Price-Increasing Competition*

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Abstract

In a discrete choice model of product differentiation, the symmetric duopoly price may be lower than, equal to, or higher than the single-product monopoly price. While the *market share effect* of competition encourages a firm to charge less than the monopoly price because a duopolist serves fewer consumers, the *price sensitivity effect* of competition motivates a higher price when more consumer choice steepens the firm's demand curve. The joint distribution of consumer values for the two conceivable products determines the relative strength of these effects, and whether presence of a symmetric competitor results in a higher or lower price compared to single-product monopoly. The analysis reveals that price-increasing competition is unexceptional from a theoretical perspective.

Keywords: product differentiation, entry, duopoly, monopoly

JEL classification: D4, L1

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1. INTRODUCTION

A fundamental insight of economics is that competition usually lowers prices. The strength of this conclusion, however, is called into question by scattered evidence from assorted industries. For example, Pauly and Sattherthwaite (1981) argue that physician services are priced higher in urban areas with more physicians per capita; Bresnahan and Reiss (1991) present survey evidence showing that automobile tire prices were somewhat higher in local markets with two dealers rather than one, although the difference is not statistically significant;[1] Bresnahan and Reiss (1990) infer from the structure of local auto retail markets that profit margins might be higher under duopoly than monopoly; Perloff, Suslow, and Seguin (2005) conclude that new entry raised prices in the anti-ulcer drug market;[2] Ward et al. (2002) provide evidence that new entry of private labels raised prices of national brands in the food industry; Goolsbee and Syverson (2004) show that airlines raised route prices when Southwest opened new routes to the same destination from a nearby airport; and Thomadsen (2005) simulates with estimated parameters from the fast food industry how prices may rise with closer geographic positioning of competitors. A theoretical re-consideration of how competition affects prices thus seems appropriate.

In this paper, we study a discrete choice model of product differentiation in which consumers' values for two substitute products have an arbitrary symmetric joint distribution. Each firm produces a single product, and the market structure is either monopoly or duopoly. We characterize under fairly weak assumptions a necessary and sufficient condition for the symmetic duopoly price to be higher than, equal to, or lower than monopoly price. This condition balances two economic effects. At the monopoly price, a duopoly firm sells to fewer consumers than the monopolist. The larger is this difference, the greater is the incentive of a duopolist to reduce price below the monopoly level. We call this the *market share effect*. On the other hand, under product differentiation a duopoly firm's demand

[1] The survey evidence also shows that tire prices are lower in markets with three or more competitors, and that this difference is statistically significant.

[2] See also Caves, Whinston, and Hurwicz (1991) and Grabowski and Vernon (1992) for evidence that generic entry triggers higher prices for corresponding brand-name drugs in the U.S. pharmaceutical industry.

curve may be steeper than the monopolist's, because consumers have a choice of products in the duopoly case, and, therefore, are less keen to purchase the duopolist's product in response to a price cut. The steeper is the duopolist's demand curve, relative to the monopolist's, the greater is the incentive of the duopolist to raise price above the monopoly level. We call this the *price sensitivity effect*. When the second effect dominates, as, for example, if consumer values for the two products are drawn from a (Gumbel) bivariate exponential distribution, duopoly competition increases price compared to monopoly.

We derive from the general necessary and sufficient condition some particular conditions under which price is higher under symmetric single-product duopoly than under single-product monopoly. If consumer values for the two products are independent, then a decreasing hazard rate is sufficient for a higher duopoly price.[3] In the more general case, a sufficient condition for price-increasing competition is that the conditional hazard rate is decreasing over a relevant range. The necessary and sufficient condition, however, also implies that price-increasing competition can occur when the hazard rate is non-monotonic.

We also consider the implications of negative dependence of consumer preferences when the marginal distribution of consumer values for a product is exponential. In the independent exponential case the monopoly price is the same as the symmetic duopoly price. Against this benchmark, we show how price-increasing competition depends on the dependence properties of consumer preferences by using copulas to characterize symmetric bivariate distribution functions. In particular, the duopoly price is higher (lower) than the monopoly price if consumers' value for one product is stochastically decreasing (increasing) in their value for the other product, meaning that a higher value for one product shifts downward (upward) the conditional distribution for the second product according to first order stochastic dominance, or, equivalently, that the corresponding copula function is concave with respect to each of its arguments. The utility of the copula approach for studying product differentiation is that it allows us to vary the dependence relationship of

[3] This observation is related to Bulow and Klemperer's (2002) result for (almost) common-value auctions that, if bidders' private signals are independent draws from a distribution with a decreasing hazard rate, then revenue per unit is higher when two units are for sale than when one unit is for sale.

two random variables while holding the marginal distributions constant.

The trade off between the market share effect and the price sensitivity effect can be illustrated in the standard Hotelling model (Hotelling, 1929) in which consumer preferences are perfectly negatively correlated. Suppose consumers seeking to purchase one unit are uniformly distributed on a unit line and firms with zero costs are at the endpoints, with each consumer valuing a firm's product at v minus a transportation cost equal to the consumer's distance from the firm. If $v = 1\frac{1}{2}$, then a monopolist located at one end of the line optimally sells to $\frac{3}{4}$ of the market at a price equal to $\frac{3}{4}$, while duopolists at either end each sell to $\frac{1}{2}$ of the market at a price of 1. The reason why price is higher under duopoly is that a duopolist's demand is less response to a price cut because the marginal consumer has the alternative of purchasing the other product. The price sensitivity effect prevails even though a duopolist has fewer unit sales and therefore sacrifices less revenue with a price cut.

We analyze a generalized Hotelling model in which consumers also have different values of v as in Böckem (1994). In this model, consumer preferences for two products have a joint uniform distribution on a varying support allowing different degrees of negative or positive correlation. The standard Hotelling model is a limiting case for which the preferences are perfectly negatively correlated, and the Bertrand duopoly model is a limiting case for which the preferences are perfectly positively correlated. We show that duopoly competition raises price if consumers' preferences for the two products are sufficiently diverse and negatively correlated.[4] Furthermore, the adverse price effect can be strong enough that aggregate consumer welfare goes down even though consumers are better served and the market expands under duopoly.

The standard view of relationship between market structure and price has been challenged

[4] This characterization suggests some empirically relevant market conditions. In markets for high-speed Internet access through either cable modem or DSL service, Chen and Savage (2007) find empirical support for the hypothesis that DSL competition reduced prices for cable modem service in places where preference diversity is low. In markets for television programming ditristribution, Goolsbee and Petrin (2004) concluded that satellite entry reduced cable prices based on an estimated positive correlation of unobserved preferences for cable service and satellite service, .

previously by several other theoretical studies. For instance, when consumers must search to find firms' prices, the presence of more firms makes it more difficult to find the lowest price in the market, reducing consumers' incentives to search. This can cause the equilibrium market price to increase with the number of firms (Satterthwaite, 1979; Stiglitz, 1987; Schulz and Stahl, 1996; Janssen and Moraga-González, 2004).[5] An alternative approach assumes that each seller faces two groups of consumers, a captured loyal group and a switching group. With more sellers, each seller's share of the switching group is reduced, increasing its incentive to exploit the captured consumers through a higher price; but equilibrium prices under competition are in mixed strategies (Rosenthal, 1980).[6] In contrast, in our analysis here, consumers have perfect information, and firms' prices are in pure strategies. While Chen and Riordan (2007) and Perloff, Suslow, and Seguin (2005) have also shown that competition can increase price under perfect information and pure strategies, these papers rely on specific spatial models for which consumer valuations for two products are perfectly negatively correlated.[7] Our present analysis goes further by developing conditions for the symmetric duopoly price to exceed the monopoly price from a much broader perspective.

In the context of a symmetric oligopoly model with independently and identically distributed consumer preferences for alternative goods, Perloff and Salop (1985) recognize that the effect of more competitors on the symmetric equilibrium price is ambiguous, and Gabaix, Laibson, and Li (2005) study this relationship for particular distributions. Our analysis differs from Gabaix, Laibson, and Li (2005) in other ways beside relaxing the independence assumption. First, their main result approximates the equilibrium markup for a general

[5] Relatedly, Anderson and Renault (1999) show that equilibrium price can be higher when products are less differentiated because of consumer search.

[6] In a similar setting with high- and low-valuation types of consumers, entry and endogenous product selection can result in market segmentation with the incumbent raising its price to sell only to high-valuation customers, if the game is solved with a solution concept that is not Nash equilibrium. See Davis, Murphy, and Topel (2004).

[7] Perloff, Suslow, and Seguin (2005) study a variant of the Hotelling duopoly model, while Chen and Riordan (2007) generalize the Hotelling model to an arbitrary number of single-product firms. For an earlier literature on the effect of competition on prices in spatial models see, for example, Ohta (1981) and references therein.

class of preference distributions as a function of the number of firms. The approximation is valid if the number of firms is sufficiently large. Our analysis of monopoly and duopoly focuses on less concentrated markets. Second, following Perloff and Salop (1985), Gabaix, Laibson and Li (2005) simplify by assuming consumers always purchase one of the available products; thus firms compete only to steal market share from each other. In contrast, our model allows consumers the option to purchase none of the products. The no-purchase option clearly is necessary for a sensible study of monopoly, but also provides a second demand margin for duopoly and thus plays a key role in our comparison of the two market structures. Finally, we develop for independence case a new necessary and sufficient condition for price to be higher under duopoly than monopoly based on the hazard rate of the preference distribution, and, furthermore, characterize new conditions for price increasing competition when preferences are dependent.

More competition can mean different things. Our main results establish conditions under which the addition of a second single-product firm raises equilibrium price. It is also true, however, that the the symmetric multiproduct monopoly price exceeds the duopoly price. The consolidation of two single product firms into a multi-product monopoly raises price for the usual reason that the monopolist internalizes the profit externality. In this sense, less competition necessarily results in higher prices. Together these results demonstrate that more product variety, whether from a multi-product monopolist or from a new entrant, can result in higher prices.

The rest of the paper is organized as follows. Section 2 formulates and analyzes the general model, and compares monopoly and duopoly prices under different assumptions about the dependence properties of consumer preferences. Section 3 analyzes uniform distribution of preferences on a varying support, thus generalizing limiting cases of Hotelling duopoly and Bertrand duopoly. Section 4 shows how competition affects consumer welfare through a price effect and a variety effect, and how the balance can be either positive or negative. Section 5 compares single-product duopoly competition with multiproduct monopoly. Section 6 draws conclusions.

2. DISCRETE CHOICE MODEL OF PRODUCT DIFFERENTIATION

Preferences

Each consumer desires to purchase at most one of two goods. The preferences of a consumer are described by reservation values for the two goods, (v_1, v_2), where $v_i \in [\underline{v}, \bar{v}]$, and $0 \leq \underline{v} < \bar{v} \leq \infty$. The distribution of preferences over the population of consumers is assumed to be nondegenerate and symmetric. Thus, the population of consumers, the size of which is normalized to one, is described by a continuous marginal distribution function $F(v_1)$ and a conditional distribution function $G(v_2 \mid v_1)$. These distribution functions are assumed to be differentiable on $[\underline{v}, \bar{v}]$, with associated density functions $f(v_1)$ and $g(v_2 \mid v_1)$. The joint density function, therefore, is $h(v_1, v_2) \equiv f(v_1)\, g(v_2 \mid v_1)$. The support of the corresponding joint distribution function, $\Omega \subseteq [\underline{v}, \bar{v}]^2$, is symmetric about the 45^{o}line.

Monopoly

Consider a single firm producing one of the two goods with a constant unit cost $c \in [\underline{v}, \bar{v})$. The firm sets a price to solve the "monopoly problem":

$$\max_{p \geq c} \; (p - c)\,[1 - F(p)]\,. \tag{1}$$

<u>Assumption 1</u>. There exists a unique interior solution to the monopoly problem, $p^m \in (c, \bar{v})$.

The necessary first-order condition for the solution to the monopoly problem is

$$[1 - F(p^m)] - (p^m - c)\, f(p^m) = 0. \tag{2}$$

The first-order condition can also be written as

$$(p^m - c)\,\lambda(p^m) = 1, \tag{3}$$

where

$$\lambda(v) \equiv \frac{f(v)}{1 - F(v)} \tag{4}$$

is the hazard rate function.

Sufficient primitive conditions for Assumption 1 are: $(\bar{v} - c)\,\lambda(\bar{v}) > 1$;[8] and $\lambda(v)$ continuously increasing on $[c, \bar{v}]$. The familiar monotone hazard rate condition, however, is not necessary. For example, Assumption 1 holds if $F(v)$ is a standard exponential distribution and $\lambda(v) = 1$, and, therefore, by continuity, also holds for sufficiently small departures from the exponential case. More generally, if $\lambda(v)$ is differentiable, then sufficient conditions for Assumption 1 are: (i) $(\bar{v} - c)\,\lambda(\bar{v}) > 1$; and (ii) $\frac{d \ln \lambda(p)}{dp} > -\frac{1}{p-c}$ for $p \in [c, \bar{v})$. Thus a decreasing or non-monotonic hazard rate is consistent with our analysis, although a uniformly decreasing hazard rate requires $\bar{v} = \infty$.

Duopoly

Now consider two firms, each producing one of the two products with constant unit cost c. The two firms play a duopoly game, setting prices simultaneously and independently, and each maximizing its own profit given equilibrium beliefs about its rival's action.

Assuming its rival sets $\bar{p}$, each firm solves the "duopoly problem":

$$\max_{p} \;\; (p - c)\, q(p, \bar{p}) \tag{5}$$

with

$$q(p, \bar{p}) \equiv \int_{\underline{v}}^{\bar{p}} [1 - G(p \mid v)]\, f(v)\, dv + \int_{\bar{p}}^{\bar{v} + \min\{0, \bar{p} - p\}} [1 - G(v - \bar{p} + p \mid v)]\, f(v)\, dv. \tag{6}$$

Thus a firm's price impacts two consumer margins: those who are indifferent between the firm's product and the outside good, and those who are indifferent between the product and the rival good.

Assumption 2. There exists a unique interior symmetric equilibrium of the duopoly game, $p^d \in (c, \bar{v})$.

A best response function, $p = R(\bar{p})$, describes a solution to the duopoly problem for each $\bar{p}$. The symmetric equilibrium of the duopoly game is an interior fixed point of a best

[8] We adopt the notational convention $(\bar{v} - c)\,\lambda(\bar{v}) = \lim_{v \to \infty} (v - c)\,\lambda(v)$ if $\bar{v} = \infty$.

response function, $p^d = R\left(p^d\right)$, on the domain $[c, \bar{v}]$. Sufficient conditions for Assumption 2 are: (iii) $R(\bar{p})$ is unique for all $\bar{p} \in [c, \bar{v}]$; (iv) $R(c) > c$; (v) $R(\bar{v}) < \bar{v}$; (vi) $R(\bar{p})$ continuous; and (vii) $R(\bar{p})$ crosses the 45^{o} degree line but once (from above). Note that $R(\bar{v}) = p^m$ means that condition (v) follows from Assumption 1.

An alternative formulation of sufficient conditions for Assumption 2 is based on the function

$$\varphi(p, \bar{p}) = \frac{\left|\frac{\partial q(p,\bar{p})}{\partial p}\right|}{q(p, \bar{p})}. \tag{7}$$

The first order condition for a solution to the duopoly problem is

$$(p - c)\, \varphi(p, \bar{p}) = 1. \tag{8}$$

Therefore, $p = R(\bar{p})$ is the unique solution to this equation if, for all $c \leq \bar{p} \leq \bar{v}$: (i') $(\bar{v} - c)\, \varphi(\bar{v}, \bar{p}) > 1$; and (ii')

$$\frac{\partial \ln \varphi(p, \bar{p})}{\partial p} > -\frac{1}{p - c}$$

for $p \in [c, \bar{v}]$. These primitive conditions are analagous to those that support Assumption 1 and are sufficient for Assumption 2. In fact, the conditions are the same when $\bar{p} = \bar{v}$. Alternatively, Assumption 2 is satisfied if p^d uniquely solves $(p - c)\, \varphi(p, p) = 1$ and $(p - c)\, \varphi(p, p^d)$ is strictly increasing in p.

An important special case is the independent exponential case with $\bar{v} = \infty$ and

$$G(v_1|v_2) = F(v_1) \equiv 1 - e^{-\lambda v_1}$$

for some $\lambda > 0$.[9] In this case,

$$\varphi(p, \bar{p}) = \frac{\lambda^2 e^{-\lambda(p-\bar{p})} \int\limits_{\bar{p}}^{\infty} e^{-2\lambda v} dv + \lambda e^{-\lambda p}\left(1 - e^{-\lambda \bar{p}}\right)}{\lambda \int\limits_{\bar{p}}^{\infty} e^{-\lambda(v-\bar{p}+p)} e^{-\lambda v} dv + e^{-\lambda p}\left(1 - e^{-\lambda \bar{p}}\right)} = \lambda.$$

Therefore, conditions (i')-(ii') hold strictly, and $p^d = p^m = c + \frac{1}{\lambda}$. Because conditions (i')-(ii') hold strictly for the case of a constant hazard rate, it is possible to perturb $h(v_1, v_2)$

[9] The generalized exponential distribution function is $F(v) = 1 - e^{-\lambda(v-\mu)}$ with $\lambda\mu < 1$ required for an interior solution of the monopoly problem. We normalize the location parameter $\mu = 0$ without loss of generality.

around the independent exponential case and still satisfy the assumptions. The independent exponential case is a key benchmark for analyzing how the properties of the distribution of (v_1, v_2) matters for the relationship between the monopoly price and the symmetric duopoly price.

Comparison

To compare the duopoly price and the monopoly price, it is useful to restate of the first-order condition of the monopoly problem as

$$\int_{\underline{v}}^{\bar{v}} \left[1 - G\left(p^m|v\right)\right] f\left(v\right) dv - \left(p^m - c\right) \int_{\underline{v}}^{\bar{v}} g\left(p^m|v\right) f\left(v\right) dv = 0. \tag{9}$$

For the purpose of comparing monopoly and duopoly, it is useful to define the function

$$\begin{aligned}\Psi\left(p\right) \equiv & \left\{\int_{\underline{v}}^{p} \left[1 - G\left(p|v\right)\right] f\left(v\right) dv + \int_{p}^{\bar{v}} \left[1 - G\left(v|v\right)\right] f\left(v\right) dv\right\} \\ & - \left(p - c\right) \left\{\int_{\underline{v}}^{p} g\left(p|v\right) f\left(v\right) dv + \int_{p}^{\bar{v}} g\left(v|v\right) f\left(v\right) dv\right\}.\end{aligned} \tag{10}$$

The equilibrium condition $p^d = R\left(p^d\right)$ implies $\Psi\left(p^d\right) = 0$, which is the equilibrium first-order condition for the duopoly problem. The comparison of monopoly and duopoly relies on the following additional assumption, which states that $\Psi\left(p\right)$ crosses the zero line once from above. Assumption 3 holds for the same sufficient conditions stated above for Assumption 2. In particular, the assumption holds if $R\left(\bar{p}\right)$ crosses the 45$^{\text{o}}$ degree line once from above.

<u>Assumption 3</u>. $\Psi\left(p\right) \geq 0$ if and only if $c \leq p \leq p^d$, and $\Psi\left(p\right) \leq 0$ if and only if $p^d \leq p \leq \bar{v}$.

If follows from $\Psi\left(p^d\right) = 0$ and Assumption 3 that $p^d \geq p^m$ if $\Psi\left(p^m\right) \geq 0$, and, conversely, $p^d \leq p^m$ if $\Psi\left(p^m\right) \leq 0$. Note further that the first-order condition for the monopoly problem implies

$$\Psi\left(p^m\right) = \left\{-\int_{p^m}^{\bar{v}} \left[G\left(v|v\right) - G\left(p^m|v\right)\right] f\left(v\right) dv\right\} - \left(p^m - c\right) \left\{\int_{p^m}^{\bar{v}} \left[g\left(v|v\right) - g\left(p^m|v\right)\right] f\left(v\right) dv\right\}. \tag{11}$$

Therefore, we have the following comparison.

Theorem 1 *Under Assumptions 1-3, $p^d \geq p^m$ if and only if*

$$\left\{ \int_{p^m}^{\bar{v}} \left[G\left(v|v\right) - G\left(p^m|v\right) \right] f\left(v\right) dv \right\} \leq \left(p^m - c\right) \left\{ \int_{p^m}^{\bar{v}} \left[g\left(p^m|v\right) - g\left(v|v\right) \right] f\left(v\right) dv \right\}, \quad (12)$$

and the converse.

The theorem is explained in Figure 1. The set of possible preferences, Ω, is represented by the shaded circle. The solid line dividing the circle, labelled p^m, represents the monopoly price. The area of the circle above this line (region A + region B) represents the market share[10] of the monopolist selling good 1. Region A alone represents the market share of a duopolist selling product 1 when both duopolists charge p^m. Thus, the wedge-shaped region B is equal to difference in market share for a monopolist charging p^m and for a duopolist when both firms charge p^m. The larger is this difference, the greater is the incentive of a duopolist to reduce price. Call this the "market share effect" of competition. The probability that preferences lie within region B is $\int_{p^m}^{\bar{v}} \left[G\left(v \mid v\right) - G\left(p^m \mid v\right) \right] f\left(v\right) dv$, which is the left-hand side of the expression in Theorem 1. Next consider the two straight edges of the wedge. Difference in the density of preferences along these edges is $\int_{p^m}^{\bar{v}} \left[g\left(v \mid v\right) - g\left(p^m \mid v\right) \right] f\left(v\right) dv$, which is part of the right-hand side of the expression in Theorem 1. This amount is difference in the slope of the duopolist's (residual) demand curve and the monopolist's demand curve at price p^m. The steeper is the duopolist's demand curve, relative to the monopolist's, the greater is the incentive of the duopolist to raise price above p^m. Call this the "price sensitivity effect" of competition. In order for $p^d > p^m$, it is thus necessary, but not sufficient, that the duopolist's demand curve is steeper than the monopolist's. This necessary condition can often hold under product differentiation, because consumers have a choice of products in the duopoly case, and, therefore, are less keen to purchase the duopolist's product in response to a price cut.[11] To sum up, Theorem 1 states that the duopolist will have incentive to raise price above the monopoly level if the

[10] Here "market share" means the portion of the consumer population who are purchasing the product, or market coverage.

[11] This condition is more likely to hold when consumers' preferences for the two prodcuts are negatively correlated; but negative correlation is not required.

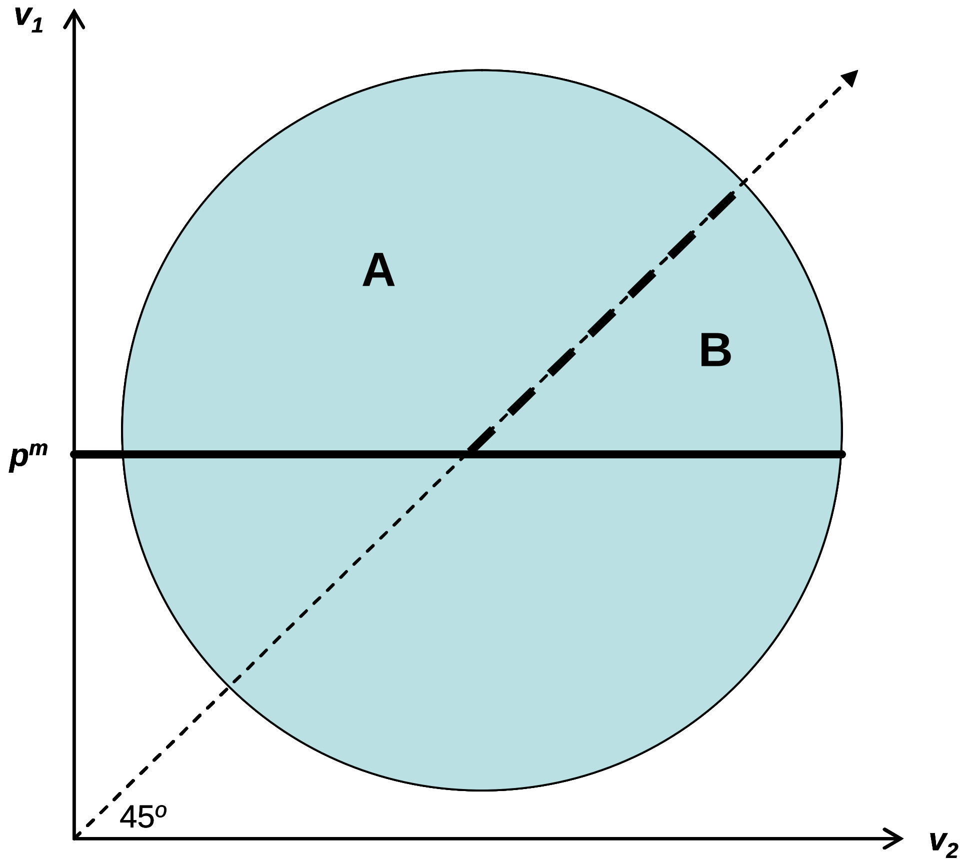

FIG. 1. Comparing single-product monopoly and symmetric duopoly

price sensitivity effect outweighs the market share effect. The theorem characterizes the balance of these effects with reference to the underlying distribution of consumer preference and the level of cost.

A special case for Theorem 1 is when v_1 and v_2 are independent. In this case, $g(v|v) = f(v)$ and $\mu(v|v) = \lambda(v)$, and condition (12) becomes

$$\int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\lambda(v)} \right] f(v)^2 dv \leq \int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\lambda(p^m)} \right] f(p^m) f(v) dv,$$

which by equation (3) is equivalent to

$$\int_{p^m}^{\bar{v}} \left[\frac{1}{\lambda(p^m)} - \frac{1}{\lambda(v)} \right] f(v)^2 dv \leq 0.$$

Therefore, we have the following comparison for the independence case. The corollary provides a necessary and sufficient condition for price-increasing competition showing precisely how departures from a constant hazard rate (i.e. the exponential distribution) matter in the independence case.[12]

Corollary 1 *If v_1 and v_2 are independent, then $p^d \geq p^m$ if and only if*

$$\int_{p^m}^{\bar{v}} \left[\frac{1}{\lambda(v)} - \frac{1}{\lambda(p^m)} \right] f(v)^2 dv \geq 0, \tag{13}$$

and the converse. Thus a sufficient condition for price-increasing (decreasing) competition is that the hazard rate is decreasing (increasing) for an appropriate range.

Examples of $p^d > p^m$ for independent valuations are provided by the Pareto distristribution: $F(x) = 1 - x^{-\alpha}$ with $\alpha > 1$, $\underline{v} = 1$, and $\bar{v} = \infty$. If $c = 1$, then $p^m = \frac{\alpha}{\alpha-1}$, and p^d satisfies $\Psi(p^d) = 0$ with

$$\Psi(p) = 1 - \frac{p^{-\alpha}}{2} + (p-1)(\alpha p^{-\alpha-1} - \alpha p^{-1} - \alpha^2 \frac{p^{-\alpha-1}}{2\alpha+1}).$$

Equilibrium prices for various values of α are given in the table below:

α	1.2	1.4	1.6	1.8	2.0	2.2	2.4
p^m	6.00	3.50	2.66	2.25	2.00	1.83	1.71
p^d	6.54	3.72	2.79	2.33	2.06	1.88	1.75
$\frac{p^d - p^m}{p^m}$	9.0%	6.2%	4.6%	3.5%	2.8%	2.3%	1.9%

Table 1. Equilibrium for the independent Pareto case

[12] Gabaix, Laibson, and Li (2005) study several distributions for which the symmetric oligopoly price increases with the number of firms, and explain that these distributions have fatter tails than the exponential distribution for which the price is constant. Our necessary and sufficient condition in Corollary 1 is not obviously a fat-tail condition, although it is consistent with fat-tailed distributions. Gabaix, Laibson, and Li (2005) characterize fat-tailed distributions using extreme value theory. While their model assumes independent preferences, it also assumes at least two firms and no outside good. Thus, their analysis does not apply directly to our model.

A provocative case of price-increasing competition is for the (Gumbel) bivariate exponential distribution:

$$h(v_1, v_2) = [(1 + \theta v_1)(1 + \theta v_2) - \theta] \exp\{-v_1 - v_2 - \theta v_1 v_2\}$$

with $0 \le \theta \le 1$. The v_i are independent for $\theta = 0$, and increasingly negatively correlated for higher θ, reaching a maximum negative correlation of 0.404 when $\theta = 1$. The marginal distributions of v_1 and v_2 are standard exponential distributions. Therefore, $p^m = 1$ if $c = 0$. In this case, a straightforward numerical analysis establishes that $p^d = 1$ if $\theta = 0$, and $p^d > 1$ if $\theta > 0$. Fig. 2 graphs the function $\Psi(p)$ for $\theta = 0$ (solid line), and $\theta = 1$ (dashed line). The curve is downward sloping as required by Assumption 3,[13] and the duopoly solution occurs where $\Psi(p) = 0$. Figure 2 shows that $\Psi(1) = 0$ for $\theta = 0$ and $\Psi(1) > 0$ for $\theta = 1$. The graph of $\Psi(p)$ for intermediate cases $0 < \theta < 1$ also crosses the horizontal access to the right of 1. Therefore, competition increases price in this case.

Another way to understand the bivariate exponential example (and others) is in terms of the duopoly reaction curve $R(p_2; \theta)$ illustrated in Figure 3. The equilibrium duopoly price is determined by the intersection of the reaction curve with the 45^{o} line. In case of independence ($\theta = 0$), the reaction curve is flat: $R(p_2; 0) = 1$; in case of negative dependence (e.g. $\theta = 1$), the reaction curve is nonmonotonic, first increasing (when p_2 is close to zero) and then decreasing. Note that the reaction curve approaches $p^m = 1$ as the rival's price (p_2) gets large, i.e. Firm 1 becomes a monopolist as Firm 2 prices itself out of the market. Intuitively, this is a general property, making clear that the duopoly reaction curve declines for at least some range of $p > p^d$ for price-increasing competition.[14]

Proceeding more generally, let

$$\mu(p|v) = \frac{g(p|v)}{1 - G(p|v)} \tag{14}$$

[13] It is straightforward to verify numerically that the profit function of duopolist is quasi-concave when the rival charges p^d satisfying $\Psi(p^d) = 1$, implying Assumption 2.

[14] Bulow, Geanakoplos, and Klemperer (1985) point out that the slope of the reaction curve depends on how the rival's price influences the price elasticity of demand, and is ambiguous in general.

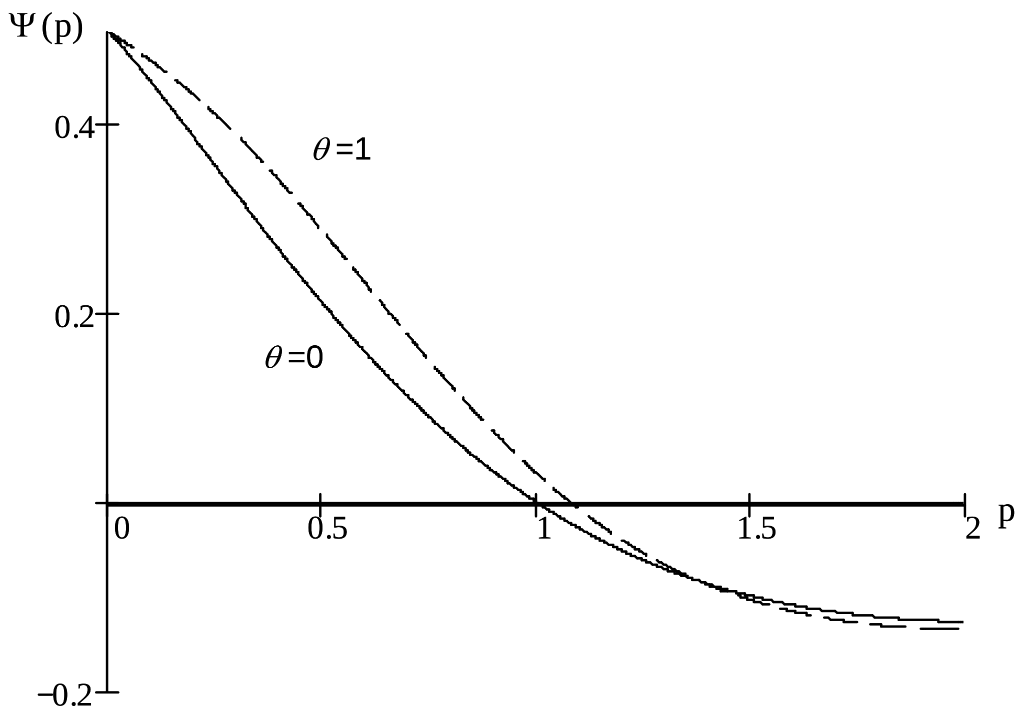

FIG. 2. Bivariate exponential case

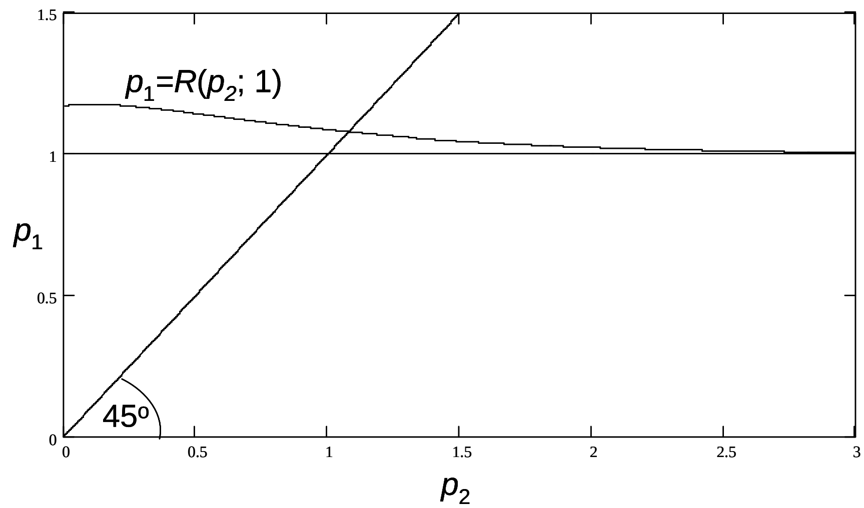

FIG. 3. Duopoly reaction curve for bivariate exponential case

denote the conditional hazard rate. Then the condition of Theorem 1 can be rewritten as

$$\int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\mu(v|v)} \right] g(v|v) f(v) dv \leq \int_{p^m}^{\bar{v}} \left[p^m - c - \frac{1}{\mu(p^m|v)} \right] g(p^m|v) f(v) dv. \qquad (15)$$

The following result follows directly from this condition and the fact that $\mu(v|v) < \mu(p^m|v)$ implies $g(v|v) < g(p^m|v)$ for $v > p^m$. The bivariate exponential distribution satisfies the requisite property for $\theta > 0$, and, therefore, provides a special case of the corollary.

Corollary 2 *Assume $\bar{v} = \infty$. Then $p^d > p^m$ if $\mu(v|v) < \mu(p^m|v)$ for $v > p^m$. Thus a sufficient condition for price-increasing competition is that the conditional hazard rate is decreasing for an appropriate range.*

We further consider how departures from independence can lead to price-increasing competition by using copulas to describe joint distribution functions. Since $F(v)$ is continuous, by Sklar's Theorem, $H(v_1, v_2) = C(F(v_1), F(v_2))$ where $C(x, y)$ is a unique symmetric copula. For simplicity we assume $C(x, y)$ is twice continuously differentiable in each argument. A symmetric copula is a symmetric bivariate distribution function on the unit square with uniform marginal distribution functions, i.e. $C(x, 0) = 0$, $C(x, 1) = x$, and conditional distribution function $C(y|x) = \frac{\partial C(x,y)}{\partial x}$. Conversely, if $C(x,y)$ is a symmetric copula, and $F(v)$ a univariate distribution function, then $H(v_1, v_2) = C(F(v_1), F(v_2))$ is a symmetric bivariate distribution function (Nelson, 2006). For example, $C(x,y) = x + y - 1 + (1-x)(1-y)e^{-\theta \ln(1-x)\ln(1-y)}$ is the copula for the bivariate exponential distribution introduced earlier.

The utility of the copula approach is that it is possible to vary the dependence properites of $H(v_1, v_2)$ by holding $F(v)$ constant and varying $C(x,y)$. The variables v_1 and v_2 are independent if $C(x,y) = xy$. More generally, a copula is "positive quadrant dependent" if $C(x,y) \geq xy$, and negative quadrant dependent if $C(x,y) \leq xy$. Positive (negative) quandrant dependence implies positive (negative) Pearson correlation of v_1 and v_2 (Nelson 2006). Stochastic monotonicity is an even stronger dependence property. Consumer preference for one product (v_1) is stochastically increasing (decreasing) in the value of the other (v_2) if $C(y|x)$ is decreasing (increasing) in x, i.e. $\frac{\partial^2 C(x,y)}{\partial x^2} \leq (\geq) 0$. Positive (negative) stochastic monotonicity implies positive (negative) quadrant dependence (Joe 1997).[15] For example, a Fairlie-Gumbel-Morgenstern (FGM) copula, $C(x,y) = xy + \alpha xy(1-x)(1-y)$ with $\alpha \in [-1, 1]$, satisfies the stochastically increasing property if $\alpha \geq 0$ and the stochastically decreasing property in the opposite case.

The following result applies the copula approach to clarify more generally how the statistical dependence of consumer preferences matters for price-increasing competition relative to the benchmark independent exponential case.

Theorem 2 *If $F(v) = 1 - e^{-\lambda v}$ and $H(v_1, v_2) \equiv C(F(v_1), F(v_2))$, then $p^m \leq (\geq) p^d$ if and only if*

$$\int_{F(p^m)}^{1} \left. \frac{\partial^2 C(x,y)}{\partial x^2} \right|_{y=x} f\left(F^{-1}(x)\right) dx \geq (\leq) 0.$$

[15] Stochastic monotonicity follows from affiliation, a dependence concept familiar from the economics literature on auctions (Milgrom and Weber 1982). A copula is positively affiliated (or positive likelihood ratio dependent) if its density is log-supermodular, and negatively affiliated if log-submodular. A positively affiliated copula is stochastically increasing, and, conversely, a negatively affiliated copula is stochastically decreasing (Nelson 2006).

Therefore, $p^m \leq p^d$ if v_1 is stochastically decreasing in v_2, and, conversely, $p^m \geq p^d$ if v_1 is stochastically increasing in v_2.

Proof. *The duopoly equilibrium condition is $\Psi(p^d) = 0$ and $p^m \geq p^d$ if $\Psi(p^m) \leq 0$; conversely, $p^d \geq p^m$ if $\Psi(p^m) \geq 0$. Using the copula formulation, and integrating by parts,*

$$\begin{aligned}\Psi(p) &= \frac{1}{2}\left[1 - C\left(F\left(p\right), F(p)\right)\right] \\ &\quad -(p-c)\left[f\left(F^{-1}\left(1\right)\right) - \int_{F(p)}^{1} C(x|x) df\left(F^{-1}\left(x\right)\right) - \int_{F(p)}^{1} \left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} f\left(F^{-1}\left(x\right)\right) dx\right].\end{aligned}$$

Therefore, $\Psi(p) \leq 0$ if and only if

$$(p-c)\left[\frac{f\left(F^{-1}\left(1\right)\right) - \int_{F(p)}^{1} C(x|x) df\left(F^{-1}\left(x\right)\right) - \int_{F(p)}^{1} \left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} f\left(F^{-1}\left(x\right)\right) dx}{\frac{1}{2}\left[1 - C\left(F\left(p\right), F(p)\right)\right]}\right] \geq 1$$

Since by assumption $F(v)$ is an exponential marginal distribution, $f\left(F^{-1}(1)\right) = 0$, and $\frac{df(F^{-1}(x))}{dv} = -\lambda$. It follows that

$$\frac{f\left(F^{-1}(1)\right) - \int_{F(p)}^{1} C(x|x) df\left(F^{-1}\left(x\right)\right)}{\frac{1}{2}\left[1 - C\left(F\left(p\right), F\left(p\right); \alpha\right)\right]} = \lambda.$$

Consequently, $\Psi(p) \leq 0$ if and only if

$$(p-c)\left[\lambda - \frac{\int_{F(p)}^{1} \left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} f\left(F^{-1}\left(x\right)\right) dx}{\frac{1}{2}\left[1 - C\left(F\left(p\right), F(p)\right)\right]}\right] \geq 1.$$

Furthermore, $p^m - c = \frac{1}{\lambda}$ *implies* $\Psi(p^m) \leq 0$ *and* $p^m \geq p^d$ *if and only if*

$$\frac{1}{\lambda}\left[\lambda - \frac{\int\limits_{F(p^m)}^{1} \left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} f\left(F^{-1}(x)\right) dx}{\frac{1}{2}\left[1 - C\left(F\left(p^m\right), F(p^m)\right)\right]}\right] \geq 1,$$

or

$$-\frac{\int\limits_{F(p^m)}^{1} \left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} f\left(F^{-1}(x)\right) dx}{\frac{1}{2}\left[1 - C\left(F\left(p^m\right), F(p^m)\right)\right]} \geq 0,$$

which holds if and only if $\int\limits_{F(p^m)}^{1} \left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} f\left(F^{-1}(x)\right) dx \leq 0$*, for which a sufficient condition is* $\frac{\partial^2 C(x,y)}{\partial x^2} \leq 0$*. The converse follows similarly.* ■

Figure 4 illustrates the symmetric equilibrium price and profit for the FGM copula family and standard exponential marginal distribution (for which $p^m = 1$). The FGM family exhibits a limited range of positive or negative dependence, i.e. Kendal's tau is $\tau = \frac{2\alpha}{9}$ (Nelson 2006, p. 162). Thus price and profit under duopoly monotonically decrease as the degree of negative dependence decreases or the degree of positive dependence increases. This is intuitive. Less negative dependence or greater positive dependence means that the two products are closer substitutes for more consumers, i.e. the products are less differentiated. Price competition is more intense with less product differentiation, and profits are lower. Notice that here duopoly price exceeds monopoly price when there is negative dependence ($\alpha < 0$), and falls below the monopoly price when there is positive dependence ($\alpha > 0$).

3. UNIFORM DISTRIBUTIONS OF PREFERENCES

We next consider an analytically more tractable model of preferences, where (v_1, v_2) are uniformly distributed, to illustrate the comparison of prices under monopoly and duopoly. This model allows each consumer's valuations for the two products to have various forms of

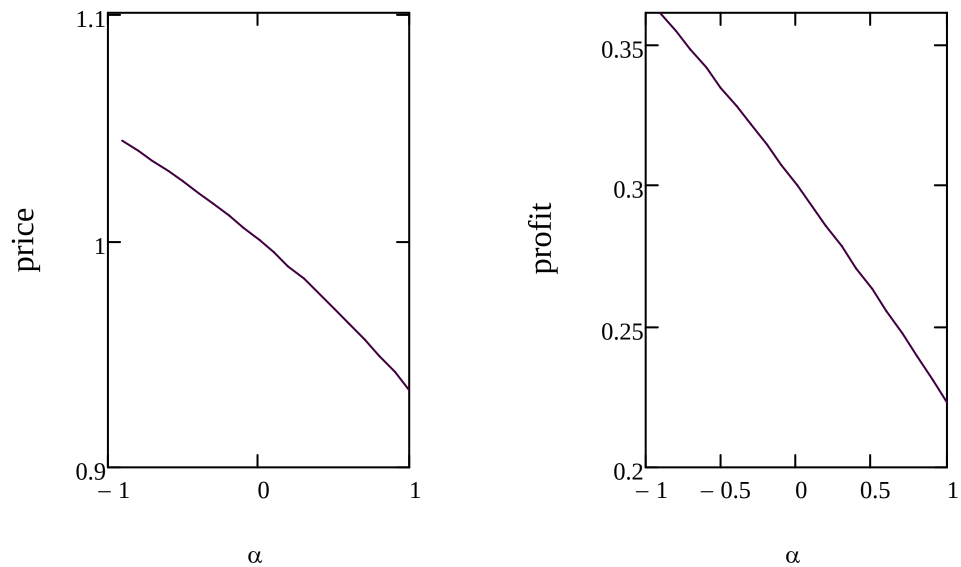

FIG. 4. Equilibrium for the FGM - exponential case

negative or positive correlations, with the familiar Bertrand and Hotelling models as limiting cases. Essentially, the model extends the Hotelling model to allow both heterogeneous consumer location (horizontal differentiation) and heterogeneous consumer value (vertical differentiation). In the absence of horizontal differentiation the model collapses to the Bertrand model with perfect positive correlation of preferences; in the absence of vertical differentiation it collapses to the Hotelling model with perfect negative correlation.

Suppose the support for (v_1, v_2), Ω, is a rectangular area on the v_1-v_2 space that is formed by segments of four lines with the following inequalities:

$$\begin{gathered} 2(1+a) \geq v_1 + v_2 \geq 2; \\ b \geq v_1 - v_2 \geq -b, \end{gathered} \tag{16}$$

where $a \in (0, \infty)$ and $b \in (0, 1]$.[16] Suppose that (v_1, v_2) is uniformly distributed on Ω. Then

$$h(v_1, v_2) = \frac{1}{2ab}, \ \ (v_1, v_2) \in \Omega. \tag{17}$$

When $b \to 0$, Ω converges to an upward sloping line; in the limit, v_1 and v_2 have perfect positive correlation and the model becomes the standard model of Bertrand competition with a downward sloping demand curve. On the other hand, when $a \to 0$, Ω converges to a downward sloping line; in the limit v_1 and v_2 have perfect negative correlation and the model becomes one of Hotelling competition with the unit transportation cost being 1, the length of the Hotelling line being b, and consumers valuing either product variety at $\frac{2+b}{2}$ (not including the transportation cost). In fact, we may consider Ω as consisting of a dense map of lines $v_1 + v_2 = x$, $x \in [2, 2(1+a)]$, each having length $\sqrt{2}b$ and being parallel to line $v_1 + v_2 = 2$. Each of these line segments corresponds to a Hotelling line with length equal to b, unit transportation cost equal to 1, and consumer valuation equal to $\frac{x+b}{2}$. Figure 4 illustrates Ω for representative values of a and $b = 1$.

We first obtain the prices under duopoly and under monopoly. We have:

Lemma 1 *Under uniform distributions with support Ω, the symmetric equilibrium price under duopoly is $p^d = b$, and the optimal price for the monopolist is*

$$p^m = \begin{cases} \frac{a+b+2}{4} & if \quad 0 < a < b - \frac{2}{3} \\ \frac{2-b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} & if \quad \max\{b - \frac{2}{3}, 0\} \le a < 1 + b \\ \frac{1+a}{2} & if \quad 1 + b \le a \end{cases}. \tag{18}$$

Notice that the equilibrium duopoly price, $p^d = b$, is simply the equilibrium price of the Hotelling model when $a \to 0$ and Ω collapses to a single downward sloping line. The Hotelling solution generalizes because Ω is essentially a collection of stacked Hotelling lines, for each of which b is a best response to b. Calculation of the monopoly price is slightly

[16] The parameter restriction $b \le 1$ is imposed to ensure that, when $a \to 0$, in the duopoly equilibrium all consumers will purchase with positive surpluses. As we will see shortly, the equilibrium duopoly price will always be $p^d = b$. If $a \to 0$, $v_1 + v_2 \to 2$; and $b \le 1$ is needed so that the consumer with $v_1 = v_2$ will still purchase with a positive surplus. When a increases, the maximum allowed value of b also increases.

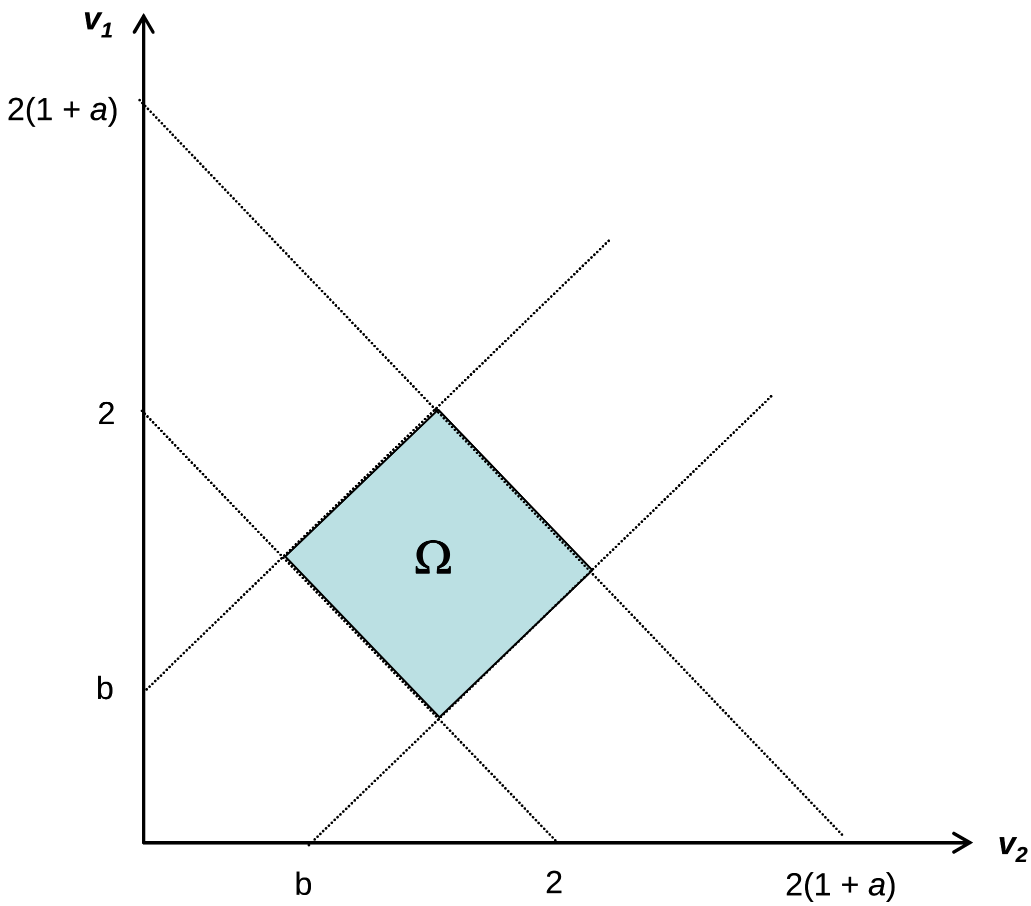

Fig. 5. Ω is an oriented rectangle

more complicated. Details of the calculations that establish Lemma 1 as well as Theorem 3 below are contained in an updated version of an appendix initially contained in Chen and Riordan (2006).

The variance and (Pearson's) correlation coefficient of v_1 and v_2 are:[17]

$$\begin{aligned} Var(v_1) &= \frac{1}{12}\left(a^2+b^2\right) = Var(v_2), \\ \rho &= \frac{(a-b)(a+b)}{a^2+b^2}. \end{aligned}$$

Therefore, given the parameter restrictions $a > 0$ and $0 < b \leq 1$, the case $a < b$ corresponds to negative correlation, $a = b$ to independence, and $a > b$ to positive correlation.

The comparison of the duopoly and monopoly prices is straightforward:

$$p^m - p^d = \begin{cases} \frac{a-3b+2}{4} & if \quad 0 < a < b - \frac{2}{3} \\ \frac{2-4b}{3} + \frac{1}{6}\sqrt{24ab - 4b + b^2 + 4} & if \quad \max\{b - \frac{2}{3}, 0\} \leq a < 1 + b \\ \frac{1+a}{2} - b & if \quad 1 + b \leq a \end{cases}. \tag{19}$$

Analysis of this equation yields the following comparison:

Theorem 3 *Under uniform distributions with support* Ω*,*

$$p^m - p^d \begin{cases} < 0 & if \quad 0 < a < \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ = 0 & if \quad 0 < a = \frac{(3b-2)(7b-2)}{8b} \text{ and } b > \frac{2}{3} \\ > 0 & if \quad \text{otherwise} \end{cases}. \tag{20}$$

We note that when $b > \frac{2}{3}$, $b - \frac{2}{3} < \frac{(3b-2)(7b-2)}{8b} < b$, and $\frac{(3b-2)(7b-2)}{8b}$ increases in b. Thus in this model of uniform distributions, duopoly price is higher than monopoly price if a is sufficiently small relative to b and b is above certain critical value, or if ρ is small and $Var(v_i)$ is high enough. This amounts to stating that competition increases price if consumer preferences are sufficiently negatively correlated and diverse. The parameter values with this

[17] Here, Pearson's correlation coefficient has a simple and intuitive parameter interpretation, and we use it to measure the preference dependence relationship. In general, dependence concepts such as positive (negative) quadratic dependence and stochastic increasing (decreasing) can be more useful than Pearson's correlation coefficient, which is a measure of the linear dependence between random variables.

feature represent a plausible but relatively small region of the preference space considered. Under uniform distributions, competition increases price only if preferences are negatively correlated ($a < b$).

4. CONSUMER WELFARE

We have so far shown when and how prices are higher under duopoly competition than under a single-product monopoly. But since there are more product varieties under duopoly, consumer welfare may still be higher under duopoly competition. It is interesting to know whether and when competition can also lower consumer welfare.[18]

Consumer welfare under monopoly is

$$W^m = \int_{p^m}^{\bar{v}} (v_1 - p^m) f(v_1) \, dv_1.$$

Consumer welfare under duopoly is

$$\begin{aligned} W^d &= \int_{p^d}^{\bar{v}} \left[\int_{\underline{v}}^{v_1} \left(v_1 - p^d \right) h(v_1, v_2) \, dv_2 \right] dv_1 + \int_{p^d}^{\bar{v}} \left[\int_{\underline{v}}^{v_2} \left(v_2 - p^d \right) h(v_1, v_2) \, dv_1 \right] dv_2 \\ &= \int_{p^d}^{\bar{v}} \left(v_1 - p^d \right) f(v_1) \, dv_1 \\ &\quad + \left[\int_{p^d}^{\bar{v}} \int_{\underline{v}}^{p^d} \left(v_2 - p^d \right) h(v_1, v_2) \, dv_1 dv_2 + \int_{p^d}^{\bar{v}} \int_{v_1}^{\bar{v}} (v_2 - v_1) h(v_1, v_2) \, dv_2 dv_1 \right]. \end{aligned}$$

The change in consumer welfare from monopoly to duopoly is:

$$\begin{aligned} W^d - W^m &= \left[\int_{p^d}^{\bar{v}} \left(v_1 - p^d \right) f(v_1) \, dv_1 - \int_{p^m}^{\bar{v}} (v_1 - p^m) f(v_1) \, dv_1 \right] \\ &\quad + \left[\int_{p^d}^{\bar{v}} \int_{\underline{v}}^{p^d} \left[v_2 - p^d \right] h(v_1, v_2) \, dv_1 dv_2 + \int_{p^d}^{\bar{v}} \int_{v_1}^{\bar{v}} [v_2 - v_1] h(v_1, v_2) \, dv_1 dv_2 \right]. \end{aligned}$$

Thus change in consumer welfare from monopoly to duopoly depends on two effects, the price effect and the variety effect: The price effect

$$\Delta W|^{price} \equiv \int_{p^d}^{\bar{v}} \left(v_1 - p^d \right) f(v_1) \, dv_1 - \int_{p^m}^{\bar{v}} (v_1 - p^m) f(v_1) \, dv_1$$

[18] If consumer welfare declines, then so does social welfare if fixed costs dissipate duopoly profits (Mankiw and Whinston 1986).

is the change in consumer welfare due to the change in the price of product 1 from monopoly to duopoly; the price effect is positive if $p^d < p^m$, zero if $p^d = p^m$, and negative if $p^d > p^m$. The variety effect

$$\Delta W|^{variety} \equiv \left[\int_{p^d}^{\bar{v}} \int_{\underline{v}}^{p^d} \left[v_2 - p^d \right] h(v_1, v_2)\, dv_1 dv_2 + \int_{p^d}^{\bar{v}} \int_{v_1}^{\bar{v}} [v_2 - v_1]\, h(v_1, v_2)\, dv_2 dv_1 \right]$$

is the change in consumer welfare due to the added option product 2 for consumers when the market structure changes from monopoly to duopoly. The variety effect is always positive, because there exist consumers who value product 2 more than product 1; some of them will change from no-purchasing to purchasing while others will switch from purchasing product 1 to purchasing product 2, thus creating new consumer surplus.

Summarizing, consumer welfare is higher under duopoly competition than under monopoly if and only if

$$\Delta W|^{price} + \Delta W|^{variety} > 0, \tag{21}$$

and the converse. A sufficient condition for competition to increase consumer welfare is that competition reduces prices, and a necessary condition for competition to reduce consumer welfare is that competition increases prices. In general, either the price effect (when it is negative) or the variety effect can dominate. Thus, depending on consumer preferences, consumer welfare can either be higher or lower under duopoly than under monopoly. This is demonstrated by the following result for the uniform distributions model, the proof for which is contained in the appendix.

Theorem 4 *Under uniform distributions with support* Ω, *if* $b \in [0.952,\ 1]$ *and* $a < \hat{a}(b)$, *where* $\hat{a}(b) = 3b - \frac{2}{7}\sqrt{3}\sqrt{28b - 7b^2 - 4} - \frac{6}{7}$ *is a positive and increasing function, competition reduces consumer welfare*; *otherwise, competition increases consumer welfare.*

Similar to the conditions leading to higher prices under duopoly, here competition reduces consumer welfare when b is above a critical value and a is sufficiently small relative to b, or if $Var(v_i)$ is high and ρ small enough. In other words, in the uniform distributions model, competition reduces consumer welfare if consumer preferences are sufficiently diverse and negatively correlated.

In contrast, in the Gumbel bivariate exponential or FGM-exponential models, consumer welfare is always higher under duopoly than monopoly. These joint distributions allow only a limited degree of negative dependence. However, the degree of negative dependence alone is not determinative of the comparison. For suppose $H(v_1, v_2) = C(F(v_1), F(v_2))$ where $C(x, y) = \min\{0, x + y - 1\}$ is the "minimum copula" exhibiting perfect negative correlation,[19] and the marginal distribution $F(v)$ is exponential. In this case, the symmetric duopoly price equals the monopoly price, the variety effect dominates, and consumer welfare is higher under duopoly.[20]

5. MULTIPRODUCT MONOPOLY

As other studies in the literature concerning the effects of market structure on prices, our main interest in this paper is to compare duopoly and monopoly prices for single-product firms. This comparion sheds light, for instance, on situations where a competitor with a differentiated product enters a market that is initially monopolized. For completeness, we now also compare the price of a multi-product monopolist who sells both products and a pair of duopolists.

A single firm producing both products solves the "multiproduct monopoly problem":

$$\max_{(p_1,p_2)\in[c,\bar{v}]^2} \ (p_1 - c)\, q(p_1, p_2) + (p_2 - c)\, q(p_2, p_1). \tag{22}$$

Assumption 4. There exists a unique interior symmteric solution to the multiproduct monopoly problem, $p^{mm} \in (c, \bar{v})$.

Recalling the definition of $\Psi(\cdot)$ from equation (10), the first-order condition for the solu-

[19] A fundamental result in the theory of copulas is that any copula is bounded below by the minimum copula (Nelson 2006).

[20] This follows because the market is not fully covered. If $F(x) = 1 - e^{-\lambda x}$ and $c = 0$, then $p^m = 1/\lambda$ and monopoly market coverage is $1 - F(p^m) = 0.368$. With duopoly the market segments: each firm sets $p^d = p^m$ and serves 0.368 of the the potential market. The result is consistent with Theorem 2 because $\left.\frac{\partial^2 C(x,y)}{\partial x^2}\right|_{y=x} = 0$ for $x \geq F(p^m)$.

tion to the multiproduct monopoly problem is equivalent to

$$\Psi\left(p^{mm}\right)+\left(p^{mm}-c\right)\left[\left[1-G\left(p^{mm}\mid p^{mm}\right)\right]f\left(p^{mm}\right)+\int_{p^{mm}}^{\bar{v}}g\left(v\mid v\right)f\left(v\right)dv\right]=0, \quad (23)$$

which implies

$$\Psi\left(p^{mm}\right)<0.$$

Since $\Psi\left(p^{d}\right)=0$, and by Assumption 3, $\Psi\left(p\right)<0$ if and only if $p>p^{d}$, we have

Theorem 5 *Under Assumption 2-4, $p^{mm}>p^{d}$.*

This result is familiar, and the intuition is well known: the price change of one product affects the profit of another product; this effect is not taken into account when the duopolists set prices independently, but the multiproduct monopolist internalizes this effect when setting prices jointly for the two products. Consequently, since the products are substitutes here, the multiproduct monopolist charges a higher price for the two products than the duopolists.[21]

Therefore, comparing prices between a multiproduct monopolist and single-product competitiors is very different from comparing prices under different market structures with single-product firms. In the former, the results are based on the familiar idea of a monopolist internalizing the externalities between different products. In the latter, the forces at work have not been well understood. The contribution of our analysis is to explain the effects determining how prices change from monopoly to duopoly for single-product firms, and identify precise conditions for price-increasing competition.

5. CONCLUSION

The relationship between market structure and price is a central issue in economics. This paper has provided a complete comparison of equilibrium prices under single-product

[21] If the two products were complements (and hence consumers might buy both products), prices would again be lower under multiproduct monopoly than under duopoly competition, More generally, whether or not prices are higher under the multiproduct monopoly depends on the nature of relations between products (e.g., Chen, 2000; and Davis and Murphy, 2000).

monopoly and symmetric duopoly in an otherwise general discrete choice model of product differentiation. The necessary and sufficient condition for price-increasing competition balances two effects of entry by a symmetric firm into a monopoly market, the market share effect and the price sensitivity effect. The market share effect is that a reduced quantity per firm under duopoly provides an incentive for the firms to cut price below the monopoly level. The price sensitivity effect is that a steeper demand curve resulting from greater consumer choice provides an incentive to raise price. Under certain conditions the price sensitivity effect outweighs the market share effect, resulting in a higher symmetric duopoly price compared to monopoly. For example, the symmetric duopoly price is higher than the single-product monopoly price if consumers' values for the two products are independently drawn from a distribution function with a descreasing hazard rate. Also, if the marginal distribution is exponential, then price is higher (lower) under duopoly than under monopoly if the joint distribution of consumer values for the two products has the stochastically decreasing (increasing) dependence property. A class of special cases is when consumer values for two products have a joint uniform distribution on a varying oriented rectangular support. This framework nests the familiar Hotelling and Bertrand models. Competition increases prices in these cases if valuations are sufficiently diverse and negatively correlated; and, with similar but tigher parameter restrictions, competition also reduces consumer welfare. In summary, our analysis shows that the consumer preferences leading to price-increasing competition are by no means exceptional. Furthermore, consumer welfare can also be lower under competition than under monopoly.

The theoretical possibility of price-increasing competition potentially has important implications for emprical industrial organization. For instance, it is a standard procedure of the new empirical industrial organization to estimate a discrete choice differentiated products demand model, and infer unobservable marginal costs from the corresponding first-order conditions for equilibrium pricing (Berry, 1994). If the demand model presumes restrictions on consumer preferences that are inconsistent with price-increasing competition, then the analysis might mistakenly conclude that marginal costs are higher in duopoly markets than monopoly markets.[22]

There are several promising directions for further theoretical research. For instance, it would be interesting to know the relationship between market structure and prices with an arbitrary number of firms. Chen and Riordan (2007) provides an analysis within a spatial setting and Gabaix, Laibson, and Li (2005) study the independence case with no outside good for specific distribution functions. More research is needed to understand the relationship in a general framework of preferences. Another direction is to consider asymmetric duopoly, for example, competition between a branded product and generic product, as is relevant in pharmaceutical markets. The market-share and price-sensitivity effects apply to this case also.

Finally, as discussed in the introduction, there is scattered empirical evidence pointing to the phenomenon of price-increasing competition in several industries. We hope that our theory stimulates new empirical research on the topic.

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