambda\alpha+\mu i\beta-\mu X_i} dc \\
&= \mu e^{\lambda\alpha+\mu i\beta-\mu X_i} \int_{\beta - x_i}^{\beta - x_{i+1}} e^{-(\lambda+\mu i)c} dc \\
&= -\frac{\mu}{\lambda+\mu i} e^{\lambda\alpha+\mu i\beta-\mu X_i} [e^{-(\lambda+\mu i)(\beta - x_{i+1})} - e^{-(\lambda+\mu i)(\beta - x_i)}] \\
&= -\frac{\mu}{\lambda+\mu i} e^{\lambda\alpha+\mu i\beta-\mu X_i} e^{-[\lambda+\mu i)]\beta} [e^{(\lambda+\mu i)x_{i+1}} - e^{(\lambda+\mu i)x_i}] \\
&= -\frac{\mu}{\lambda+\mu i} e^{\lambda(\alpha-\beta)-\mu X_i} [e^{(\lambda+\mu i)x_{i+1}} - e^{(\lambda+\mu i)x_i}] \\
&= -\frac{\mu}{\lambda+\mu i} e^{\lambda(\alpha-\beta+x_i)-\mu(X_i - i x_i)} [e^{(\lambda+\mu i)(x_{i+1}-x_i)} - 1] \\
&= \frac{\mu}{\lambda+\mu i} e^{-\lambda(\beta-\alpha-x_i)-\mu(X_i - i x_i)} [1 - e^{-(\lambda+\mu i)(x_i - x_{i+1})}]
\end{aligned}
$$

and, consequently, the differenced first-order conditions for the exponential-quadratic model imply

$$
a(x_i - x_{i+1}) = \frac{\mu}{\lambda+\mu i} e^{-\lambda(\beta-\alpha-x_i)-\mu(X_i - i x_i)} [1 - e^{-(\lambda+\mu i)(x_i - x_{i+1})}]
$$

Given x_i, this equation obviously has a solution at $x_i - x_{i+1} = 0$. Furthermore, since the slope of the LHS in $(x_i - x_{i+1})$ is a and the slope of the RHS is $\mu e^{\lambda(\alpha-\beta)-\mu(X_i - i x_i)-[\lambda+\mu i)](x_i - x_{i+1})}$ and decreasing, if $X_i = i x_i$, then there is no strictly positive solution if

$$
a \geq \mu e^{-\lambda(\beta-\alpha-x_i)}
$$

Provided this condition holds, iteration establishes that $x_i = x_1$ is the unique solution to the simplified difference equation for all $i = 1, .., n$ for an arbitrary x_1.

Notice that $\beta - \alpha - x_i \geq 0$ implies $e^{-\lambda(\beta-\alpha-x_i)} < 1$. Therefore, $a \geq \mu$ is sufficient for a symmetric solution.

Part (3) $x_k \geq \beta - \alpha \geq x_{k+1}$ **for** $n > k$. For $i = 1, ...k-1$, the analysis in Part I implies

$$a(x_i - x_{i+1}) = \frac{1}{i} e^{-\mu(X_i - ix_i)} [1 - e^{-\mu i(x_i - x_{i+1})}]$$

and for $i = k, ...n-1$, the analysis in Part II implies

$$a(x_i - x_{i+1}) = \frac{\mu}{\lambda + \mu i} e^{-\lambda(\beta-\alpha-x_i)-\mu(X_i - ix_i)} [1 - e^{-(\lambda+\mu i)(x_i - x_{i+1})}]$$

Iterating from a given x_1, $a \geq \mu$ is sufficient for a symmetric solution at each step.

Characterization

Part (1) $\beta - \alpha \geq x$ The "boundary condition" for an optimal solution is $\Psi'(x_n) = \frac{\partial S(\mathbf{x})}{\partial x_n}$, which if $\alpha - \beta \geq x_n$ in the exponential-quadratic model becomes

$$\begin{aligned}
ax_n &= \mu \int_{\alpha}^{\infty} e^{-(\lambda+\mu n)c + \lambda\alpha + \mu n\beta - \mu X_n} dc + \mu \int_{\beta - x_n}^{\alpha} e^{-\mu nc + \mu n\beta - \mu X_n} dc \\
&= \mu e^{\lambda\alpha + \mu n\beta - \mu X_n} \int_{\alpha}^{\infty} e^{-(\lambda+\mu n)c} dc + \mu e^{\mu n\beta - \mu X_n} \int_{\beta - x_n}^{\alpha} e^{-\mu nc} dc \\
&= -\frac{\mu}{\lambda + \mu n} e^{\lambda\alpha + \mu n\beta - \mu X_n} [0 - e^{-(\lambda+\mu n)\alpha}] - \frac{1}{n} e^{\mu n\beta - \mu X_n} [e^{-\mu n\alpha} - e^{-\mu n(\beta - x_n)}] \\
&= \frac{\mu}{\lambda + \mu n} e^{\mu n(\beta-\alpha) - \mu X_n} - \frac{1}{n} e^{\mu n(\beta-\alpha) - \mu X_n} [1 - e^{-\mu n(\beta - \alpha - x_n)}] \\
&= [\frac{\mu}{\lambda + \mu n} - \frac{1}{n}] e^{\mu n(\beta-\alpha) - \mu X_n} + \frac{1}{n} e^{-\mu(X_n - nx_n)}
\end{aligned}$$

Multiplying through by n, simplifying, and rearranging, the boundary condition becomes

$$e^{-\mu(X_n - nx_n)} \left[1 - \frac{\lambda}{\lambda + \mu n} e^{-\mu n(\alpha - \beta + x_n)}\right] = anx_n$$

At a symmetric solution, the boundary condition becomes

$$1 - \frac{\lambda}{\lambda + n\mu} e^{-n\mu(x_n + \alpha - \beta)} = nax_n.$$

Part (2) $x \geq \beta - \alpha$ The boundary condition is $\Psi'(x_n) = \frac{\partial S(\mathbf{x})}{\partial x_n}$, which if $x_n \geq \alpha - \beta$ in the exponential-quadratic model becomes

$$\begin{aligned} ax_n &= \mu \int_{\beta - x_n}^{\infty} e^{-(\lambda+\mu n)c+\lambda\alpha+\mu n\beta-\mu X_n} dc \\ &= \mu e^{\lambda\alpha+\mu n\beta-\mu X_n} \int_{\beta - x_n}^{\infty} e^{-(\lambda+\mu n)c} dc \\ &= -\frac{\mu}{\lambda+\mu n} e^{\lambda\alpha+\mu n\beta-\mu X_n} [0 - e^{-(\lambda+\mu n)(\beta - x_n)}] \\ &= \frac{\mu}{\lambda+\mu n} e^{\lambda\alpha+\mu n\beta-\mu X_n-(\lambda+\mu n)(\beta - x_n)} \\ &= \frac{\mu}{\lambda+\mu n} e^{-\lambda(\beta-\alpha-x_n)-\mu(X_n - n x_n)} \end{aligned}$$

At a symmetric solution, the boundary condition becomes

$$nax_n = \frac{\mu n}{\lambda+\mu n} e^{-\lambda(\beta-\alpha-x_n)}.$$

9.2 Proof of Lemma 2

For F and G exponential, a representative supplier's problem becomes

$$\max_z (b(z) - c) e^{-\lambda \max\{b(z)-\alpha, 0\} - \mu(n-1))(z+x-\beta)}.$$

Taking the derivatives with respect to z separately for the cases $b(z) \leq \alpha$ and $b(z) > \alpha$, evaluated at $z = c$, gives, respectively, the first-order conditions

$$b'(c) - (n-1)\mu[b(c) - c] = 0$$

and

$$b'(c) - [\lambda b'(c) + (n-1)\mu][b(c) - c)] = 0.$$

Imposing the boundary constraint $b(\hat{c}) = \hat{c}$ for some arbitrary $\hat{c}$ for the former gives the unique solution

$$b_I(c) = c + \frac{1}{(n-1)\mu} + k e^{(n-1)\mu c}$$

for $c \leq \hat{c}$ for some constant k (that remains to be determined) while imposing the boundary condition $\lim_{c\to\infty}(b(c) - c)/c = 0$ on the latter yields the unique solution

$$b_{II}(c) = c + \frac{1}{\lambda + (n-1)\mu}.$$

Setting $b_{II}(\hat{c}) = \alpha$ gives us $\hat{c}$ as stated in the lemma. Standard arguments imply that the equilibrium bidding function has to be continuous everywhere. As the functions b_I and b_{II} are continuous for $c \leq \hat{c}$ and $c \geq \hat{c}$, we are left to insure that $b_I(\hat{c}) = b_{II}(\hat{c})$, which will then determine k. Imposing this equality gives us

$$\frac{1}{\lambda + (n-1)\mu} = \frac{1}{(n-1)\mu} + ke^{(n-1)\mu\hat{c}},$$

implying

$$k = -\left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right] e^{-(n-1)\mu\hat{c}}.$$

Plugging k back into $b_I(c)$, we get

$$b_I(c) = c + \frac{1}{(n-1)\mu} + \left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right] e^{-(n-1)\mu(\hat{c}-c)}.$$

To see that we can write the bidding function $b(c)$ as stated in the lemma, observe that for $c \geq \hat{c}$ it is identical to $b_{II}(c)$ and for $c \leq \hat{c}$ it is identical to $b_I(c)$.

9.3 Proof of Proposition 3

9.3.1 Necessary condition with mean-shifting investment

Suppose a representative firm were to deviate and choose $x + \varepsilon$ instead of x. The deviant would have cost distribution $G(c; x + \varepsilon) = G_0(c + x + \varepsilon)$. For $c \geq \beta - x$, the deviant would still follow the equilibrium bidding strategy $b(c)$ if it expects its rivals to do likewise; similarly, rivals also would follow the equilibrium bidding strategy because the deviation is unobserved. For $\beta - x > c \geq \beta - x - \varepsilon$, the deviant would bid $b(\beta - x)$ assuming ε is not too large. Therefore, the ex post profit of the deviant with cost realization $c \geq \beta - x - \varepsilon$ is

$$U(c) = \begin{cases} [b(c) - c][1 - G(c; x)]^{n-1}[1 - F(b(c)] & \text{if} \quad c \geq \beta - x \\ [b(\beta - x) - c - \varepsilon][1 - F(b(\beta - x)] & \text{if} \quad c \leq \beta - x \end{cases}.$$

Given the optimality of $b(c)$, the envelope theorem implies

$$U'(c) = \begin{cases} -[1 - G(c; x)]^{n-1}[1 - F(b(c)] & \text{if} \quad c > \beta - x \\ -[1 - F(b(\beta - x)] & \text{if} \quad c < \beta - x \end{cases}$$

Using a change of variables, the ex ante (expected) profit of the deviant

$$\Pi(\varepsilon) = \int_{\beta - x}^{\infty} U(c - \varepsilon) dG(c; x) - \frac{a}{2}(\varepsilon + x)^2$$

where x is the symmetric equilibrium investment and a small positive deviation investment $\varepsilon > 0$. The same function also describes ex ante profit for a negative deviation $\varepsilon < 0$. Furthermore, the function is differentiable at $\varepsilon = 0$. Therefore the symmetric equilibrium first-order condition is

$$-\int_{\beta-x}^{\infty} U'(c)dG(c;x) - ax = 0$$

or, equivalently,

$$x = \frac{1}{a}\int_{\beta-x}^{\infty} [1 - G(c;x)]^{n-1}[1 - F(b(c)]dG(c;x).$$

9.3.2 Necessary conditions for a symmetric equilibrium in the exponential-quadratic model

Using the properties of exponential distributions, $b(\beta - x) - \alpha > 0$ implies

$$b(c) - c = \frac{1}{\lambda + (n-1)\mu}$$

and

$$1 - F(b(c)) = e^{-\lambda[c + \frac{1}{\lambda+(n-1)\mu} - \alpha]}.$$

Furthermore,

$$L(c) = 1 - e^{-n\mu(c+x-\beta)}$$

and

$$L'(c) = n\mu e^{-n\mu(c+x-\beta)}$$

Therefore,

$$\begin{aligned} X &= \frac{1}{a}\int_{\beta-x}^{\infty} [e^{-\lambda[c+\frac{1}{\lambda+(n-1)\mu}-\alpha]}][n\mu e^{-n\mu(c+x-\beta)}]dc \\ &= \frac{n\mu}{a}e^{-\frac{\lambda}{\lambda+(n-1)\mu}+\lambda\alpha+n\mu(\beta-x)}\int_{\beta-x}^{\infty} e^{-(\lambda+n\mu)c}dc \\ &= \frac{n\mu}{a(\lambda+n\mu)}e^{-\frac{\lambda}{\lambda+(n-1)\mu}-\lambda(\beta-\alpha-x)} \end{aligned}$$

and

$$x = \frac{\mu}{a(\lambda+n\mu)}e^{-\frac{\lambda}{\lambda+(n-1)\mu}-\lambda(\beta-\alpha-x)}$$

Observe that the term in the exponent is positive under the condition for constant markup bidding. Therefore, $x \leq \frac{\mu}{\lambda+n\mu}$ in as symmetric equilibrium.

Let $\Delta \equiv \beta - \alpha - x$. Then

$$\Delta = \beta - \alpha - \frac{\mu}{a(\lambda + n\mu)} e^{-\frac{\lambda}{\lambda+(n-1)\mu} - \lambda\Delta} \tag{1}$$

The right-hand-side increasing and concave in Δ; it equals $\beta - \alpha - \frac{\mu}{a(\lambda+n\mu)}$ when $\Delta = -\frac{1}{\lambda+(n-1)\mu}$, and goes to $\beta - \alpha$ as $\Delta \to \infty$. The equation has a unique solution on the domain $[-\frac{1}{\lambda+(n-1)\mu}, \infty)$ if

$$\beta - \alpha > \left[\frac{\mu}{a(\lambda + n\mu)} - \frac{1}{\lambda + (n-1)\mu}\right] \tag{2}$$

TBD: Confirm when this condition is also necessary, e.g.

$$\frac{\lambda\mu}{a(\lambda + n\mu)} \leq 1$$

Condition (2) is an "if and only" condition for the existence of a symmetric equilibrium with $b(\beta - x) > \alpha$ under the parameter restriction $\lambda\mu \leq \lambda a + an\mu$. (Observe that this restriction is always satisfied if we impose $\mu < a$.) To see this, observe that the LHS of (1) is less than its RHS at $\Delta = -\frac{1}{\lambda+(n-1)\mu}$ if and only if (2) is satisfied. Moreover, at $\Delta = -\frac{1}{\lambda+(n-1)\mu}$, the derivative of the RHS is $\frac{\lambda\mu}{a(\lambda+n\mu}$, which is less than 1 (the derivative of the LHS) under the additional parameter restriction. Thus, if this additional parameter restriction is satisfied but (2) is not, then there is no point of intersection of the LHS and the RHS on the domain $[-\frac{1}{\lambda+(n-1)\mu}, \infty)$.

9.3.3 Sufficient conditions for a symmetric equilibrium in the exponential-quadratic model

The best response of a representative firm (x) to a symmetric investment by rivals ($\bar{x}$) satisfies

$$\begin{aligned} ax &= \int_{\beta-x}^{\infty} [1 - F(b(c))][1 - G_0(c + \bar{x})]^{n-1} dG_0(c + x) \\ &= \int_{\beta}^{\infty} [1 - F(b(c - x))][1 - G_0(c + \bar{x} - x)]^{n-1} dG_0(c) \\ &= e^{-[\lambda+(n-1)\mu]x} \int_{\beta}^{\infty} [1 - F(b(c))][1 - G_0(c + \bar{x})]^{n-1} dG_0(c) \end{aligned}$$

which is equivalent to

$$axe^{[\lambda+(n-1)\mu]x} = \int_{\beta}^{\infty} [1 - F(b(c))][1 - G_0(c + \bar{x})]^{n-1} dG_0(c)$$

The left-hand-side is a non-negative and increasing convex function of x, equal 0 at $x = 0$ and going to ∞ as $x \to \infty$. The right-hand-side is a positive and decreasing function of $\bar{x}$. If follows that a unique best response is a continuously decreasing function of $\bar{x}$. Furthermore, a fixed point of this response function exists and is a symmetric Nash equilibrium.

9.4 Proof of Lemma 4

Notice that in a first-price auction, $b(c) \geq c$ will always hold. Therefore, there are a priori four cases to consider:
i) $b(\beta - x_2) > \max\{\alpha, \beta - x_1\}$
ii) $\alpha > b(\beta - x_2) > \beta - x_1$
iii) $\beta - x_1 > b(\beta - x_2) > \alpha$
iv) $\min\{\alpha, \beta - x_1\} > b(\beta - x_2)$
However, assuming $x_1 \geq x_2$, we are left we cases i) and ii).\

The proof proceeds along similar lines as the one of Lemma 2. For F and G exponential and $x_1 \geq x_2$, a representative non-integrated supplier's problem is

$$\max_z (b(z) - c)e^{-\lambda \max\{b(z)-\alpha,0\} - \mu(b(z)+x_1-\beta) - \mu(n-2))(z+x_2-\beta)}.$$

Taking the derivatives with respect to z separately for the cases $b(z) \leq \alpha$ and $b(z) > \alpha$, evaluated at $z = c$, gives, respectively, the first-order conditions

$$b'(c) - [b'(c)\mu + (n-2)\mu][b(c) - c)] = 0$$

and

$$b'(c) - [(\lambda + \mu)b'(c) + (n-2)\mu][b(c) - c)] = 0.$$

Imposing the boundary constraint $b(\hat{c}) = \hat{c}$ for some arbitrary $\hat{c}$ for the former gives the unique solution

$$b_I(c) = c + \frac{1}{(n-1)\mu} + ke^{(n-1)\mu c}$$

for $c \leq \hat{c}$ for some constant k (that remains to be determined) while imposing the boundary condition $\lim_{c\to\infty}(b(c) - c)/c = 0$ on the latter yields the unique solution

$$b_{II}(c) = c + \frac{1}{\lambda + (n-1)\mu}.$$

Setting $b_{II}(\hat{c}) = \alpha$ gives us $\hat{c} = \alpha - \frac{1}{\lambda+(n-1)\mu}$ as in Lemma 2. Standard arguments imply that the equilibrium bidding function has to be continuous everywhere. As the functions b_I and b_{II} are continuous for $c \leq \hat{c}$ and $c \geq \hat{c}$, we are left to insure that $b_I(\hat{c}) = b_{II}(\hat{c})$, which will then determine k. Imposing this equality gives us

$$\frac{1}{\lambda + (n-1)\mu} = \frac{1}{(n-1)\mu} + ke^{(n-2)\mu\hat{c}},$$

implying

$$k = \left[\frac{1}{\lambda + (n-1)\mu} - \frac{1}{(n-1)\mu}\right] e^{-(n-1)\mu\hat{c}}.$$

Plugging k back into $b_I(c)$, we get

$$b_I(c) = c + \frac{1}{(n-1)\mu} + \left[\frac{1}{(n-1)\mu} - \frac{1}{\lambda + (n-1)\mu}\right] e^{-(n-1)\mu(\hat{c}-c)}.$$

To see that we can write the bidding function $b(c)$ as stated in second part of the lemma, observe that for $c \geq \hat{c}$ it is identical to $b_{II}(c)$ and for $c \leq \hat{c}$ it is identical to $b_I(c)$. Finally, notice that for $x_2 \leq \beta - \alpha + \frac{1}{\lambda+(n-1)\mu}$, $c \geq \hat{c}$ so $b(c)$ is as stated in the first part of the lemma in this case.

9.5 Proof of Proposition 5

9.5.1 Investment incentives of integrated firm

Marginal increase in procurement value Assuming mean-shifting investments, assuming $b(\beta - x_2) \geq \alpha \geq \beta - x_1$, the equilibrium gross value of procurement to the customer is

$$V(x_1, x_2) = \int_{b(\beta-x_2)}^{\infty} v\{1-[1-G_0(v+x_1)][1-G_0(b^{-1}(v)+x_2)]^{n-1}\}dF(v) + \int_{\alpha}^{b(\beta-x_2)} vG_0(v+x_1)dF(v)$$

and the marginal value of investment is

$$\begin{aligned} V_x(x_1, x_2) &= \int_{b(\beta-x_2)}^{\infty} v[1 - G_0(b^{-1}(v) + x_2)]^{n-1} g_0(v + x_1)df(v) + \int_{\alpha}^{b(\beta-x_2)} vg_0(v + x_1)dF(v) \\ &= \int_{b(\beta-x_2)}^{\infty} v[1 - G_0(b^{-1}(v) + x_2)]^{n-1} g_0(v + x_1)dF(v) + \int_{\alpha}^{b(\beta-x_2)} vg_0(v + x_1)dF(v) \end{aligned}$$

Marginal reduction in procurement cost

Conditional marginal effects The expected procurement cost conditional of $v \geq b(\beta - x)$ is

$$\begin{aligned}
\theta(v; x_1, x_2) &= \int_{\beta - x_2}^{b^{-1}(v)} b(c) dP(c; x_1, x_2) + \int_{\beta - x_1}^{b(\mu_N)} c dG_0(v + x_1) \\
&= (n-1) \int_{\beta - x_2}^{b^{-1}(v)} b(c)[1 - F(b(c))][1 - G_0(v + x_2)]^{n-2} dG_0(v + x_2) \\
&\quad + \int_{\mu_N}^{b^{-1}(v)} b(c)[1 - G_0(v + x_2)]^{n-1} dG_0(b(v) + x_1) + \int_{\mu_I}^{b(\mu_N)} c dG_0(v + x_1) \\
&= (n-1) \int_{\beta - x_2}^{b^{-1}(v)} b(c)][1 - G_0(v + x_2)]^{n-2} dG_0(v + x_2) \\
&\quad - \int_{\mu_N}^{b^{-1}(v)} b'(c)[1 - G_0(v + x_2)]^{n-1} G_0(b(v) + x_1) dc \\
&\quad + v[1 - G_0(b^{-1}(v) + x_2)]^{n-1} G_0(v + x_1) - \int_{\mu_I}^{b(\mu_N)} G_0(c + x_1) dc
\end{aligned}$$

where

$$P(c; x_1, x_2) = 1 - [1 - G_0(b(c) + x_1)][1 - G_0(v + x_2)]^{n-1}$$

Therefore, the conditional marginal return of investment by the integrated firm is

$$\begin{aligned}
\theta_x(v;x_1,x_2) &= -\int\limits_{\beta-x_2}^{b^{-1}(v)} b'(c)[1-G_0(b^{-1}(v)+x_2)]^{n-1}g_0(b(v)+x_1)dc \\
&\quad +v[1-G_0(b^{-1}(v)+x_2)]^{n-1}[1-g_0(b^{-1}(v)+x_2)]^{n-1}-\int\limits_{\mu_I}^{b(\mu_N)} g_0(c+x_1)dc \\
&= -\int\limits_{\beta-x_2}^{b^{-1}(v)} [1-G_0(b^{-1}(v)+x_2)]^{n-1}dG_0(b(v)+x_1) \\
&\quad +v[1-G_0(b^{-1}(v)+x_2)]^{n-1}g_0(c+x_1)-G_0(b(\beta-x_2)+x_I) \\
&= -(n-1)\int\limits_{\mu_N}^{b^{-1}(v)} G_0(b(c)+x_I)[1-G_0(v+x_2)]^{n-2}dG_0(v+x_2)- \\
&\quad G_0(v+x_1)[1-G_0(b^{-1}(v)+x_2)]^{n-1}+v[1-G_0(b^{-1}(v)+x_2)]^{n-1}g_0(v+x_1) \\
&= -(n-1)\int\limits_{\mu_N}^{b^{-1}(v)} G_0(b(c)+x_I)[1-G_0(v+x_2)]^{n-2}dG_0(v+x_2) \\
&\quad -[1-G_0(b^{-1}(v)+x_2)]^{n-1}[G_0(v+x_1)-vg_0(v+x_1)]
\end{aligned}$$

The expected procurement cost conditional on $b(\beta-x)\geq v\geq\alpha$ is

$$\begin{aligned}
\theta(v;x_1,x_2) &= \int\limits_{\beta-x_1}^{v} cdG_0(v+x_1) \\
&= vG_0(v+x_1)-\int\limits_{\beta-x_1}^{v} G_0(v+x_1)dc
\end{aligned}$$

and the corresponding marginal return to investment is

$$\theta_x(v;x_1,x_2)=vg_0(v+x_1)-G_0(v+x_1)$$

Unconditional marginal effect Therefore, if $b(\beta-x_2)>\alpha>\beta-x_1$, equilibrium expected procurement cost is

$$\Theta(x_1,x_2)=\int\limits_{\alpha}^{\infty}\theta(v;x_1,x_2)dF(v)$$

and the marginal effect of investment by the integrated supplier is

$$
\begin{aligned}
\Theta_x(x_1, x_2) &= \int_{\alpha}^{\infty} \theta_x(v) dF(v) \\
&= -(n-1) \int_{b(\beta-x_2)}^{\infty} \int_{\beta-x_2}^{b^{-1}(v)} G_0(b(c)+x_1)[1-G_0(c+x_2)]^{n-2} dG_0(c+x_2) dF(v) \\
&\quad - \int_{b(\beta-x_2)}^{\infty} [1-G_0(b^{-1}(v)+x_2)]^{n-1}[G_0(v+x_1) - vg_0(v+x_1)]\} dF(v) \\
&\quad - \int_{\alpha_d}^{b(\mu_N)} [G_0(v+x_1) - vg_0(v+x_1)] dF(v) \\
&= -(n-1) \int_{\mu_N}^{\infty} [1-F(b(c))] G_0(b(c)+x_1)[1-G_0(c+x_2)]^{n-2} dG_0(c+x_2) \\
&\quad - \int_{b(\beta-x_2)}^{\infty} [1-G_0(b^{-1}(v)+x_2)]^{n-1}[G_0(v+x_1) - vg_0(v+x_1)]\} dF(v) \\
&\quad - \int_{\alpha_d}^{b(\beta-x_2)} [G_0(v+x_1) - vg_0(v+x_1)] dF(v) \\
&\quad = -(n-1) \int_{\mu_N}^{\infty} [1-F(b(c))] G_0(b(c)+x_1)[1-G_0(c+x_2)]^{n-2} dG_0(c+x_2) \\
&\quad - \int_{b(\beta-x_2)}^{\infty} [1-G_0(b^{-1}(v)+x_2)]^{n-1} G_0(v+x_1) dF(v) - \int_{\alpha}^{b(\beta-x_2)} G_0(v+x_1) dF(v) \\
&\quad + v[1-G_0(b^{-1}(v)+x_2)]^{n-1} g_0(v+x_1)] dF(v) + \int_{\alpha}^{b(\mu_N)} vg_0(v+x_1)] dF(v)
\end{aligned}
$$

Marginal return from investment Therefore, the net marginal value of investment for the integrated supplier is

$$
\begin{aligned}
& V_x(x_1,x_2) - \Theta_x(x_1,x_2) \\
= \; & (n-1)\int_{\beta-x_2}^{\infty}[1-F(b(c))]G_0(b(c)+x_I)[1-G_0(c+x_2)]^{n-2}dG_0(c+x_2) \\
& + \int_{b(\beta-x_2)}^{\infty}[1-G_0(b^{-1}(v)+x_2)]^{n-1}G_0(v+x_1)dF(v) + \int_{\alpha}^{b(\beta-x_2)}G_0(v+x_1)dF(v) \\
= \; & (n-1)\int_{\beta-x_2}^{\infty}[1-F(b(c))]G_0(b(c)+x_I)[1-G_0(c+x_2)]^{n-2}dG_0(c+x_2) \\
& + \int_{\beta-x_2}^{\infty}[1-G_0(v+x_2)]^{n-1}G_0(b(c)+x_I)dF(b(v) + \int_{\alpha}^{b(\beta-x_2)}G_0(v+x_1)dF(v) \\
= \; & (1-F(b(\beta-x_2)))G_0(b(\beta-x_2)+x_I) \\
& + \int_{\beta-x_2}^{\infty}[1-F(b(c))][1-G_0(v+x_2)]^{n-1}dG_0(b(c)+x_1) \\
& + \int_{\alpha}^{b(\beta-x_2)}G_0(b(c)+x_1)dG(v) \\
= \; & \int_{\beta-x_2}^{\infty}[1-F(b(c))][1-G_0(v+x_2)]^{n-1}dG_0(b(c)+x_1) \\
& + \int_{\alpha}^{b(\beta-x_2)}[1-G_0(v+x_1)]dF(v) + G_0(\alpha+x_1)
\end{aligned}
$$

In equilibrium, this is equated to marginal investment cost $\Psi'(x_1)$.

9.5.2 Investment incentives of independent suppliers

By analogy to our previous analysis for a symmetric non-integrated equilibrium, each of the $n-1$ independent suppliers symmetrically equates its marginal cost of investment to

$$\int_{\beta-x_2}^{\infty}[1-F(b(c))][1-G(b(c)][1-G_0(c+x_2)]^{n-2}dG_0(c+x_2)$$

9.5.3 Exponential-quadratic model

For the exponential case, the first-order condition for an independent supplier simplifies to

$$x_2 = \frac{1}{a}\frac{\mu}{n\mu+\lambda}e^{-\lambda(\beta-\alpha-x_2)-\mu(x_1-x_2)-\frac{\lambda+\mu}{\lambda+(n-1)\mu}} \tag{3}$$

Furthermore, it is not hard to show that

$$\begin{aligned} & \int_{\beta-x_2}^{\infty}[1-F(b(c))][1-G_0(v+x_2)]^{n-1}dG_0(b(c)+x_1) \\ = & \int_{\beta-x_2}^{\infty}[1-F(b(c))][1-G_0(b(c)+x_1)][1-G_0(c+x_2)]^{n-2}dG_0(c+x_2) \end{aligned}$$

which means for the exponential case

$$\begin{aligned} x_1-x_2 &= \frac{1}{a}\left[\int\limits_{\alpha}^{b(\beta-x_2)}[1-G_0(v+x_1)]dF(v)+G_0(\alpha+x_1)\right] \\ &= \frac{\mu}{a(\lambda+\mu)}\left[1-e^{-\lambda(\beta-\alpha-x_2)-\frac{\lambda+\mu}{\lambda+(n-1)\mu}+\mu(x_2-x_1)}\right] \\ &+ \frac{\lambda}{a(\lambda+\mu)}\left[1-e^{\mu(\beta-\alpha-x_1)}\right]. \end{aligned} \tag{4}$$

Numerical Computations For the numerical computations, it is useful to re-arrange (3) to get

$$e^{-\mu(x_1-x_2)}=x_2\frac{a(n\mu+\lambda)}{\mu}e^{\lambda(\beta-\alpha-x_2)+\frac{\lambda+\mu}{\lambda+(n-1)\mu}}.$$

which after taking logs gives us

$$x_1-x_2=-\frac{1}{\mu}\left[\ln(x_2)+\ln\left(\frac{a(n\mu+\lambda)}{\mu}\right)+\lambda(\beta-\alpha-x_2)+\frac{\lambda+\mu}{\lambda+(n-1)\mu}\right]. \tag{5}$$

Inserting this expression for x_1-x_2 into (4) we get an equation that depends on x_2 only:

$$\begin{aligned} - & \frac{1}{\mu}\left[\ln(x_2)+\ln\left(\frac{a(n\mu+\lambda)}{\mu}\right)+\lambda(\beta-\alpha-x_2)+\frac{\lambda+\mu}{\lambda+(n-1)\mu}\right] \\ = & \frac{\mu}{\mu+\lambda}\left[1-x_2\frac{a(n\mu+\lambda)}{\mu}\right]+\frac{\lambda}{\mu+\lambda}\left[1-x_2\frac{a(n\mu+\lambda)}{\mu}e^{-(\mu+\lambda)(\alpha-\beta+x_2)+\frac{\lambda+\mu}{\lambda+(n-1)\mu}}\right]. \end{aligned}$$

Uniqueness of the best response functions In analogy to the arguments used in the proof of Proposition 3, the best response of a representative nonintegrated supplier (x) to a symmetric investment by the nonintegrated rivals ($\bar{x}$)

and the integrated supplier (x_1) satisfies

$$\begin{aligned}
ax &= \int_{\beta-x}^{\infty} [1-F(b(c))][1-G_0(b(c)+x_1)][1-G_0(c+\bar{x})]^{n-1} dG_0(c+x) \\
&= \int_{\beta}^{\infty} [1-F(b(c-x))][1-G_0(b(c-x)+x_1)][1-G_0(c-x+\bar{x}-x)]^{n-1} dG_0(c) \\
&= e^{-[\lambda+(n-1)\mu]x} \int_{\beta}^{\infty} [1-F(b(c))][1-G_0(b(c)+x_1)][1-G_0(c+\bar{x})]^{n-1} dG_0(c)
\end{aligned}$$

which is equivalent to

$$axe^{[\lambda+(n-1)\mu]x} = \int_{\beta}^{\infty} [1-F(b(c))][1-G_0(b(c)+x_1)][1-G_0(c+\bar{x})]^{n-1} dG_0(c)$$

The left-hand-side is a non-negative and increasing convex function of x, equal 0 at $x = 0$ and going to ∞ as $x \to \infty$. The right-hand-side is a positive and decreasing function of $\bar{x}$. It follows that a unique best response is a continuously decreasing function of $\bar{x}$. Furthermore, a fixed point $x = \bar{x}$ of this response function exists and is a symmetric best response.

To establish uniqueness of the best response of the integrated firm to any $x_2 \geq 0$, observe first that the lefthand side of (4) is trivially linear in x_1 and equal to 0 at $x_1 = x_2$. The righthand side, in contrast, is increasing and concave in x_1 and greater than x_2 at $x_1 = x_2$. This can be established, for example, by inspecting the equality preceding (4). Moreover, at $x_1 = 0$, the lefthand side is $-x_2 \leq 0$ while the righthand side is positive. This follows from inspecting the righthand side of the equality preceding (4). Therefore, for any $x_2 \geq 0$ there is a unique best response $x_1(x_2)$. Moreover, this best response satisfies $x_1(x_2) > x_2$.

Equilibrium Existence The proof that an equilibrium of the form described in the proposition exists given the parameter restrictions imposed in the proposition rests on the following properties of the best response functions:

1. $x_2(\beta - \alpha) \geq \beta - \alpha$
2. $x_2(x_1)$ is decreasing in x_1 for $x_1 \in [\beta - \alpha, 1/a]$
3. $x_2(1/a) > 0$
4. $x_1(x_2) \geq x_2$ for all $x_2 \in [0, \beta - \alpha + \frac{1}{\lambda+(n-1)\mu}]$
5. $x_1(0) \leq 1/a$.

Given that the best response functions $x_1(x_2)$ and $x_2(x_1)$ derived above are continuous, these properties then imply that the best response functions have a fixed point on the set $[0, \beta - \alpha + \frac{1}{\lambda+(n-1)\mu}] \times [\beta - \alpha, 1/a]$. We now establish these properties in turn.

1. Evaluating the lefthand side and the righthand side of the equation that defines x_2 at $x_2 = \beta - \alpha$ and $x_1 = \beta - \alpha$, we get

$$\beta - \alpha \leq \frac{1}{a} \frac{\mu}{n\mu + \lambda} e^{-\frac{\lambda+\mu}{\lambda+(n-1)\mu}} \tag{6}$$

under the parameter restrictions we imposed. Moreover, taking derivatives we get

$$1 > \frac{\lambda + \mu}{n\mu + \lambda} \frac{\mu}{a} e^{-\frac{1}{\lambda+(n-1)\mu}},$$

where 1 is the derivative of the lefthand side of (6) and $\frac{\lambda+\mu}{n\mu+\lambda} \frac{\mu}{a} e^{-\frac{1}{\lambda+(n-1)\mu}}$ is the derivative of the righthand side of (6) evaluated at $x_1 = x_2 = \beta - \alpha$. This means that $x_2(\beta - \alpha) > \beta - \alpha$ because the conditions imply that at $\beta - \alpha$ we are to the left of the first point of intersection of the linear lefthand side and the increasing and convex righthand side.

2. Taking the total derivative of $x_2(x_1)$ defined in (3) with respect to x_1 gives

$$\frac{\partial x_2}{\partial x_1} = -\mu \frac{x_2}{1 - (\lambda + \mu) x_2}. \tag{7}$$

This is negative if $x_2(x_1) < \frac{1}{\lambda+\mu}$. Evaluating next $x_2(x_1)$ at the lower bound for x_1, i.e. at $x_1 = \beta - \alpha$, we get exactly the same conditions that emerge in Proposition 3. By the same logic as there it now follows that the best response x_2 is smaller than $\beta - \alpha + \frac{1}{\lambda+(n-1)\mu}$. Thus, $\beta - \alpha + \frac{1}{\lambda+(n-1)\mu} < \frac{1}{\lambda+\mu}$ is sufficient for $x_2(x_1)$ to be decreasing in x_1 for $x_1 \geq \beta - \alpha$.

3. $x_2(1/a) > 0$ follows because x_2 is positive.

4. $x_1(x_2) = x_2 + \frac{1}{a} \frac{\mu}{\lambda+\mu} e^{\mu(\beta-\alpha-x_1(x_2)} \left[1 - e^{-(\lambda+\mu)(\beta-\alpha-x_2+\frac{1}{\lambda+(n-1)\mu})}\right]$, where the term in brackets is positive if and only if $x_2 \leq \beta - \alpha + \frac{1}{\lambda+(n-1)\mu}$.

5. Plugging $x_2 = 0$ into $x_1(x_2)$ gives $x_1(0) = \frac{1}{a} \frac{\mu}{\lambda+\mu} e^{-\mu(x_1(0)-(\beta-\alpha)}[1 - e^{-\mu(x_1(0)-(\beta-\alpha))} \left[1 - e^{-(\lambda+\mu)(\beta-\alpha+\frac{1}{\lambda+(n-1)\mu}}\right]$, which is less than $1/a$.

9.6 Proof of Proposition 6

Observe that

$$\Phi(n, \mu) = \frac{1}{n\mu} \left[1 - (n-1)\Delta_I \left(1 - \mu + \frac{\mu}{2} \left(\frac{n-1}{n} \Delta_I + \frac{2}{n}\right)\right)\right]$$

because $\Delta_I = \frac{n}{n-1} \left(x_I - \frac{1}{n}\right)$. Evaluated at $\mu = 1$, we get

$$\Phi(n, 1) = \frac{1}{2n^2} \left[2n - (n-1)\Delta_I[(n-1)\Delta_I + 2]\right].$$

Observe that $\Phi(n,1)$ is decreasing in Δ_I and equal to 0 at $\Delta_I = \frac{1-n+\sqrt{1-3n^2+2n^3}}{(n-1)^2} := \hat{\Delta}(n)$. Therefore, vertical integration is stable if and only if $\Delta_I < \hat{\Delta}(n)$. For $n = 2$, we get $\hat{\Delta}(2) > 1$, which is sufficient to prove the result because $\Delta_I \in [0,1]$ for any n. (Moreover, for $n = 3$, one gets $\Delta_I = 0.73 < 0.82 = \hat{\Delta}(3)$.)

The following argument will be used in the remainder of this proof: Because the lefthand side of the equation $\Delta_I = 1 - e^{-\mu\Delta_I - \frac{1}{n-1}}$ is linearly increasing in Δ_I while the righthand side is increasing and concave in Δ_I, the positive root Δ_I of this equation is larger than y whenever $y < 1 - e^{-y-\frac{1}{n-1}}$ holds and smaller than y whenever $y > 1 - e^{-y-\frac{1}{n-1}}$ holds. Therefore, showing that $\hat{\Delta}(n) < \Delta_I$ is equivalent to showing that $\hat{\Delta}(n) < 1 - e^{-\hat{\Delta}(n)-\frac{1}{n-1}}$. This is, for example, the case for $n = 9, 10, 11, ...$

To prove the statement for μ close to 0, observe first that the sign of $\Phi(n,\mu)$ is the same as the sign of $\frac{1}{n} - (1-\mu)(x_I - 1/n) - \frac{\mu}{2}(x_I - 1/n)(x_I + 1/n)$, whose sign, as μ goes to 0, is the same as the sign of $1 - (n-1)\Delta_I$. It thus suffices to show that $\lim_{\mu\to 0} \Delta_I < \frac{1}{n-1}$. But this is certainly the case because $\frac{1}{n-1} > 1 - e^{-1/(n-1)}$ holds for any $n \geq 2$. This completes the proof.

Figure 1:
Profitability of vertical divestiture with inelastic demand

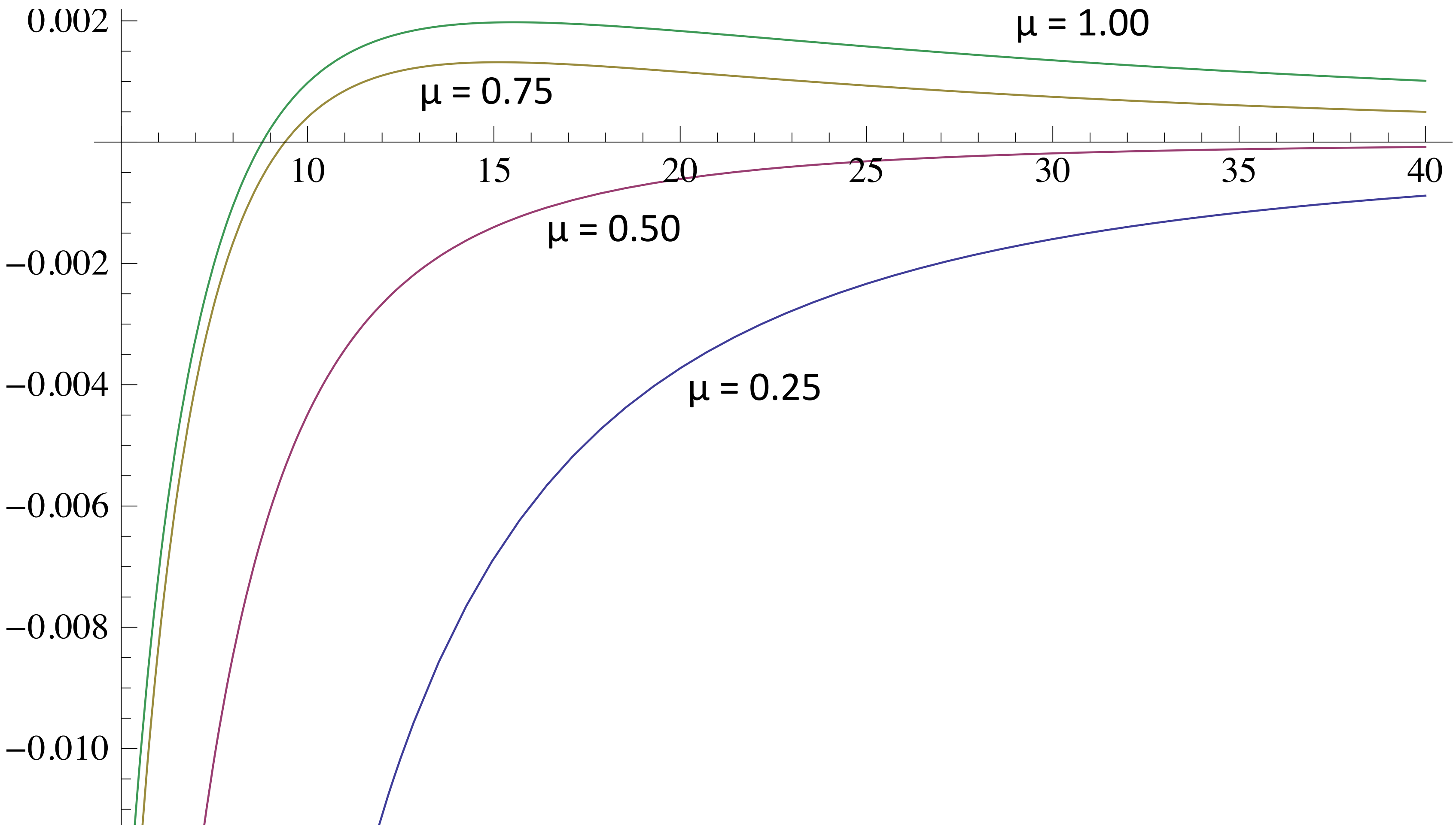

Figure 2:
Critical α for symmetric non-integration equilibrium with b(c) ≥ α

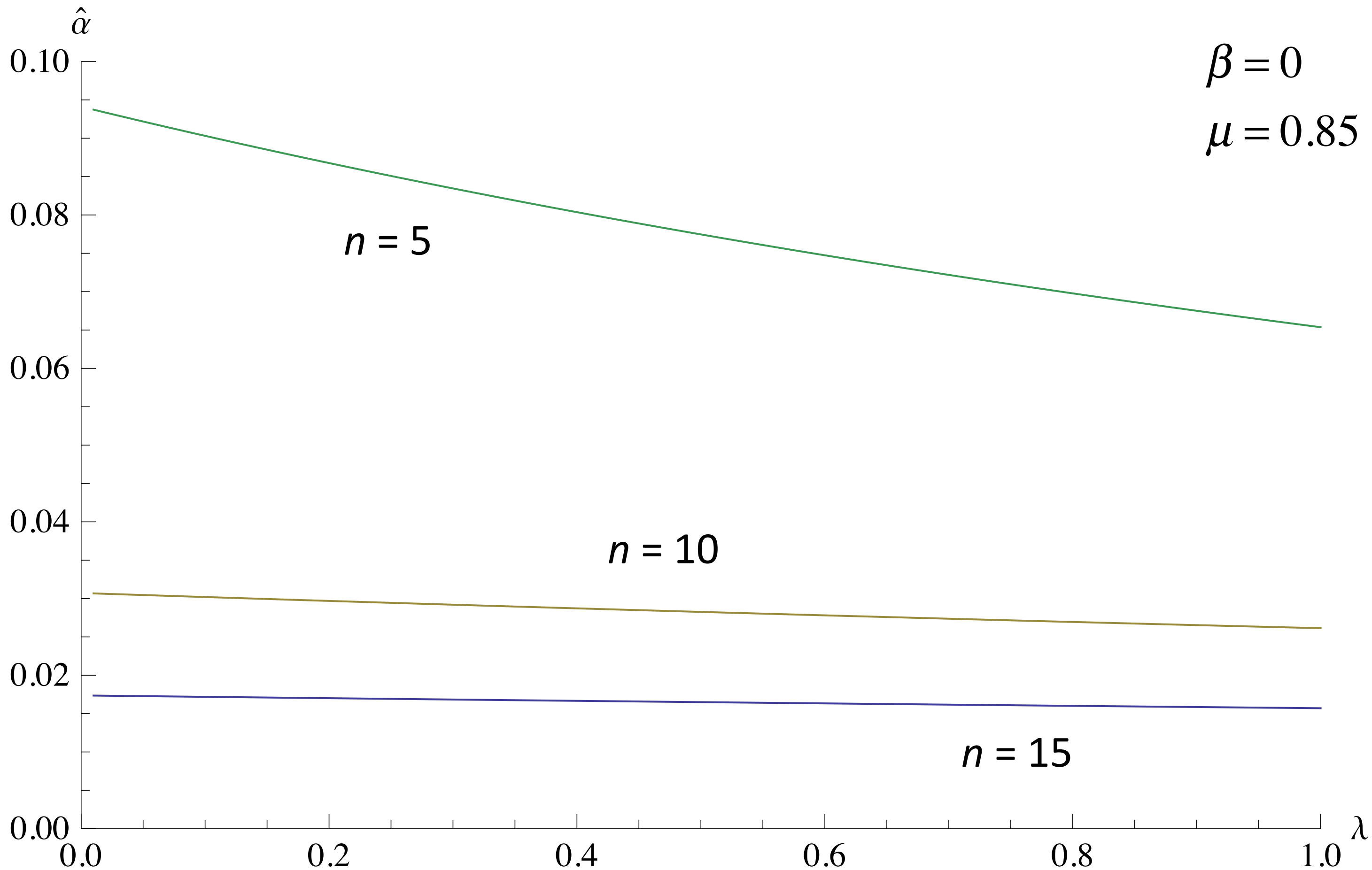

Figure 3:
Profitability vertical integration with elastic demand