

Vertical Integration, Exclusive Dealing, and *Ex Post* Cartelization*

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Abstract

This paper uncovers an unnoticed connection between exclusive contracts and vertical organization. A vertically integrated firm can use exclusive dealing to foreclose an equally efficient upstream competitor and to cartelize the downstream industry. Neither vertical integration nor exclusive dealing alone achieves these anticompetitive effects. The cartelization effect of these two practices may be limited when downstream firms are heterogeneous and supply contracts are not contingent on uncertain market conditions. The extent of cartelization also depends on the degree of downstream market concentration and on the degree to which downstream competition is localized.

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1. INTRODUCTION

Antitrust scholars have devoted much ink to the competitive effects of vertical mergers (Riordan and Salop, 1995). For the most part, the economics literature focuses on how vertical integration *per se* alters pricing incentives in relevant upstream and downstream markets. The Chicago school of antitrust, represented by Bork (1978), emphasizes that the efficiencies of vertical integration are likely to cause lower prices to final consumers, while a more recent strategic approach to the subject, represented by Ordoover, Salop and Saloner (1990) and Hart and Tirole (1990), shows how vertical integration lacking any redeeming efficiencies might have the opposite purpose and effect. Ma (1997), Choi and Yi (2000), Church and Gandal (2000), and Chen (2001) consider richer models that feature trade-offs between anticompetitive effects and efficiencies. The debate is far from settled, in no small part because workable *indicia* of harmful vertical mergers are lacking except in special cases (Riordan, 1998).

The use of exclusive contracts by customers and suppliers in intermediate product markets is equally controversial. The courts and antitrust agencies historically have treated exclusive dealing harshly, finding in many cases such practices illegally to foreclose competition. The Chicago school disputes this approach, advising instead that exclusive contracts are presumptively efficient, because usually it is unprofitable to foreclose competition *via* exclusive contracts without good efficiency reasons (Bork, 1978). More recently, industrial organization economists have studied alternative models that demonstrate equilibrium incentives to foreclose more efficient potential entrants with exclusive contracts (Aghion and Bolton, 1987; Bernheim and Whinston, 1988; Rasmusen, Ramseyer and Wiley, 1991; Segal and Whinston, 2000; Spector, 2004).

An important institutional feature of some intermediate product markets is the coexistence of vertical integration and exclusive contracts. For instance, in *Standard Oil Co. v. U.S.* (1949), Standard Oil sold about the same amount of gasoline through its own service stations as through independent retailers with which it had exclusive dealing contracts. In *Brown Shoe Co. 62 F.T.C. 679* (1963), Brown Shoe had vertically integrated into the re-

tailoring sector while using exclusive dealing contracts with independent retailers. In *U.S. v. Microsoft* (D.D.C. 2000), Microsoft's had license agreements with competing online service providers, requiring them to promote and distribute Microsoft's Internet Explorer to the exclusion of competitive browsers. This institutional feature is potentially important because, as we shall show, the incentive for and effects of exclusive contracts may depend on whether an upstream supplier is vertically integrated, and, conversely, the returns to vertical integration may depend on the possibility of exclusive contracting.

While the existing economics literatures on vertical integration and exclusive contracts yield important insights on the competitive effects of these practices used in isolation, the literatures generally ignore incentives for and effects of these practices in combination. The purpose of this paper is to uncover an unnoticed connection between exclusive contracts and vertical integration, and to develop a model for analyzing how these practices complement each other to achieve an anticompetitive effect. More specifically, we argue that a vertically integrated upstream firm has the ability and incentive to use exclusive contracts to exclude equally efficient upstream competitors and control downstream prices.¹ The *ex post* effect is a cartelization of the downstream industry. Neither exclusive dealing nor vertical integration alone has this anticompetitive effect.

The paper is organized as follows. Section 2 previews our basic ideas. We illustrate the relationship between vertical integration and exclusive dealing in a simple model of industrial organization with two identical upstream and two identical downstream firms. We then discuss complications that arise with heterogeneous downstream firms when competitive advantage is uncertain and non-contractible, thus providing a transition to our main model of bilateral duopoly. The main model studied in Section 3 combines upstream requirements contracting with downstream bidding to serve final customers. In this model, one or the other downstream firm has an *ex post* competitive advantage in selling to a

¹As discussed later, the Hart and Tirole (1990) model explains the exclusion of only a less efficient competitor. While the Ordover, Salop, and Saloner (1990) model does demonstrate the equilibrium exclusion of an equally efficient competitor, some controversial assumptions of the model limit its applicability (Hart and Tirole, 1991; Reiffen, 1992; Ordover, Salop and Saloner, 1992).

particular customer, but these downstream advantages are not contractible *ex ante* when upstream requirements contracts are struck. We demonstrate in this context the ability of a vertically integrated firm profitably to employ an exclusive contract that raises input prices and partially cartelizes the downstream duopoly. We further show that exclusive contracts do not achieve this anticompetitive effect if the industries are vertically separated, and discuss extensions to bilateral oligopolies, and to private rather than public requirements contracts. Section 4 reviews some key features of our model of downstream competition, discusses the robustness to other models of downstream competition, relates our results to previous economics literature, discusses two relevant antitrust cases, and discusses policy implications.

There are two appendices. Appendix A proves the formal results of Section 3. Appendix B studies two alternative spatial models of downstream markets with multiple independent competitors: a hub-and-spokes model of non-localized competition, and a circle model of localized competition. The results obtained earlier extend naturally to these models of bilateral oligopoly, with additional insights that the extent of upstream foreclosure and downstream cartelization depends on the nature of competition and on concentration in the downstream market. In these more general models, vertical integration *cum* exclusive dealing has anticompetitive effects compared to the competitive equilibrium outcome of a vertically separated market structure. Our conclusion that vertical integration is necessary for anticompetitive effects of exclusive dealing in the case of public contracting, however, depends on a restriction that the upstream industry is not too concentrated relative to the downstream industry. This restriction rules out additional equilibria that might display anticompetitive effects of exclusive dealing even under vertical separation.

2. BASIC IDEAS

That vertical integration and exclusive dealing can combine to foreclose an equally efficient upstream competitor and to raise downstream prices is easy to demonstrate in a simple model of industrial organization. Suppose there are two identical upstream firms, $U1$ and

$U2$, and two identical downstream firms, $D1$ and $D2$. The downstream firms require one unit of an intermediate good to produce one unit of the final good, for which identical consumers have a known reservation price V . Upstream costs are normalized to zero and downstream costs per unit of production are equal to $C < V$. If the firms are independent, then Bertrand competition in the upstream market followed by Bertrand competition in the downstream market results in a final goods price equal to C . Against this backdrop, a vertically integrated $U1-D1$ has an incentive to purchase an exclusive right to serve the downstream market and charge final consumers a price equal to V . For example, $U1-D1$ might pay $D2$ to withdraw from the market, or, alternatively, acquire $D2$. Such blatant monopolization likely would meet objections from antitrust authorities. More benign in appearance is an exclusive requirements contract that achieves the same anticompetitive effect. A contract that requires $D2$ to purchase from $U1$ at a price of $V - C$ fully extracts monopoly rents from the downstream market. Firm $U2$ is excluded from the upstream market, and final consumers pay V to purchase from either $D1$ or $D2$.²

It is interesting that $D2$ does not need much persuasion to agree to purchase its requirements exclusively from $U1-D1$ on non-competitive terms. If $D2$ were to decline an exclusive requirements contract with $U1-D1$, and instead to deal with $U2$ on competitive terms, then vigorous competition from $D1$ would squeeze out downstream profits to the point where $D2$ would be happy to have fallen into $U1$'s exclusive arms for a small concession, e.g. a small fixed payment. The Chicago school correctly observes that a downstream firm must be compensated to agree to forgo the benefits of upstream competition (Bork, 1978), but the above simple model shows that the necessary compensation need not be large if the firm has little to lose because of vigorous downstream competition.³ An exclusive contract effectively monopolizes the downstream industry, and the monopoly rents can be shared in some measure by all concerned firms.

²The presence of additional equally efficient stand-alone upstream firms does not change the argument. Furthermore, if there are additional equally efficient stand-alone downstream firms, $U1-D1$ can offer the same requirement contracts to all stand-alone downstream firms and achieve the monopoly outcome.

³In formalizing and qualifying Bork's argument, Bernheim and Whinston (1998) ignore downstream competition and vertical integration in their models of exclusive dealing.

It also is interesting that neither vertical integration nor exclusive dealing alone can be counted on to achieve these anticompetitive effects if contracts are bilateral. The vertically integrated $U1-D1$ could not persuade the independent $D2$ to pay a supra-competitive price for the intermediate good without an exclusive contract, because $D2$ would retain an *ex post* incentive to purchase from $U2$ on competitive terms and cut its retail price to steal business from $D1$. Similarly, unable to commit to a multilateral contract that binds both $D1$ and $D2$, a vertically-separated $U1$ is unable to pay both $D1$ and $D2$ enough to induce them independently to forego the competitive alternative. Thus, vertically-separated upstream firms in equilibrium maximize bilateral profits by offering each downstream firm an efficient two-part tariff that sets the unit price of the intermediate good equal to marginal cost.⁴

Matters are more complicated if downstream market conditions are uncertain and non-contractible. Suppose that C is a random variable, and that the realization of C becomes known after contracting for the intermediate good, but before setting downstream prices. Suppose further that requirements contracts take the form of uncontingent two-part tariffs. Then monopolization of the downstream industry by $U1-D1$ is accomplished with an exclusive requirements contract that excludes $D2$ by setting the marginal price of the intermediate good above all possible values of $V - C$. Otherwise, competition from $D2$ would drive the downstream price below the monopoly level in some states of the world. Thus, under conditions of uncertainty and non-contractibility, $U1-D1$ can use an exclusive contract effectively to purchase a monopoly right. The contract is hardly subtle, and such blatant exclusion likely would catch the attention of antitrust authorities.

Matters are complicated further by downstream heterogeneity. If some consumers prefer $D2$'s product, or are more cheaply served by $D2$, then a requirements contract that excludes $D2$ obviously cannot fully maximize industry joint profits. Rather a fully effective *ex post*

⁴If upstream contracts are public commitments, then there exists a continuum of subgame perfect equilibria in which $U1$ and $U2$ both offer the intermediate good to $D1$ at a price equal to 0 (marginal cost) and to $D2$ at a price $W \geq 0$, and $D1$ sells the final good at a price $\min\{W + C, V\}$. The competitive equilibrium with $W = 0$ seems the most natural one. This multiplicity of equilibria is an artifact of the homogeneous downstream market structure, and vanishes in our main model with heterogeneous downstream firms. The multiplicity also vanishes in the case of private bilateral contracts (Chen and Riordan, 2003).

cartelization of the downstream industry would require coordinated pricing that divides the downstream market efficiently. For example, if random downstream costs have different realizations for $D1$ and $D2$, then it is efficient to assign final consumers to the low cost firm. But, if these uncertain downstream market conditions are non-contractible, then $U1-D1$ would have the conflicting incentives both to exclude and not to exclude $D2$. $U1-D1$ generally is unable both to divide the market efficiently and to fully extract rents with a two-part tariff that $D2$ would accept. Thus, the combination of uncertainty, non-contractibility, and heterogeneity appear to create difficulties for *ex post* cartelization *via* vertical integration and exclusive dealing.

To understand fully the relationship between vertical integration and exclusive dealing, therefore, it is important to go beyond the simple case of homogeneous downstream firms and to study the relationship under conditions of downstream heterogeneity, uncertainty, and noncontractibility. In what follows, we analyze a game-theoretic model of an industry possessing these features. This analysis will make clear several points. First, the synergistic relationship between vertical integration and exclusive dealing is not due to the extremely vigorous nature of potential downstream competition between identical producers; rather, it holds more generally in the presence of heterogeneous downstream firms who possess some degree of market power. Second, while the vertically integrated firm has the incentive and ability to exclude upstream competition and cartelize the downstream market, its ability to do so may be reduced with downstream heterogeneity and noncontractible uncertainty. In particular, the fixed payment needed to persuade $D2$ to enter the exclusive contract may not be small when downstream firms are heterogeneous,⁵ and only partial cartelization of the downstream industry is feasible when downstream monopoly prices vary with non-contractible market conditions. Third, extending the model to multiple independent downstream competitors, while maintaining the assumption of bilateral contracting, reveals that the degree of *ex post* cartelization of the downstream industry depends on market con-

⁵This is despite a hidden bonus to $D2$: Because the integrated firm treats foregone wholesale revenues as an opportunity cost, both of the downstream firms offer the final good at supra-competitive prices, which provides another source of compensation to $D2$ for agreeing to the exclusivity.

centration and on whether or not competition is localized. Fourth, the exclusive contracts that a vertically integrated firm uses to cartelize the downstream industry are not blatant antitrust violations. The vertically integrated firm subtly employs the marginal wholesale price of a two part tariff to raise the downstream price, and judicially employs the fixed fee to distribute the rents from cartelization.

3. HETEROGENEOUS DOWNSTREAM FIRMS

In this section, we study the main model of the paper. After introducing the model, we consider a benchmark case in which an upstream monopolist is vertically integrated with one of the downstream duopolists. We then introduce an equally efficient non-integrated upstream competitor, and prove that in equilibrium the vertically integrated firm profitably employs an exclusive contract to achieve the same market outcome as in the upstream monopoly case, except for the distribution of rents between the upstream and downstream industries. We further show that exclusive contracts are irrelevant if the industries are vertically separated. We complete this section by discussing what happens if the model is extended to more general bilateral oligopolies and if contracts are private rather than public.

3.1. Main Model

We model a vertically organized industry with M customers purchasing a product from two downstream firms $D1$ and $D2$. Shortly, we will simplify without loss of generality by setting $M = 1$. The downstream firms manufacture differentiated products by combining a component input with other inputs whose cost is normalized to zero. There are two upstream firms $U1$ and $U2$ supplying the component input at the same unit cost $c \geq 0$. For example, the customers could be construction companies purchasing ready-mixed concrete from a downstream industry that procures cement from an upstream industry. Alternatively, the customers could be automobile manufacturers purchasing batteries from a downstream industry that procures battery separators (a component input) from an up-

stream industry. Still another example could be firms purchasing health services on behalf of employees from health maintenance organizations who use upstream hospital facilities. We will refer to a customer alternatively as a "consumer", even though the model perhaps best applies to business-to-business transactions.

The model is patterned roughly on markets for cement and concrete markets. Cement is a fixed proportions input into the production of concrete, and concrete producers typically procure cement supplies under requirements contracts. The demand for ready-mixed concrete is located at constructions sites that are difficult to predict or specify in contracts. Since delivered ready-mixed concrete requires a cement truck, transportation costs evidently are important and idiosyncratic to the location of the construction sites. The model captures these cost characteristics with a number of simplifying assumptions. We revisit cement and concrete markets at the end, when we discuss applications.

A consumer located at $x \in [0, 1]$ is interested in purchasing one unit of the downstream product. The consumer's uncertain reservation value v has a cumulative distribution function $F(v)$ on support $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v} < \bar{v} < \infty$. The corresponding probability density function is $f(v) > 0$ for $v \in [\underline{v}, \bar{v}]$. The consumer's uncertain reservation value gives rise to a downward-sloping expected demand curve.⁶ The corresponding expected marginal revenue function is also smooth and downward sloping under the following maintained familiar technical assumption:

$$A1. \quad \frac{d\left(\frac{1-F(p)}{f(p)}\right)}{dp} \leq 0.$$

Each downstream firm combines a component input with other inputs whose cost is normalized to zero. Additionally, to sell to the consumer $D1$ incurs transportation costs τx and $D2$ incurs $\tau(1 - x)$, where $\tau > 0$ is a fixed parameter, measuring the degree of *ex post* cost heterogeneity. Thus, the transportation costs of the two firms are negatively correlated. For simplicity the consumer's location x is uniformly distributed on $[0, 1]$. This

⁶We could replace the assumption of a random v with the assumption that the consumer has a conventional downward sloping demand curve.

simple spatial cost structure captures adequately the more general idea of uncertain cost heterogeneity.

The downstream firms “bid” prices to the consumer, P_1 and P_2 . The consumer purchases the lower priced product as long as that price is below the consumer’s realized reservation value v , and nothing otherwise. At the time of bidding, the firms know x but do not know v . Thus, each firm understands the extent to which it is advantaged or disadvantaged in competing for each consumer, while remaining uncertain about the magnitude of the consumer’s demand.⁷ As explained below, in equilibrium the advantaged downstream firm predictably wins the competition. In the symmetric case $x = \frac{1}{2}$, we select the equilibrium in which $D1$ wins.

Now suppose that $U1$ and $D1$ are vertically integrated. $U1$ and $U2$ each offer $D2$ a contract requiring $D2$ to purchase exclusively from $U1$ or $U2$. The requirements contracts are assumed to take the form of a two-part tariff specifying a fixed transfer payment from $D2$ to U_i , t_i , and a unit price r_i that $D2$ pays contingent on actual production. $t_i > 0$ means that $D2$ pays a fee to U_i , while $t_i < 0$ means the opposite. If a contract is accepted, t_i is paid irrespective of whether any sale is made, but r_i is paid only if $D2$ actually makes a sale. Thus the component is produced to order, i.e. the exclusive supplier produces the component only if $D2$ succeeds in the downstream market.

At the time of contracting between the upstream and downstream industry, customer characteristics are uncertain. A consumer’s location x becomes known after $D2$ commits to an exclusive supply relationship, but before downstream price competition. Moreover, customer characteristics, x and v , are not contractible. The implicit assumption justifying this approach is that the transaction costs for determining the realization of x , and making

⁷The model can be reinterpreted as a discrete choice model of consumer demand in which the “transportation cost” is incurred directly by the consumer. Suppose each consumer has a discrete demand for the downstream product determined by a pair of values $(v_1, v_2) = (v - \tau x, v - \tau(1 - x))$. Then, the variable x determines product differentiation from the perspective of the customer, while the variable v determines the customer’s average value for the downstream product. It is standard in discrete choice oligopoly models to decompose consumer valuations into a component that is known to firms and a component that is unknown. The distinguishing feature of our model is that the unknown component is a common value.

the contract depend on this determination are prohibitively high.⁸ The consumer's reservation value is never observed publicly, although it is easy to write a contract contingent on production resulting from the consumer's purchase decision. Furthermore, *U1-D1* cannot commit to any internal transfer price that is not *ex post* jointly optimal, nor can anyone commit to a retail price through the supply contracts.

So far, we have assumed an arbitrary finite number of customers M . It is sufficient, however, to focus on the case $M = 1$. This is because the downstream firms compete separately for the business of each customer, and the only thing that links these competitions is the contract price per unit that determines the marginal cost of *D2*. Because the customers are identical *ex ante*, the equilibrium r_i are independent of M , and the equilibrium t_i are proportional to M . Therefore, we simplify the exposition henceforth by setting $M = 1$. The main results extend to an arbitrary M by interpreting the t_i as payments per potential customer.⁹

To summarize, the timing of the game (assuming $M = 1$) when *U1* and *D1* are vertically integrated is as follows:

- Stage 1. *U1* and *U2* offer contracts (t_1, r_1) and (t_2, r_2) .
- Stage 2. *D2* chooses a contract.
- Stage 3. x is realized.
- Stage 4. *D1* and *D2* bid prices.
- Stage 5. v is realized and the consumer makes a purchase decision.

Remark 1 *The game form ignores the possibility that *D2* might decline any exclusive contract and instead purchase on a spot market after learning x . A spot market is irrelevant because, as will be seen shortly, in equilibrium *U2* offers a requirements contract on terms*

⁸A conceivable possibility is that contract terms depend on messages exchanged after x is realized, in the spirit of the Nash implementation literature (Maskin, 1985). We implicitly assume that the transactions costs associated with the necessary message game are prohibitively burdensome. Alternatively, such communication between downstream competitors might be construed to violate the antitrust laws. In any case, the usual message game *a la* Maskin would violate the assumption that contracts are bilateral.

⁹We could also assume M is random, and interpret t_i as the transfer payment divided by the expected number of customers in the market.

that are the same as would prevail in the spot market.

We refine equilibrium by requiring that $D1$ and $D2$ do not set prices below their costs at Stage 4, and $U2$ does not offer a contract at Stage 1 that would be unprofitable if accepted by $D2$. Thus we confine our attention to equilibrium strategies with the property that a player never strictly prefers her offer to be rejected, whether in Stage 1 or in Stage 4 of the game. The refinement prevents firms from "squeezing" the profits of rivals by making an offer that are not credible. The refinement is implied by the stronger requirement that players do not use weakly dominated strategies, which is too strong for our purposes because it would eliminate all pure strategy equilibria.¹⁰

Several features of the model are worth previewing and highlighting. First, cost heterogeneity and uncertainty are captured by the random location variable x . The cost heterogeneity is manifest in an asymmetric Bertrand model of downstream competition in which one firm or another has a clear advantage. When a firm enjoys a small advantage, the rival imposes a competitive constraint by setting price equal to its opportunity cost of supplying the consumer. The advantaged firm must meet that price in order to make the sale. A firm with a sufficiently large advantage, however, is effectively a monopolist. Second, the asymmetric Bertrand model of downstream competition brings into sharp focus the idea that foregone upstream revenues are an opportunity cost of $U1-D1$ when $U1$ has an exclusive supply contract with $D2$. In this case, an increase in r_1 softens downstream competition whenever the disadvantaged rival imposes a competitive constraint by increasing dollar for dollar the marginal opportunity cost of both $D2$ and $U1-D1$. By eliminating this opportunity cost for $U1-D1$, a decision by $D2$ to contract with $U2$ instead of $U1$ causes a "price war" in the downstream market. Third, the non-contractibility of downstream costs is captured by the assumption that requirements contracts are not contingent on the realization of x . This non-contractibility creates a conflict between horizontal control and vertical control for the vertically integrated firm. Horizontal control means that $U1-D1$ would like to raise r_1 in order to soften competition when the rival poses a competitive

¹⁰This is a familiar problem in games with infinitely many strategies, e.g. the Bertrand duopoly with cost asymmetry (Kreps, 1990, p. 419, footnote d).

constraint. Vertical control means that *U1-D1* would like to lower r_1 in order to avoid double marginalization when the independent firm has monopoly power. The conflict means that *U1-D1* cannot perfectly cartelize the downstream industry with an exclusive requirements contract. Fourth, the random v is merely a device to construct a downward sloping demand curve, which is important for "cartelization" to have a harmful effect on social welfare. Fifth, for convenience we assume that contracts are public. Our main results hold under private bilateral contracting as well, as we demonstrated in our earlier working paper (Chen and Riordan, 2003).¹¹

3.2 Upstream monopoly

We begin by considering a benchmark situation where *U1* is the only supplier in the upstream market, and modify the game accordingly. Thus *U1* offers (t, r) to *D2* at Stage 1, and *D2* accepts or rejects at Stage 2. This benchmark model establishes some preliminary results for our analysis of upstream duopoly.

It is useful to characterize unconstrained monopoly prices in the downstream market. If *D2* rejects *U1*'s contract offer at Stage 1, or if the realization of x at Stage 3 is sufficiently favorable, then *D1* operates as an unconstrained monopolist and chooses $p = P_1^m(x)$ to maximize $\{(p - c - \tau x)[1 - F(p)]\}$. Given regularity assumption *A1*, $P_1^m(x)$ exists uniquely and satisfies:

$$P_1^m(x) - c - \tau x = \frac{1 - F(P_1^m(x))}{f(P_1^m(x))}. \quad (1)$$

Similarly, if *D2* accepts *U1*'s contract offer and is sufficiently advantaged by the realization of x , then *D2* offers the monopoly price $P_2^m(x, r)$ satisfying

$$P_2^m(x, r) - r - \tau(1 - x) = \frac{1 - F(P_2^m(x, r))}{f(P_2^m(x, r))}, \quad (2)$$

assuming r is not prohibitively high, i.e. $r < \bar{v} - \tau(1 - x)$. It can be easily verified that $P_1^m(x)$ increases in x and $P_1^m(x) - c - \tau x$ decreases in x . Similarly, $P_2^m(x, r)$ increases in r and $(1 - x)$; and $P_2^m(x, r) - r - \tau(1 - x)$ decreases in r and $(1 - x)$.

¹¹If binding multilateral contracts were feasible, then vertical integration would not be a necessary ingredient of cartelization (Mathewson and Winter, 1984).

For convenience, we make the following additional technical assumption:

$$A2. \quad P_1^m(0) \geq c + \tau.$$

$A2$ is satisfied if the likely values of V are not too small relative to $c + \tau$. The assumption implies that, if $r = c$, then $U1$ - $D1$ and $D2$ always pose a competitive constraint on each other. This innocuous assumption simplifies our arguments in certain places, but is not crucial for our main results.

Now suppose that $D2$ accepts $U1$'s contract offer (t, r) , and competes against $D1$ for customer x . The price subgame at Stage 4 is an asymmetric Bertrand game. When $x < \frac{1}{2}$, $D1$ has the advantage, and Bertrand competition means that $D1$ wins the competition by setting its price either at its monopoly level or at the marginal cost of $D2$, whichever is less. When $x > \frac{1}{2}$, $D2$ is the more efficient supplier. $D1$ stands ready to sell at its marginal opportunity cost $r + \tau x$, or, if $r + \tau x > P_1^m(x)$, at its monopoly price $P_1^m(x)$ so that the probability of a sale is not unprofitably low. In response, $D2$ sets its price either at $P_2^m(x, r)$ or at $\min\{P_1^m(x), r + \tau x\}$, whichever is smaller. Accordingly, equilibrium pricing strategies are as follows. Define:

$$P_1(x, r) = \min\{P_1^m(x), r + \tau(1 - x)\}, \quad (3)$$

$$P_2(x, r) = \min\{P_2^m(x, r), \min\{P_1^m(x), r + \tau x\}\}. \quad (4)$$

Lemma 1 *Assume that $U1$ is the sole upstream supplier. (1) Suppose that $P_1^m(\frac{1}{2}) \geq r + \frac{1}{2}\tau$. Then the following is a Nash equilibrium of the $D1$ - $D2$ pricing subgame: If $x \leq \frac{1}{2}$, then $D1$ offers $P_1(x, r)$, $D2$ offers $r + \tau(1 - x)$, and the customer selects $D1$. If $x > \frac{1}{2}$, then $D2$ offers $P_2(x, r)$, $D1$ offers $\min\{P_1^m(x), r + \tau x\}$, and the customer selects $D2$. (2) If (t, r) is an equilibrium contract between $U1$ and $D1$, then $P_1^m(\frac{1}{2}) \geq r + \frac{1}{2}\tau$.*

P roof. See Appendix A.

The equilibrium prices when competitive constraints bind are those for a standard Bertrand duopoly with different constant marginal costs, say $c_1 < c_2$, where the equilibrium price is c_2 . Although both sellers charging a price $p \in (c_1, c_2)$ can also be supported as a Nash

equilibrium, seller 2 would prefer not to be selected as the supplier at such a price. Thus, requiring that a seller should not strictly prefer to be rejected at the price it bids, the only equilibrium in our pricing game between $D1$ and $D2$ is the one characterized in Lemma 1. In what follows, we consider this as the unique (refined) equilibrium in the pricing subgame.

From Lemma 1, in equilibrium $P_1^m(\frac{1}{2}) \geq r + \frac{1}{2}\tau$. This holds because otherwise r would be so high that $D2$ is unable to serve some of the consumers who are located closer to $D2$ than to $D1$, which means that industry profit is not maximized. We note that this also holds if $D2$ has some outside option for obtaining the input. This extension is relevant for the case of upstream competition considered later.

We next define:

$$\begin{aligned} \Pi(r) = & \int_0^{\frac{1}{2}} [P_1(x, r) - \tau x - c] [1 - F(P_1(x, r))] dx \\ & + \int_{\frac{1}{2}}^1 [P_2(x, r) - \tau(1-x) - c] [1 - F(P_2(x, r))] dx \end{aligned} \quad (5)$$

$$t(r) = \int_{\frac{1}{2}}^1 [P_2(x, r) - \tau(1-x) - r] [1 - F(P_2(x, r))] dx \quad (6)$$

$\Pi(r)$ is the joint upstream-downstream industry profit when $D2$ contracts to purchase from $U1$ at unit price r , and $t(r)$ is the transfer price that fully extracts rents from $D2$. We now fully characterize the equilibrium of the game.

Proposition 1 *The game where $U1$ is the only upstream supplier has a unique equilibrium. At this equilibrium, $U1$ offers $D2$ contract (\hat{t}, \hat{r}) , which is accepted by $D2$, where*

$$\hat{r} = \arg \max_{c \leq r \leq \bar{v}} \{\Pi(r)\}, \quad \hat{t} = t(\hat{r}).$$

$D1$ is the seller with price $P_1(x, \hat{r})$ if $x \leq \frac{1}{2}$, and $D2$ is the seller with price $P_2(x, \hat{r})$ if $x > \frac{1}{2}$. Furthermore, $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$.

P proof. See Appendix A.

The equilibrium contract has a cartelizing effect. By charging $D2$ a wholesale markup $(\hat{r} - c)$, $U1$ raises $D2$'s marginal cost directly, creating an incentive for $D2$ to raise its prices.

Thus, $D2$ sells at a higher price when $x \geq 1/2$, and poses less of a competitive constraint on $D1$ when $x < 1/2$. The markup also raises $U1-D1$'s opportunity cost of winning the downstream competition, creating an incentive for $D1$ to raise its prices and be less of a competitive constraint on $D2$ when $x \geq 1/2$ and $P_2^m(x, \hat{r}) > \hat{r} + \tau x$. The overall effect is to lessen horizontal competition in the downstream market and to reduce consumer welfare, relative to the situation where the wholesale price for $D2$ is c .¹²

The cartelization of the industry, however, is only partial, due to the assumption that x is not contractible and the resulting conflict between horizontal and vertical control. Full cartelization requires an *es post* monopoly price for all values of x . To see this, first consider the consumer at $x = 1$, where

$$P_2(1, \hat{r}) = \min \{P_2^m(1, \hat{r}), \min\{P_1^m(1), \hat{r} + \tau\}\} > P_1^m(0)$$

since $P_2^m(1, \hat{r}) > P_2^m(1, c) = P_1^m(0)$, $P_1^m(1) > P_1^m(0)$, and $\hat{r} + \tau > P_1^m(0)$. Therefore, for consumers sufficiently close to $x = 1$, $P_2(x, \hat{r}) > P_1^m(1 - x)$, or the price is above the vertically-integrated industry monopoly level. Thus, there is a problem of double marginalization (Spengler, 1950) when cost heterogeneity is greatest. Next, consider consumers at or slightly below $x = \frac{1}{2}$. For these consumers, since $\hat{r} < P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$ from Proposition 1, we have $P_1(x, \hat{r}) < P_1^m(x)$, or the price is below the vertically-integrated industry monopoly level, i.e. there is a problem of excessive horizontal competition when the downstream firms have similar costs.

The obstacle to full cartelization is non-contractibility, i.e. contract terms do not vary with the location of the final consumer. This fact creates a tension between improving vertical efficiency in some circumstances and intensifying horizontal competition in others. The conflict arises in our model from the downward-sloping expected demand curve generated by the consumer's uncertain reservation price. A lower value of \hat{r} causes lower

¹²It is important for our result that $U1-D1$ takes an integrated view of its operations and coordinates its upstream-downstream prices to maximize the integrated firm's expected profit. In our context, if this were not true, there would be no difference between a pair of vertically integrated or separated firms. The strategic incentives and effects can still be present, albeit to a less extent, if the interests of $U1$ and $D1$ are not completely harmonized under vertical integration.

downstream prices by reducing $D2$'s marginal cost as well as $U1-D1$'s marginal opportunity cost. Thus, $U1$ faces a trade-off in setting the price to $D2$. Reducing \hat{r} alleviates $D2$'s double marginalization problem at some locations, but also intensifies horizontal price competition elsewhere. If \hat{r} is reduced, neither $U1-D1$ nor $D2$ can commit not to undercut each other for the consumer that is located closer to the rival. The problem is that downstream monopoly prices vary with the location of the consumer; and the single instrument \hat{r} cannot achieve these prices in all circumstances.

3.3. Upstream Duopoly

We now return to the model where the upstream market is a duopoly. Recall that the contracts offered by $U1$ and $U2$ are denoted by (t_1, r_1) and (t_2, r_2) . We argue in several steps that in equilibrium $U1$ wins the contract competition on terms that effectively cartelize the downstream industry. The first step shows that $U2$ always seeks to contract with $D2$ with $r_2 = c$. The second step establishes that $U1-D1$ always can profitably outbid $U2$ with $r_1 = \hat{r}$. The last step fully characterizes equilibrium contracts, confirming that $U1-D1$ is willing and able to compensate $D2$ for declining to contract with $U2$ on competitive terms.

The following lemma establishes that $U2$ and $D2$ have a joint incentive to negotiate $r_2 = c$. Raising r_2 above c would cause $D2$ to price less aggressively, but would have no salutary strategic effect of softening $D1$'s competition. The reason why the strategic effect is absent in our model is that $D1$ prices either at cost or at its own monopoly price whenever $D1$ is a competitive constraint, and these clearly do not depend on r_2 . Therefore, the only consequence for $U2$ and $D2$ of raising r_2 above c is the adverse one of causing $D2$ to succeed less often.

Lemma 2 $r_2 = c$ maximizes the joint profits of $U2$ and $D2$ when $U2$ is the contracted supplier of $D2$.

P proof. See Appendix A.

Second, maximized industry profits are higher when $D2$ contracts with $U1-D1$, implying that $U1-D1$ can always profitably outbid $U2$ for $D2$'s business. The maximized joint profits

of $U1-D1$ and $D2$ is $\Pi(\hat{r})$. If $D2$ were to contract with $U2$ with $r_2 = c$, then industry profit would be $\Pi(c)$, half of which would go to $U1-D1$, and half shared by $U2$ and $D2$. Notice that, from Proposition 1, $\Pi(\hat{r}) > \Pi(c)$. The reason for this drop in industry profits from a mere switch of equally efficient suppliers is that $U1-D1$ no longer internalizes the opportunity cost of upstream's sales to $D2$, and therefore has a discrete incentive to price more aggressively in the downstream market. Furthermore, since $\Pi(\hat{r}) - \frac{1}{2}\Pi(c) > \frac{1}{2}\Pi(c)$, $U1-D1$ can compensate $D2$ for agreeing to purchase at $r_1 = \hat{r}$ instead of $r_2 = c$ (even with $t_2 = 0$), and still extract an additional monopoly profit. The credible threat of a price war helps discipline $D2$ from contracting with $U2$.

These observations lead directly to the conclusion that in equilibrium $U1-D1$ cartelizes the downstream industry with an exclusive requirements contract. It is optimal for $U1-D1$ to set $r_1 = \hat{r}$ and for $U2$ to set $r_2 = c$. Given these variable price terms, $U1-D1$ and $U2$ compete by bidding t_1 and t_2 respectively. This bidding competition has the essential structure of an asymmetric Bertrand game. Consequently, in equilibrium $D2$ contracts with $U1$, with $U2$ offering $(0, c)$ and $U1$ offering (t_1^*, \hat{r}) , and

$$t_1^* = \int_{\frac{1}{2}}^1 [P_2(x, \hat{r}) - \tau(1-x) - \hat{r}] [1 - F(P_2(x, \hat{r}))] dx - \int_{\frac{1}{2}}^1 \tau(2x-1) [1 - F(c + \tau x)] dx. \quad (7)$$

The transfer t_1^* extracts the downstream profits from $D2$ except for the amount $D2$ could earn by declining the contract offer and procuring from $U2$ on competitive terms. By assumption $A2$, this amount, $t(c)$, is equal to the second integration in equation (7). Finally, it is straightforward that $t_1^* < 0$ in compensation for $\hat{r} > c$.

Proposition 2 *The game where the upstream market is a duopoly has a unique equilibrium. In equilibrium $U2$ offers $D2$ $(0, c)$ and $U1$ offers $D2$ (t_1^*, \hat{r}) with $t_1^* < 0$, $D2$ contracts with $U1$, and the downstream equilibrium outcome is the same as under upstream monopoly.*

P proof. See Appendix A.

Thus, a vertically integrated firm is able to outbid a equally efficient stand-alone supplier for an exclusive relationship with a downstream competitor. When the integrated firm

supplies $D2$ at a price above marginal cost, the former has less incentive to undercut $D2$ because of the opportunity cost of foregone input sales to $D2$.¹³ This dampening of horizontal competition explains $U1$'s advantage and ability to preempt $U2$ (Gilbert and Newbery, 1982). Because of downstream heterogeneity, the profitable exclusion of $U2$ may nevertheless cost $U1$ - $D1$ a substantial amount. However, this cost approaches zero as the difference between $D1$ and $D2$ disappears, i.e. $t_1^* < 0$ and $\lim_{\tau \rightarrow 0} \int_{\frac{1}{2}}^1 \tau(2x - 1) [1 - F(c + \tau x)] dx = 0$ imply $\lim_{\tau \rightarrow 0} t_1^* = 0$.¹⁴

The exclusion of upstream competition leads to higher downstream prices compared to when $U2$ supplies $D2$. The exclusivity of the contract clearly is important for the cartelization outcome under vertical integration. Since $\hat{r} > c$, $D2$ would want to purchase from $U2$ *ex post* as long as $r_2 < \hat{r}$, and $U2$ would be willing to cut r_2 to as low as c to gain $D2$'s business. This implies that, if upstream firms cannot sign exclusive contracts with downstream firms, perhaps due to legal restrictions or to difficulties in contract enforcement, then the input price to $D2$ must be set at $r_1 = r_2 = c$, with $t_1 = t_2 = 0$. Therefore:

Remark 2 *In the game where the upstream market is a duopoly, the cartelization of the downstream market can be achieved only if exclusive requirements contracts are feasible.*

3.4. Vertical Separation

We have shown that exclusive contracts used by a vertically integrated firm can achieve the market outcome of an upstream monopolist. To see that vertical integration is important for the cartelization effect of the exclusive contracts, we next consider a variation of our

¹³This "opportunity cost" idea appeared in Chen (2001), who finds that a vertically integrated firm can only exclude a less efficient supplier. A key reason for our more dramatic result is that we allow two-part tariff contracts in the input market while Chen considers only uniform price contracts. In addition, Chen's model is based on non-exclusive contracts, although he allows some possibility of lock-in due to switching costs.

¹⁴While we have assumed for simplicity that $U1$ and $U2$ are equally efficient, the same logic would hold, and so would Proposition 2, if $U2$ had a small efficiency advantage. In this case, however, $U1$ - $D1$ would have an incentive to "outsource" supplies of the input from the more efficient $U2$.

model in which $U1$ and $D1$ are vertically separated independent firms. We shall show that exclusive contracts are irrelevant in this case: the equilibrium input price for both downstream firms is c .

The timing of the modified game is as follows:

Stage 1. $U1$ and $U2$ each offer separate contracts to $D1$ and $D2$.¹⁵

Stage 2. $D1$ and $D2$ choose contracts.

Stage 3. x is realized.

Stage 4. $D1$ and $D2$ bid prices.

Stage 5. v is realized and the consumer makes a purchase decision.

A contract offer from U_i to D_j is a transfer payment and intermediate goods price, (t_{ij}, r_{ij}) for $i, j = 1, 2$. Adapting our notation, we let (t_j, r_j) now denote any contract that D_j accepts, whether offered by $U1$ or $U2$. When the input prices for $D1$ and $D2$ are (r_1, r_2) , let the joint profits of D_j with its supplier U_i be $\Pi_{ij}(r_1, r_2)$, $i, j = 1, 2$. We further let

$$P(x, r_1, r_2) = \min \{P^m(x, r_1), r_2 + \tau(1 - x)\}$$

where $P^m(x, r)$ is the monopoly price for $D1$ to serve a consumer at marginal cost $(r + \tau x)$, defined implicitly by

$$P^m(x, r) - r - \tau x = \frac{1 - F(P^m(x, r))}{f(P^m(x, r))}.$$

If $[r_2 + \tau(1 - x)]$ is the marginal cost of $D2$, then equilibrium prices are $\max\{P(x, r_1, r_2), r_1 + \tau x\}$ for $D1$, and $\max\{P(1 - x, r_2, r_1), r_2 + \tau(1 - x)\}$ for $D2$. Bertrand competition implies that the downstream firm with the lowest marginal cost wins the customer. Thus, if $(r_1 + \tau x) \leq [r_2 + \tau(1 - x)]$, the equilibrium outcome is for $D1$ to serve consumer x at price $P(x, r_1, r_2)$.

Proposition 3 *The game under vertical separation has a unique equilibrium with $(t_j^*, r_j^*) = (0, c)$ for both $j = 1, 2$.*

¹⁵To be consistent with our earlier analysis, we again assume that these are exclusive contracts requiring a downstream firm to purchase only from a certain upstream firm, although exclusive contracts are not necessary for our result that the intermediate-good price will be equal to c under vertical separation.

P roof. See Appendix A.

If both $D1$ and $D2$ were to contract only with $U1$ at input prices above c , then downstream prices would be higher and joint upstream-downstream industry profits would also be higher. Therefore, one might conjecture that in equilibrium $U1$ would be able to use exclusive contracts to cartelize the downstream industry as in the case of vertical integration. So why is this not the case in the absence of vertical integration? The reason is that one of the downstream firms can pair with $U2$ at a lower input price and obtain a joint profit that is more than its joint profit with $U1$ under the higher input price. This competitive option would frustrate any attempt by $U1$ to use exclusive contracts to cartelize the downstream industry, because it makes it too costly for an independent $U1$ to gain the compliance of both downstream firms.¹⁶ This reasoning is made precise in Appendix A.

Since $r_i^* = c$ for $i = 1, 2$, there is no need for exclusive contracts in equilibrium, and firms have equilibrium incentives to negotiate supply arrangements on competitive terms, i.e. exactly as they would in spot markets.

Remark 3 *When $U1$ and $D1$ are vertically separated, exclusive contracts are irrelevant in equilibrium.*

3.5. Bilateral Oligopoly

Our spatial model of downstream price competition is restrictive in that it only suits the case of bilateral duopoly. The logic of our results, however, is more general. In Appendix B, we first introduce a generalization of the model, in which n downstream competitors are located at terminal nodes of a symmetric hub-and-spokes network and customers are randomly located on the connected spokes. The downstream firms incur transportation costs

¹⁶In a repeated game setting, a nonintegrated $U1$ might use the threat of reversion to competitive pricing to cartelize the downstream market with exclusive contracts. If $D2$ were to reject $U1$'s exclusive deal, and contract with $U2$ instead on competitive terms, then $U1$ could punish the defection by contracting with $D1$ on competitive terms in the next period. If the period length were sufficiently short, then the subgame perfect equilibrium threat could support cartelization even in a vertically separated setting. Even in a repeated game setting, however, vertical integration can facilitate collusion (Nocke and White, 2003).

to deliver the intermediate good to a customer at a particular location. Thus a firm located at the terminal node of a customer’s spoke has a cost advantage over other competitors. This hub-and-spokes model is interesting because it exhibits a strong form of non-localized competition;¹⁷ each downstream firm possesses market power over nearby customers, but is constrained by all other competitors who are equidistant.¹⁸ As in Chamberlinian monopolistic competition, each downstream firm perceives itself as competing against the rest of the market. Appendix B also introduces a circle model of localized competition, which has the structure that downstream firms are symmetrically located around a circle as in Salop (1979), while a customer’s location of the circle is drawn from a uniform distribution. In this case, each downstream firm again has market power over a nearby customer, and is constrained only by the next nearest downstream firm. In each of these generalizations, the timing of decisions is the same as in our main model, except that there are an arbitrary number of downstream firms. Both models also allow for $m \geq 2$ equally efficient upstream firms.

Our main results generalize readily to the hub-and-spokes model.¹⁹ With $n > 2$ downstream competitors, vertical integration combines with exclusive dealing to foreclose equally efficient upstream competitors and to raise downstream prices. There is, however, an addi-

¹⁷Non-localized competition means in general that a consumer may have first-choice preference over downstream products, but no strong second-choice preference, or, alternatively, a consumer has a most-efficient supplier of the downstream product, but other suppliers are equally efficient. For example, consider a case in which a consumer can buy either from a single local supplier or over the Internet from more distant suppliers. Non-localized competition also applies naturally to markets with consumer switching costs.

¹⁸This property is reminiscent of Chamberlinian monopolistic competition; individual firms have power over price while competing against “the market”. See also Hart (1985a, 1985b) and Perloff and Salop (1985).

¹⁹A key observation for the extension of our results to the hub-and-spokes model of downstream oligopoly is that prices are strategic complements (Bulow, Geanakoplos, and Klemperer, 1985). Thus, an exclusive contract that raises the marginal input price to a downstream competitor has the benefit of encouraging other downstream rivals to raise their prices also. These infectious effects enable a vertically integrated cartel organizer to achieve higher downstream prices by bringing the entire downstream industry under exclusive contracts. The argument is related to Davidson and Deneckere’s (1985) analysis of incentives to form coalitions.

tional result from the spokes model: the equilibrium upstream price under vertical integration decreases in the number of downstream firms. This suggests that market concentration in the downstream market can be important for the evaluation of the combined effects of vertical integration and exclusive contracts.

Our results also extend to the circle model of localized competition. In the circle model, the vertically integrated upstream firm only brings under exclusive contract its immediate downstream neighbors, while contracting efficiently with more distant downstream firms. Thus, in the case of four or more downstream firms, upstream competitors are excluded only from supplying the portion of the downstream market that is local to the integrated firm. Nevertheless, the combination of vertical integration and exclusive dealing has an anticompetitive effect in this local market segment.

Taken together, the hub-and-spokes model and the circle model indicate that the extent of upstream foreclosure and downstream cartelization depends on the nature (localized versus non-localized) of competition. The main complication in the more general models of bilateral oligopoly is the possibility that exclusive contracts have a cartelizing effect even under vertical separation. This possibility is eliminated with a parameter restriction that the upstream industry is not too concentrated compared to the downstream industry. The parameter restriction, which rules out the possibility of multiple equilibria, qualifies our conclusion that vertical integration is necessary for anticompetitive effects of exclusive dealing in the case of public contracting.

3.6. Private Contracts

To keep the exposition of our results simple, we have maintained the assumption that bilateral requirements are public. In our earlier working paper (Chen and Riordan, 2003), however, we arrive at essentially the same conclusions under the alternative assumption that the contracts are private information. In business-to-business relationships especially, private contracting may be the more realistic assumption. The alternative assumption of private bilateral contracts has been an important one in the new vertical contracting

literature (McAfee and Schwartz, 1994; O'Brian and Shaffer, 1992; Rey and Tirole, 2003; and Rey and Vergé, 2004). The privacy of contracts clearly is irrelevant for our upstream monopoly case, because there are only two contracting parties and both necessarily observe offers and acceptances. The assumption potentially matters for the vertically integrated upstream duopoly case only if $D2$ were to reject $U1-D1$'s equilibrium offer. In this circumstance, $U1-D1$ must form some conjecture about a possible deviation contract that has offered by $U2$ and accepted by $D2$. The natural conjecture, however, is $r_2 = c$ because that always maximizes bilateral profits.²⁰ The distinction between public and private contracts, however, matters much more in the vertical separation case. In that case, if $U1$ makes an out-of-equilibrium offer to $D1$, then $D1$ must form a belief of what offer $U1$ has made to $D2$. The arbitrariness of these beliefs in response to out-of-equilibrium contract offers allows for multiple equilibria. The new vertical contracting literature has focussed on the selection of equilibria by postulating particular structures of out-of-equilibrium beliefs.

Our results under the alternative assumption of private bilateral contracts are as follows; see Chen and Riordan (2003) for details. First, Proposition 2 holds under private contracts if $U1-D1$ always believes that $r_2 = c$, which, as mentioned above, is the natural belief. Second, Proposition 3 holds under symmetry beliefs and the additional refinement $r_i \geq c$. Symmetry beliefs means that if $D1$ receives an out-of-equilibrium offer from $U1$, then $D1$ believes that $U1$ has offered the same r_1 to $D2$ and, furthermore, that $D2$ will accept the offer. While we have not ruled out equilibria with $r_i < c$, any such equilibrium would be Pareto dominated for the industry by an equilibrium with $r_2 = c$. More importantly, the proposition is sufficient for our conclusion that vertical integration is important for the cartelization of the downstream industry.²¹ Finally, these results also extend naturally to the hub-and-spokes and circle models. Therefore, our conclusions do not depend on the

²⁰The concern about out-equilibrium-beliefs can be avoided entirely in this case by interpreting $U2$ as competitive fringe that always stands ready to supply at $r_2 = c$.

²¹The result also holds under passive beliefs and the strategy restriction $r_i \geq c$. Under passive beliefs, Dj maintains the belief that Di has accepted an equilibrium contract even after receiving an out-of-equilibrium offer. Without the strategy restriction, a passive belief equilibrium does not exist except when $c = 0$, for the similar reason as in McAfee and Schwartz (1995).

assumption of public contracts. Furthermore, with private contracts, we do not need the parameter restriction on market structure to conclude that vertical integration is necessary for exclusive requirements contracts to have a cartelizing effect.

4. DISCUSSION

Our analysis has revealed a relationship between vertical integration and exclusive dealing that has gone unnoticed in the economics literature. A vertically integrated firm may have a special ability and incentive to use exclusive requirements contracts to exclude equally efficient upstream competitors and effect a cartelization of the downstream industry. The vertically integrated firm convinces independent downstream rivals to accept exclusive supply contracts on supracompetitive terms with a carrot and stick. The carrot takes the familiar form of some direct compensation to the downstream independent firm for forgoing a competitive supply alternative. The stick is the credible threat of a price war if the competitor declines the exclusive deal. This price war threat is credible because, under the exclusive contract, the vertically integrated firm internalizes the opportunity cost of foregone input sales, and consequently has a clear incentive to price less aggressively in the downstream market. This opportunity cost would disappear if $D2$ were to contract with $U2$ instead. The special advantage of a vertically integrated firm for cartelizing the industry is that the price war threat reduces the necessary size of the carrot.

We developed this point in the context of a "bidding market" on downstream competition, in which there is a separate winner-take-all competition to supply each customer (Klemperer, 2005). Additional important elements of the downstream market structure are that the good is produced to order, firms have heterogeneous costs for serving different customers,²² and demand for the good has some price sensitivity. Such features may better describe business-to-business transactions than mass consumer markets. For example, markets for ready-mixed concrete used in large construction projects have these features. While we developed a specific model of a bidding market, we expect our main insight about

²²Alternatively, the downstream products are differentiated and customers have heterogeneous preferences.

vertical integration *cum* exclusive dealing applies to other models of downstream competition. The key idea is simple: a vertically integrated firm who supplies a downstream rival under an exclusive contract also raises its own opportunity cost when it raises the supply price, thus effectively committing to softer competition.

In other models of downstream competition, however, performance comparisons of vertically-integrated and vertically-separated market structures may be less sharp. An important property of our model is that an optimal contract between an independent upstream firm and independent downstream firm sets the supply price equal to cost and redistributes profits with a lump sum transfer. This property arises from the assumption that firm heterogeneity is common knowledge at the time of downstream competition, i.e. the *Di* perfectly understand who has the advantage in any particular competition. A consequence of this property is that the input is supplied competitively in a non-integrated environment. This competitive pricing principle may not hold in other models of differentiated price competition in which bidders are less certain of their relative positions at the time of setting downstream prices. For example, consider a duopoly in which downstream competitors have upward sloping reaction curves that intersect once, and assume that an increase in the input price shifts up a firm's reaction curve at the point of intersection. In this case, an independent upstream and independent downstream firm may have an incentive to negotiate an exclusive contract with a supracompetitive supply price as a way to soften competition. Thus, exclusive contracts in a non-integrated environment may also have a partially cartelizing effect. Such a strategic commitment to softer competition is lost when a vertically integrated firm supplies only itself at cost, resulting in a more competitive downstream market compared to the non-integrated environment. The integrated firm nevertheless may still have an incentive negotiating exclusive contracts that raise the supply price to the independent downstream firm. Moreover, it seems plausible that the end result is an even less competitive downstream market than vertical separation, assuming the opportunity cost effect of vertical integration *cum* exclusive dealing is sufficiently strong. Demonstrating this result formally, however, would require a different analysis for a more general class of models. Our bidding model is simpler to analyze because there is no strategic incentive to

use exclusive contracts to soften competition in the vertically-separated environment.²³

We have also shown that the cartelizing ability of the vertically integrated firm may be limited when downstream firms are heterogeneous and contracts cannot be contingent on uncertain market conditions. In particular a complete cartelization remains elusive when downstream monopoly prices vary with non-contractible market conditions. In such circumstances, the extent to which a vertically integrated supplier is able to cartelize the downstream industry depends on the degree of concentration in the downstream market and on the degree to which downstream competition is localized. We demonstrate this in Appendix B in the context of alternative spatial models of downstream competition that allow an arbitrary number of competitors, the hub-and-spokes model and the circle models, which otherwise have the similar "bidding market" characteristics as our main model.

Hart and Tirole (1990) made an important contribution to the vertical integration literature by showing that vertical integration enables an upstream monopolist to overcome a commitment problem when bilateral contracts are private, and achieve an *ex post* monopoly outcome in the downstream market. Rey and Tirole (2003) felicitously refer to this result as "restoring" monopoly power. The essential logic is that a vertically integrated firm better internalizes the opportunity cost of cutting supply prices to downstream rivals. The same logic carries over if the upstream firm competes against inferior upstream rivals, although the ability to achieve a full monopoly outcome is constrained by potential competition from the less efficient suppliers. The Hart-Tirole-Rey theory does not explain an incentive for partial vertical integration if the upstream rivals are equally efficient. Our analyses show that such an incentive does exist if a vertically-integrated upstream firm has recourse to exclusive contracts. By charging a higher marginal supply price to downstream rivals, the vertically integrated supplier engineers a "more collusive" downstream outcome. The resulting increase in industry profits is shared among market participants *via* lump sum transfers.²⁴ While Hart and Tirole (1990) study quantity-setting in the downstream

²³Interestingly though, this would not be an issue if requirements contracts were private as in Chen and Riordan (2004). Private contracts, however, complicate the analysis in other ways.

²⁴Alternatively, the upstream monopolist could solve the commitment problem by contracting with down-

market, Rey and Vergé (2004) demonstrate a similar problem of opportunism in a model of downstream price-setting, creating a similar role for vertical integration to restore monopoly power.

Aghion and Bolton (1987) made an important contribution to the literature on exclusive contracting by showing how penalty contracts could exclude an equally or more efficient entrant. Our analysis complements theirs by showing how a vertically integrated firm can use exclusive contracts to exclude an equally or more efficient firm who is already in the market. As suggested by the Chicago School, the exclusion of the upstream competitor is costly to the integrated firm, i.e. transfer payments are needed to gain the acquiescence of the downstream industry. But the necessary transfer payments are not so large as to make *ex post* cartelization unprofitable for the vertically integrated upstream firm. Interestingly, this cost approaches zero when the heterogeneity between downstream firms disappears: the vertically integrated firm relies on cutting its downstream prices as a (hidden) threat to persuade the independent downstream firms to accept the exclusive contract; this threat provides the most powerful incentive, and hence there is little need for explicit transfer payment, when the downstream producers become perfect substitute for each other.

We briefly discuss two antitrust cases that show the empirical relevance of our theory. One case is *Kodak v. F.T.C.* (1925). Kodak had a 90% market share for raw cinematic film that it supplied to downstream picture-makers. Kodak acquired capacity to enter the downstream industry, and reached essentially an exclusive-dealing agreement with picture-makers in which it agreed not to deploy the capacity if picture-makers would refrain from purchasing imported raw film. The Court found this agreement to be an illegal restraint of trade. Another case is *TEKAL/ITALCEMENTI* (A76), brought up by the Italian Antitrust Authority against Italcementi, the main cement manufacturer in Sardinia, Italy.²⁵ Faced with lower-priced competition from imported cement, Italcementi acquired ten concrete stream firms exclusively. Such contracts, however, can only exclude less efficient upstream competitors (Hart and Tirole, 1990; Rey and Tirole, 2003).

²⁵The discussion of this case is based on Italian Antitrust Authority Annual Report 1994, published on April 30, 1995. We thank Pierluigi Sabbatini of the Italian Antitrust Authority for directing our attention to this case.

production facilities between April and June 1993, and began to sell its concrete at prices below variable cost, with the intention of dissuading the independent concrete producers from purchasing their cement from importers. It was then able to enter into contractual agreements with some main concrete purchasing companies that effectively excluded other cement producers. The Italian Antitrust Authority ruled that the conduct of Italcementi was part of an overall plan to restrict access to the Sardinian cement market and constituted an abuse of dominant position, and it fined the company 3.75 billion lire.

While these two cases occurred in different times, countries, and industries,²⁶ the strategic considerations involved in both of them are remarkably similar to those in our theory. In both cases, a vertically integrated upstream producer entered into exclusive contracts with independent downstream firms that excluded other upstream firms from market access. The independent downstream firms appeared to be willing to accept such arrangements because the integrated upstream producer used its downstream facilities to threaten and discipline the independents: if the independents purchased inputs from the vertically integrated upstream producer, the vertically integrated downstream producer would compensate the independents by reducing or refraining from competition; otherwise it would aggressively cut prices. In other words, the credible threat of a price war encouraged downstream competitors to enter into an exclusive supply arrangement on supracompetitive terms - the opportunity cost effect identified in our model. As a result, the vertically integrated firm was able to exclude upstream competitors and likely also raised downstream prices. We also notice that the key features of our model are possibly present in the cases. In particular, for TEKAL/ITALCEMENTI (A76), the different downstream concrete producers likely had different shipping costs for consumers at different locations; downstream market condition was likely to be uncertain in that the location and the demand of a final customer might be unknown ex ante; and pricing contracts between a cement (upstream) producer and a

²⁶Interestingly, there is a case similar to TEKAL/ITALCEMENTI (A76) in New Zealand, concerning a vertically integrated cement/concrete company, Fletcher Concrete and Infrastructure Limited. In 2002, the New Zealand Commerce Commission concluded that company's conduct had the purpose and effect of excluding competition in the cement market and raising concrete prices, and issued a warning.

concrete (downstream) producer did not appear to be contingent on the locations of final consumers. Although the details of the two cases are different from our theoretical model, they do illustrate the empirical relevance of our argument that vertical integration raises heightened concerns about exclusive dealing and *vice versa*.

We close by discussing policy implications at a more general level. If our theory is to be useful for policies concerning vertical mergers and/or exclusive contracts, it must be supported by evidence on market structure. Our analysis suggests the following relevant evidence:

- Sole source requirements contracting is a normal industry practice or at least has some industry precedent. Otherwise, the theory might be judged as too speculative about post-merger industry conduct.
- Upstream price competition is “tough” before the merger or the use of exclusive contracts by a vertically integrated firm, as would be the case if the firms have similar capabilities and were not colluding tacitly (Sutton, 1991). Otherwise, there may be little to gain from cartelization *via* exclusive contracts, or the vertically-integrated firm might be unable to exclude a more efficient upstream competitor.
- The vertically-integrated firm is likely to have substantial excess capacity or can expand capacity easily. Otherwise, the integrated firm is unlikely to be able to supply other downstream firms on competitive terms.
- The downstream market is concentrated, and there are barriers to entry. Otherwise, the cartelization effect is small relative to the size of the market, or would be undone by new entry.²⁷
- Finally, evidence in favor of a plausible efficiency theory should be weighed against evidence in support of an anticompetitive effect (Riordan and Salop, 1995).²⁸

²⁷Market definition is a key issue when competition is localized. Sales to customer groups with few real alternatives may constitute a distinct product market.

²⁸For example, if the upstream competition were “soft”, as would be the case if the upstream firms colluded

APPENDIX A: PROOFS

Appendix A contains proofs for the main results of Section 3.

Proof of Lemma 1: We organize our proof in two parts:

(1): If $P_1^m\left(\frac{1}{2}\right) \geq r + \frac{1}{2}\tau$, then the proposed pricing strategies by $D1$ and $D2$ constitute a Nash equilibrium.

First consider the cases where $x \leq \frac{1}{2}$. Notice that $\tau x \leq \tau(1-x)$. From standard arguments in Bertrand competition, $P_1(x, r)$ maximizes the joint profits of $U1-D1$ given $D2$'s offer, $D2$'s offer is optimal for $D2$ given $P_1(x, r)$, and the consumer will select the firm with the lower cost, which is $D1$ here. The consumer will make the actual purchase if $P_1(x, r) \leq v$.

Next consider the cases where $x > \frac{1}{2}$. Notice that $\tau x > \tau(1-x)$ in these cases. Notice also that, since $P_1^m(x) - c - \tau x$ decreases in x , we may possibly have $P_1^m(x) < r + \tau x$ even though $P_1^m\left(\frac{1}{2}\right) \geq r + \frac{1}{2}\tau$. We proceed with two possible situations:

(i) Suppose $P_1^m(x) > r + \tau x$. At $P_2(x, r) = \min\{P_2^m(x, r), r + \tau x\}$, with the customer selecting $D2$, the expected profit of $U1-D1$ is $[r - c][1 - F(P_2(x, r))]$.

If $D1$ undercuts $D2$ so that it would be selected by the customer, the expected profit of $U1-D1$ is less than

$$[r + \tau x - (c + \tau x)][1 - F(r + \tau x)] \leq [r - c][1 - F(P_2(x, r))].$$

On the other hand, given $D1$'s offer, it is optimal for $D2$ to charge $P_2(x, r)$ and to be selected by the customer. Thus the proposed strategies constitute a Nash equilibrium.

(ii) Suppose instead $P_1^m(x) \leq r + \tau x$. We have $r + \tau(1-x) < r + \frac{1}{2}\tau \leq P_1^m\left(\frac{1}{2}\right) < P_1^m(x)$. With the same logic as above, competition between $D1$ and $D2$ must drive the price down to $P_1^m(x)$, and the consumer selects $D2$.

(2): If (t, r) is an equilibrium contract, then indeed $P_1^m\left(\frac{1}{2}\right) \geq r + \frac{1}{2}\tau$.

Suppose that, to the contrary, there is an equilibrium contract (t, r) such that $P_1^m\left(\frac{1}{2}\right) < r + \frac{1}{2}\tau$. We shall show that the expected industry profit is higher under an alternative contract (t', r') , or simply under r' , where $P_1^m\left(\frac{1}{2}\right) = r' + \frac{1}{2}\tau$. Since t and t' will be chosen expressly or tacitly, and if uniform pricing were the normal pre-merger industry practice, then the merger arguably might increase economic efficiency by eliminating a double markup.

such that the expected profits of $D2$ are zero under the respective contracts, it follows that the expected profit for $U1-D1$ must be higher under contract (t', r') than under contract (t, r) , which produces a contradiction.

First consider the cases where $x \leq \frac{1}{2}$. Since $\tau x \leq \tau(1-x)$ and

$$P_1^m(x) \leq P_1^m\left(\frac{1}{2}\right) \leq r' + \tau(1-x) < r + \tau(1-x),$$

the equilibrium price will be $P_1(x, r) = P_1^m(x)$, under either r or r' , and the customer will select $D1$. Therefore for $x \leq \frac{1}{2}$, both contracts produce the same expected industry profits.

Now consider the cases where $x > \frac{1}{2}$. Then $P_1^m(x) < r + \tau x$ from $P_1^m\left(\frac{1}{2}\right) < r + \frac{1}{2}\tau$ and the fact that $P_1^m(x) - \tau x$ decreases in x . Thus

$$r' + \tau(1-x) < r' + \frac{1}{2}\tau = P_1^m\left(\frac{1}{2}\right) < P_1^m(x) < r' + \tau x.$$

Let $\hat{x} > \frac{1}{2}$ be such that either \hat{x} uniquely solves

$$P_1^m(\hat{x}) = r + \tau(1 - \hat{x}),$$

or $\hat{x} = 1$ if $P_1^m(1) < r$. Then for $\frac{1}{2} < x < \hat{x}$, $P_1^m(x) < r + \tau(1-x)$.

Hence, under r , the equilibrium price will be $P_1^m(x)$ but $D1$ will be selected by the customer for $\frac{1}{2} < x < \hat{x}$; while under r' the equilibrium price will also be $P_1^m(x)$ but $D2$ will always be selected by the customer for $\frac{1}{2} < x \leq 1$. Therefore, for $\frac{1}{2} < x \leq 1$, industry profits will be higher under r' than under r , since $\tau(1-x) < \tau x$.

Thus expected industry profits are higher under r' than under r , contradicting that (t, r) is an equilibrium contract. ■

Proof of Proposition 1: We only need prove that $P_1^m(0) - \tau < \hat{r} < P_1^m\left(\frac{1}{2}\right) - \frac{1}{2}\tau$; everything else follows directly from Lemma 1 and from Assumption A2.

We first show that $P_1^m(0) - \tau < \hat{r}$. Suppose to the contrary $P_1^m(0) - \tau \geq \hat{r}$. Then, $P_1^m(0) > \hat{r} + \tau x$ and $P_1^m(0) > \hat{r} + \tau(1-x)$, for all $x \in (0, 1)$. We thus have

$$\begin{aligned} P_1(x, \hat{r}) &= \hat{r} + \tau(1-x) < P_1^m(0) < P_1^m(x) \text{ for } 0 < x \leq \frac{1}{2}, \\ P_2(x, \hat{r}) &= \min\{P_2^m(x, \hat{r}), \hat{r} + \tau x\} < P_1^m(0) < P_1^m(1-x) \text{ for } \frac{1}{2} < x < 1. \end{aligned}$$

By raising \hat{r} slightly above $P_1^m(0) - \tau$, both $P_1(x, \hat{r})$ and $P_2(x, \hat{r})$ will be closer to $P_1^m(x)$ and $P_1^m(1-x)$, respectively, for all $0 < x < 1$, which would lead to a higher expected industry profit than under $\hat{r} \leq P_1^m(0) - \tau$. This implies that it cannot be optimal for $U1$ to offer $\hat{r} \leq P_1^m(0) - \tau$; and therefore $\hat{r} > P_1^m(0) - \tau$.

We next show that $\hat{r} < P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$. It suffices to show that $\hat{r} \neq P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$, since from Lemma 1 $\hat{r} \leq P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$. Now, from the proof of Lemma 1, if $\hat{r} = P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$, the equilibrium prices would be $P_1(x, \hat{r}) = P_1^m(x)$ for $x \leq \frac{1}{2}$ and

$$P_2(x, \hat{r}) = \min\{P_2^m(x, \hat{r}), \min\{P_1^m(x), \hat{r} + \tau x\}\} > P_1^m(1-x) \quad \text{for } \frac{1}{2} < x \leq 1.$$

That is, $P_1(x, \hat{r})$ is optimal for $x \leq \frac{1}{2}$ while $P_2(x, \hat{r})$ is inefficiently too high for $x > \frac{1}{2}$. A slight reduction in \hat{r} would reduce both $P_1(x, \hat{r})$ and $P_2(x, \hat{r})$ for x that is close to $\frac{1}{2}$, causing a first-order increase in industry profits for those x that are to the right of $\frac{1}{2}$ and a second-order decrease in industry profits for those x that are to the left of $\frac{1}{2}$. Therefore, in equilibrium $\hat{r} \neq P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$. ■

Proof of Lemma 2: First, when $D2$ contracts with $U2$ as its supplier, let \tilde{x} be such that

$$c + \tau\tilde{x} = r_2 + \tau(1 - \tilde{x}),$$

or $\tilde{x} = \frac{r_2 - c}{2\tau} + \frac{1}{2}$. Then \tilde{x} is the marginal customer for $D1$ and $D2$. In the downstream market competition, any customer with $x \leq \tilde{x}$ will select $D1$ as the potential seller, and any consumer with $x > \tilde{x}$ will select $D2$ as the potential seller. We can restrict our attention to $\tilde{x} \in [0, 1]$, since it is not optimal for $U2$ to offer some r_2 that results in \tilde{x} outside of this interval.

Second, given any r_2 and for any $x > \tilde{x}$, the equilibrium price for $D2$ must be

$$\tilde{P}_2(x, r_2) = \min\{c + \tau x, P_2^m(x, r_2)\}.$$

But since

$$P_2^m(x, r_2) \geq P_2^m(x, c) = P_1^m(1-x) > P_1^m(0) > c + \tau x,$$

we have

$$\tilde{P}_2(x, r_2) = c + \tau x \quad \text{for } x > \tilde{x}.$$

The joint profits of $U2$ and $D2$ are

$$\begin{aligned}\Pi_2(r_2) &= \int_{\tilde{x}}^1 [(c + \tau x) - (c + \tau(1 - x))] [1 - F(c + \tau x)] dx \\ &= \int_{\tilde{x}}^1 \tau(2x - 1) [1 - F(c + \tau x)] dx,\end{aligned}$$

where recall that $\tilde{x} = \frac{r_2 - c}{2\tau} + \frac{1}{2}$. Thus,

$$\Pi_2'(r_2) = -\tau(2\tilde{x} - 1) [1 - F(c + \tau\tilde{x})] \frac{1}{2\tau},$$

which is positive if $\tilde{x} < \frac{1}{2}$, or equivalently if $r_2 < c$; and is negative if $\tilde{x} > \frac{1}{2}$, or equivalently if $r_2 > c$. Thus $\tilde{r}_2 = c$. ■

Proof of Proposition 2: A Pareto optimal contract between $U1-D1$ and $D2$ (i.e. a contract that maximizes the profit of $U1-D1$ given a profit constraint for $D2$) maximizes $\Pi(r_1)$, while a Pareto optimal contract between $U2$ and $D2$ has $r_2 = c$ by Lemma 2. Therefore, we can restrict attention to contracts with $r_1 = \hat{r}$ and $r_2 = c$. Given this restriction, and given the expected profits determined according to Lemma 1, competition at Stage 1 is reduced to an asymmetric procurement auction in which $U1-D1$ and $D2$ bid t_1 and t_2 for the exclusive right to supply $D2$ at prices \hat{r} and c respectively. Since $\Pi(\hat{r}) > \Pi(c)$ by Proposition 1, standard reasoning implies that in (the unique refined) equilibrium $U2$ bids $t_2 = 0$ and $U1-D1$ wins with t_1^* defined in equation (7). Since $r_1 = \hat{r}$, the downstream equilibrium outcome is the same as under upstream monopoly by Proposition 1. It remains to show that $t_1^* < 0$. Notice that when r_1 increases, $P_2(x, r_1)$ is either unchanged when $P_2(x, r_1) = P_1^m(x)$, or increases otherwise. In addition, $P_2(x, r_1) - \tau(1 - x) - r_1$ is non-increasing in r_1 , and is strictly decreasing in r_1 if $P_2(x, r_1) = P_1^m(x)$. Therefore,

$$t_1^* < \int_{\frac{1}{2}}^1 [P_2(x, c) - \tau(1 - x) - c] [1 - F(P_2(x, c))] dx - \int_{\frac{1}{2}}^1 \tau(2x - 1) [1 - F(c + \tau x)] dx = 0,$$

where the last equality is a consequence of Assumption $A2$; that is, when $r_1 = c$, $D1$'s willingness to sell at cost always constrains $D2$'s market power. ■

Proof of Proposition 3:

We organize the proof in two steps, noticing that we need only be concerned with contracts in which $r_j \geq c$. In step 1, we show that any $r_i > c$ cannot occur in equilibrium. We then

show in step 2 that there exists an equilibrium where $(t_j^*, r_j^*) = (0, c)$, for $j = 1, 2$. Since step 1 implies that at any possible equilibrium $r_j^* = c$, and hence $t_j^* = 0$, combining step 1 and step 2 completes our proof.

Step 1. *There can be no equilibrium where $r_i > c$ for any i .*

Suppose to the contrary that there is some equilibrium where $r_i > c$ for at least one i . Without loss of generality, suppose that $r_1 > c$ and $r_2 \geq c$. There are two possible cases to consider.

Case 1: r_1 and r_2 are offered by the two different upstream firms. Without loss of generality, suppose r_1 is from $U1$ and r_2 is from $U2$. The marginal consumer between $D1$ and $D2$, \tilde{x} , satisfies

$$\tilde{x}(r_1, r_2) \equiv \frac{r_2 - r_1}{2\tau} + \frac{1}{2}.$$

It is easy to see that $\tilde{x}(r_1, r_2) \in (0, 1)$, since otherwise one of the upstream-downstream pair would have zero joint profit; and by a contract with $r_i = c$ the pair can have a positive expected profit. So suppose that $\tilde{x}(r_1, r_2) \in (0, 1)$. The joint profits of $U1-D1$ under r_1 , given r_2 , is

$$\begin{aligned} \Pi_{11}(r_1, r_2) &= \int_0^{\tilde{x}} [\min\{P^m(x, r_1), r_2 + \tau(1-x)\} - (c + \tau x)] \\ &\cdot [1 - F(\min\{P^m(x, r_1), r_2 + \tau(1-x)\})] dx. \end{aligned}$$

A reduction of r_1 to c would increase \tilde{x} ; in addition, it would either increase or have no effect on

$$[\min\{P^m(x, r_1), r_2 + \tau(1-x)\} - (c + \tau x)][1 - F(\min\{P^m(x, r_1), r_2 + \tau(1-x)\})].$$

Therefore a reduction of r_1 to c would increase the joint profits of $U1-D1$, or $\Pi_{11}(c, r_2) > \Pi_{11}(r_1, r_2)$ for $r_1 > c$. This shows that there can be no equilibrium where the downstream firms are supplied by the two separate upstream firms and at least one downstream firm contracts to receive the input at a unit price above c .

Case 2: r_1 and r_2 are offered by a single upstream firm, say, $U1$. Let t_1 and t_2 be the transfer payments from $D1$ and $D2$ to $U1$ at the proposed possible equilibrium. Let $\Pi(r_1, r_2)$

denote the joint profits of $U1$, $D1$, and $D2$ when $D1$ and $D2$ contract with $U1$ under r_1 and r_2 . Then

$$\begin{aligned}\Pi(r_1, r_2) &= \int_0^{\tilde{x}(r_1, r_2)} [P(x, r_1, r_2) - (c + \tau x)] [1 - F(P(x, r_1, r_2))] dx \\ &\quad + \int_{\tilde{x}(r_1, r_2)}^1 [P(1 - x, r_2, r_1) - (c + \tau(1 - x))] [1 - F(P(1 - x, r_2, r_1))] dx \\ &= \Pi_{11}(r_1, r_2) + \Pi_{12}(r_1, r_2) = \Pi_{21}(r_1, r_2) + \Pi_{22}(r_1, r_2).\end{aligned}$$

Given (t_1, r_1) accepted by $D1$, an offer from $U2$ to $D2$ with $r_2' = c$ will produce the following joint profit between $U2$ - $D2$:

$$\begin{aligned}\Pi_{22}(r_1, c) &= \int_{\max\{\tilde{x}(r_1, c), 0\}}^1 [\min(P^m(1 - x, c), r_1 + \tau x) - (c + \tau(1 - x))] \\ &\quad \cdot [1 - F(\min(P^m(1 - x, c), r_1 + \tau x))] dx.\end{aligned}$$

Thus, in order to prevent $D2$ from deviation, we must have

$$-t_2 \geq \Pi_{22}(r_1, c) \geq \Pi_{22}(r_1, r_2) = \Pi_{12}(r_1, r_2),$$

where we notice that the $U2$ - $D2$ pair would maximize its joint profit by setting $r_2 = c$, given any $r_1 \geq c$. On the other hand, given (t_2, r_2) accepted by $D2$, an offer from $U2$ to $D1$ with $r_1' = c$ will produce the following joint profit between $U2$ - $D1$:

$$\begin{aligned}\Pi_{21}(c, r_2) &= \int_0^{\min\{\tilde{x}(c, r_2), 1\}} [\min(P^m(x, c), r_1 + \tau(1 - x)) - (c + \tau x)] \\ &\quad \cdot [1 - F(\min(P^m(x, c), r_1 + \tau(1 - x)))] dx.\end{aligned}$$

Thus, in order to prevent $D1$ from deviating, we must have

$$-t_1 \geq \Pi_{21}(c, r_2) > \Pi_{21}(r_1, r_2) = \Pi_{11}(r_1, r_2),$$

since $r_1 > c$. Therefore for the proposed contracts to be part of an equilibrium, it is necessary that

$$-(t_1 + t_2) \geq \Pi_{22}(r_1, c) + \Pi_{21}(c, r_2) > \Pi_{22}(r_1, r_2) + \Pi_{21}(r_1, r_2) = \Pi(r_1, r_2),$$

which means that $U1$ will have to receive a negative profit. Therefore there can be no equilibrium where the downstream firms are supplied by a single upstream firm and at least one downstream firm contracts to receive the input at a unit price above c .

We have thus shown that there can be no equilibrium where $r_i > c$ for any i .

Step 2. *There exists an equilibrium in which $(t_{ij}, r_{ij}) = (0, c)$ for $i, j = 1, 2$, $D1$ accepts the contract offered by $U1$, and $D2$ accepts the contract offered by $U2$.*

We need to show that an upstream firm cannot benefit from deviating offers to either one downstream firm or to both downstream firms.

First, given the contract offered by $U2$ and that $D2$ accepts $U2$'s contract, $r_{11} = c$ maximizes the joint profits of $U1-D1$, and it is optimal for $U1$ to set $t_{11} = 0$ given that $(t_{21}, r_{21}) = (0, c)$, and optimal for $D1$ to accept $U1$'s offer. Similarly, given the contract offered by $U1$ and that $D1$ accepts $U1$'s contract, $r_{22} = c$ maximizes the joint profits of $U2-D2$, and it is optimal for $U2$ to set $t_{22} = 0$ given that $(t_{12}, r_{12}) = (0, c)$, and optimal for $D2$ to accept $U2$'s offer.

Next, suppose that an upstream firm, say $U1$, offers both $D1$ and $D2$ a deviating contract that involves some $r_j > c$ and is accepted. With the same logic in Case 2 of Step 1 above, either one of the Dj will receive a higher payoff by staying with $U2$ under $r = c$, or $U1$ will receive a negative payoff. Thus any such deviation cannot be successful. ■

APPENDIX B: BILATERAL OLIGOPOLY

Appendix B extends our analysis to bilateral Oligopoly with two models of downstream competition.

Hub and Spokes Model

We first develop a new model of price competition by multiple downstream firms that is a natural extension of the duopoly model. In addition to extending our results, the model may also have independent interest in suggesting a new way of modeling non-localized price competition by differentiated oligopolists. To save space, we shall make our arguments mostly informally; and, we continue to assume that contracts are bilateral and public.

Suppose that the downstream has $n \geq 2$ firms, $D1, D2, \dots, Dn$. As before, $D1$ and $U1$ are vertically integrated. Each Di is associated with a line of length $\frac{1}{2}, l_i$. The two ends of l_i are

called origins and terminals, respectively. Firm D_i is located at the origin of l_i , and the lines are so arranged that all the terminals meet at one point, the center. This forms a network of lines connecting competing firms (“spokes”), and a firm can supply the consumer only by traveling on the lines. *Ex ante*, the consumer is located at any point of this network with equal probabilities. The realized location of the consumer is fully characterized by a vector (l_i, x_i) , which means that the consumer is on l_i with distances of x_i to D_i and of $\frac{1}{2} - x_i + \frac{1}{2} = 1 - x_i$ to D_j , $j \neq i$.²⁹ Obviously, the linear duopoly model is a special case of the hub-and-spokes model with $n = 2$.

As in our earlier analysis, consider first the case where U_1 is a monopolist in the upstream market. A contract offered by U_1 to D_j , $j = 2, \dots, n$, can be written as (t_j, r_j) . Modifying equations (1) and (2), we can define $P_1^m(x_1)$ and $P_j^m(x_j, r_j)$ as satisfying

$$\begin{aligned} P_1^m(x_1) - c - \tau x_1 &= \frac{1 - F(P_1^m(x_1))}{f(P_1^m(x_1))}, \\ P_j^m(x_j, r_j) - r_j - \tau x_j &= \frac{1 - F(P_j^m(x_j, r_j))}{f(P_j^m(x_j, r_j))}, \quad j = 2, \dots, n. \end{aligned}$$

Let $\bar{r} \equiv \min\{r_j : j = 2, \dots, n\}$. Modifying equations (3) and (4) in Section 3, for $i = 1, \dots, n$ and $j = 2, \dots, n$, we can define

$$P_1((l_i, x_i), \bar{r}) = \begin{cases} \min\{P_1^m(x_1), \bar{r} + \tau(1 - x_1)\} & \text{if } i = 1 \\ \min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\} & \text{if } i \neq 1 \end{cases},$$

$$P_j((l_i, x_i), r_j, \bar{r}) = \begin{cases} \min\left\{P_j^m(x_j, r_j), \max\{r_j + \tau x_j, \min\{P_1^m(1 - x_j), \bar{r} + \tau(1 - x_j)\}\}\right\} & \text{if } i = j \\ r_j + \tau(1 - x_i) & \text{if } i \neq j \end{cases}.$$

Then, extending Lemma 1, in any downstream pricing game following any $\{(t_j, r_j) : j = 2, \dots, n\}$, there is a unique (refined) equilibrium outcome,³⁰ in which D_1 sets $P_1((l_i, x_i), \bar{r})$ and D_j sets

²⁹For the consumer located at the center, we shall denote her by $(l_1, \frac{1}{2})$.

³⁰As in standard Bertrand competition with more than two firms, the strategy profile supporting the unique equilibrium outcome may not be unique.

$P_j((l_i, x_i), r_j, \bar{r})$, with the equilibrium price for consumer (l_i, x_i) being

$$P^*((l_i, x_i), r_i, \bar{r}) = \begin{cases} \min\{P_1^m(x_1), \bar{r} + \tau(1 - x_1)\} & \text{if } i = 1 \\ \min\{P_i^m(x_i, r_i), \max\{r_i + \tau x_i, \min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\}\}\} & \text{if } i \neq 1 \end{cases};$$

consumer (l_i, x_i) selects $D1$ if $i = 1$ or if $i \neq 1$ but $\min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\} < r_i + \tau x_i$; and consumer (l_i, x_i) selects Di if $i \neq 1$ and $\min\{P_1^m(1 - x_i), \bar{r} + \tau(1 - x_i)\} \geq r_i + \tau x_i$. As in Lemma 1, $P_1^m(\frac{1}{2}) \geq r_i + \frac{1}{2}\tau$ for any equilibrium contract (t_i, r_i) .

The presence of additional downstream firms introduces several issues that we must consider in extending the analysis leading to Proposition 1.

First, it is now possible that $r_j \neq r_k$ for some $j, k = 2, \dots, n$ and $j \neq k$. Suppose that $r_k = \bar{r} < r_j$ for some $j = 2, \dots, n$; i.e., Dk has a cost advantage in supplying (l_j, x_j) when $r_k + \tau(1 - x_j) < r_j + \tau x_j$. But Dk cannot benefit from selling to such a consumer, since the competition from $D1$ will drive the price down to $\min\{P_1^m(1 - x_j), r_k + \tau(1 - x_j)\} \leq r_k + \tau(1 - x_j)$. This is because the perceived marginal cost for $D1$ in supplying such a consumer when Dk is the other potential supplier and purchases from $U1$ at r_k , is $c + r_k - c = r_k$.

Second, it immediately follows that to maximize joint upstream-downstream industry profits, we must have $(t_j, r_j) = (t, r)$ for $j = 2, \dots, n$; because, if $r_k < r_j$ for some $j \neq k$, then slightly lowering r_j has no effect on the competition for consumer (l_i, x_i) , $i \neq j$ but increases the expected industry profit from consumer (l_j, x_j) . This allows us to generalize equations (5) and (6) and define

$$\begin{aligned} \Pi(r) &= \frac{2}{n} \int_0^{\frac{1}{2}} [P_1(x, r) - \tau x - c] [1 - F(P_1(x, r))] dx + \\ &\quad \frac{n-1}{n} 2 \int_0^{\frac{1}{2}} [P_2(x, r) - \tau x - c] [1 - F(P_2(x, r))] dx, \\ t(r) &= \frac{2}{n} \int_0^{\frac{1}{2}} [P_2(x, r) - \tau x - r] [1 - F(P_2(x, r))] dx, \end{aligned}$$

where $\Pi(r)$ is the joint industry profits when $(t_j, r_j) = (t(r), r)$ for all $j = 2, \dots, n$. The transfer $t(r)$ fully extracts rents from the downstream industry.

Notice that an increase in r has the similar trade off here as in the downstream duopoly case: it affects positively the profit for $D1$ due to relaxed competition, but affects negatively the profits for each Dj if it worsens the double mark-up distortion. Since the second effect is more important with a higher n , we conclude that \hat{r} decreases in n , where

$$\hat{r} = \arg \max_{c \leq r \leq \bar{v}} \{\Pi(r)\}.$$

As in Proposition 1, we will have $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$, and define $\hat{t} = t(\hat{r})$.

We can thus extend Proposition 1 to the hub-and-spokes model with $n \geq 2$ downstream competitors.

Proposition 1' *The game where $U1$ is the only upstream supplier has a unique equilibrium, in which $U1$ offers Dj contract (\hat{t}, \hat{r}) , which is accepted by Dj , $j = 2, \dots, n$. Di is the potential seller with price $P^*((l_i, x_i), \hat{r}, \hat{r})$ if the consumer is located at (l_i, x_i) , $i = 1, \dots, n$. Furthermore, $c \leq P_1^m(0) - \tau < \hat{r} < P_1^m(\frac{1}{2}) - \frac{1}{2}\tau$, and \hat{r} decreases in n .*

Thus, just as in the downstream duopoly model, the firm that is nearest to the consumer will bid the lowest price and will make the sale if this price does not exceed the consumer's valuation. The equilibrium \hat{r} is above c for the same reason as in the duopoly case: it reduces downstream competition and thus raises industry profits.

Returning to upstream duopoly, when Dj contracts to purchase from $U2$ at $(0, c)$, $D1$ will charge $c + \tau(1 - x_j) < \min\{P_1^m(1 - x_j), \hat{r} + \tau(1 - x_j)\}$ if the consumer is located at (l_j, x_j) and $j \neq 1$, and thus the (expected) joint profit of $U2$ - Dj is

$$\frac{2}{n} \int_0^{\frac{1}{2}} \tau(1 - 2x) [1 - F(c + \tau(1 - x))] dx,$$

which is lower than the joint $U1$ - Dj profit under \hat{r} .

Since the expected profit of $D2$ when it contracts with $U1$, excluding any transfer payment, is

$$\begin{aligned} & \frac{2}{n} \int_0^{\frac{1}{2}} [P^*((l_2, x), \hat{r}, \hat{r}) - \tau x - \hat{r}] [1 - F(P^*((l_2, x), \hat{r}, \hat{r}))] dx \\ &= \frac{2}{n} \int_0^{\frac{1}{2}} [P_2(x, \hat{r}) - \tau x - \hat{r}] [1 - F(P_2(x, \hat{r}))] dx, \end{aligned}$$

we can modify equation (7) to define

$$t^* = \frac{2}{n} \int_0^{\frac{1}{2}} [P_2(x, \hat{r}) - \tau x - \hat{r}] [1 - F(P_2(x, \hat{r}))] dx - \frac{2}{n} \int_0^{\frac{1}{2}} \tau(1-2x) [1 - F(c + \tau(1-x))] dx,$$

where $t^* < 0$.

We can thus extend Proposition 2 as follows:

Proposition 2' *The game where the upstream market is a duopoly has an equilibrium in which U2 offers Dj contract $(0, c)$ and U1 offers Dj contract (t^*, \hat{r}) , and Dj contracts with U1, $j = 2, \dots, n$. This downstream outcome is the same as under upstream monopoly.*

The intuition here is the same as in the downstream duopoly case: When the integrated firm supplies $D2, \dots, Dn$ at a price above marginal cost, the former has less incentive to undercut the latter because of the opportunity cost of foregone input sales to Dj . This dampening of horizontal competition explains $U1$'s advantage and ability to preempt $U2$. The r that is optimal under upstream monopoly is again chosen to maximize the joint industry profits, and t^* is chosen so that each stand-alone firm is willing to enter the exclusive contract with $U1$. If any $Dj, j = 2, \dots, n$, deviates and contracts with $U2$ at $(0, c)$, $D1$ will reduce its price to $c + \tau(1 - x_i)$ for any consumer located at (l_i, x_i) , $i \neq 1$, making the expected joint profit between $U2$ - Dj lower than the expected joint profit between $U1$ - Dj under \hat{r} , which implies that no deviation would occur.³¹

Since $\hat{r} > c$, just as in the downstream duopoly case, the use of exclusive contracts is crucial for $U1$ to be able to exclude $U2$ and to raise the downstream prices.

We can further show that, as in the downstream duopoly case, if $U1$ and $D1$ are vertically separated, there exists an equilibrium in which exclusive contracts are irrelevant due to competitive (marginal cost) contracting for the intermediate good. For purposes of this discussion we suppose there are m equally efficient upstream firms, indexed $i = 1, \dots, m$, and n downstream firms, indexed $j = 1, \dots, n$. Proposition 2' extends readily to this generalization, with $Ui, i = 3, \dots, m$ acting the same as $U2$. The discussion below pertains to vertically-separated oligopolies.

³¹Notice that since in equilibrium $U2$ offers $(0, c)$, adding additional upstream firms that are the same as $U2$ will not change the results.

First, we can argue that it is an equilibrium for all U_i to offer $(0, c)$ to all downstream firms and U_1 's offer is accepted by all D_j , $j = 1, \dots, n$. It suffices to consider deviations by an upstream firm, say U_2 , that offer any (t, r) , $r > c$, to all D_j , $j = 1, \dots, n$.³² For D_j to be willing to accept the deviation contract, it is necessary that D_i receives a payment that compensates it for the loss in profit due to $r > c$, or

$$\begin{aligned} -t &\geq \frac{2}{n} \int_0^{\frac{1}{2}} \tau(1 - 2x_i) [1 - F(c + \tau(1 - x_i))] dx_i \\ &\quad - \frac{2}{n} \int_0^{\max\{0, \frac{1}{2} - \frac{r-c}{2\tau}\}} [(c + \tau(1 - x_i)) - (r + \tau x_i)] [1 - F(c + \tau(1 - x_i))] dx_i \\ &> \frac{2}{n} \int_0^{\frac{1}{2}} (r - c) [1 - F(c + \tau(1 - x_i))] dx_i. \end{aligned}$$

With the deviation, U_2 's revenue from D_j is

$$(r - c) \frac{2}{n} \int_0^{\frac{1}{2}} [1 - F(\min\{P_2^m(x_i, r), r + \tau(1 - x_i)\})] dx_i.$$

Therefore, with n firms,

$$\begin{aligned} &n \left[(r - c) \frac{2}{n} \int_0^{\frac{1}{2}} [1 - F(\min\{P_2^m(x_i, r), r + \tau(1 - x_i)\})] dx_i \right] - n(-t) \\ &< (r - c) 2 \left[\int_0^{\frac{1}{2}} [1 - F(c + \tau(1 - x_i))] dx_i - \int_0^{\frac{1}{2}} [1 - F(c + \tau(1 - x_i))] dx_i \right] = 0, \end{aligned}$$

which implies that there can be no profitable deviation from the candidate equilibrium.

Second, we can rule out equilibria in which any downstream firm, say D_j , contracts to purchase at $r_j > c$ with the additional parameter restriction $m \geq \frac{n}{2} + 1$. Recall that the basic intuition for this result under downstream duopoly is the following: if U_1 contracts with D_1 and U_2 contracts with D_2 , then each pair would maximize its joint profit by setting the price from U_i to D_i at c ; if one of the upstream firms, say U_1 , contracts with both D_1 and D_2 at some price above c , U_2 can offer contracts to either D_1 or D_2 with price c and achieve a higher joint profit with either of them than the joint profit between either U_1 - D_1 or U_1 - D_2 . This intuition extends to multiple downstream competitors. Since $m \geq \frac{n}{2} + 1$, at

³²A deviation aimed at a strict subset of downstream firms is even less profitable, because the upstream firm must compensate for sales lost to the remaining downstream competitors for whom $r_i = c$.

least one upstream firm, say $U2$, is either not contracting with any downstream firm or is contracting with only one downstream firm at any possible equilibrium. If $U2$ is contracting with only one downstream firm, say $D2$ at r_2 , then it is optimal for $r_2 = c$, which implies that it would not be optimal for any other pair of upstream and downstream firms to have the intermediate good price above c . If $U2$ is not contracting with any downstream firm and some upstream firm, say $U1$, is contracting with one or several Dj with $r_j > c$, then $U2$ can offer $r_j = c$ to one of the Dj , which will be accepted. (It is important to see why the cartelization equilibrium can be sustained under $U1-D1$ integration but not under vertical separation. Under $U1-D1$ integration, a deviation to $r = c$ for some Dj would be met with a reduction of $D1$'s perceived marginal cost from \hat{r} to c when $D1$ competes with Dj , which makes the deviation unprofitable; while under $U1-D1$ separation, the other downstream's marginal costs are taken as given when Dj considers deviation.)

Notice that competitive contracting is still an equilibrium under vertical separation even without the parameter restriction on m . Thus this restriction is not crucial for our main insight about the effect of vertical integration. However, for $m < \frac{n}{2} + 1$, we have not ruled out the possibility of another vertical-separation equilibrium in which each upstream firm contracts with several downstream firms at some price above c . In such a situation, an under-cutting upstream firm must balance the considerations of its own part of the downstream market and the rest of the downstream market. This complication is avoided with the assumption $m \geq \frac{n}{2} + 1$.³³

Circle Model

We next consider an alternative way of extending our model to multiple downstream firms. Instead of considering non-localized competition in the downstream market, we consider localized competition, adopting the circular city model of Salop (1979). Assume that the consumer is located with equal chance at any point of a circle with a perimeter

³³The restriction is unnecessary if contracts are private (Chen and Riordan, 2003). The same is true for the circle model that follows.

equal to 1. Firms are located equidistant from each other on the circle. With $n > 2$ firms, $D1, D2, \dots, Dn$, the distance between any two neighboring firms is simply $\frac{1}{n}$. Let $D1$ be located at the bottom of the circle, followed clockwise by $D2, \dots, Dn$. Thus, $D1$'s neighboring firms on the left and on the right are denoted as $D2$ and Dn , respectively. The realized location of the consumer is denoted as $x \in [0, 1]$, where $x = 0$ if the consumer is at the bottom of the circle (the position of $D1$), and x increases clockwise (so, for instance, $x = \frac{1}{2}$ if the consumer is located at the top point of the circle). In what follows we shall only sketch our analysis, under the same contracting assumptions as in the hub-and-spokes model and assume $m \geq \frac{n}{2} + 1$. As in the hub-and-spokes model, the parameter restriction rules out the possibility of additional equilibria.

If $U1$ and $D1$ are vertically separated, then again the only equilibrium outcome is for all downstream firms to purchase the input at price c , same as in our basic model with rather similar reasoning.³⁴ In what follows we thus assume that $U1$ and $D1$ are vertically integrated. For convenience, we shall focus on the case $n = 4$, and will in the end discuss the cases $n > 4$ and $n = 3$.

With $n = 4$, $D1$ competes with $D2$ and $D4$ respectively when $x \in [0, \frac{1}{4}]$ and $x \in [\frac{3}{4}, 1]$, $D2$ competes with $D3$ when $x \in [\frac{1}{4}, \frac{1}{2}]$, and $D3$ competes with $D4$ when $x \in [\frac{1}{2}, \frac{3}{4}]$. Notice that the only firm $D1$ does not compete with directly is $D3$. Denote the contract Dj accepts by (t_j, r_j) , $j = 2, 3, 4$.

As before, if $U1$ were the only upstream producer, then in equilibrium $(t_j, r_j) = (t^*, r^*)$ and $r^* > c$, and we can extend Proposition 1 to the following:

The game where $U1$ is the only upstream supplier has a unique equilibrium. At this equilibrium, $r_i^ = r^*$, $i = 2, 3, 4$, for some $r^* > c$. $D1$ is the potential seller when $x \in [0, \frac{1}{8}] \cup [\frac{7}{8}, 1]$, $D2$ is the potential seller when $x \in [\frac{1}{8}, \frac{3}{8}]$, $D3$ is the potential seller when $x \in [\frac{3}{8}, \frac{5}{8}]$, and $D4$ is the potential seller when $x \in [\frac{5}{8}, \frac{7}{8}]$.*

More interesting is what happens under upstream competition ($m \geq 2$), to which we now return. We sketch our argument in two parts:

³⁴Without the restriction $m \geq \frac{n}{2} + 1$, this equilibrium outcome is still valid, but we do not rule out other possible equilibria. The restriction avoids the complication.

(1) $r_3^* = c$ in equilibrium.

Because $U1$ is integrated with $D1$, and $D1$ competes directly with $D2$ and $D4$, $U1$ has an advantage over Ui , $i = 2, \dots, m$, in achieving any potential downstream collusive outcome. It thus suffices to argue that if in equilibrium $U1$ contracts with all three independent downstream firms, Dj , $j = 2, 3, 4$, we must have $r_3^* = c$. To make this argument, we notice that r_3 only affects the competition for $x \in [\frac{1}{4}, \frac{3}{4}]$, or the top half of the circle. In equilibrium, we must have $r_2^* = r_4^*$, and due to symmetry we can focus on the segment $x \in [\frac{1}{4}, \frac{1}{2}]$ and consider profits on that segment. For any given r_2^* , if $r_3^* > c$, Ui could offer a contract to $D3$ at $r_3 = c$ that maximizes their joint profit, and this profit, same as the joint profit of $U1-D3$ if they contract under $r_3 = c$, is higher than the joint profit of $U1-D3$ with $r_3^* > c$. Furthermore, an offer from Ui to $D2$ with $r_2 = c$ would enable $Ui-D2$ to earn a higher joint profit when $r_3^* > c$ than when $r_3^* = c$. Therefore, to prevent $D2$ and $D3$ to accept a competitive contracting offer from Ui , it costs $U1$ more under $r_3^* > c$ than under $r_3^* = c$. Thus, it is optimal for $U1$ to contract with $D3$ at $r_3^* = c$. Notice that if it is an equilibrium for $U1$ to contract with $D3$ at $r_3^* = c$, it is also an equilibrium for Ui to contract with $D3$ at $r_3^* = c$.

(2) In equilibrium, $U1$ is able to raise the input price of its neighbors; i.e., $r_2^* > c$ and $r_4^* > c$, and to raise the final price for the consumer.

We shall look for r_2 and r_4 such that the joint profits of $U1-D1-D2$ are maximized when the consumer is located on the left half of the circle and the joint profits of $U1-D1-D4$ are maximized when the consumer is located on the right half of the circle. (Note that we already know $r_3^* = c$.) Because of symmetry, the equilibrium r_2^* and r_4^* would be equal.

For consumer x located between $D1$ and $D2$ ($x \in [0, \frac{1}{4}]$), the consumer's distances from $D1$ and $D2$ are x and $\frac{1}{4} - x$, respectively. Since the distance of consumer x from $D3$ is $\frac{1}{2} - x$, in order for the consumer to be served by either $D1$ or $D2$, we need

$$r_2 + \left(\frac{1}{4} - x\right) \tau \leq c + \left(\frac{1}{2} - x\right) \tau,$$

or³⁵ $r_2 \leq c + \frac{1}{4}\tau$. But since $c + \frac{1}{4}\tau < c + \tau \leq P_1^m(0)$, it follows that, for any $x \in [0, \frac{1}{4}]$,

³⁵If this condition is not satisfied, then $D3$ would compete with $D1$ for consumer $x \in [0, \frac{1}{4}]$. By lowering

in equilibrium $D1$ and $D2$ will charge prices that are below their unconstrained monopoly prices. The equilibrium prices for consumer x are thus equal to $\max\{r_2 + \tau(\frac{1}{4} - x), r_2 + \tau x\}$, and $D1$ and $D2$ each serves the consumer located between $[0, \frac{1}{8}]$ and $[\frac{1}{8}, \frac{1}{4}]$, respectively.

For consumer $x \in [\frac{1}{4}, \frac{1}{2}]$, for whom $D2$ and $D3$ compete, the marginal consumer is $\hat{x}_2 = \frac{c-r_2}{2\tau} + \frac{3}{8}$, where $D2$ serves if $x \in [\frac{1}{4}, \hat{x}_2]$ with price $c + (\frac{1}{2} - x)\tau$ and $D3$ serves if $x \in [\hat{x}_2, \frac{1}{2}]$.

Therefore, the expected joint profit of $U1-D1-D2$ when the consumer is located on the left half of the circle is

$$\begin{aligned} \Pi(r_2) &= 2 \int_0^{\frac{1}{8}} \left[r_2 + \left(\frac{1}{4} - x\right)\tau - (c + x\tau) \right] \left[1 - F\left(r_2 + \left(\frac{1}{4} - x\right)\tau\right) \right] dx \\ &\quad + \int_{\frac{1}{4}}^{\hat{x}_2} \left[c + \left(\frac{1}{2} - x\right)\tau - \left(c + \left(x - \frac{1}{4}\right)\tau\right) \right] \left[1 - F\left(c + \left(\frac{1}{2} - x\right)\tau\right) \right] dx. \end{aligned}$$

Let

$$\hat{r}_2 \equiv \arg \max_{c \leq r_2 \leq c + \frac{1}{4}\tau} \Pi(r_2).$$

Then, since

$$2 \int_0^{\frac{1}{8}} \left[r_2 - c + \left(\frac{1}{4} - 2x\right)\tau \right] \left[1 - F\left(r_2 + \left(\frac{1}{4} - x\right)\tau\right) \right] dx$$

is strictly increasing in r_2 at $r_2 = c$, while

$$\begin{aligned} &\frac{d \left[\int_{\frac{1}{4}}^{\hat{x}_2} \left(\frac{3}{4} - 2x\right)\tau \left[1 - F\left(c + \left(\frac{1}{2} - x\right)\tau\right) \right] dx \right]}{dr_2} \Bigg|_{r_2=c} \\ &= \left(\frac{3}{4} - 2\hat{x}_2\right)\tau \left[1 - F\left(c + \left(\frac{1}{2} - \hat{x}_2\right)\tau\right) \right] \left(-\frac{1}{2\tau}\right) \Bigg|_{r_2=c} = 0, \end{aligned}$$

we must have $\Pi'(r_2)|_{r_2=c} > 0$, and thus $\hat{r}_2 > c$.

If $D2$ were to contract with Uj , the contract that would maximize the joint profit of $Uj-D2$ and give all this profit to $D2$ is $(0, c)$. The joint profit of $U1-D1-D2$ when the consumer is located on the left half of the circle would then be $\Pi(c) < \Pi(\hat{r}_2)$. Notice that $D2$'s profit when it accepts $(0, c)$ from $U2$ is $\frac{2}{3}\Pi(c)$, and $U1-D1$'s profit from this part of the circle is $\frac{1}{3}\Pi(c)$.

r_2 to $c + \frac{1}{4}\tau$, the price for x is not changed but the profits to $D3$ would go to $D2$. Thus, to look for the optimal r_2 , we need to restrict to $r_2 \leq c + \frac{1}{4}\tau$.

Now let t_2^* be such that $D2$'s profit when it accepts (t_2^*, \hat{r}_2) from $U1$ is $\frac{2}{3}\Pi(c)$. Then, $D2$'s profit when it accepts (t_2^*, \hat{r}_2) from $U1$ is the same as that when it accepts $(0, c)$ from Uj , and $U1$ will indeed offer (t_2^*, \hat{r}_2) to $D2$ since $\Pi(\hat{r}_2) - \frac{2}{3}\Pi(c) > \frac{1}{3}\Pi(c)$. Therefore, corresponding to Proposition 2, we have:

The game where the upstream market has $m \geq 2$ equally efficient firms has a unique equilibrium outcome, where $U1$ contracts with $D2$ and $D4$ at (t_2^, \hat{r}_2) , while $D3$ contracts with either $U1$ or Uj , $j \neq 1$, at $(0, c)$.*

Importantly, however, now the downstream equilibrium outcome is *different* from under upstream monopoly. The vertically integrated firm is able to raise input prices only for its neighbors, using exclusive contracts. If $U1-D1$ attempts to contract with the non-neighboring firm, $D3$, at $r_3 > c$, and if $D3$ instead accepts $U2$'s offer at c , $U1-D1$ cannot "punish" $D3$ with a reduction of $D1$'s opportunity cost from r_3 to c , since $D1$ does not compete directly with $D3$ and r_2^* is given. Localized downstream competition thus reduces the vertically integrated firm's ability to cartelize the downstream market. More generally, if $n > 4$, in equilibrium we have $r_2^* = r_n^* > c$ and $r_j^* = c$ for $j = 3, \dots, n-1$.³⁶

The $n = 3$ case is different because $D2$ and $D3$ compete directly both with $U1$ and with each other. Consequently the joint profit of $U1-D1-D2$ depends on r_3 . By the theorem of the maximum there exists a continuous bounded function $\sigma(r_3)$ such that $r_2 = \sigma(r_3) \geq c$ maximizes the joint profit of $U1-D1-D2$ given any $r_3 \geq c$, and by Brouwer's theorem there exists a fixed point $r^* = r_2(r^*)$ that defines a symmetric equilibrium $r_3^* = r_2^* = r^*$. Finally, the joint profit of $U1-D1-D2$ is increasing in r_2 when $r_2 = c$, which implies $r^* > c$.

Therefore, our main results also hold in the circle model of downstream competition: vertical integration in combination with exclusive contracts excludes an equally efficient supplier and partially cartelizes the downstream industry; neither of these practices alone can be counted to achieve these effects. However, the extent of upstream foreclosure and downstream cartelization depends on the nature of competition—whether it is localized or non-localized, in addition to on the level of concentration in the downstream market.

³⁶The restriction $m \geq \frac{n}{2} + 1$ rules out the possibility of other equilibria that involve non-competitive contracting for firms beyond $D1$'s two neighbors.

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