Preferences, Prices, and Performance in Multiproduct Industries*

Yongmin Chen† and Michael H. Riordan‡

July 22, 2011

Abstract: This paper develops a new approach to discrete choice demand for multiproduct industries, using copulas to separate the marginal distribution of consumer values for each product from their dependence relationship. The comparative statics of demand strength and preference diversity, both properties of the marginal distribution, are remarkably similar across market structures, revealing unifying principles of industry conduct and performance. Preference dependence, disentangled from preference diversity as a distinct indicator of product differentiation, is a key determinant of how prices differ between multiproduct industries and single-product monopoly. Sufficient conditions are found under which multiproduct monopoly or symmetric single-product oligopoly prices are above or below the single-product monopoly price.

Keywords: Product differentiation, multiproduct industries, discrete choice demand, preference dependence, copula.

†University of Colorado at Boulder; yongmin.chen@colorado.edu
‡Columbia University; mhr21@columbia.edu

*Earlier versions, circulated under the titles "Preference and Equilibrium in Monopoly and Duopoly" and "Preferences, Prices, and Performance in Monopoly and Duopoly", were presented at the 2009 Summer Workshop in Industrial Organization (University of Auckland), the 2009 Summer Workshop on Antitrust Economics and Competition Policy (SHUFE, Shanghai), 2009 IO Day Conference (NYU), Segundo Taller de Organización Industrial (Chile, 2009), the 2010 Choice Symposium (Key Largo), and seminars at Ecole Polytechnique/CREST, Fudan University, Lingnan University, Mannheim University, National University of Singapore, University of Melbourne, and University of Rochester. The authors thank conference and seminar participants, especially discussants Simon Anderson and Barry Nalebuff, and two referees for useful comments.

1. INTRODUCTION

This paper develops a new framework for studying how consumer preferences determine equilibrium conduct and performance in multiproduct industries. At the heart of this new framework is the copula approach to modelling the distribution of consumer preferences in a discrete choice model of product differentiation. This approach separates the effects of the marginal distributions of consumer values for a product variety from the dependence relations between varieties captured by a copula. Our analysis uncovers several unifying principles of equilibrium pricing and market performance, indentifies an important new dimension along which product differentiation can be measured and fruitfully analyzed, and establishes new general results on how prices differ between multiproduct industries and single product monopolies.

We consider a horizontally differentiated industry with symmetric varieties of a good. Consumer values for the varieties are distributed according to a continuous multivariate distribution. The theory of copulas in statistics allows us to represent the joint distribution by the marginal distribution and a copula. We focus on three dimensions of consumer preferences: demand strength, preference diversity, and preference dependence. Demand strength and preference diversity are measured respectively by the mean (μ) and variance (σ) of the marginal distribution of consumer values for each variety. Preference dependence is captured by the copula, which may have the property of positive dependence, independence, or negative dependence. When a copula family is ordered by a parameter (θ) according to conditional stochastic dominance, we employ this parameter to measure preference dependence. Loosely speaking, greater preference dependence means more consumers regard the different varieties to be close substitutes. Thus preference dependence is an intuitive measure of the degree of product differentiation.

The copula framework advances the standard approach to product differentiation in discrete choice demand models by relaxing in a neat way the typical assumption of independent consumer values for alternative products. Under general conditions of preference dependence, we re-examine firm conduct and market performance in multiproduct industries, considering both monopoly and symmetric oligopoly market structures. We also characterize outcomes under single-product monopoly, both as a benchmark for comparisons and to elucidate our results for multiproduct industries.

The strategic variables are prices, which firms choose simultaneously under oligopoly competition.

We show how demand strength and preference diversity affect equilibrium prices, profits, and consumer welfare under general conditions of preference dependence. First, prices, profits and consumer surplus are all higher with stronger demand. Second, firm profits increase in preference diversity for "low-demand" products ($\mu \leq 0$); while for "high-demand" products ($\mu > 0$) profits exhibit a U-shaped relationship with σ, first decreasing and then increasing. Third, prices and consumer surplus both increase in preference diversity if $\mu \leq 0$. These comparative-static results are similar across multiproduct industry structures and under various preference dependence conditions, providing unifying principles of industry conduct and performance.[1] Moreover, the effects of preference diversity on profits clarify a key result in Johnson and Myatt (2006) for "variance-ordered distributions" for a single-product monopoly, and extend their insights to horizontally-differentiated multiproduct monopoly and to horizontally-differentiated price-setting oligopoly allowing for general preference dependence.[2]

We also consider how preference dependence affects prices and profits in multiproduct industries. The standard approach to discrete-choice models of differentiated oligopoly pioneered by Perloff and Salop (1985) typically considers preference diversity as an indicator of product differentiation under the assumption of independent consumer values for different varieties.[3] In our model, σ and θ are distinct indicators of product differentiation, and it is important to disentangle their effects in a general theory of product differentiation. Our analysis advances the theory on product differentiation by disentangling these two effects, and by providing sufficient conditions under which prices and profits in multiproduct industries decrease as preferences

[1] The results require appropriate regularity conditions for each market structure. The comparative statics of oligopoly prices and profits require somewhat stronger sufficient conditions because of the strategic interdependence of price decisions.

[2] Johnson and Myatt (2006) show that firm profit is maximized with either minimum or maximum preference diversity for a single-product monopolist, and, under the assumption of preference independence between varieties, for a multiproduct monopoly with vertically differentiated products and for a quantity-setting oligopoly. They develop insightful interpretations of this result for business strategy in areas such as advertising, marketing, and product design.

[3] Anderson, dePalma, and Thisse (1992) provides an excellent overview of discrete-choice models of product differentiation.

become more positively dependent or less negatively dependent.[4]

Our analysis further leads to two new results concerning how prices differ between multiproduct industries and single-product monopoly. First, extending Chen and Riordan (2008), we find that the single-product monopoly price is higher than the symmetric oligopoly price if the hazard rate of the marginal distribution is *non-decreasing* and preferences are *positively* dependent, but lower if the hazard rate is *non-increasing* and preferences are *negatively* dependent. Second, the symmetric multiproduct monopoly price is higher than the single-product monopoly price, provided that preferences possess a uniform dependence property, i.e., if preferences are either positively dependent, or independent, or negatively dependent.

The copula approach is more powerful than simply assuming a particular joint distribution for consumer values for alternative products. The bivariate normal distribution, for example, does have the virtue of neatly separating preference diversity (variance) and preference dependence (correlation) but is restrictive in part because the marginal normal distribution has a particular shape with an increasing hazard rate. Consequently, the bivariate normal distribution implies that duopoly price is below the monopoly price in the neighborhood of independence. The copula approach is able to retain the dependence propoerties of the bivariate normal distribution by assuming a Gaussian copula (Nelsen 2006) while allowing for alternative marginal distributions that lead to a different conclusion. While it is known that more product variety can sometimes result in higher or lower prices, the copula approach clarfies conditions under which this must be the case. By separating the properties of the marginal distribution from the dependence properties of the joint distribution, the more general copula approach clarifies which properties of the structure of preferences matter for particular comparative static results.

We formulate our main model in Section 2, which for expositional clarity and convenience contains only two product varieties. Section 3 establishes the comparative statics of preferences under various market structures, and Section 4 compares prices across market structures. Section 5 illustrates our findings and additional results by numerically analyzing a particular class of preferences described by exponential marginal distributions and the Fairlie-Gumbel-Morgenstern (FGM) family of copulas.

[4] As we noted above, preference diversity, the usual measure of product differentiation used in the literature under preference independence, has non-monotonic relations with profits if $\mu > 0$.

Section 6 concludes. An appendix contains proofs under the more general assumption of $n \geq 2$ product varieties, proving the results in the main model (where $n = 2$) and generalizing them to an arbitrary number of varieties.

2. PREFERENCES AND DEMAND

Each consumer is assumed to purchase at most one unit of two possible symmetric varieties of a good, referred to as X and Y. A consumer's value for X is $w_X = w(x)$ and for Y is $w_Y = w(y)$, where x and y is each uniformly distributed on $[0, 1]$, and $w(\cdot)$ is a strictly-increasing and twice-differentiable function with a subinterval on which $w(x) > 0$. The utility of the "outside good" is normalized to zero. If p is the price of X and r is the price of Y, then a type (x, y) consumer purchases X if $w(x) - p \geq \max\{w(y) - r, 0\}$ and Y if $w(y) - r > \max\{w(x) - p, 0\}$. If only X is available, then the consumer purchases it if $w(x) - p \geq 0$.

The population of consumers, whose size is normalized to 1, is described by a symmetric copula $C(x, y)$, which is a bivariate uniform distribution satisfying $C(x, 1) = x = C(1, x)$, and $C(x, 0) = 0 = C(0, x)$. For convenience, $C(x, y)$ is assumed to be twice differentiable on $[0, 1]^2 \equiv I^2$ with a joint density given by $C_{12}(x, y) \equiv \partial^2 C(x, y)/\partial x \partial y$. The copula determines the statistical dependence of consumer values for the two varieties. In particular, $C_1(x, y) \equiv \partial C(x, y)/\partial x$ is the conditional distribution of y given x, and $C_{11}(x, y) \equiv \frac{\partial^2 C(x,y)}{\partial x^2} < 0$ (> 0) indicates positive (negative) stochastic dependence. The independence copula is $C(x, y) = xy$. Positive (negative) stochastic dependence implies positive (negative) quadrant dependence, i.e. $C(x, y) > xy$ $(C(x, y) < xy)$.[5]

We parameterize the preference distribution along three dimensions: demand strength, preference diversity, and preference dependence. Let

$$\mu = \int_0^1 w(x) dx$$

and

$$\sigma^2 = \int_0^1 [w(x) - \mu]^2 \, dx$$

[5] Positive stochastic dependence in turn is implied by positive likelihood ratio dependence (called "affliation" in the economics literature), or, in the differentiable case, $\frac{\partial \ln C_{12}(u,v)}{\partial u \partial v} > 0$ (Nelsen, 2006).

denote the mean and variance of consumer values for each variety, and define the normalized utility $u(x) = \frac{w(x)-\mu}{\sigma}$. For given parameters μ and σ, the marginal distribution of consumer values for X is $F(\frac{w_X-\mu}{\sigma}) \equiv u^{-1}\left(\frac{w_X-\mu}{\sigma}\right)$, and similarly for Y.[6] The joint distribution of values can then be written as $C\left(F(\frac{w_X-\mu}{\sigma}), F(\frac{w_Y-\mu}{\sigma})\right)$.[7] A family of copulas, $C(x, y; \theta)$, indexed by parameter θ, satisfies the monotonic dependence ranking property (**MDR**) if $\partial C_{11}(x, y; \theta)/\partial\theta \equiv C_{11\theta}(x, y; \theta) < 0$ for interior (x, y). **MDR** implies $C_\theta(x, y; \theta) \equiv \partial C(x, y; \theta)/\partial\theta > 0$ (Nelsen, 2006).

Summarizing, the distribution of consumer preferences for the two goods is completely characterized by the marginal distribution function $F(u)$, the copula $C(x, y; \theta)$,[8] and parameters μ and σ which respectively measure demand strength and preference diversity. The copula indicates the nature of preference dependence, and, for copula families satisfying **MDR,** the parameter θ measures the degree of preference dependence. Introduced by Chen and Riordan (2008), the copula approach to product variety has the defining property of disentangling the effects of the marginal distribution of consumer values for each variety from their dependence relationship. The key feature of the copula approach is that a copula and marginal distributions completely characterize the joint distribrution of consumer values for varieties. Later in Section 5, we study numerically a class of bivariate exponential distributions based on the Failie-Gumbel-Morgenstern (FGM) copula family, which satisfies **MDR**.[9]

[Insert Figure 1 about here]

The demand for X and Y is illustrated in Figure 1, which partitions the consumer type space (I^2) into an acceptance set for X, an acceptance set for Y, and an acceptance set for the outside good, given normalized prices $\bar{p} \equiv \frac{p-\mu}{\sigma}$ and $\bar{r} \equiv \frac{r-\mu}{\sigma}$. A type

[6] The support for $F(\cdot)$ is extended in the usual way, i.e., $F(u) = 0$ for $u < u(0)$ and $F(u) = 1$ for $u > u(1)$, unless $u(0) = -\infty$ and/or $u(1) = \infty$.

[7] By Sklar's Theorem (Nelsen, 2006), it is without loss of generality to represent joint distribution of consumers' values for two products by a copula and marginal distributions. However, our symmetry assumption confines our analysis to symmetric joint distributions with the same marginal distributions for X and Y.

[8] The parameter θ is suppressed notationally for results that hold θ fixed or that do not require the **MDR** property.

[9] Although we assume a one-parameter family of copulas for expositional convenience, our analytic comparative results regarding dependence only require that copulas are ordered by stochastic dependence.

(x, y) consumer prefers X to the outside good if

$$x \geq F(\bar{p})$$

and X to Y if

$$x \geq F(u(y) + \bar{p} - \bar{r}).$$

The boundaries of the acceptance set for X are defined by replacing these inequalities with equalities over the relevant ranges. Accordingly, the demand function for X is calculated by integrating over the acceptance set:

$$\begin{aligned} Q(\bar{p}, \bar{r}; \theta) &= \int_{F(\bar{p})}^{1} \int_{0}^{F(u(x)+\bar{r}-\bar{p})} C_{12}(x, y; \theta)\, dy dx \\ &= \int_{F(\bar{p})}^{1} C_1(x, F(u(x) + \bar{r} - \bar{p}); \theta)\, dx. \end{aligned} \tag{1}$$

The demand for Y is calculated similarly. Thus, the two good are always substitutes because

$$\frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial \bar{r}} = \int_{F(\bar{p})}^{1} C_{12}(x, F(u(x) + \bar{r} - \bar{p}); \theta)\, f(u(x) + \bar{r} - \bar{p}) dx > 0. \tag{2}$$

If only good X is available, its demand is obtained by setting $F(\bar{r}) = 1$ in (1), or

$$Q^o(\bar{p}) = 1 - F(\bar{p}). \tag{3}$$

As discussed in Chen and Riordan (2008), the introduction of product Y at the same price has two effects on demand for product X. The "market share effect" shifts down the demand for product X as some consumers switch to a more attractive alternative, and the "price sensitivity effect" tilts the slope of the demand curve for product X as the identity of marginal consumers changes. These two effects combine to determine how the introduction of a second variety changes the price elasticity of demand for X at any given price.

The copula approach has the advantage of linking endogenous demand elasticities of classical demand theory to more fundamental properties of preference distributions. First, at any $p > 0$, the price elasticity of demand for a single available product is a function of demand strength and preference diversity:

$$\eta^X \equiv \left| \frac{p}{Q^o(\bar{p})} \frac{\partial Q^o(\bar{p})}{\partial p} \right| = \lambda \left(\frac{p-\mu}{\sigma} \right) \frac{p}{\sigma}, \tag{4}$$

where $\lambda(\bar{p}) \equiv \frac{f(\bar{p})}{1-F(\bar{p})}$ is the hazard rate (of the marginal distribution). η^X decreases with μ if and only if $\lambda(\bar{p})$ is increasing; and η^X decreases with σ if $p \geq \mu$ and $\lambda(\bar{p})$ is non-decreasing, or if $p \leq \mu$ and $\lambda(\bar{p})$ is non-increasing. Second, with both varieties available and with $\bar{r} = \bar{p}$, the cross- and own-price elasticities are

$$\eta^{XY*} = \left| \frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial r} \frac{r}{Q(\bar{p}, \bar{r}; \theta)} \right|_{\bar{r}=\bar{p}} = \frac{2p \int_{F(\bar{p})}^{1} C_{12}(x, x; \theta) f(u(x))\, dx}{\sigma \left[1 - C(F(\bar{p}), F(\bar{p}); \theta)\right]},$$

$$\eta^{XX*} = \left| \frac{\partial Q(\bar{p}, \bar{r}; \theta)}{\partial p} \frac{p}{Q(\bar{p}, \bar{r}; \theta)} \right|_{\bar{r}=\bar{p}} = \eta^{XY*} + \frac{2pC_1(F(\bar{p}), F(\bar{p}); \theta) f(\bar{p})}{\sigma \left[1 - C(F(\bar{p}), F(\bar{p}); \theta)\right]}.$$

Note that η^{XX*} depends on price sensitivity at both the intensive and extensive margins, i.e. both the density of consumers indifferent between X and Y and those indifferent between X and the outside good, whereas η^{XY*} only depends on price sensitivity at the intensive margin. A sufficient condition for η^{XY*} to increase as θ increases is

$$\int_t^1 C_{12\theta}(x, x; \theta) f(u(x))\, dx \geq 0 \tag{5}$$

for any $t \in [0, 1)$, where $C_{12\theta}(x, y; \theta) \equiv \partial C_{12}(x, y; \theta)/\partial\theta$. Condition (5) holds if greater preference dependence is associated with greater density of consumer preferences along the diagonal of the type space, meaning that more purchasing consumers value the two goods similarly and therefore are sensitive to a unilateral price cut starting from a symmetric situation. While $C_{12\theta}(x, x; \theta) \geq 0$ seems an intuitive property, it does not appear to be a general implication of **MDR**. The condition does hold for the FGM copula family studied in Section 5.[10]

3. COMPARATIVE STATICS

The copula approach enables us to derive new results on how prices, profits, and consumer welfare vary with the strength (μ), diversity (σ) and dependence (θ) of the distribution of consumer preferences. To proceed, we make a couple of additional simplifying assumptions for both this and the next section. First, the average cost of production for each variety is constant, and without loss of generality normalized to zero. An appropriate interpretation of the normalization is that consumers reimburse the firm for the cost of producing the product in addition to paying a markup p. Consequently, μ can be interpreted as standing for mean demand minus the constant average variable cost, and naturally can be either positive or negative.[11] Second, equilibrium prices exist uniquely and are interior under all market structures.[12] Together with the symmetry of $C(\cdot,\cdot)$, this simplifying assumption implies that equilibrium is symmetric. These maintained assumptions facilitate comparative statics and comparisons of outcomes for different market structures.

Demand Strength and Preference Diversity

We start with the familiar case of a single-product monopolist who produces X. While this does not require a consideration of preference dependence, our analyses of various cases nevertheless have a common structure in key respects, and the single-product monopoly case serves as a point of comparison for firm conduct. For these reasons, it is worthwhile to elucidate carefully the single-product monopoly problem.

[10]To understand the ambiguity, observe that $C_{12\theta}(x,x;\theta) = dC_{1\theta}(x,x;\theta)/dx - C_{11\theta}(x,x;\theta)$. While $-C_{11\theta}(x,x;\theta) > 0$ by MDR, $dC_{1\theta}(x,x;\theta)/dx$ may be either positive or negative. In the FGM case, $dC_{1\theta}(x,x;\theta)/dx < 0$ in an intermediate range of x, although $-C_{11\theta}(x,x;\theta) > 0$ still dominates in this range.

[11]This interpretation requires an appropriate adjustment of the elasticity formulas. Suppose consumers pay $p + c$ where $c \geq 0$ is the constant marginal cost of production. Then the single-product price elasticity of demand becomes $\lambda(\bar{p})\frac{p+c}{\sigma} \equiv \frac{p+c}{p}\eta^X$ with η^X given by (4). The other elasticity formulas require similar adjustment.

[12]For convenience, we refer to optimal prices under monopoly as equilibrium prices. An interior price satisfies $p \in (w(0), w(1))$, so the market neither shuts down nor is fully covered. Consequently, profit functions are differentiable at equilibrium prices.

Furthermore, our analysis of this base case clarifies and expands aspects of Johnson and Myatt (2006)'s seminal analysis of the effect of preference diversity on monopoly profit.

The structure of our analysis hinges on the price normalization introduced in our discussion of demand. The single-product monopolist's (gross) profit function is

$$\pi^m(\bar{p}) = \sigma(\bar{p} + \bar{\mu}) [1 - F(\bar{p})] \tag{6}$$

where $\bar{p}$ is the normalized price and $\bar{\mu} \equiv \frac{\mu}{\sigma}$ is the strength-diversity ratio. The profit-maximizing normalized price ($\bar{p}^m$) satisfies

$$(\bar{p}^m + \bar{\mu})\lambda(\bar{p}^m) = 1 \tag{7}$$

at an interior solution, where $\lambda(\bar{p})$ is the hazard rate determining the elasticity of demand. The following regularity condition guarantees a unique local maximum.

A1: $d[(\bar{p} + \bar{\mu})\lambda(\bar{p})]/d\bar{p} > 0$.

Thus the familiar assumption of a monotonically increasing hazard rate ($\lambda'(u) \geq 0$) is sufficient but not necessary for **A1**. The maximum profit is $\pi^m \equiv \pi^m(\bar{p}^m)$ and the consumer welfare is

$$w^m = \sigma \int_{\bar{p}^m}^{u(1)} [1 - F(\bar{p})] \, d\bar{p}. \tag{8}$$

Defining normalized profits and consumer surplus as $\overline{\pi}^m = \pi^m/\sigma$ and $\overline{w}^m = w^m/\sigma$, we first establish how normalized price, profit, and consumer welfare vary with the strength-diversity ratio.

[Insert Figure 2 about here]

Lemma 1 *Given **A1**: (i) $\frac{d\bar{p}^m}{d\bar{\mu}} < 0$, and there exists some $\bar{\mu}^m > 0$ such that $\bar{p}^m \gtreqless 0$ if $\bar{\mu} \lesseqgtr \bar{\mu}^m$; (ii) $\frac{d\overline{\pi}^m}{d\bar{\mu}} > 0$; and (iii) $\frac{d\overline{w}^m}{d\bar{\mu}} > 0$.*

The lemma characterizes the normalized price, profit, and consumer welfare effects of a shift in the strength-diversity ratio. Part (i) is apparent from Figure 2. From

A1, the function $(\bar{p}+\bar{\mu})\,\lambda(\bar{p})$ increases in $\bar{p}$ and intersects 1 once from below; an increase in $\bar{\mu}$ shifts up the entire function and moves the intersection point to the left; furthermore, as $\bar{\mu}$ increases, the intersection point eventually becomes negative. Part (ii) follows easily from applying the envelope theorem to (6), and part (iii) follows from simple differentiation and part (i).

Lemma 1 leads immediately to the following proposition establishing how single-product monopoly conduct and performance depend on demand strength and preference diversity.

Proposition 1 *Given **A1**: (i)* $\frac{dp^m}{d\mu} \gtreqqless 0$ *if* $\lambda'(p) \gtreqqless 0$; $\frac{d\pi^m}{d\mu} > 0$; $\frac{dw^m}{d\mu} > 0$*; (ii)* $\frac{dp^m}{d\sigma} > 0$ *if* $\bar{\mu} \leq \bar{\mu}^m$ *and* $\lambda'(\cdot) \geq 0$; $\frac{d\pi^m}{d\sigma} \gtreqqless 0$ *if* $\bar{\mu} \lesseqqgtr \bar{\mu}^m$; $\frac{dw^m}{d\sigma} > 0$ *if* $\mu \leq 0$.

As one might expect, profit and consumer welfare under single-product monopoly are both increasing in demand strength, and so is monopoly price if the hazard rate is increasing. If we interpret μ as mean consumer value net of constant marginal cost, then $-\frac{dp^m}{d\mu}$ is the pass-through rate, i.e. the rate at which an increase in cost translates to a higher markup. The amount of pass-through depends on the slope of the hazard rate, i.e. $\lambda' > (<)\ 0$ corresponds to less (more) than full pass-through as discussed by Weyl and Fabinger (2009).

Johnson and Myatt (2006) studied families of "variance-ordered" distributions for which demand strength is a differentiable function of preference diversity, i.e. $\mu = \mu(\sigma)$. An important result of their analysis under this assumption is that π^m is a quasi-convex function of σ if $\mu(\sigma)$ is weakly convex, which implies that a single-product monopolist seeks either to maximize or minimize preference diversity. Part (ii) of Proposition 1 details this result for the special case of constant demand strength, i.e. $\mu'(\sigma) = 0$, and clarifies for this case that the monopolist seeks to increase preference diversity when demand is relatively weak $(\mu < \sigma\bar{\mu}^m)$ but seeks to decrease diversity when demand is relatively strong $(\mu > \sigma\bar{\mu}^m)$. Thus, since $\bar{\mu}^m > 0$ from Lemma 1, π^m is increasing in σ for $\mu \leq 0$, but is a U-shaped function of σ when $\mu > 0$, first decreasing and then increasing.[13]

[13] Johnson and Myatt (2006) establishes a preference for extremes for an even broader family of distributions ordered by a decreasing sequence of "rotation" points. Johnson and Myatt (2006) also showed that q^m is a convex function of σ if $\mu(\sigma)$ is weakly convex. By equation (3), however, q^m is a decreasing function of $\bar{p}^m$. Therefore, because $dq^m/d\sigma = f(\bar{p}^m)\,[d\bar{p}^m/d\bar{\mu}]\,[\mu/\sigma^2]$, part (i) of

Our analysis goes further than Johnson and Myatt (2006) by explicitly considering the consumer welfare effects of demand strength and preference dispersion. Part (ii) of Proposition 1 shows that the firm's incentive to increase σ for weak-demand products ($\mu \leq 0$) coincides with the consumer interests. In those cases, even though higher σ leads to higher prices, it also leads to higher output and to higher average values for consumers who actually purchase. As a result, consumer welfare goes up. If $\mu > 0$, higher σ will still increase the values of consumers who purchase the product, but it now also reduces output, so the effect on consumer welfare is no longer clear cut. Nevertheless, if $\mu > 0$, σ is sufficiently large, and $\lambda'(\cdot) \geq 0$, it can be shown that a further increase in σ increases both profit and consumer welfare.

We next turn to a price-setting multiproduct monopoly producing both varieties of the good,[14] who optimally charges the same price for the two symmetric variants. Defining $\bar{p}$ and $\bar{\mu}$ as before, the multiproduct monopolist's profit function is

$$\pi^{mm}(\bar{p}) = \sigma(\bar{p} + \bar{\mu})\left[1 - C\left(F(\bar{p}), F(\bar{p})\right)\right]. \tag{9}$$

The profit-maximizing normalized price $\bar{p}^{mm}$ satisfies

$$(\bar{p}^{mm} + \bar{\mu})\,\lambda^C(\bar{p}^{mm}) = 1 \tag{10}$$

where

$$\lambda^C(\bar{p}) \equiv \frac{2C_1(F(\bar{p}), F(\bar{p}))}{1 - C(F(\bar{p}), F(\bar{p}))} f(\bar{p}). \tag{11}$$

A key observation is that $\lambda^C(\bar{p})$ is the hazard rate for the cumulative distribution function $F^C(\bar{p}) \equiv C(F(\bar{p}), F(\bar{p}))$ on support $[u(0), u(1)]$. This is the distribution function for the maximum order statistic for the pair of normalized consumer values, i.e. the probability that both $u(x)$ and $u(y)$ are less than $\bar{p}$.

The appropriate regularity condition, which is satisfied in the FGM-exponential case of Section 5, serves the same role as for the single-product monopoly case:

Lemma 1 clarifies for the constant mean case that q^m increases with σ if $\mu < \bar{\mu}^m$ and, conversely, q^m decreases with σ if $\mu > \bar{\mu}^m$.

[14] Johnson and Myatt (2006) studied a quantity-setting monopoly selling a line of vertically-differentiated products to a one-dimensional population of consumers. In contrast, we study a price-setting monopoly selling horizontally-differentiated products to a two-dimensional population.

A2: $d\left[\left(\bar{p}+\bar{\mu}\right)\lambda^{C}(\bar{p})\right]/dp>0.$

Note that the first-order condition (10) is the same as (7) for single-product monopoly, except for the different hazard rate function. Therefore, the comparative statics of $\bar{p}^{mm}$, $\overline{\pi}^{mm}$, and $\overline{w}^{mm}$ with respect to $\bar{\mu}$ follow from Lemma 1, with the maximum profit being $\pi^{mm}\equiv\sigma\overline{\pi}^{mm}\equiv\pi^{mm}\left(\bar{p}^{mm}\right)$, and consumer surplus

$$w^{mm}\equiv\sigma\bar{w}^{mm}=\int_{\bar{p}^{mm}}^{u(1)}\left[1-C\left(F(\overline{p}),F(\overline{p})\right)\right]d\bar{p}.$$

The comparative statics with respect to μ and σ are therefore the same as under single-product monopoly:

Proposition 2 *Given* ***A2****: there exists* $\bar{\mu}^{mm}>0$ *such that (i)* $\frac{dp^{mm}}{d\mu}\gtreqless 0$ *if* $\lambda^{C\prime}(p)\gtreqless 0$; $\frac{d\pi^{mm}}{d\mu}>0$; $\frac{dw^{mm}}{d\mu}>0$*; (ii)* $\frac{dp^{mm}}{d\sigma}>0$ *if* $\bar{\mu}\leq\bar{\mu}^{mm}$ *and* $\lambda^{C\prime}(\cdot)\geq 0$; $\frac{d\pi^{mm}}{d\sigma}\gtreqless 0$ *if* $\bar{\mu}\lesseqgtr\bar{\mu}^{m}$; $\frac{dw^{mm}}{d\sigma}>0$ *if* $\mu\leq 0$.

It is also apparent that Proposition 2 extends to the case of a multiproduct monopolist selling an arbitrary number of symmetric product varieties. If u_i is the normalized consumer value of variety i, with $i=1,...n$, then the joint distribution of $(u_1,...u_n)$ can be written as $C(F(u_1),...F(u_n))$ by Sklar's Theorem (Nelsen, 2006), where $C\left(x_1,...,x_n\right)$ is an n-dimension symmetric multivariate copula. The hazard rate for the distribution of the maximum order statistic then becomes

$$\lambda^{C}(\bar{p})\equiv\frac{nC_1(F\left(\bar{p}\right),...F\left(\bar{p}\right))}{1-C(F\left(\bar{p}\right),...F\left(\bar{p}\right))}f\left(\bar{p}\right). \tag{12}$$

Applying **A2** to this function yields the generalization to arbitrary $n\geq 2$.

We now suppose the two varieties are sold by symmetric single-product firms. The profit function of Firm X is $\pi^{d}\left(\bar{p},\bar{r}\right)=\sigma(\bar{p}+\bar{\mu})Q\left(\bar{p},\bar{r}\right)$. In equilibrium, $\bar{p}=\bar{r}=\bar{p}^{d}$, satisfying

$$\left(\bar{p}^{d}+\bar{\mu}\right)h(\bar{p}^{d})=1, \tag{13}$$

where we define the adjusted hazard rate under duopoly competition as

$$h(\bar{p}) \equiv \lambda^C(\bar{p}) + \frac{2}{1 - C(F(\bar{p}), F(\bar{p}))} \int_{F(\bar{p})}^{1} C_{12}(x, x) f(u(x)) \, dx, \tag{14}$$

which is the hazard rate under multiproduct monopoly adjusted by an extra term. The extra term measures the business-stealing effect when both firms charge the same price, i.e. the percentage demand increase from a price cut resulting from customers who change allegiance. Notice that if $\bar{p} \leq u(0)$, then $\lambda^C(\bar{p}) = 0$ and $h(\bar{p}) = 2\int_0^1 c(x, x) f(u(x)) \, dx$. Thus $u(0)$ is the critical price separating the equilibrium regimes of fully covered versus non-fully covered markets.

We modify the regularity condition for unique comparative statics:

A3: $d[(\bar{p} + \bar{\mu}) h(\bar{p})] / dp > 0$.

A3 is implied by $h'(\bar{p}) \geq 0$ and is also satisfied in the FGM-exponential case in Section 5. With the appropriate regularity condition in hand, the comparative static for symmetric duopoly are similar to those for multiproduct monopoly, thus further extending insights of Johnson and Myatt (2006) to horizontally-differentiated price-setting duopoly. Each firm's equilibrium profit and consumer welfare are, respectively:

$$\pi^d \equiv \sigma \bar{\pi}^d = \frac{1}{2} \sigma \left(\bar{p}^d + \bar{\mu}\right) \left[1 - C\left(F\left(\bar{p}^d\right), F\left(\bar{p}^d\right)\right)\right], \tag{15}$$

$$w^d \equiv \sigma \bar{w}^d = \sigma \int_{\bar{p}^d}^{u(1)} \left[1 - C(F(\bar{p}), F(\bar{p}))\right] d\bar{p}. \tag{16}$$

The equilibrium condition (13) has the same form as (7). Thus, the effects of $\bar{\mu}$ on $\bar{p}^d$ and $\bar{w}^d$ under duopoly is qualitatively similar to those for monopoly, whereas the effect on profits is similar under the sufficient condition of a non-decreasing adjusted hazard rate $h(\cdot)$:

Lemma 2 *Given **A3**: (i)* $\frac{d\bar{p}^d}{d\bar{\mu}} < 0$, *and there exists some* $\bar{\mu}^d > 0$ *such that* $\bar{p}^d \gtreqless 0$ *if* $\bar{\mu} \lesseqgtr \bar{\mu}^d$. *(ii)* $\frac{d\bar{\pi}^d}{d\bar{\mu}} > 0$ *if* $h'(\cdot) \geq 0$. *(iii)* $\frac{d\bar{w}^d}{d\bar{\mu}} > 0$.

The reason for the additional condition on $h(\cdot)$ to ensure $\frac{d\bar{\pi}^d}{d\bar{\mu}} > 0$ is the following. An increase in $\bar{\mu}$ directly increases demand, with a positive effect on profits; but it also has an indirect effect through adjustment in the rival's price that can potentially lower equilibrium price p^d.[15] If $h'(\cdot) \geq 0$, then $\frac{dp^d}{d\bar{\mu}} = \frac{d(\bar{p}^d + \bar{\mu})}{d\bar{\mu}} \geq 0$; hence higher $\bar{\mu}$ leads to higher p^d and higher π^d. The comparative statics of μ and σ follow straightforwardly with a similar form as for the monopoly cases.

Proposition 3 *Given **A3**: (i)* $\frac{dp^d}{d\mu} \gtreqless 0$ *if* $h'(p) \gtreqless 0$; $\frac{d\pi^d}{d\mu} > 0$ *if* $h'(\cdot) \geq 0$; $\frac{dw^d}{d\mu} > 0$. *(ii)* $\frac{dp^d}{d\sigma} > 0$ *if* $\bar{\mu} \leq \bar{\mu}^d$ *and* $h'(\cdot) \geq 0$; $\frac{d\pi^d}{d\sigma} \gtreqless 0$ *if* $\bar{\mu} \lesseqgtr \bar{\mu}^d$ *and* $h'(\cdot) \geq 0$; $\frac{dw^d}{d\sigma} > 0$ *if* $\mu \leq 0$.

This result also readily generalizes to $n \geq 2$ varieties by defining

$$h(\bar{p}) = \lambda^C(\bar{p}) + \frac{n(n-1)\int_{F(\bar{p})}^{1} C_{12}(x, ...x) f(u(x))\, dx}{1 - C(F(\bar{p}), ...F(\bar{p}))} \tag{17}$$

where $\lambda^C(\bar{p})$ is given by (12), and the business-stealing effect now reflects price sensitivity at the $n-1$ margins with the other varieties and a symmetric market share $\frac{1-C(F(\bar{p}),...F(\bar{p}))}{n}$ when all varieties are priced the same.

Preference Dependence

We further consider how equilibrium outcomes are affected by preference dependence under multiproduct monopoly and oligopoly. For this purpose we consider copula families satisfying **MDR**.

First consider a multiproduct monopoly offering two varieties. A useful property of an **MDR** copula family is that the function $C_1(x, x; \theta)$ increases (decreases) in θ when x is small (large). This implies that greater positive dependence shifts up the hazard rate for the multiproduct monopolist when market coverage is high enough.

Lemma 3 *Given **MDR**, there exists some* $u^* \in (u(0), u(1)]$ *such that* $\frac{\partial \lambda^C(\bar{p};\theta)}{\partial \theta} > 0$ *if* $\bar{p} \leq u^*$.

[15] For a multiproduct monopolist, this second effect is zero due to the envelope theorem because the business-stealing effect is internalized.

Furthermore, it is straightforward that the market is fully covered, or nearly so, if the strength-diversity ratio is sufficiently high.[16] This consideration leads to the conclusion that prices decrease with preference dependence if demand is sufficiently strong. The profit of the multiproduct monopolist, however, always decreases with greater dependence, whether or not price increases because of the resulting downward shift in demand.

Proposition 4 *Given **A2** and **MDR**: (i) for any θ, there exists some $\bar{\mu}^*$ such that $p^{mm} > u(0)$ when $\bar{\mu} = \bar{\mu}^*$ and $\frac{dp^{mm}}{d\theta} < 0$ if $\bar{\mu} \geq \bar{\mu}^*$; and (ii) $\frac{d\pi^{mm}}{d\theta} < 0$.*

Thus, a multiproduct monopolist prefers that consumer values for its two products are less positively (more negatively) dependent, when the strength-diversity ratio is sufficiently high. This is intuitive, since the more similar are product varieties the less valuable is a choice. Thus a higher θ reduces output at any given price and hence reduces equilibrium profit, while the effect of θ on equilibrium price is more subtle. The lower output under a higher θ motivates the firm to lower price, but the slope of the demand curve also changes with θ, possibly having an opposing effect on price. Both effects work in the same direction if demand is sufficiently strong. It is possible, however, that p^{mm} increases with θ if demand is sufficiently weak. For example, in the bivariate exponential case introduced above, numerical analysis shows that p^{mm} increases in θ if $\bar{\mu}$ is below a critical value.

The effect of greater preference dependence on consumer welfare (w^{mm}) is also ambiguous in general. On the one hand, a higher θ shifts down the demand curve, thus reducing consumer surplus at any price. On the other hand, a higher θ might result in a lower price, as when demand is sufficiently strong, which increases consumer surplus given the demand curve. However, if more dependence leads to higher prices, as it is sometimes the case when $\bar{\mu}$ is low, then greater dependence reduces consumer welfare. This general ambiguity persists even in the neighborhood of independence and for strong demand. For the bivariate exponential special case, however, numerical analysis in Section 5 shows that consumer welfare increases with preference dependence when demand is sufficiently strong.

[16] Let $\bar{\mu}^o = \frac{1}{f(u(0))} - u(0)$. Then the market is fully covered for $\bar{\mu} \geq \bar{\mu}^o$ and almost fully covered for $\bar{\mu} = \bar{\mu}^o - \epsilon$ and ϵ a small positive number. Our maintained interiority assumption implicitly assumes $\bar{\mu} < \bar{\mu}^o$.

Next consider duopoly. It is intuitive to expect that duopoly competition intensifies with more preference dependence, as more consumers regard the two varieties to be close substitutes. In general, however, the effect of preference dependence on prices and profits is ambiguous. As under multiproduct monopoly, the regularity condition is not enough to ensure that prices monotonically decrease with θ. For while a higher θ results in a lower output, motivating a lower price (market share effect), it also affects the slope of the residual demand curve, potentially providing an incentive to raise price (price sensitivity effect). Under duopoly, a unilateral marginal reduction in price affects the slope of a duopolist's residual demand on both an extensive margin (market expansion) and the intensive margin (business stealing). The ambiguity of the price sensitivity effect on the extensive margin explains why more substitutability between the two goods (e.g. $C_{12\theta}(x,x) \geq 0$) may not be sufficient to conclude that p^d decreases with θ. We next identify sufficient conditions under which p^d and π^d decrease with θ.

We first identify a lemma providing technical conditions that are sufficient for $\frac{\partial h(\bar{p})}{\partial \theta} > 0$, which, together with **A3,** immediately implies $\frac{d\bar{p}^d}{d\theta} < 0$.

Lemma 4 *Given **A2, A3** and **MDR**: $\bar{p}^d$ decreases in θ if*

$$h(u) + \frac{f'(u)}{f(u)} \geq 0 \tag{18}$$

and

$$\frac{d^2 \ln f(u)}{du^2} \geq \frac{2f(u(x))^2}{C_\theta(x,x;\theta)} C_{11\theta}(x,x;\theta). \tag{19}$$

Using the technical lemma, the next proposition identifies simple plausible sufficient conditions for $\frac{dp^d}{d\theta} < 0$ and $\frac{d\pi^d}{d\theta} < 0$. Part (i) extends Proposition 4 to the duopoly case when the market is fully or almost fully covered. A sufficiently strong demand ensures that higher θ increases price sensitivity on the extensive margin (as for multiproduct monopoly), and condition (5), under which the two varieties are closer substitutes for higher θ, ensures that the same on the intensive margin. The other parts of the proposition use Lemma 4 to verify more basic sufficient conditions. Part (ii) invokes positive dependence and limited log-curvature of the marginal density (e.g. when f is approximately uniform or exponential). Part (iii) invokes stronger log-curvature restrictions on the marginal density (e.g. when f is approximately uniform) without imposing restrictions on the copula.

Proposition 5 *Given* ***A2, A3*** *and* ***MDR****:* p^d *and* π^d *decrease in* θ *if one of the following conditions is satisfied: (i)* $\bar{\mu}$ *is sufficiently large and* (5) *holds at* $t = 0$*; (ii)* $C_{11} < 0$ *and* $\left|\frac{d^2 \ln f(x)}{dx^2}\right|$ *is sufficiently small; or (iii)* $\frac{d \ln f(x)}{dx}$ *and* $\frac{d^2 \ln f(x)}{dx^2}$ *both are not too negative.*

The standard discrete choice oligopoly theory of product differentiation, pioneered by Perloff and Salop (1985), typically assumes independence of values for different varieties. We contribute to this literature by showing that preference dependence is a useful indicator of product differentiation, separately from the effect of preference diversity. In fact, while profits increase in σ when μ is relatively small ($\mu < \sigma\bar{\mu}^d$), profits always monotonically decrease in θ under multiproduct monopoly, and profits also monotonically decrease in θ under duopoly for all μ when f is approximately uniform or when f is approximately exponential and C is positively dependent. The effect of more preference dependence on consumer welfare appears to be ambiguous generally, with a higher θ lowering both consumer demand and equilibrium prices. In the bivariate exponential special case, however, w^d increases in θ.

The comparative statics of preference dependence also generalize to cases of a multimarket monopoly selling $n \geq 2$ product varieties and n symmetric single-product oligopolists. For these generalizations, a multivariate copula $C(\mathbf{x};\theta)$ satisfies **MDR** if both $\partial C_{11}(\mathbf{x};\theta)/\partial\theta \equiv C_{11\theta}(\mathbf{x};\theta) < 0$ and $C_\theta(\mathbf{x}) \equiv \partial C(\mathbf{x};\theta)/\partial\theta > 0$ for interior $\mathbf{x} = (x_1, ...x_n)$.[17] With this modification, the only other differences in the statement of the results are that order statistic $\lambda^C(\bar{p};\theta)$ is defined according to (12), p^d and π^d are replaced with p^n and π^n to indicate n firm symmetric oligopoly, condition (19) generalizes to

$$\frac{d^2 \ln f(u)}{du^2} \geq \frac{nf(u(x))^2}{C_\theta(x,...x;\theta)} C_{11\theta}(x,...x;\theta) \tag{20}$$

and condition (5) generalizes to

$$\int_t^1 C_{12\theta}(x,...,x;\theta) f(u(x))\, dx \geq 0 \tag{21}$$

[17]If $n = 2$, $C_{11\theta}(\mathbf{x};\theta) < 0$ implies $C_\theta(x,y;\theta) > 0$ (Nelsen, 2006).

for any $t \in [0, 1)$, where $C_{12\theta}(x, ..., x; \theta) \equiv \partial C_{12}(x, ..., x; \theta)/\partial\theta$. The proofs in the appendix are developed accordingly.

Discussion

To summarize, there are several reasons why the comparative-statics of monopoly and oligopoly in a setting of general preference dependence are interesting.

First, the effects of preference strength (μ) and diversity (σ) on prices, profits and consumer welfare are remarkably similar across single-product monopoly and multiproduct market structures for any n and any dependence relationship, thus suggesting broad unifying principles of firm conduct and market performance in single and multiproduct industries. Additionally, the effects of σ on profits extend the seminal work of Johnson and Myatt (2006).

Second, the effects of preference dependence (θ) on prices and profits suggest a new way to think about product differentiation. Both σ and θ can be interpreted as indicators of the degree of product differentiation: higher σ indicates more heterogeneity of consumer values for each product, while higher θ indicates greater similarity of these values *between* products for a randomly chosen consumer.[18] They have different economic meanings and it is important to disentangle their effects in a general theory of product differentiation.

Third, the key preference parameters have intuitive interpretations. As argued by Johnson and Myatt (2006), firms can use advertising, marketing, and product design strategies to influence preference diversity (σ), and by extension demand strength (μ) and preference dependence (θ). The comparative statics of μ, σ, and θ can then have implications for business strategies in these and possibly other areas. For example, Proposition 5 loosely suggests that two competing single-product firms might have a mutual incentive to design or promote their products so that consumer values are less positively dependent or more negatively dependent. Furthermore, since government regulation and other interventions can influence firm strategies, the comparative statics may have implications for public policies. Finally, as discussed earlier, these

[18]Preference dispersion sometimes is interpreted as an indicator of horizontal differentiation under the assumption of independent values, becuase with greater dispersion more consumers regard the two goods to be substantially different.

fundamental preference parameters link to the price elasticities of classical demand theory, thus providing richer interpretations of the comparative static of consumer demand.

4. PRICE AND MARKET STRUCTURE

The copula approach also enables us to derive new results on how prices differ across market structures, disentangling the roles of the marginal distributions and the dependence relationship. This will provide important insights on how market structure affects firm conduct.

We start with comparing equilibrium prices under single-product monopoly and under duopoly. While Chen and Riordan (2008) finds a sufficient conditions for $p^m \gtreqless p^d$ when the marginal distribution is exponential (i.e. $\lambda'(\cdot) = 0$), it has been an open question how the prices compare for arbitrary marginal distributions, which we can now answer with the following result:

Proposition 6 *Given A**1** and **A3**: if $C_{11} < 0$ and $\lambda'(p) \geq 0$, then $p^m > p^d$; and if $C_{11} > 0$ and $\lambda'(p) \leq 0$, then $p^m < p^d$.*

Therefore, positive dependence and a non-decreasing hazard rate for the marginal distribution ensures that duopoly competition lowers prices; conversely, negative dependence and a non-increasing hazard rate ensures that competition raises prices.[19] It is noteworthy that these results depend on the marginal distribution of consumer values only through the normalized hazard rate, and are independent of demand strength and preference diversity.

[Insert Figure 3 about here]

Figure 3 illustrates the price-increasing competition result. With negative dependence and a non-increasing hazard, the function $h(\bar{p})$ is below $\lambda(\bar{p})$, $(\bar{p} + \bar{\mu})h(\bar{p})$ intersects the dotted horizontal line to the right of $(\bar{p} + \bar{\mu})\lambda(\bar{p})$, and, consequently $\bar{p}^d > \bar{p}^m$. The economic intuition for the result is as follows. A duopolist sells less output at the monopoly price, p^m, and thus a slight price reduction at p^m is less costly

[19] Chen and Riordan (2007) and Perloff, Suslow, and Sequin (1995) present more specific models of product differentiation in which entry can result in higher prices.

to the duopolist since it applies to a smaller output. This "market share effect" is a standard reason why one expects more competition to lower price. However, as Chen and Riordan (2008) discuss, there is a potentially offsetting "price sensitivity effect" when products are differentiated. Since a duopolist sells on a different margin from a monopolist, the slope of a duopolist's (residual) demand curve differs from the slope of the single-product monopolist's demand curve. Furthermore, greater negative dependence makes it more difficult for the duopolist to win over marginal consumers who value its own product less but its rival's product more. Similarly, a non-increasing hazard rate tends to put less consumer density on the duopolist's intensive margin, further reducing price sensitivity.[20] Together, negative dependence and a non-increasing hazard rate are sufficient for the price sensitivity effect to dominate the market share effect, resulting in a higher price under duopoly competition.[21]

The conditions specified in Proposition 6 obviously are not exhaustive. With sufficient negative dependence, price increasing competition can occur even if the the hazard rate is increasing. For example, the familiar Hotelling model with consumers uniformly distributed on a line, which features perfect negative dependence and an increasing hazard, exhibits a duopoly price above the single monopoly price when the market is not fully covered under monopoly. Furthermore, generalizations of the Hotelling model to an arbitrary number of firms (Chen and Riordan 2007; 2008) also exhibit price increasing competition under a similar condition.

Next, we compare the prices for the multiproduct monopoly with those under single-product monopoly and symmetric duopoly.

Proposition 7 *Given **A1-A3:** (i)* $p^{mm} > p^d$*; and (ii)* $p^{mm} > p^m$ *if* $C_{11}(x,x)$ *has a uniform sign for* $x \in (0,1)$*, i.e., for* $x \in (0,1)$*, either* $C_{11}(x,x) \geq 0$ *or* $C_{11}(x,x) = 0$*, or* $C_{11}(x,x) \leq 0$.

[20] The argument in the proof of Proposition 6 can be adapted to show more formally that, with $\lambda'(\cdot) \leq 0$, the (residual) demand curve of a duopolist, given by (1), is indeed steeper than that of the monopolist if $C(\cdot,\cdot)$ is negatively dependent, independent, or has sufficiently limited positive dependence.

[21] We emphasize that our result is about when the entry of a competitor with a differentiated product lowers or raises market price. As the next proposition will clarify, for the *same* number of products, competition *always* lowers price. Also, even when the introduction of a differentiated product by a competitor raises market price, consumers may still often benefit from the competition due to the increase in product variety (Chen and Riordan, 2009).

As one might expect, $p^{mm} > p^d$, or prices for two substitutes are higher under monopoly than under competition, as shown in Chen and Riordan (2008). The familiar intuition is that a multiproduct monopolist internalizes the negative effects of reducing one product's price on profits from the other product. The comparison of prices under multiproduct monopoly (p^{mm}) and single-product monopoly (p^m) is more subtle. The multiproduct monopolist has higher total output at p^m than the single-product monopolist, which motivates it to raise its symmetric price above p^m. But, as with the duopoly comparison, the marginal consumers of the multiproduct monopolist differ from those of the single-product monopolist, which can potentially make the slope of the multiproduct monopolist's demand curve steeper than that of the single-product monopolist. Interestingly, the market share effect unambiguously dominates, provided that $C_{11}(x, x)$ has a uniform sign for $x \in (0, 1)$, which of course is ensured if $C(\cdot, \cdot)$ is negatively dependent, independent, or positively dependent. Consequently, the hazard rate corresponding to the pricing problem of the multiproduct monopolist is below that of the single-product monopolist, and $\bar{p}^{mm} > \bar{p}^m$.

There is some relationship between Proposition 7 and Proposition 4. A change from two varieties to one variety for a monopolist is the same as increase from limited dependence to perfect positive dependence for the two varieties. Therefore, by continuity Proposition 7 implies that price must decrease for a sufficiently large increase in dependence at least under a uniform dependence condition. Furthermore, price must decrease with preference dependence at least over some range.

Under general preference distributions, Propositions 6 and 7 largely settle the question of how prices differ across market structures under monopoly and duopoly. These results also extend to n varieties as follows. First, under a uniform dependence property, the symmetric multiproduct monopoly price is above the single-product monopoly price. Second, the symmetric oligopoly price is below (above) the single-product monopoly price if preferences are positively (negatively) dependent and the hazard rate is increasing (decreasing). Third, the multiproduct monopoly price exceeds the oligopoly price. These more general results are proved in the appendix.

5. FGM-EXPONENTIAL CASE

We illustrate numerically our results for the bivariate exponential distribution based on the FGM copula.[22] Thus we adopt the following parametric functions:

$$u(x) = -\left[1 + \ln(1-x)\right], \tag{22}$$

$$C(x, y; \theta) = xy + \theta xy(1-x)(1-y). \tag{23}$$

In this case, the normalized utility $u = u(x)$ has an exponential distribution

$$F(u) = 1 - e^{-u-1} \tag{24}$$

with the properties that $E\{u\} = 0$, $Var(u) = 1$, $f(u)$ is loglinear, and $\lambda(u)$ is constant. The copula belongs to the Fairlie-Gumbel-Morgenstern (FGM) family for which $\theta \in [-1, 1]$. Members of the FGM family exhibit positive stochastic dependence if $\theta > 0$, negative dependence if $\theta < 0$, and independence if $\theta = 0$.[23] The FGM copula density is

$$C_{12}(x, y) = 1 + \theta(2x-1)(2y-1). \tag{25}$$

The density function has the property that an increase in preference dependence increases density of consumers along the diagonal:

$$C_{12\theta}(x, x; \theta) = (2x-1)^2 \geq 0. \tag{26}$$

Consequently, as discussed early, an increase in preference dependence always increases the cross-price elasticity of demand for this case. Our numerical analysis focuses on the extreme cases of positive and negative dependence ($\theta = 1$ and $\theta = -1$) and independence ($\theta = 0$). We illustrate graphically the comparative static properties and the comparisons of conduct and performance across markets.

[Insert Figure 4 about here]

[22] Gumbel (1960) introduced several bivariate exponential distributions. This is the second one.

[23] The FGM family of copulas has a limited range of positive and negative dependence; so the extreme cases, $\theta = 1$ and $\theta = -1$, do not indicate perfectly positive and negative dependence.

Figures 4 shows how normalized price ($\bar{p}$) varies with preferences and across market structures for the FGM-exponential case. There are three panels, corresponding to negative dependence ($\theta = -1$), independence ($\theta = 0$), and positive dependence ($\theta = 1$). The horizontal axis in each graph is measured with respect to $q^m = 1 - F(\bar{p}^m)$, the profit-maximizing single-product monopoly market share. From Lemma 1 there is a monotonic negative relationship between $\bar{p}^m$ and $\bar{\mu}$, and, therefore, a positive monotonic relationship between q^m and $\bar{\mu}$. Consequently, the graphs effectively describe how industry outcomes vary with the strength-diversity ratio ($\bar{\mu}$) over the relevant range.[24] Confirming our analytical results, we observe: (1) $\bar{p}$ decreases with $\bar{\mu}$ under all three market structures; (2) $\bar{p}^d$ decreases in θ; (3) $\bar{p}^{mm}$ decreases in θ only for $\bar{\mu}$ sufficiently high;[25] and (4) $\bar{p}^{mm}$ is always the highest, whereas $\bar{p}^m < \bar{p}^d$ for $\theta = -1$ (negative dependence), $\bar{p}^m = \bar{p}^d$ for $\theta = 0$ (independence), and $\bar{p}^m > \bar{p}^d$ for $\theta = 1$ (positive dependence).

[Insert Figures 5 and 6 about here]

Figure 4 and Figure 5 conduct the same exercises for normalized profit ($\bar{\pi}$) and consumer welfare ($\bar{w}$). Again confirming our analytical results, we observe: (1) $\bar{\pi}$ and $\bar{w}$ increase with $\bar{\mu}$ under all three market structures;[26] (2) $\bar{\pi}^{mm}$ and $\bar{\pi}^d$ decrease as θ increases; (3) $\bar{\pi}^{mm} > \bar{\pi}^m > \bar{\pi}^d$. Numerical analysis also confirms that the effects of θ on consumer welfare is ambiguous: $\bar{w}^{mm}$ is decreasing in θ except for large values of $\bar{\mu}$, in which case greater positive dependence can deliver more consumer welfare than independence because price is lower Duopoly competition creates the most consumer welfare, while a multiproduct monopoly creates more consumer welfare than single-product monopoly except when demand is sufficiently strong. The case of very strong demand is interesting. The higher price of a multiproduct monopoly inefficiently reduces the quantity demanded, reducing welfare to more than offset the benefits of greater product variety for consumers. Even then, however, total welfare

[24] In this case, $\overline{p}^m = 1 - \overline{\mu}$, $q^m = e^{-(2-\overline{\mu})}$, and $\overline{\mu}$ ranges between $-\infty$ and 2 as q^m ranges between 0 and 1.

[25] While the effect of θ on $\bar{p}^{mm}$ is quantitatively small, a close comparison of the panels shows that that $\bar{p}^{mm}$ decreases in θ for $\bar{\mu}$ above a critical value but increases in θ for $\bar{\mu}$ below. Thus the numerical analysis shows that if is difficult to go beyond Proposition 4.

[26] Since $\overline{p}^d < \overline{p}^m$ when $\theta = -1$, the market is fully covered in duopoly when $q^m < 1$ but sufficiently high. In this range, $\overline{\pi}^d$ is flat.

clearly is higher under multiproduct monopoly because the widening profit gap offsets the consumer surplus loss. Not surprisingly, the adverse consumer welfare effect of greater variety is more pronounced, and occurs in a wider range of circumstances, when preference dependence is more negative.

6. CONCLUSION

Using copulas to describe the distribution of consumer preferences is a convenient and intuitive approach to discrete choice demand in multiproduct industries. The approach leads to several sets of conclusions about how preferences matter for multiproduct industries. First, with certain qualifications, prices, profits, and consumer welfare all increase in demand strength, and they also all increase in preference diversity when demand is low ($\mu \leq 0$); but profit first decreases and then increases in preference diversity when demand is high ($\mu > 0$). These comparative statics are robust to varying degrees of preference dependence across market structures. Second, preference dependence can be disentangled from preference diversity as a distinct indicator of product differentiation in multiproduct industries, in the sense that greater dependence leads to lower prices and profits under certain conditions. Third, for an initially monopolized market, the entry of one or several horizontally differentiated competitors lowers (raises) market prices if preferences are positively (negatively) dependent and the hazard rate of the marginal distribution is non-decreasing (non-increasing); and, under a uniform dependence condition, a single-product monopolist will raise price when it adds one or several horizontally differentiated products.

There are many possible directions for future research using the copula approach. One important direction is to further extend our analysis to oligopoly markets with arbitrary numbers of firms and product varieties and to relax symmetry For example, whereas we have found that the symmetric n-firm oligopoly price is above or below the single-product monopoly price depending on preference dependence and the hazard rate, it would also be interesting to find out under what conditions the symmetric oligopoly price might be decreasing or increasing in the number of firms. Similarly, it would be interesting to know more generally the conditions under which a product line expansion by a multiproduct monopolist results in higher or lower prices. Relaxing the symmetry assumption is also important. For example, a symmetric model is inappropriate for understanding conditions under which generic entry results in higher or lower branded drug prices as in Perloff, Suslow, and Seguin (1995).

An important application of the copula approach to product variety is to endogenize market structure, along the lines of Shaked and Sutton (1990), but to employ a discrete-choice demand model instead of a representative consumer model.[27] Following Shaked and Sutton (1990), we may assume that two firms first simultaneously decide which products to offer by incurring a fixed cost (K) for each, and then simultaneously choose prices in a subgame perfect equilibrium. Chen and Riordan (2009) includes some results for this extension and for the case of two possible product varieties. Shaked and Sutton's (1990) analysis of how the level of fixed costs influences equilibrium market structure extends readily to the discrete choice demand model, and it is similarly possible to examine the effects of demand strength, preference diversity, and preference dependence. As in Shaked and Sutton (1990), it is possible that multiproduct monopoly and horizontally differentiated duopoly equilibria co-exist, with the interpretation that product line expansion by a monopolist can foreclose welfare improving competition.[28]

A potentially interesting extension is to apply the copula approach to a Lancasterian model of consumer demand in which consumers have preferences over the characteristics that compose products,[29] and to endogenize product design and market structure in a two-stage game in which rival firms in the first stage select product characteristics, and in the second stage set prices, thus further extending Shaked and Sutton (1990) to endogenize the nature of product differentiation. Chen and Riordan (2009) contains some preliminary analyses along these lines, including an example demonstrating how horizontal competition between low-quality duopolists can foreclose a monopoly with a high-quality product to the detriment of industry

[27] Early work on the economics of multiproduct firms considered the incentive and ability of an incumbent to use brand proliferation to deter entry (e.g., Schmalensee, 1978; Judd, 1985), as well as the role of cost factors such as economies of scope in giving rise to multiproduct firms (Panzar and Willig, 1981). Shaked and Sutton (1990) directed the literature toward considering how demand may affect the nature of equilibrium with multiproduct firms, focusing especially on empirically testable relationships between market characteristics and market structure.

[28] Schmalensee (1978) originally examined this argument in the context of a circle model of consumer preferences.

[29] The characteristics approach to demand goes back to Lancaster (1971). See Berry and Pakes (2007) for a more recent treatment.

profit, consumer welfare, and social welfare.

The copula approach can also be applied to other areas of applied microeconomics, such as the economics of search (e.g., Anderson and Renault 1999; Schultz and Stahl 1996; Bar Isaac, Caruana, and Cunat 2010) and the economic analysis of horizontal mergers. Finally, the copula approach to discrete choice demand models, and the rich set of predictions that our analysis has already generated and that can be further developed concerning how market characteristics affect prices, profits, consumer surplus, and market structures, might open up interesting new directions for empirical industrial organization research.[30]

REFERENCES

Anderson, S.P., A. de Palma, and J-F Thisse. *Discrete Choice Theory of Product Differentiation*. Cambridge: The MIT Press, 1992.

Anderson, S.P. and R. Renault. "Pricing, Product Diversity, and Search Costs: A Bertrand-Chamberlin-Diamond Model." *RAND Journal of Economics* 1999, 30(4), 719-35.

Bar-Isaac, H. G. Caruana and V. Cuñat, "Search, Design, and Market Structure," 2010, working paper.

Berry, S. and A. Pakes. "The Pure Characteristics Demand Model." *International Economic Review*, 2007, 48(4), Special Issue *Economics to Econometrics: Contributions in Honor of Daniel L. McFadden*, Charles F. Manski and Whitney Newey, eds., pp. 1193-1225.

Chen, Y. and M.H. Riordan. "Price-increasing Competition." *RAND Journal of Economics*, 2008, 39(4), pp.1042-1058.

___________. "Prices and Variety in the Spokes Model." *Economic Journal*, 2007, 117 (522), 897-921.

___________. "Preferences, Prices, and Performance in Monopoly and Duopoly."

[30]See, for example, Chen and Savage (forthcoming) for an empirical analysis of how preference dispersion affects prices and price differences between monopoly and duopoly markets for Internet services.

Columbia University Department of Economics Discussion Paper No. 0910-03, November 2009.

Chen, Y. and S. Savage. "The Effects of Competition on the Price for Cable Modem Internet Access." *Review of Economics and Statistics*, 2011, 93, pp. 201-217.

Johnson, J.P. and D.P. Myatt. "On the Simple Economics of Advertising, Marketing, and Product Design." *American Economic Review*, 2006, 96(3), pp. 756-784.

Judd, K.L. "Credible Spatial Preemption." *RAND Journal of Economics*, 1985, 16(2), pp. 153-166.

Lancaster, K.J. "A New Approach to Consumer Theory." *Journal of Political Economy*, 1966, 74(2), pp. 132-157.

Nelsen, R.B. *An Introduction to Copulas.* New York: Springer, 2006.

Nalebuff, B. "Bundling as an Entry Barrier." *Quarterly Journal of Economics*, 2004, 119(1), pp. 159-87.

Patton, A.J. "Copula-Based Models for Financial Time Series." Oxford Financial Center Economics Series, #2008fe21, Oxford University, 2008.

Panzar, J. and R.D. Willig. "Economies of Scope." *American Economic Review*, 1981, 71, pp. 268-272.

Perloff, J.M. and S.C. Salop. "Equilibrium with Product Differentiation." *Review of Economic Studies*, 1985, pp. 107-120.

Perloff, J.M., V.Y. Suslow, and P.M. Seguin. "Higher prices from Entry: Pricing of Brand-name Drugs." W.P. No. CPC-99-03, Competition Policy Center, University of California at Berkeley, 1995.

Schmalensee, R. "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry." *Bell Journal of Economics*, 1978, 9, pp. 305-327.

Schmalensee, R. "Gaussian Demand and Commodity Bundling." *Journal of Business*, 1984, 57, pp. 58-73.

Schultz, N. and K. Stahl. "Do Consumers Search for Higher prices? Oligopoly Equilibrium and Monopoly Optimum in Differentiated-Products Markets." *RAND Journal of Economics*, 1996, 27(3), pp. 542-62.

Shaked, A. and J. Sutton, “Multiproduct Firms and Market Structure.” *RAND Journal of Economics*, 1990, 21(1), pp. 45-62.

Weyl, E.G. and M. Fabinger, "Pass-through as an Economic Tool," working paper, 2009.

APPENDIX: PROOFS

This appendix contains the proofs for Propositions 1, 4, 5, 6, 7 and for Lemmas 3 and 4. All other results have already been shown in the text. In all proofs to follow, except that of Proposition 1 which deals with single-product monopoly, we consider $n \geq 2$ product varieties, describe consumer preferences by marginal distribution $F(u_i)$ and symmetric multivariate copula $C(x_1, ..., x_n)$, and define $\lambda^C(\cdot)$ and $h(\cdot)$ accordingly from (12) and (17). Hence, the proofs for the corresponding results in our main model are obtained when $n = 2$ and $(x, y) = (x_1, x_2)$.

Proof of Proposition 1. (i) First, from (7) and part (i) of Lemma 1,

$$\frac{dp^m}{d\mu} = \frac{1}{\sigma}\frac{dp^m}{d\bar{\mu}} = \frac{d(\bar{p}^m + \bar{\mu})}{d\bar{\mu}} \gtreqless 0 \text{ if } \lambda'(p) \gtreqless 0. \tag{27}$$

Next, $\frac{d\pi^m}{d\mu} = \frac{d\bar{\pi}^m}{d\bar{\mu}} > 0$ and $\frac{dw^m}{d\mu} = \frac{d\bar{w}^m}{d\bar{\mu}} > 0$, from Lemma 1.

(ii) First, from part (i) of Lemma 1:

$$\frac{dp^m}{d\sigma} = \bar{p}^m + \sigma\frac{d\bar{p}^m}{d\sigma} = \bar{p}^m - \bar{\mu}\frac{d\bar{p}^m}{d\bar{\mu}} = \frac{1}{\sigma}\left[p^m - \mu\left(1 + \frac{d\bar{p}^m}{d\bar{\mu}}\right)\right] = \frac{1}{\sigma}\left[p^m - \mu\left(\frac{d(\bar{p}^m + \bar{\mu})}{d\bar{\mu}}\right)\right].$$

Therefore, from (27), since $p^m > 0$ at an interior optimum, $\frac{dp^m}{d\sigma} > 0$ if $\mu \leq 0$ and $\lambda'(p) \geq 0$.If $\mu > 0$, then Lemma 1 implies $1+\frac{d\bar{p}^m}{d\bar{\mu}} < 1$ and $\frac{dp^m}{d\sigma} > \frac{1}{\sigma}(p^m - \mu) \equiv \bar{p}^m \geq 0$ if $\bar{\mu} \leq \bar{\mu}^m$. Next, that $\bar{p}^m \gtreqless 0$ if $\bar{\mu} \lesseqgtr \bar{\mu}^m$ from Lemma 1 implies

$$\frac{d\pi^m}{d\sigma} = \bar{\pi}^m + \sigma\frac{d\bar{\pi}^m}{d\sigma} = \bar{\pi}^m + \sigma\frac{\partial\bar{\pi}^m}{\partial\bar{\mu}}\frac{d\bar{\mu}}{d\sigma} = \bar{\pi}^m - \bar{\mu}\left[1 - F(\bar{p}^m)\right] = \bar{p}^m\left[1 - F(\bar{p}^m)\right] \gtreqless 0 \text{ if } \bar{\mu} \lesseqgtr \bar{\mu}^m.$$

Finally, since $\frac{d\bar{w}^m}{d\bar{\mu}} > 0$ from part (iii) of Lemma 1, if $\mu \leq 0$:

$$\frac{dw^m}{d\sigma} = \bar{w}^m + \sigma\frac{d\bar{w}^m}{d\sigma} = \int_{\bar{p}^m}^{u(1)}\left[1 - F(\bar{p})\right]d\bar{p} - \bar{\mu}\frac{d\bar{w}^m}{d\bar{\mu}} > 0.$$

■

Proof of Lemma 3. Assume there are $n \geq 2$ symmetric varieties, and define **MDR**

and $\lambda^C(\bar{p};\theta)$ accordingly. For any $\bar{p} > F^{-1}(0)$ and for all θ,

$$\int_0^{F(\bar{p})} C_1(x,...,x;\theta)\,dx = \frac{1}{n}\int_0^{F(\bar{p})} \frac{dC(x,...,x;\theta)}{dx}dx = \frac{1}{n}C(F(\bar{p}),...,F(\bar{p});\theta)$$

increases in θ by **MDR**, which is possible only if $C_{1\theta}(x,...,x;\theta) \equiv \partial C_1(x,...,x;\theta)/\partial\theta > 0$ for all θ if x is sufficiently close to zero. Similarly, $C_{1\theta}(x,...,x;\theta) < 0$ for all θ if x is sufficiently close to 1. Thus there must exist $x' > 0$ such that, for all θ, $C_{1\theta}(x',...,x';\theta) = 0$ and $C_{1\theta}(x,...,x;\theta) > 0$ if $x < x'$.[31] Since

$$\begin{aligned}&\frac{\partial\lambda^C(\bar{p};\theta)}{\partial\theta}\\ =\;& n\left[\frac{C_{1\theta}(F(\bar{p}),...,F(\bar{p});\theta)}{1-C(F(\bar{p}),...,F(\bar{p});\theta)} + \frac{C_1(F(\bar{p}),...,F(\bar{p});\theta)\,C_\theta(F(\bar{p}),...,F(\bar{p});\theta)}{[1-C(F(\bar{p}),...,F(\bar{p});\theta)]^2}\right] f(\bar{p}),\end{aligned}$$

there exists some $u^* \in [F^{-1}(x'),\ u(1)]$ such that $\partial\lambda^C(\bar{p};\theta)/\partial\theta > 0$ for all θ if $\bar{p} \le u^*$.
■

Proof of Proposition 4. (i) From (10)**,** for any θ, let $\bar{\mu}^*$ be such that $[u^* + \bar{\mu}^*]\lambda^C(u^*;\theta) = 1$, where $u^* \ge F^{-1}(x') > u(0)$ is defined in Lemma 3 Then, $\bar{p}^{mm} = u^* > u(0)$ if $\bar{\mu} = \bar{\mu}^*$. If $\bar{\mu} \ge \bar{\mu}^*$, $\bar{p}^{mm} \le u^*$ and Lemma 3 implies $\frac{\partial\lambda^C(\bar{p}^{mm};\theta)}{\partial\theta} > 0$. It follows from (10) that $\frac{d\bar{p}^{mm}}{d\theta} < 0$ and hence $\frac{dp^{mm}}{d\theta} < 0$. (ii) holds from application of the envelope theorem to (9) and $C_\theta(\cdot;\theta) > 0$. ■

Proof of Lemma 4. Assume there are $n \ge 2$ symmetric varieties, and define **MDR** and $h(\bar{p};\theta)$ accordingly. Notice first that

$$\frac{dC_1(x,...,x)}{dx} = C_{11}(x,...x) + ...C_{1n}(x,...x) = C_{11}(x,...x) + (n-1)\,C_{12}(x,...x),$$

or

$$(n-1)\,C_{12}(x,...x) = \frac{dC_1(x,...x)}{dx} - C_{11}(x,...x).$$

Thus,

$$h(\bar{p}) \equiv \lambda^C(\bar{p}) + n(n-1)\frac{\int_{F(\bar{p})}^1 C_{12}(x,...x)\,f(u(x))\,dx}{1-C(F(\bar{p}),...F(\bar{p}))}$$

[31] Similarly, there exists $x'' \ge x'$ such that $C_{1\theta}(x',...,x';\theta) = 0$ and $C_{1\theta}(x,...,x;\theta) < 0$ if $x > x''$. For the FGM family with $n = 2$, $x' = x'' = 1/2$.

$$= \frac{nC_1(F(\bar{p}),...F(\bar{p}))f(\bar{p})}{1-C(F(\bar{p}),...F(\bar{p}))} + n\frac{\int_{F(\bar{p})}^1 \left[\frac{dC_1(x,...x)}{dx} - C_{11}\right] f(u(x))\,dx}{1-C(F(\bar{p}),...F(\bar{p}))}$$

$$= \frac{nC_1(F(\bar{p}),...F(\bar{p}))f(\bar{p})}{1-C(F(\bar{p}),...F(\bar{p}))} + n\frac{\int_{F(\bar{p})}^1 \frac{dC_1(x,...x)}{dx} f(u(x))\,dx - \int_{F(\bar{p})}^1 C_{11}(x,...x) f(u(x))\,dx}{1-C(F(\bar{p}),...F(\bar{p}))}.$$

Since

$$\begin{aligned}
&\int_{F(\bar{p})}^1 \frac{dC_1(x,...x)}{dx} f(u(x))\,dx \\
&= f(u(1)) - C_1(F(\bar{p}),...F(\bar{p}))f(\bar{p}) - \int_{F(\bar{p})}^1 C_1(x,...x)\frac{f'(u(x))}{f(u(x))}dx,
\end{aligned}$$

$$\begin{aligned}
h(\bar{p}) &= n\frac{f(u(1)) - \int_{F(\bar{p})}^1 C_1(x,...x)\frac{f'(u(x))}{f(u(x))}dx - \int_{F(\bar{p})}^1 C_{11}(x,...x) f(u(x))\,dx}{1-C(F(\bar{p}),...F(\bar{p}))} \\
&= n\frac{f(u(1)) + \int_{F(\bar{p})}^1 \frac{f'(u(x))}{f(u(x))}\frac{1}{n}\frac{d[1-C(x,...,x)]}{dx} - \int_{F(\bar{p})}^1 C_{11}(x,...x) f(u(x))\,dx}{1-C(F(\bar{p}),...F(\bar{p}))}
\end{aligned}$$

$$= -\frac{f'(\bar{p})}{f(\bar{p})} + n\frac{f(u(1)) - \frac{1}{n}\int_{F(\bar{p})}^1 [1-C(x,...x)]\frac{d\frac{f'(u(x))}{f(u(x))}}{dx} - \int_{F(\bar{p})}^1 C_{11}(x,...x) f(u(x))\,dx}{1-C(F(\bar{p}),...F(\bar{p}))}.$$

It follows that, $\frac{\partial h(\bar{p};\theta)}{\partial\theta} =$

$$\frac{\int\limits_{F(\bar{p})}^{1} \left[\frac{C_\theta(x,...x)}{f(u(x))}\frac{d^2\ln f(u)}{du^2} - nC_{11\theta}(x...x)f(u(x))\right]dx + \left[h(\bar{p}) + \frac{f'(\bar{p})}{f(\bar{p})}\right]C_\theta(F(\bar{p}),...F(\bar{p}))}{1-C(F(\bar{p}),...F(\bar{p}))}$$

and, since $C_\theta > 0$ and $C_{11\theta} < 0$ by **MDR,** (20) implies $\frac{\partial h(\bar{p};\theta)}{\partial\theta} > 0$. Finally, since the equilibrium oligopoly price satisfies

$$(\bar{p}^n + \bar{\mu})\,h(\bar{p}^n;\theta) = 1, \tag{28}$$

from **A3** we have $\frac{d\bar{p}}{d\theta} < 0$. ■

Proof of Proposition 5. Assume there are $n \geq 2$ symmetric varieties, and define **MDR** and $h(\bar{p};\theta)$ accordingly. First, observe that $\frac{dp^n}{d\theta}$ has the same sign as $\frac{d\bar{p}^n}{d\theta}$. Second, observe that π^n decreases in θ when $\frac{d\bar{p}^n}{d\theta} < 0$, because

$$\begin{aligned}\frac{d\pi^n}{d\theta} &= \frac{\partial \pi^n}{\partial \theta} + \frac{\partial \pi^n}{\partial \bar{p}^n}\frac{d\bar{p}^n}{d\theta} \\ &= -\frac{1}{n}\left(\bar{p}^n\sigma + \mu\right) C_\theta\left(F\left(\bar{p}^d\right),...,F\left(\bar{p}^d\right);\theta\right) + \frac{\partial \pi^n}{\partial \bar{p}^n}\frac{d\bar{p}^n}{d\theta} < 0,\end{aligned}$$

where $C_\theta\left(F\left(\bar{p}^d\right),...,F\left(\bar{p}^d\right);\theta\right) > 0$ by **MDR** and $\frac{\partial \pi^n}{\partial \bar{p}^n} > 0$ by the envelope theorem and the fact that a firm's demand increases in the other firm's price. Given these observations we focus on sufficient conditions for $\frac{d\bar{p}^d}{d\theta} < 0$ for each part of the proposition.

(i) From the proof of Proposition 4, $\lambda^C(\bar{p};\theta)$ increases with θ if $\bar{\mu}$ is sufficiently large so that $\bar{p}$ is sufficiently close to $u(0)$. Therefore, from **MDR** and (17), if (21) holds for $t = 0$, then $h(\bar{p};\theta)$ increases with θ; and from (28) $\bar{p}^n$ and hence p^n decrease with θ.

(ii) If $\frac{d^2 \ln f(u)}{du^2} \to 0$ and $C_{11} < 0$, then

$$h(\bar{p};\theta) + \frac{f'(\bar{p})}{f(\bar{p})} \to n\frac{f(u(1)) - \int_{F(p)}^{1} C_{11}(x,...,x;\theta) f(u(x))\,dx}{1 - C\left(F(\bar{p}),...,F(\bar{p});\theta\right)} > 0$$

and $\frac{d^2 \ln f(u)}{du^2} > \frac{2f^2(u(x))}{C_\theta(x,...x;\theta)} C_{11\theta}(x,...x;\theta)$ since $C_{11\theta} < 0$, thus satisfying the n-variant generalization of Lemma (4).

(iii) Finally, if $\frac{d \ln f(u)}{du}$ and $\frac{d^2 \ln f(u)}{du^2}$ both are not too negative, then $h(u) + \frac{f'(u)}{f(u)} \geq 0$ and $\frac{d^2 \ln f(u)}{du^2} > \frac{2f^2(u(x))}{C_\theta(x,x)} C_{11\theta}(x,...x;\theta)$ by **MDR**, thus satisfying the generalization of Lemma (4). ■

Proof of Proposition 6. Assume there are $n \geq 2$ symmetric varieties, and define **MDR** and $h(\bar{p})$ accordingly. It suffices to show that (i) $h(\bar{p}) > \lambda(\bar{p})$ if $C_{11} < 0$ and $\lambda'(\bar{p}) \geq 0$; and (ii) $h(\bar{p}) < \lambda(\bar{p})$ if $C_{11} > 0$ and $\lambda'(\bar{p}) \leq 0$.

(i) Suppose that $\lambda'(\bar{p}) \geq 0$. Then, since

$$\frac{dC_1(x_1,...,x_1)}{dx_1} - C_{11}(x_1,...,x_1) = (n-1)C_{12}(x_1,...,x_1) > 0,$$

$$\begin{aligned}
h(\bar{p}) &= \frac{nC_1(F(\bar{p}),...F(\bar{p}))f(\bar{p})}{1-C(F(\bar{p}),...F(\bar{p}))} + n\frac{\int_{F(\bar{p})}^1 (1-x)(n-1)C_{12}(x,...x)\frac{f(u(x))}{1-F(u(x))}dx}{1-C(F(\bar{p}),...F(\bar{p}))} \\
&= \frac{nC_1(F(\bar{p}),...F(\bar{p}))f(\bar{p})}{1-C(F(\bar{p}),...F(\bar{p}))} + n\frac{\int_{F(\bar{p})}^1 (1-x)\left[\frac{dC_1(x,...,x)}{dx} - C_{11}(x,...x)\right]\frac{f(u(x))}{1-F(u(x))}dx}{1-C(F(\bar{p}),...F(\bar{p}))} \\
&\geq \frac{nC_1(F(\bar{p}),...F(\bar{p}))f(\bar{p})}{1-C(F(\bar{p}),...F(\bar{p}))} + n\frac{f(\bar{p})}{1-F(\bar{p})}\frac{\int_{F(\bar{p})}^1 (1-x)\left[\frac{dC_1(x,...x)}{dx} - C_{11}(x,...x)\right]dx}{1-C(F(\bar{p}),...F(\bar{p}))}.
\end{aligned}$$

After substituting into above the following:

$$\begin{aligned}
&\frac{\int_{F(\bar{p})}^1 (1-x)\left[\frac{dC_1(x,...x)}{dx} - C_{11}\right]dx}{1-C(F(\bar{p}),...F(\bar{p}))} \\
&= \frac{(1-x)C_1(x,...x)|_{F(\bar{p})}^1 + \int_{F(\bar{p})}^1 C_1(x,...x)\,dx - \int_{F(\bar{p})}^1 (1-x)C_{11}(x...x)dx}{1-C(F(\bar{p}),...F(\bar{p}))}
\end{aligned}$$

$$= \frac{-(1-F(\bar{p}))C_1(F(\bar{p}),...F(\bar{p})) + \frac{1}{n}(1-C(F(\bar{p}),...F(\bar{p}))) - \int_{F(\bar{p})}^1 (1-x)C_{11}(x,...x)dx}{1-C(F(\bar{p}),...F(\bar{p}))}$$

and simplifying, we obtain:

$$h(\bar{p}) \geq \lambda(\bar{p})\left[1 - n\frac{\int_{F(\bar{p})}^1 (1-x)C_{11}(x,...x)dx}{1-C(F(\bar{p}),...F(\bar{p}))}\right].$$

Hence $h(\bar{p}) > \lambda(\bar{p})$ if $C_{11} < 0$.

(ii) Suppose that $\lambda' \leq 0$. By analogous derivations, we have

$$h(\bar{p}) < \lambda(\bar{p})\left[1 - n\frac{\int_{F(\bar{p})}^1 (1-x)C_{11}(x,...x)dx}{1-C(F(\bar{p}),...F(\bar{p}))}\right].$$

Hence $h(\bar{p}) < \lambda(\bar{p})$ if $C_{11} > 0$. ■

Proof of Proposition 7. Assume there are $n \geq 2$ symmetric varieties, and define **MDR,** $\lambda^C(\bar{p})$, and $h(\bar{p})$ accordingly. (i) Since $h(\bar{p}) > \lambda^C(\bar{p})$ from (14), comparing (10) and (13) leads to $\bar{p}^n < \bar{p}^{mm}$.

(ii) It suffices to show that $\lambda^C(\bar{p}) < \lambda(\bar{p})$ for all $\bar{p} \in (u(0), u(1))$ if $C_{11}(x, ..., x) \leq 0, C_{11}(x, ..., x) = 0$, or $C_{11}(x, ..., x) \geq 0$ for all $x \in (0, 1)$.

First,

$$\frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} = \frac{\frac{nC_1(F(\bar{p}),...F(\bar{p}))}{1-C(F(\bar{p}),...F(\bar{p}))} f(\bar{p})}{\frac{f(\bar{p})}{1-F(\bar{p})}} = \frac{nC_1(F(\bar{p}), ...F(\bar{p}))}{1 - C(F(\bar{p}), ...F(\bar{p}))} [1 - F(\bar{p})].$$

If $C_{11} \geq 0$ or if $C_{11} = 0$, then, since

$$\int_{F(\bar{p})}^{1} (1-x)\, dC_1(x, ...x) = -(1 - F(\bar{p}))\, C_1(F(\bar{p}), ...F(\bar{p})) + \frac{1}{n} [1 - C(F(\bar{p}), ...F(\bar{p}))]$$

and $C_{1k}(x, ..., x) = C_{12}(x, ..., x) > 0$ for $x \in (0, 1)$ and for all $k \neq 1$,

$$\begin{aligned}
\frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} &= \frac{[1 - C(F(\bar{p}), ...F(\bar{p}))] - n\int_{F(\bar{p})}^{1} (1-x)\, dC_1(x, ...x)}{1 - C(F(\bar{p}), ...F(\bar{p}))} \\
&= \frac{[1 - C(F(\bar{p}), ...F(\bar{p}))] - n\int_{F(\bar{p})}^{1} (1-x)\, [C_{11} + C_{12} + ... + C_{1n}]\, dx}{1 - C(F(\bar{p}), ...F(\bar{p}))} \\
&= 1 - \frac{n\int_{F(\bar{p})}^{1} (1-x)\, [C_{11} + (n-1)\, C_{12}]\, dx}{1 - C(F(\bar{p}), ...F(\bar{p}))} < 1.
\end{aligned}$$

Next, suppose $C_{11}(x, ..., x) \leq 0$ for all $x \in (0, 1)$. Then, $C_1(x, ..., x) \leq \frac{C(x,...,x)}{x}$ for all $x \in (0, 1)$ since

$$C(x, ..., x) = \int_0^x C_1(t, x, ...x)\, dt \geq \int_0^x C_1(x, ...x)\, dt = C_1(x, ...x)\, x.$$

Hence, letting $x = F(\bar{p})$,

$$\frac{\lambda^C(u(x))}{\lambda(u(x))} = \frac{n(1-x)\, C_1(x, ...x)}{1 - C(x, ...x)} \leq \frac{n(1-x)\, C(x, ...x)}{x[1 - C(x, ...x)]}.$$

Now, suppose to the contrary that $\frac{\lambda^C(u(x))}{\lambda(u(x))} = \frac{n(1-x)C_1(x,...x)}{1-C(x,...x)} \geq 1$. Then $\frac{n(1-x)C(x,...x)}{x[1-C(x,...x)]} \geq 1$. But since

$$\lim_{x \to 1} \frac{n(1-x)C(x,...x)}{x[1-C(x,...x)]} = n \lim_{x \to 1} \frac{-C(x,...x) + (1-x)nC_1(x,...x)}{1 - C(x,...x) - nxC_1(x,...x)} = 1$$

and, letting $\mathbf{x} = (x, ...x)$,

$$\begin{aligned} & \frac{d\left[\frac{(1-x)C(\mathbf{x})}{x[1-C(\mathbf{x})]}\right]}{dx} \\ = \; & \frac{[-C(\mathbf{x}) + n(1-x)C_1(\mathbf{x})]\{x[1-C(\mathbf{x})]\} - (1-x)C(\mathbf{x})[1-C(\mathbf{x}) - nxC_1(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2} \\ = \; & \frac{n(1-x)C_1(\mathbf{x})x[1-C(\mathbf{x})] - C(\mathbf{x})[1-C(\mathbf{x})] + (1-x)C(\mathbf{x})nxC_1(\mathbf{x})}{\{x[1-C(\mathbf{x})]\}^2} \end{aligned}$$

$$\begin{aligned} = \; & \frac{n(1-x)C_1(\mathbf{x})x - C(\mathbf{x})[1-C(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2} \geq \frac{[1-C(\mathbf{x})]x - C(\mathbf{x})[1-C(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2} \\ = \; & \frac{[1-C(\mathbf{x})][x - C(\mathbf{x})]}{\{x[1-C(\mathbf{x})]\}^2} > 0 \text{ for } x \in (0,1), \end{aligned}$$

we have

$$\frac{n[1-x]C(\mathbf{x})}{x[1-C(\mathbf{x})]} < 1,$$

a contradiction. Therefore $\frac{2[1-x]C_1(x,...,x)}{1-C(x,...,x)} < 1$ for any $x \in (0,1)$, or $\frac{\lambda^C(\bar{p})}{\lambda(\bar{p})} < 1$ for any $\bar{p} \in (u(0), u(1))$. ■

Figure 1
Acceptance Sets

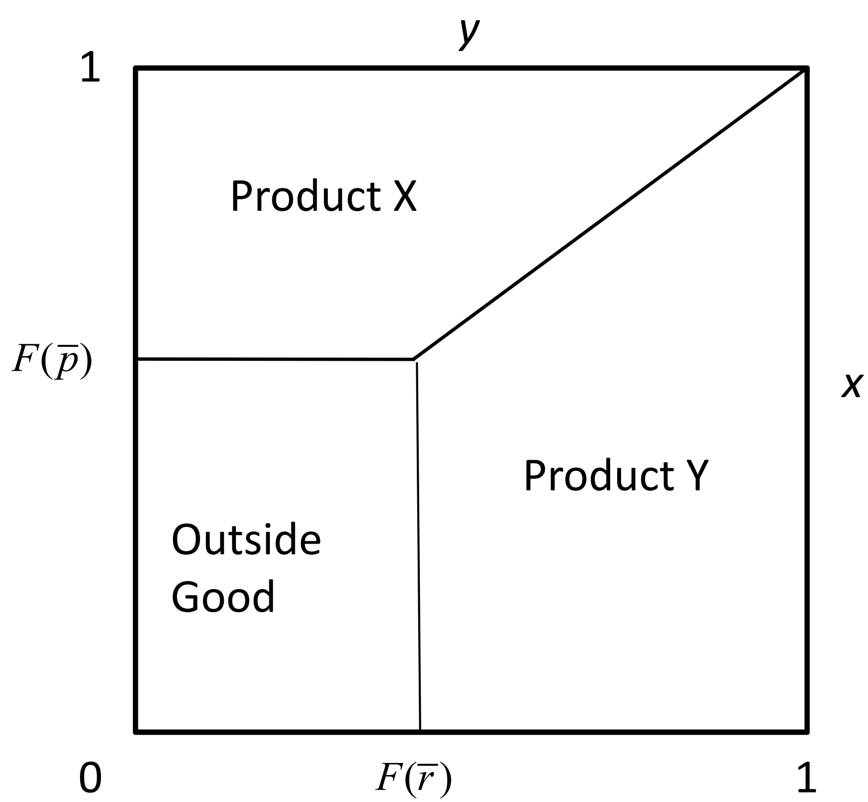

Figure 2
Determination of Single-product Monopoly Price

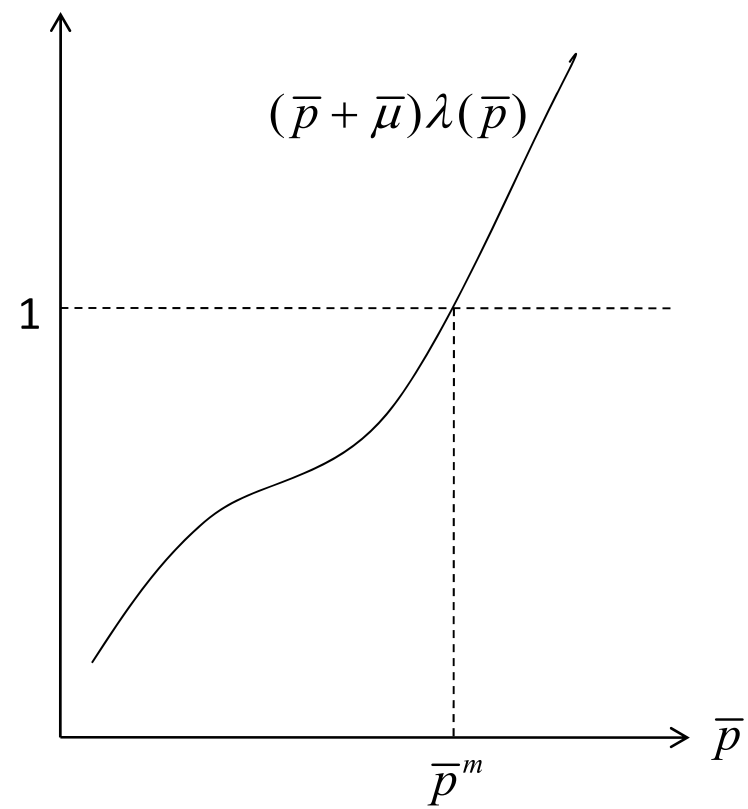

Figure 3
Price-Increasing Competition

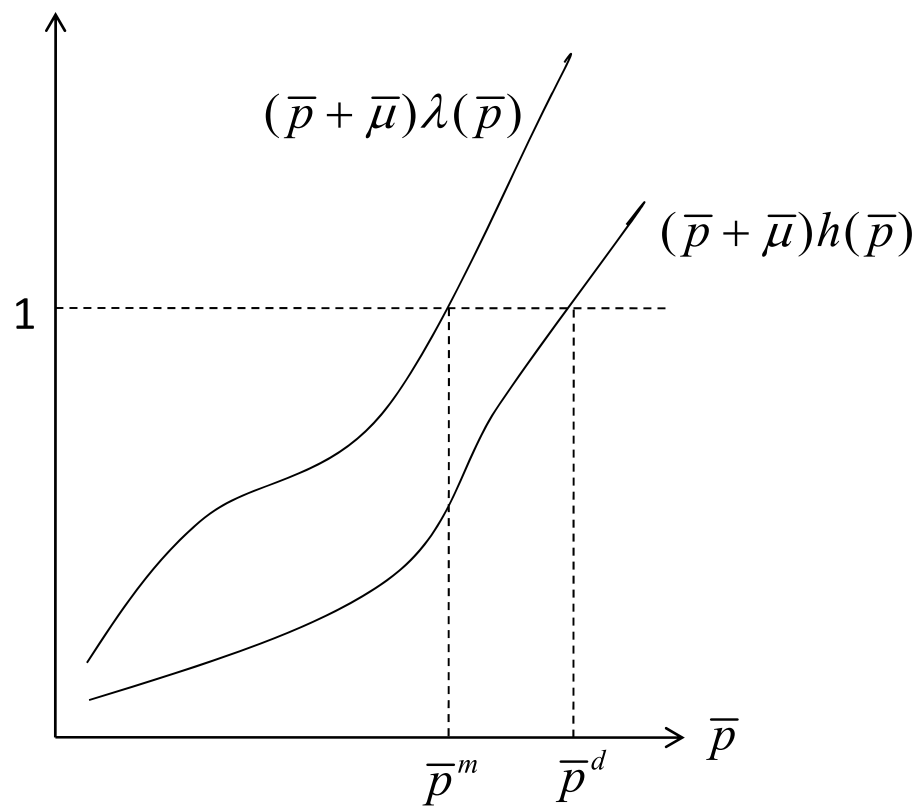

Figure 4
Prices in the Bivariate Exponential Case

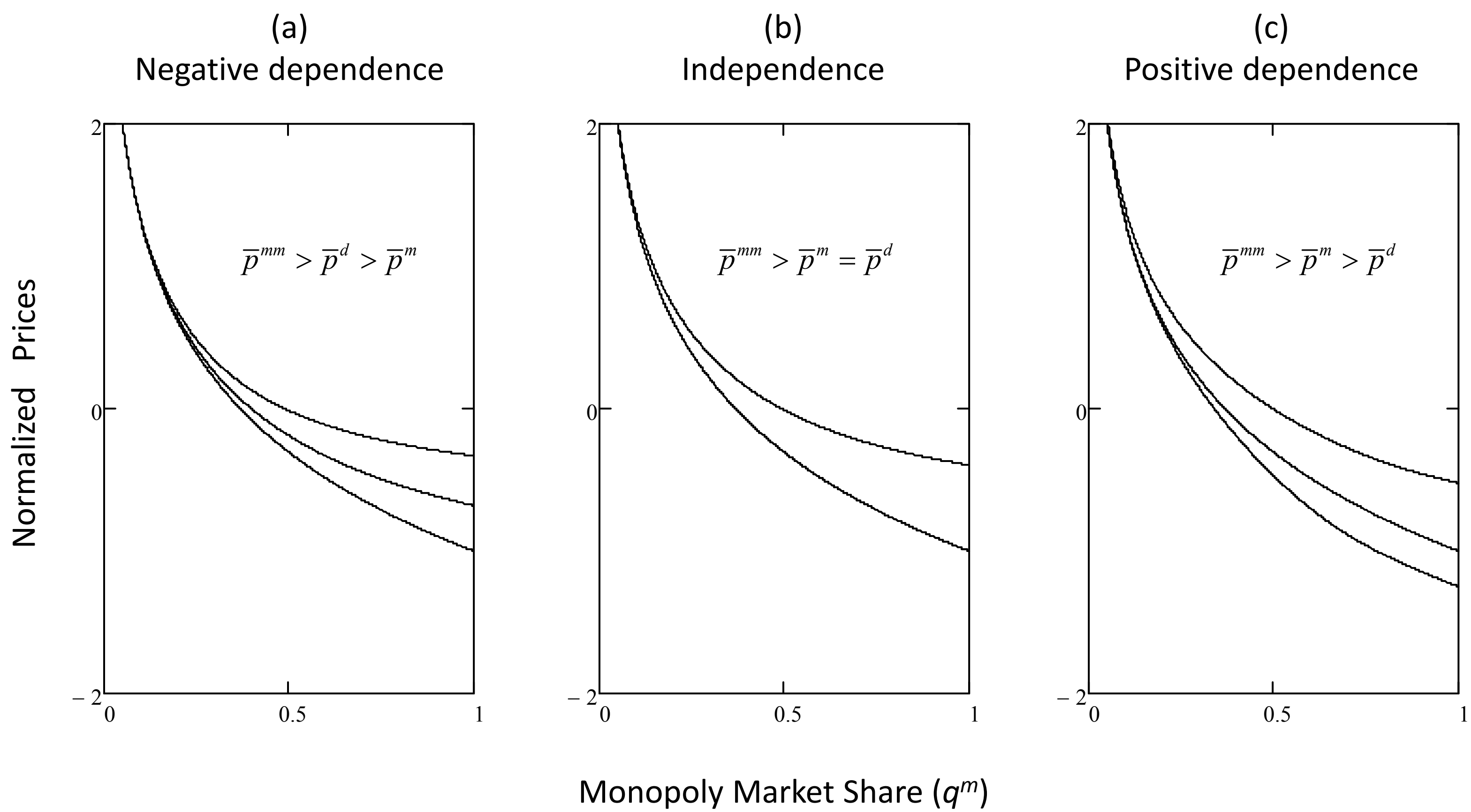

Figure 5
Profits in the Bivariate Exponential Case

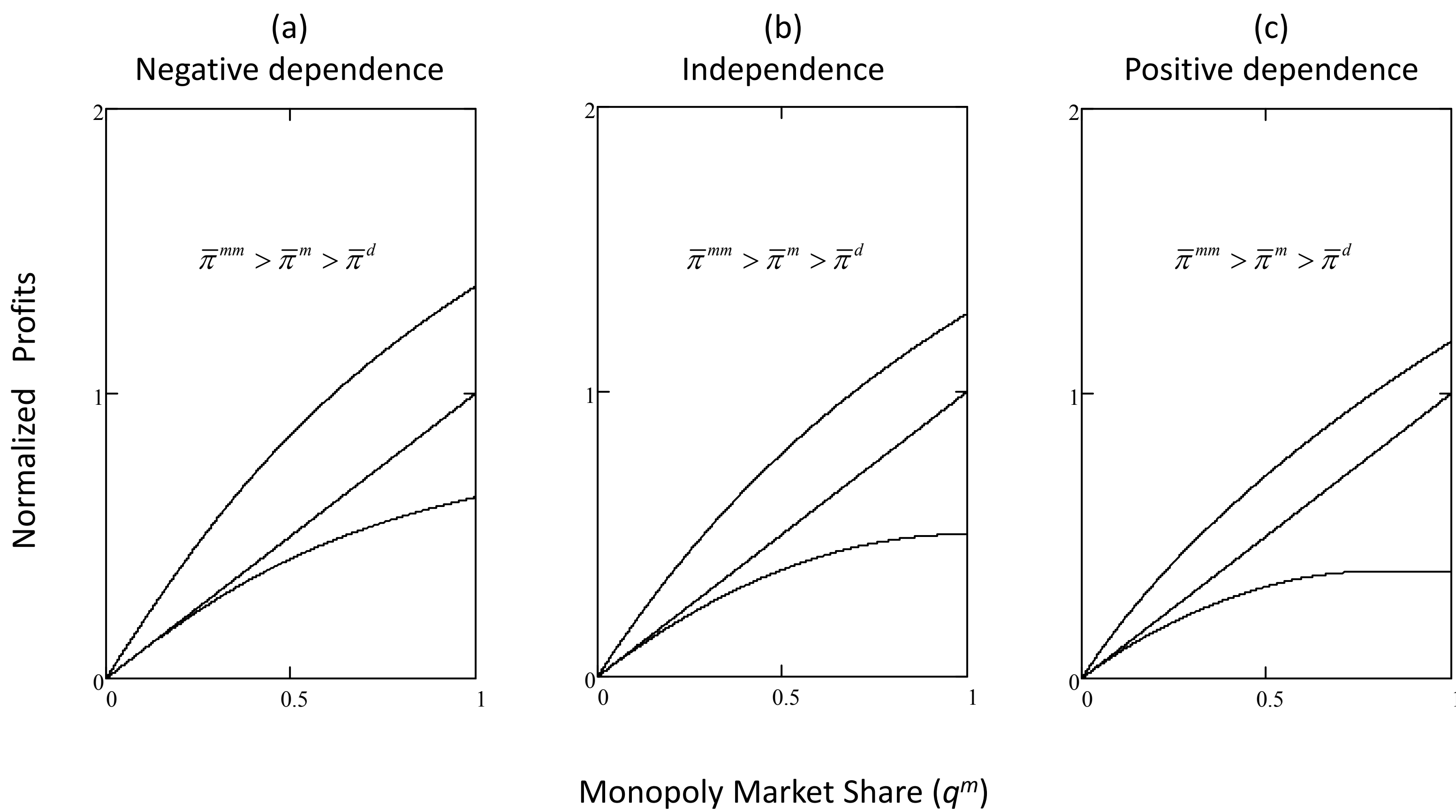

Figure 6
Consumer Welfare in the Bivariate Exponential Case