Dynamics of price regulation

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We study the dynamics of price regulation for an industry adjusting to exogenous technological progress. First, we characterize the optimal capacity path and replacement cycles in a neoclassical investment model. Second, we show that naive rate-of-return regulation, which ignores components of economic depreciation, eventually results in a deficient level of capacity due to excessively high retail prices burdened by the need to recover the underdepreciated costs of historical investments. Third, we explain how price-cap regulation leads to more efficient capital replacement decisions compared to naive rate-of-return regulation, and we show how finite price cap horizons distort capital replacement. Finally, we interpret recent regulatory reforms in telecommunications markets.

1. Introduction

The price regulation of natural monopolies in the United States and elsewhere has gone through various phases of rate-of-return regulation, price cap regulation, and now, especially in telecommunications, new forms of wholesale price regulation. These regulatory changes have occurred against a backdrop of rapid technological change. In telecommunications, capital equipment costs and operating costs have declined steadily with the development of microwave, fiber optic, and digital switching technologies, and these costs are poised to drop further with packet-switching and Internet-protocol

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1 The Federal Communications Commission adopted price-cap regulation for AT&T in 1989, although retail long distance prices have since been deregulated. The FCC currently regulates interstate access fees for long distance service with price caps. Approximately 30 states had adopted some form of price-cap regulation as of 1996 (see Sappington and Weisman (1996) and Tardiff and Taylor (1996)).

2 The regulation of long distance access fees is a form of wholesale price regulation. Wholesale price regulation is also being applied to unbundled elements of local telephone networks under the Telecommunications Act of 1996.
technologies. Similar regulatory changes have occurred in electricity, where costs have dropped with the development of new technologies for generation, including gas turbine technology.

We study price regulation in a model of industry structure emphasizing the dynamic paths of prices and investments induced by technological progress in the capital equipment industry. The most important element of our model is that exogenous technological progress lowers both capital equipment and operating costs. Consequently, the optimal path of regulated prices reflects the value of waiting for further technological improvements. In particular, the optimal price at each date recovers operating costs, the user cost of capital, and a contribution to fixed costs. The user cost of capital includes economic depreciation resulting from the declining price of new capacity plus the value of operating cost improvements that are forgone by investing sooner rather than later. Nonrecurring fixed costs are recovered over time in a manner that reflects Ramsey pricing principles for output sold at different dates. Investments in new capacity adjust to meet demand at these prices, and capacity becomes economically obsolete when the long-run marginal cost of replacement capacity drops below the short-run operating cost of the old capacity.

Rate-of-return regulation has been used to regulate public utilities for much of the century. Rate-of-return regulation is designed to recover historical investment costs. It is not designed to adjust prices to the changing long-run marginal cost of a dynamic market structure. In particular, the regulated price recovers an allowed rate of return and an allowed depreciation of a rate base equaling cumulative undepreciated investments. Naïve rate-of-return regulation, which underdepreciates assets by ignoring technological progress, distorts the path of prices relative to the optimal path. Initially the regulated prices are too low because they fail to recover economic depreciation. However, regulated prices eventually become too high because of the firm’s entitlement to the allowed rate of return on a bloated rate base. Correspondingly, naïve rate-of-return regulation eventually results in an economically deficient level of capacity. Moreover, naïve rate-of-return regulation lacks incentives to retire old capacity and replace it with new capacity with lower operating costs.

Price-cap regulation provides better incentives for capital replacement but may still leave prices too high if the initial price cap is set to recover the historical costs inherited from rate-of-return regulation. Moreover, there is a sense in which the regulator gets only one bite from the apple. Even in the case of deterministic technological progress, if the parameters of regulation are not set optimally initially, then a problem of stranded costs may prevent the subsequent achievement of full efficiency by more enlightened regulation. Stranded costs arise when a change in regulatory structure prevents recovery of a fair rate of return on previous investments. A stranded-costs problem can be expected in a transition from a regime of naïve rate-of-return regulation when the allowed depreciation rate has been set below economic depreciation.\(^3\) We argue that with uniform pricing it is optimal to recover stranded costs by setting a markup above the optimal price level according to Ramsey pricing principles. If the demand elasticity is constant over time, then the optimal markup is proportional to the declining retail price, and a constant sales tax can implement the second-best solution.

In practice, price-cap regulation has been implemented only over limited time horizons. Thus, regulators’ commitments to replace rate-of-return regulation with price-cap regulation have been only temporary. We explain how a temporary price-cap regime provides the regulated firm distorted incentives for capacity replacement. If the

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\(^3\) Sidak and Spulber (1997) argue forcefully that the firm is legally entitled to recover stranded costs that result from a change of regulatory regime.
price-cap horizon is excessively short, then price-cap regulation provides no better incentives for capacity replacement than does rate-of-return regulation. For longer price-cap horizons, capacity replacements are concentrated at the beginning of the price-cap period, and the firm has no incentive to replace capacity near the end of the price-cap period. Moreover, there is a tendency for many capacity replacements to be “rushed.” Repeated intervals of price-cap regulation can be expected to generate cycles in capacity replacement.

There has been a great deal of research dealing with the properties of rate-of-return and price cap regulation. Most of this research has dealt with static models. One set of models deals with the “Averch-Johnson effect”; see, e.g., Averch and Johnson (1962) and Joskow (1972), where rate-of-return regulation distorts a firm’s incentive to minimize costs because it receives a supracompetitive return on capital. A second line of research deals with a firm’s incentives to produce in a market where it faces competition when it is regulated in another market; see, e.g., Braeutigam and Panzar (1989). A third line deals with the cases where firms must be provided incentives for cost reduction or have private information about costs or demand; see, e.g., Cabral and Riordan (1989) and Lewis and Sappington (1989). There has been some work done on dynamic models. Bradley and Price (1988), Vogelsang (1988), Brennan (1991), and Pint (1992) examine complete-information models, while Sibley (1989) investigates a model where the firm has private information about costs and demand. These articles generally ignore technological progress.

There are several dynamic regulatory models that examine investment issues by a regulated firm in a political economy framework. In a symmetric-information model, where regulators are short lived and care about consumers only during their term, whereas the firm is long lived, Lewis and Sappington (1991) show that on average investment is lower than the first-best level. Besanko and Spulber (1992) and Laffont and Tirole (1992) examine how a regulator’s commitment ability affects a firm’s investment decisions when the firm has private information about its productivity. Using an infinite time horizon model under uncertainty, Gilbert and Newbery (1994) show that rate-of-return regulation using the “used and useful criteria” is able to support efficient investment under a wider range of parameters than under standard rate-of-return regulation. Other articles with regulatory review include Lyon (1991) and Salant and Woroch (1992). Technological progress is not a feature of these models.

The rest of the article is organized as follows. In Section 2 we study optimal prices, investment, and capital replacement for the case of deterministic, exogenous technological progress and constant demand. We briefly discuss the implications of uncertain technological progress and growing and uncertain demand, providing more detailed results in appendices. In Sections 3 and 4 we examine the relationship between the optimal solution and rate-of-return and price-cap regulation, respectively. We discuss our results in relation to the telecommunications industry in Section 5.

2. Optimal prices

Our model assumes the following market structure for a particular service. Demand is stable and described by a downward-sloping inverse demand curve \( p = \phi(X) \). The corresponding revenue function is \( R(X) = \phi(X)X \). For simplicity, we assume that the demand curve has a constant elasticity over relevant ranges of output. Thus, marginal revenue is \( R'(X) = [1 - (1/\epsilon)]\phi(X) \), where \( \epsilon > 0 \) is the price elasticity of demand. Consumers’ willingness to pay for \( X \) units of output, equal to the area under the demand curve, is
Our model allows several components of costs: capacity costs, operating costs, and fixed costs. The first two components decline with technological progress, which is embodied in new capital equipment. In the short run, output is constrained by capacity. Thus, $X$ units of output require at least $X^*$ units of capacity. Capacity is a capital asset that depreciates at a physical rate $\delta \geq 0$. The investment cost of a unit capacity is $q$, and it declines at a deterministic rate $\mu \geq 0$. The operating cost, $c(t)$, of new (state of the art) capacity declines at rate $\theta \geq 0$ and for a given vintage is constant over the life of the asset. There is a nonrecurring fixed cost of operation, $F \geq 0$, which is paid upfront.

The optimal investment path maximizes the present discounted value of social welfare subject to a break-even constraint. The discount rate is equal to $r$. Social welfare depends on the path of investment in new capacity and the utilization of capacity over time. Investment in new capacity at date $t$ is equal to $x(t)$, and the utilization rate of vintage $\tau$ capacity at date $t \geq \tau$ is $u(t, \tau)$. Obviously, $u(t, \tau)$ must lie between zero and one. If $e^{-\delta(t-\tau)}x(\tau)$ is the amount of vintage $\tau$ capacity available at date $t$, then the total amount of capacity utilized at date $t$, and hence total output, is$^5$

$$X(t) = \int_0^t u(t, \tau)e^{-\delta(t-\tau)}x(\tau) \, d\tau + u(t, 0)e^{-\delta t}X(0)$$

and the market-clearing price at date $t$ is

$$p(t) = \Phi(X(t)).$$

Similarly, the firm’s total operating cost at date $t$ is

$$C(t) = \int_0^t u(t, \tau)e^{-\delta(t-\tau)}x(\tau)c(\tau) \, d\tau + u(t, 0)e^{-\delta t}X(0)c(0).$$

Thus, the social welfare is

$$\int_0^\infty e^{-rt}[\Phi(X(t)) - C(t) - x(t)q(t)] \, dt - X(0)q(0).$$

The break-even constraint requires the present discounted value of the stream of revenues resulting from investment and utilization paths to cover the present discounted value of all costs. This constraint is

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$^4$ This is akin to what Jorgenson (1963) calls the replacement ratio. It is the rate at which capacity of each vintage physically wears out. But as we shall see, in our framework, capacity is also replaced due to economic obsolescence.

$^5$ This formulation allows for a discrete initial capacity investment of $X(0)$ and a subsequent investment path $x(t)$. 

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Optimal investment and utilization paths maximize (3) subject to (4). This constrained maximization is assumed to be well defined, which will be the case if \( r \) is sufficiently large and \( F \) is not too large. If demand is inelastic over relevant ranges, then the constrained maximization problem is well defined for any \( r \). The case of inelastic demand is of course the case for which monopoly price regulation is most urgent, as an unregulated monopolist will always raise price when demand is inelastic.

The solution to the social planning problem can be derived by invoking the Kuhn-Tucker theorem to define a Lagrangian, by substituting the definition of output (1) and cost (2) into objective function (3) and maximizing point-wise with respect to \( \{X(0), x(t), u(t, \tau)\} \). Let \( \lambda \geq 0 \) denote the Lagrangian multiplier for the break-even constraint (4). The following theorem characterizes a declining price path corresponding to the optimal investment and capital retirement plan, and it characterizes the optimal economic life of capacity for each vintage, i.e., \( \Delta(t) \) such that \( u(t, \tau) = 1 \) for \( \tau \leq t \leq \Delta(t) \) and \( u(t, \tau) = 0 \) for \( t > \Delta(t) \). A declining price would certainly seem to be the normal case, in view of declining operating and capital equipment costs. However, the possibility of \( \Delta(t) \) declining complicates proving this. The subsequent corollary gives a simpler, complete characterization for the special case in which capacity and operation costs decline at the same rate, i.e., \( \theta = \mu \); this is due to the stationarity of \( \Delta(t) \). These results are proved in Appendix A.

**Theorem 1.** A declining optimal price path satisfies

\[
P(t) = c(t) + (r + \delta + \mu)q(t) + \theta c(t) \int_0^{\Delta(t)} e^{-(r+\delta)\tau} d\tau,
\]

where \( \lambda \geq 0 \) is a fixed constant and \( \Delta(t) \) satisfies

\[
c(t) - c(t)e^{-\Delta(t)} = (r + \delta + \mu)q(t)e^{-\Delta(t)} + \theta c(t)e^{-\Delta(t)} \int_0^{\Delta(t)} e^{-(r+\delta)\tau} d\tau.
\]

We interpret the theorem in steps. First, consider the base case in which there are no operating costs and no fixed costs, i.e., \( c(t) = 0 \) for all \( t \) and \( F = 0 \). In this case, (6) does not apply. The break-even constraint is slack, i.e., \( \lambda = 0 \), it is never efficient to retire capacity, i.e., \( \Delta(t) = \infty \), and the optimal pricing formula (5) becomes simply

\[
p(t) = (r + \delta + \mu)q(t).
\]

\[\text{In this case, social surplus and profit remain bounded as output increases, i.e., the integrals in (3) and (4) necessarily converge. Moreover, a policy satisfying (4) necessarily exists, as revenue can be made arbitrarily large by shrinking output.}\]

\[\text{This is a very transparent method for deriving necessary conditions for a solution. An alternative approach is to treat the problem as an optimal control problem and invoke the maximum principle or Bellman’s principle. See Malcomson (1975) and Appendix B.}\]

\[\text{It is easy to establish that prices are declining for the corollary, but it requires some technicalities that we avoid for the general case. Actually, equations (5) and (6) only require a slack nonnegativity constraint on investment, } x(t) > 0 \], which must be the case if \( p(t) \) is declining.\]

\[\text{See, for example, Nickell (1978) for a derivation of a variation of this formula in a two-factor neo-classical investment model.}\]
Along the efficient investment path, the retail price equals the user cost of capital, which equals a competitive return on investment plus economic depreciation. The rate of economic depreciation equals the rate of physical depreciation, \( \delta \), plus the rate of productivity improvement of new capacity, \( \mu \). Retail price falls at the rate of technological advance, which equals the rate of decline of capacity acquisition costs (\( \mu \)). It is easily seen that the efficient investment path can be implemented in a competitive equilibrium in which investments at each date are exactly recovered by the cash flow generated by the investments (see, for example, Jorgenson (1967)):

\[
\int_{t}^{\infty} e^{-(r+\delta)t-0} p(s) \, ds = q(t).
\]

From this it follows that the break-even constraint is slack in this case.

The next step is to consider the case in which both capital equipment and operating costs are falling but there are no fixed costs (\( F = 0 \)). The economic life of capacity is defined by equation (6), which is implied by the condition that vintage \( t \) capacity is optimally retired when price falls below its operating cost, i.e., \( p(t) \leq c(t) \). Setting \( \lambda = 0 \), the optimal pricing formula of equation (5) becomes

\[
p(t) = c(t) + (r + \delta + \mu)q(t) + \lambda c(t) \int_{0}^{\Delta(t)} e^{-(r+\delta)\tau} \, d\tau.
\]

Equation (8) differs from equation (7), when operating costs were not present, by the term \( \lambda c(t) \int_{0}^{\Delta(t)} e^{-(r+\delta)\tau} \, d\tau \). This term equals the present discounted value of the reductions in operating costs that would be obtained by delaying investment slightly. This is properly considered an opportunity cost of current investment and therefore is a component of the user cost of capital. This solution has a natural interpretation in terms of Bellman’s Principle of Optimality, which we present in Appendix A. This solution can also be implemented in a competitive equilibrium.

The optimal capacity replacement condition (6) defines a cost-minimizing replacement schedule. The left-hand side is the instantaneous cost savings from replacing vintage \( t \) at date \( t + \Delta(t) \). This benefit from optimal replacement is set equal to the user cost of capital on the right-hand side, which, as discussed just above, accounts for all aspects of economic depreciation. This difference equation determines an optimal replacement schedule: vintage \( t_0 \) is replaced at date \( t_1 = t_0 + \Delta(t_0) \) by an asset that in turn is replaced at date \( t_2 = t_1 + \Delta(t_1) \), and so on. This optimal replacement schedule in fact applies to any declining path of prices, not just the optimal path; indeed, we will use this optimal replacement condition as a benchmark when we study price-cap regulation.

Finally, consider the general case with positive fixed costs. The optimal price path (5) can be rewritten as

\[
p(t) = \left[ c(t) + (r + \delta + \mu)q(t) + \lambda c(t) \int_{0}^{\Delta(t)} e^{-(r+\delta)\tau} \, d\tau \right] / p(t) = \frac{\lambda}{(1 + \lambda)\epsilon}.
\]

From standard Ramsey principles, the optimal price-cost margin when the firm must

\[10\] This is essentially the problem solved by Malcomson (1975), who used optimal control techniques.
recover fixed costs is inversely related to the elasticity of demand.\textsuperscript{11,12} In calculating the Ramsey formula, the appropriate measure of long-run marginal cost is operating cost plus the user cost of capital, the latter including a term for forgone operating improvements as a component of economic depreciation. The optimal markup can be interpreted as a sales tax equal to $\psi = \lambda / (1 + \lambda \epsilon)$. The value of $\psi$ is determined implicitly by substituting (5) into the binding constraint (4).\textsuperscript{13} This outcome cannot be implemented competitively because of the scale economies arising from fixed costs.\textsuperscript{14}

The following corollary is a special case of Theorem 1, where the rate of technological progress is the same for capital and operating costs. It provides a particularly clean representation of our results, since the optimal life of an asset is the same for all vintages. It is straightforward in this case that the optimal price path is declining. By continuity, the optimal price path will be close to the one in the corollary, and hence declining, if $\mu$ is not too different from $\theta$.

\textbf{Corollary 1.} If $\theta = \mu$ and $c(t)/q(t) = \gamma > 0$, then the optimal price path solves

$$
\left[ 1 - \frac{\lambda}{(1 + \lambda \epsilon)} \right] p(t) = \left[ \gamma + r + \delta + \mu + \mu \gamma \int_{0}^{\Delta} e^{-(r+\delta)\tau} \, d\tau \right] q(t),
$$

where $\lambda \geq 0$ is a fixed constant and $\Delta$ and uniquely satisfies

$$
\gamma [e^{\mu \Delta} - 1] = (r + \delta + \mu) + \theta \gamma \int_{0}^{\Delta} e^{-(r+\delta)\tau} \, d\tau.
$$

This model of deterministic technological progress is highly tractable, but of course not realistic. Technological progress is uncertain and often quite discrete. Appendix B extends the basic model so that uncertain technological progress follows a discrete Poisson arrival process. In this case, the path of optimal prices reflects essentially the same opportunity costs as in the deterministic model, except that appropriate allowance must be made for the uncertain economic life of assets. In particular, the user cost of capital reflects the expected decline of capital equipment costs and the expected present discounted value of forgone operating improvements. To our knowledge, this extension to technological uncertainty is new to the neoclassical investment literature.

The assumption of stable demand is easily generalized to allow for growing demand. The statement of Theorem 1 does not rely on the structure of demand, only on the structure of costs. A growing demand, like increasing productivity, assures that nonnegativity constraints on new investments remain slack, which is what leads to the

\textsuperscript{11} We assumed for simplicity that the price elasticity is constant. If this were not the case, then the Ramsey markup would vary over time with the price elasticity of demand. In particular, it is economically efficient to recover a larger portion of the fixed costs when demand elasticity is relatively low; see Baumol and Bradford (1970).

\textsuperscript{12} Previous analyses of price dynamics for a regulated firm have focused on investment adjustment costs; see Mitchell and Vogelsang (1991).

\textsuperscript{13} The exact condition determining $\psi$ is that the present value of receipts from the sales tax covers the fixed cost. The condition determining $\psi$ has a unique solution, if $R(p)$ is increasing in $p$, which is true if $\epsilon \leq 1$. If $\epsilon > 1$, then a solution exists only if $F$ is not too large.

\textsuperscript{14} A two-part tariff generally is more efficient than the Ramsey pricing rule. In particular, if the demand for the first unit of consumption by all consumers is quite inelastic, then recovery of fixed cost is best done by the fixed portion of the tariff. For various economic, distributional, and political reasons, this may not be possible.

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cost-based pricing rule of the theorem. Uncertain demand is more problematic. If demand can shrink with some probability, which of course is not unrealistic given business cycles, then the characterization of optimal investment must take account of the option value of waiting to invest in the light of new demand information. Presumably, this requires an upward correction to the optimal price path. Appendix B draws on results from Dixit and Pyndyck (1994) to show for a special case how demand uncertainty is reflected in the optimal price path. This issue is potentially quite important, as is evident from large stranded costs in electricity markets that resulted from overly optimistic demand forecasts.

3. Rate-of-return regulation

Traditional rate-of-return regulation calculated a rate base in each period, and regulates the retail price to yield a fair return on the rate base. Operating costs and depreciation of the rate base are expensed. That is, price is set to recover operating costs, depreciation, and a return on the capital base. In this section we consider a stylized form of continuous rate-of-return regulation that establishes an allowed rate of return on investment, \( \bar{\eta} \), an allowed rate of depreciation, \( \bar{\delta} \), at each date, and a required time at which the asset must be removed from the rate base, \( \bar{\Delta} \). For simplicity, we ignore the fixed cost \( F \) in this section.

The rate base, \( K(t) \), equals the undepreciated value of all previous investments. The initial rate base is \( K(0) = g(0)\xi(0) \). The rate base subsequently decreases with depreciation and increases with new investment. Thus, the adjustment of the rate base is given by

\[
\dot{K}(t) = -\bar{\delta}K(t) - e^{-\bar{\delta}s}q(t-\bar{\Delta})x(t-\bar{\Delta}) + q(t)x(t).
\]

The first term in (11) represents the reduction in the rate base due to allowed depreciation. The second term represents the assets older than \( \bar{\Delta} \) periods that are retired from the rate base. The last term is the value of new investment. Thus, the value of the rate base at date \( t \) is

\[
K(t) = \int_{t-\bar{\Delta}}^{t} e^{-\bar{\delta}(t-s)}q(s)x(s) \, ds.
\]

The revenue requirement of a regulated monopolist at date \( t \geq \bar{\Delta} \) is

\[
R(X(t)) = C(t) + (\bar{\eta} + \bar{\delta})K(t),
\]

where \( R(X(t)) = X\phi(X(t)) \) is the revenue function. The revenue requirement covers operating costs, an allowed return on the rate base, and an allowed depreciation of the rate base.\(^{16}\)

Rate-of-return regulation can be interpreted as a series of loans made to consumers. The loan maturity is \( \bar{\Delta} \). The rate base is the outstanding loan balance. Each period,
consumers pay interest on the loan balance \((\bar{r}K(t))\) and repay part of the principal \(\delta K(t)\). These payments are in the form of allowed revenues above operating costs. New investments in capacity are financed by additional loans.

In theory, rate-of-return regulation could track optimal prices if the allowed rate of return is equal to the competitive rate \((\bar{r} = r)\), the allowed depreciation rate is equal to economic depreciation as characterized by Theorem 1 \((\delta(t) = \delta + \mu + \theta c(t) \int_0^t e^{-(r+\delta)t} d\tau)\), and capacity is replaced whenever it becomes economically obsolete as characterized by the theorem \((\Delta(t) = \Delta(t))\). It should be noted that, owing to the different rates of technological progress for capital and operating costs, depreciation rates and accounting lives must be vintage dependent to obtain efficiency. In practice, rate-of-return regulation generally has not taken proper account of technological progress in setting depreciation schedules (see Kahn, 1988).

To study the consequences of ignoring economic depreciation resulting from technological progress, we examine a model of naive rate-of-return regulation with \(\bar{r} = r, \quad \delta = \delta, \quad \Delta = \infty\). This form of rate-of-return regulation allows for full recovery of the investment costs if and only if the firm never retires the asset and therefore provides no incentive for the efficient retirement of old capacity. In this case

\[
R(X(t)) = C(t) + (r + \delta)K(t).
\]

(12)

The rate of change of the revenue requirement is

\[
R'(X(t))X(t) = \dot{C}(t) + (r + \delta)\dot{K}(t),
\]

(13)

where

\[
\dot{C}(t) = -\delta C(t) + c(t)[\dot{X}(t) + \delta X(t)] \quad \text{and} \quad (14)
\]

\[
\dot{K}(t) = -\delta K(t) + q(t)[\dot{X}(t) + \delta X(t)].
\]

(15)

The initial conditions for the system are

\[
\phi(X(0)) = c(0) + (r + \delta)q(0) \quad \text{and} \quad K(0) = q(0)X(0).
\]

(16)

Substituting (12), (14), and (15) into (13) and rearranging gives

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17 Rate-of-return regulation is not flexible enough to mimic the efficient price path when there is technological uncertainty, see Appendix B. One could obtain efficiency if the rate base were reduced at the actual rate of economic depreciation, while expected current economic depreciation were treated as an expense. Rate-of-return with two different depreciation rates could mimic the efficient price path, but this is not how it is practiced.

18 Schmalensee (1989) shows that a regulated firm prefers slow depreciation schedules if the allowed rate of return is above the competitive level. This preference is perhaps one reason why rate-of-return regulation may result in underdepreciation of capital assets. Another possible reason is that myopic regulators may be attracted to the temporarily lower prices that resulted from a low allowed depreciation rate.

19 Admittedly, this is a caricature of rate-of-return regulation. The economic obsolescence in setting the accounting lives of assets has been noticed (Phillips, 1993), although there is not much attention given to other consequences of technological progress for economic depreciation. In any case, accounting lives have remained quite long.
\[ [R' (X(t)) - c(t) - (r + \delta)q(t)] X(t) = -\delta X(t) [\phi(X(t)) - c(t) - (r + \delta)q(t)]. \] (17)

Equation (17) is a first-order differential equation in the quantity of output \( X(t) \). It follows from the initial condition (16) that \( R_q (X(0)) - c(0) - (r + \delta)q(0) < 0 \) and \( X(0) = 0 \). However, it is straightforward to show that \( \dot{X}(t) > 0 \) because \( q(t) < 0 \) and \( c(t) < 0 \). It follows from equation (17) that, after the initial date, \( X(t) \) evolves such that \( \phi(X(t)) - c(t) - (r + \delta)q(t) > 0 \) and \( R_q (X(t)) - c(t) - (r + \delta)q(t) < 0 \). Thus, \( \dot{X}(t) > 0 \) after the initial date, i.e., price declines under naive rate-of-return regulation. This result is not surprising. The initial investment in capacity is financed by a markup of price above marginal operating cost. When the capacity depreciates and is replaced, the replacement cost is lower because of technological progress, and the requisite markup is less. Moreover, for a given markup, price declines with marginal operating cost.

To characterize the price distortions that arise under naive rate-of-return regulation, it is useful to rewrite the differential equation in terms of \( Y(t) = \frac{p(t)}{q(t)} \). Since

\[
\frac{\dot{Y}(t)}{Y(t)} = -\frac{1}{\epsilon} \frac{\dot{X}(t)}{X(t)} + \mu,
\]

the substitution of (17) into (18) and simplification gives

\[
\frac{\dot{Y}(t)}{Y(t)} = \frac{\delta [Y(t) - \gamma(t) - r - \delta]}{(\epsilon - 1)Y(t) - \epsilon[\gamma(t) + \epsilon r + r + \delta]} + \mu, \quad (19)
\]

where \( \gamma(t) = c(t)/[q(t)] \). Letting \( \overline{Y} \) denote the limiting value of \( \gamma(t) \) as \( t \to \infty \), the path of \( Y(t) \), governed by (19), converges to

\[
\overline{Y} = \frac{\delta + \mu \epsilon}{\delta + \mu (\epsilon - 1)} (r + \delta + \overline{\gamma})
\]

if \( \delta + \mu (\epsilon - 1) > 0 \), or equivalently if

\[
\epsilon > \frac{\mu - \delta}{\mu}. \quad (20)
\]

If the demand elasticity is too low, then \( \dot{Y}(t) \) increases without bound, and the price-capital cost ratio goes to infinity. We assume (20) holds to avoid this unrealistic result.

We compare the limiting value of \( Y(t) \) under naive rate-of-return regulation to its corresponding optimal value. Using equations (5) and (6), the optimal price path converges to

\[
\overline{Y}^* = \overline{Y} + (r + \delta + \mu) + \theta \overline{\gamma} \int_0^{\overline{X}^*} e^{-(r+\delta)\tau} d\tau
\]

with

\[
\overline{\gamma} = \left[ 1 + \theta \int_0^{\overline{X}^*} e^{-(r+\delta)\tau} d\tau \right] \overline{X}^* e^{-\delta \overline{X}^*} + (r + \delta + \mu) e^{-\mu \overline{X}^*}.
\]

where \( \overline{X}^* \) is the limiting value of the efficient replacement schedule. Initially, we have \( Y < \overline{Y}^* \), \( \dot{Y} = 0 \), and \( \dot{\overline{Y}} > 0 \). In the limit,
\[
\bar{Y} - \bar{Y}^* = \left[ \frac{\mu}{\delta + \mu(\epsilon - 1)} \right] (r + \delta + \bar{Y}) - \mu - \theta \bar{Y} \int_{0}^{\infty} e^{-(r+\delta)\tau} d\tau. \tag{21}
\]

Equation (21) is positive if
\[
\frac{\mu(r + \delta + \bar{Y})}{\mu + \theta \bar{Y} \int_{0}^{\infty} e^{-(r+\delta)\tau} d\tau} > \delta + \mu(\epsilon - 1). \tag{22}
\]

If \( \theta > \mu \), then \( \bar{Y} \to 0 \), \( \bar{Y}^* \to \infty \),
\[
\bar{Y} = \frac{\delta \mu e}{\delta + \mu(\epsilon - 1)} (r + \delta), \quad \text{and} \quad \bar{Y}^* = (r + \delta + \mu).
\]

Thus, \( \bar{Y} > \bar{Y}^* \) if and only if \( (r + \mu)/\mu > \epsilon > (\mu - \delta)/\mu \).

If \( \theta = \mu \), then (22) becomes
\[
\frac{(r + \delta + \bar{Y})}{1 + \gamma \int_{0}^{\infty} e^{-(r+\delta)\tau} d\tau} > \delta + \mu(\epsilon - 1). \tag{23}
\]

Note that the right-hand-side is increasing in \( \epsilon \). Letting \( \epsilon = (r + \mu)/\mu \), (23) is equivalent to
\[
\gamma > \gamma [1 - e^{-(r+\delta)\bar{Y}^*}],
\]
which holds for any \( \bar{Y}^* > 0 \). Therefore, \( \bar{Y} > \bar{Y}^* \) if \( (r + \mu)/\mu > \epsilon > (\mu - \delta)/\mu \).

If \( \theta < \mu \), then \( \bar{Y} \to \infty \), \( \bar{Y}^* \to 0 \),
\[
\bar{Y} = \frac{\delta \mu e}{\delta + \mu(\epsilon - 1)} \gamma, \quad \text{and} \quad \bar{Y}^* = \gamma.
\]

Thus, \( \bar{Y} > \bar{Y}^* \) if and only if \( \epsilon > (\mu - \delta)/\mu \).

Summarizing, a sufficient condition for \( \bar{Y} > \bar{Y}^* \) is \( (r + \mu)/\mu > \epsilon > (\mu - \delta)/\mu \).

In particular, this condition holds if we assume inelastic demand and require that the price-capital cost ratio does not go to infinity. In this case, \( Y \) starts out too low and eventually becomes too high. This implies that investment is initially too high and is eventually too low relative to the first best. One way to interpret this result is that early vintages of capacity crowd out later vintages. The reason for this bias is that the firm collects “too much” on old capital because it is not retired efficiently, and the resulting higher prices in the future reduce the use of newer capital. We summarize our results in the following theorem:

**Theorem 2.** If \( (\mu - \delta)/\mu < \epsilon < (\mu + r)/\mu \), then under naive rate-of-return regulation
\[
p(0) < c(0) + (r + \delta + \mu)q(0)
\]
and as \( t \to \infty \), \( \theta \to c(t) + (r + \delta + \mu)q(t) + \theta c(t) \int_{0}^{\Delta(t)} e^{-(r+\delta)\tau} d\tau \), where \( \Delta(t) \) satisfies...
Thus, investment in capacity is at first excessive, but eventually becomes and stays deficient.

4. Price-cap regulation

Rate-of-return regulation has gradually given way to price-cap regulation as the preferred method of regulating public utility prices. Under naive rate-of-return regulation, the firm has an incentive to keep old capital in its rate base indefinitely. Given excessively long depreciation schedules, this is potentially a huge inefficiency of the sort that price-cap regulation aims to correct.

A price cap is a ceiling below which the regulated firm has price flexibility. Thus, price-cap regulation can be defined by a price path \( \bar{p}(t) \) and a constraint \( p(t) \leq \bar{p}(t) \). Since the price cap is generally below the monopoly price, the price cap will typically be binding for a regulated monopolist, necessarily so if demand is inelastic. In our deterministic model, there is no loss of generality in assuming the price cap is binding.

Ignoring fixed costs (i.e., assuming \( F = 0 \)), it is immediate from Theorem 1 that the optimal binding price cap is equal to long run marginal cost, given by equation (8), i.e.,

\[
\bar{p}(t) = c(t) + (r + \delta + \mu)q(t) + \theta c(t) \int_0^{\Delta(t)} e^{-(r+\delta)t} d\tau
\]

for all \( t \), with \( \Delta(t) \) satisfying equation (6) in Theorem 1. Given a regulatory commitment to this price path, the regulated firm maximizes profits by making cost-minimizing capacity replacement decisions, i.e., the firm will replace capacity after \( \Delta(t) \) periods. Thus, by construction, the price path is efficient and induces optimal investments in new capacity. However, the optimal price cap does not translate into the kind of simple rules used in practice.

Problems potentially arise in a transition from rate-of-return regulation to price-cap regulation. Suppose that the transition occurs at a date \( t^* \) when price is above the optimal \( \bar{p}(t) \) and the rate base exceeds the value of capacity due to underdepreciation; that is,

\[
p(t^*) = \phi(X(t^*)) > (r + \delta + \mu)q(t^*) + c(t^*) \left( 1 + \theta \int_0^{\Delta(t^*)} e^{-(r+\delta)t} d\tau \right)
\]

and

\[K(t^*) > q(t^*)X(t^*).\]

In these circumstances, a move to optimal price-cap regulation would lead to stranded costs equal to \( K(t^*) - q(t^*)X(t^*) \). That is, under the optimal price path the firm would earn a competitive return only on a rate base value of \( q(t^*)X(t^*) \), and the balance of its rate base would remain uncompensated.

A naive approach to price-cap regulation is to set the initial price cap at the prevailing level, and to require it to decline at a fixed rate corresponding to technological progress. Such a naive approach to price-cap regulation also potentially creates a stranded-costs problem. The problem is illustrated by a transition from naive rate-of-return regulation for the special case in which operating costs decline at the same rate of capital acquisition costs (i.e., \( \theta = \mu \)). In this case, the naive price-cap regime sets \( p(t^*) = c(t^*) + (r + \delta) [K(t^*)/X(t^*)] \) and has price declining at rate \( \mu \). If naive rate-of-return regulation would have continued, then the ratio of the retail price to the price...
of new capacity would have risen continuously over time, approaching an upper bound that exceeded the efficient price level. A move to naive price-cap regulation essentially freezes the ratio of the retail price to the price of new capacity, thereby forcing the retail price below the level it would have achieved under continued rate-of-return regulation. Consequently, in the absence of other benefits, the firm is deprived of the return on capital that it anticipated from its previous investment.

An important potential benefit to the firm under price-cap regulation is the returns to cost reduction from the replacement of obsolete capacity. We show below that the value of this investment depends on the duration of the price-cap period and the age distribution of the firm’s capital stock. To the extent that this benefit is insufficient to compensate the firm for past investments, the need to recover remaining stranded costs creates a second-best pricing problem. The optimal approach is to treat the stranded cost as a fixed cost and to apply Theorem 1 to the calculation of the price-cap path. Thus, stranded costs should be recovered by a Ramsey markup on retail prices or, equivalently, by an appropriate revenue tax.\(^{20}\) Obviously, any unrecovered fixed costs should be treated similarly.\(^{21,22}\)

Our discussion of price caps so far has presumed a permanent commitment to them. This is necessary if price-cap regulation is to provide optimal incentives for cost-minimizing decisions on capacity replacement. Typically, though, price-cap regulation has been implemented in a more limited way over a finite planning horizon. At the end of the planning horizon, there could be a reversion to rate-of-return regulation, reinitialization to a new price-cap regime, or adoption of a new regulatory regime. In the remainder of this section, we discuss the consequences of a fixed horizon for the regulated firm’s capacity replacement decisions when there is a reversion to rate-of-return regulation or, equivalently for our purposes, a new price-cap regime that holds the firm indifferent between the new regime and renewed rate-of-return regulation. Obviously, this is a restrictive assumption that may not always hold in the “real world,” where the price-cap horizon is not necessarily fixed in advance. However, we expect our main qualitative results to apply more generally to models in which the firm’s expected profit at the end of the price-cap period depends positively on its end-of-period rate base. The fact that the rate base is typically kept track of during the price-cap period suggests that it may be realistic to assume that the firm’s capital base at the beginning of a new price cap affects the parameters of the new price cap.\(^{23}\)

We consider the following stylized characterization of price-cap regulation. The price-cap regime begins at date 0 and ends in future period \(T\); so if the current period is \(t\), then the length of the remaining price-cap horizon is \((T - t)\). At the end of the price-cap period there is a reversion to rate-of-return regulation. This means that the rate base is kept track of during the price-cap period, and the firm is entitled to a competitive rate of return on the rate base at date \(T\). We first analyze the “terminal replacement problem,” i.e., the case in which the firm is deciding whether to replace

\(^{20}\) Even with this correction for stranded-costs recovery, price-cap regulation is flawed in an environment with technological uncertainty, as in Appendix B.

\(^{21}\) As currently practiced, a price cap is reduced at a fixed rate of productivity growth. However, to track efficient prices, a properly initialized price cap must fall at the rate of actual productivity growth. This could be accomplished by indexing the price cap for a firm to the industry rate of productivity growth. If a firm is small relative to the industry, this preserves the incentive properties of price-cap regulation.

\(^{22}\) The problem of adjusting price caps to uncertain costs has been studied in a different context by Armstrong and Vickers (1995).

\(^{23}\) Many price caps have a termination period where there is a planned review of the price cap. In the United Kingdom the price cap is reinitialized based on the average unit costs and the capital/output ratios since the last rate hearing; see Pint (1992).
a capacity vintage when the replacement vintage will be kept until the end of the price-cap period. This focus enables us to demonstrate that there is no capacity replacement near the end of the price-cap period, and that some capacity replacement is deferred inefficiently. We also characterize the optimal timing of terminal replacements, and we show that many vintages are replaced too soon relative to the optimal path. Finally, we take up the case where new vintages may be adopted and replaced within the same price-cap period, and we show that there is a cumulative effect to premature replacement.

We first show that some replacements are deferred inefficiently. Consider the firm’s decision whether or not to scrap a unit of older capacity of vintage $s$. It is profitable to scrap this asset and replace it with state-of-the-art capacity that will be kept until at least the end of the price-cap period if

$$1 - e^{-(r + \delta)(T - t)} \frac{[c(s) - c(t)] - q(t)}{r + \delta} \geq e^{-(r + \delta)(T - t)}[q(s) - q(t)].$$

(24)

The first term on the left-hand side is the operating cost savings over the price-cap period from replacing the unit of capacity, while the second term is the acquisition cost of the replacement capacity. The right-hand term is the reduction in the value of the rate base at the end of the price-cap horizon. The net benefit must exceed the direct cost of the replacement. It is immediate from (24) that the horizon effect may cause inefficient delay because an optimal replacement schedule would set its left-hand side equal to zero. The reason for the delay is the incentive to increase the value of the price-cap rate base at the end of the price-cap horizon. In the case in which the price-cap period is reinitialized at the end of the period, we would expect to see a burst of cost-reducing capacity replacement at the start of the new cycle.

We next show that no capacity is replaced if the price-cap horizon is too short and that, in any case, there is no scrapping near the end of the price-cap period. Taking account of the deterministic decline in operating and capital costs, condition (24) is equivalent to

$$\frac{(1 - e^{-(r + \delta)(T - t)})y(0)e^{-\theta(t - s)}}{r + \delta} \left[1 - e^{-\theta(t - s)}\right] - e^{-(r + \delta)(T - t)}[1 - e^{-\theta(t - s)}] - e^{-\mu(t - s)} \geq 0. \quad (25)$$

Inequality (25) cannot be satisfied if $(T - t)$ is small, but for sufficiently large $(T - t)$, the equality defines a critical strictly positive $(t - s)$, which is the age of capacity such that replacement is profitable for all older vintages. Similarly, given $T$, there is a critical $(T - s)$, such that no more scrapping occurs within the price-cap period. It is noteworthy that the incentive to scrap capacity does not depend on the price path itself, only on the price-cap horizon.\footnote{This depends on the assumption of a fixed price-cap horizon. The price-cap path could influence investment incentives if the horizon is endogenous. For example, Cabral and Riordan (1989) show that, if the price cap is not sufficiently remunerative, then the firm may forgo investing and request a rate hearing.} If the price-cap horizon is not long enough, i.e., if $T$ is small, then it will not be profitable to replace any capacity at all. This can be seen by inspection of inequality (25). Thus, a price-cap horizon that is too short will not have the intended effect of stimulating cost reduction through replacement of obsolete capacity.

We next examine the optimal timing of a terminal replacement, and show that a price cap can hasten capacity replacement inefficiently. Suppose that the firm has a
current technology \( t_1 \) and is contemplating changing it with a technology \( t_2 \). The firm will keep \( t_2 \) in its rate base until date \( T \), when the price cap is reinitialized. Obviously, in light of the preceding discussion, this makes sense only if \( T - t_2 \) is not too large. Otherwise, it would be profitable to replace \( t_2 \) before the end of the price-cap horizon. Thus we are implicitly restricting attention to a terminal replacement within the price-cap period. With this caveat in mind, we consider the change in present discounted profit (looking forward from \( t = 0 \)) between this replacement strategy and the strategy of not scrapping technology \( t_1 \) at all:

\[
\frac{e^{-(r+\delta)t_2} - e^{-(r+\delta)T}[c(t_1) - c(t_2)]}{r + \delta} - e^{-(r+\delta)t_2}q(t_2) - e^{-(r+\delta)T}[q(t_1) - q(t_2)].
\]

The first-order condition for the optimal \( t_2 \) is

\[
c(t_1) - c(t_2) = (r + \delta + \mu)q(t_2) + \theta c(t_2) \int_0^{T-t_2} e^{-(r+\delta)\tau} d\tau - e^{-(r+\delta)(T-t_2)}\mu q(t_2). \tag{26}
\]

This condition is similar to the optimal replacement condition of Theorem 1 (equation (6)). This can be seen by replacing \( t_1 \) with \( t \) and \( T - t_2 \) with \( \Delta(t_2) \) in equation (26).\(^{25}\)

With these substitutions, the only difference is the additional term \( -\mu e^{-(r+\delta)\Delta(t_2)} q(t_2) \), which accounts for the effect of the capacity replacement on the value of the rate base at the end of the price-cap period. By itself, the effect of the extra term is to hasten the optimal replacement date for vintage \( t_2 \) relative to the optimal path, i.e., \( t_2 - t_1 < \Delta(t_1) \).

Thus, as long as \( T - t_2 \) does not exceed \( \Delta(t_2) \) substantially, the terminal rate base effect causes an inefficiently early replacement of \( t_2 \); i.e., there is a large range of vintages that are replaced too soon relative to the optimal path.\(^{26}\) In this sense, price-cap regulation provides an excessive incentive for cost reduction.

Finally, we examine a case where it is possible that a vintage put into place under the current price-cap regime may also be replaced in that same regime, showing that there is a cumulative effect of hastened capacity replacement. Suppose that the firm has a current technology \( t_1 \) and is contemplating changing it with a technology \( t_2 \), which subsequently will be terminally replaced at date \( t_3 \). The change in profit between this replacement strategy and the strategy of not scrapping technology \( t_1 \) at all is

\[
\left[ \frac{e^{-(r+\delta)t_2} - e^{-(r+\delta)t_3} [c(t_1) - c(t_2)]}{r + \delta} - e^{-(r+\delta)t_3}q(t_2) \right] + \frac{e^{-(r+\delta)t_3} - e^{-(r+\delta)T}[c(t_1) - c(t_3)]}{r + \delta} - e^{-(r+\delta)t_3}q(t_3) - e^{-(r+\delta)T}[q(t_1) - q(t_3)]. \tag{27}
\]

The first bracketed term in expression (27) is the change in the firm’s profit from scrapping \( t_1 \) at date \( t_2 \) and holding \( t_2 \) until date \( t_3 \). The second bracketed term is the

\(^{25}\) Note that this equation can be used to “work backward.” That is, given the price-cap length \( T \), equation (26) expresses the vintage to be replaced \( t_2 \) implicitly as a function of the replacement date \( t_2 \).

\(^{26}\) By varying \( T - t_2 \), we derive a range of vintages that are replaced too soon relative to the optimal path. If it is large, then equation (26) implies that corresponding vintages are replaced too late relative to the optimal path. It is unclear whether such a large \( T - t_2 \) is consistent with the condition for terminal replacement that places an upper bound on \( T - t_2 \); unfortunately, we have not found a convenient way to analyze this.

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change in profits from replacing $t_2$ at date $t_3$ and holding technology $t_3$ until date $T$. The last term is the change in the rate base at date $T$. Maximizing (27), the firm’s first-order conditions with respect to $t_2$ and $t_3$ respectively imply

$$c(t_1) - c(t_2) = (r + \delta + \mu)q(t_2) + \theta c(t_2) \int_0^{t_3-t_2} e^{-(r+\delta)\tau} d\tau$$

and

$$c(t_2) - c(t_3) = (r + \delta + \mu)q(t_3) + \theta c(t_3) \int_0^{T-t_3} e^{-(r+\delta)\tau} d\tau - e^{-(r+\delta)(T-t_3)} \mu q(t_3).$$

Equation (29) is similar to the terminal scrapping formula discussed above. Clearly, from (28) vintage $t_1$ is replaced too quickly if $t_3 - t_2 < \Delta(t_2)$. Thus, if vintage $t_2$ is replaced “too soon” relative to the optimal path, then so is vintage $t_1$. This is the sense in which hastened capacity replacement is cumulative. The replacement date for $t_1$ is efficient given $t_3$, but $t_3$ is too early. Thus, there is a range of $t_1$ such that both $t_1$ and its replacement $t_2$ are scrapped too quickly.

We summarize our key results in the following theorem:

**Theorem 3.** If the length of a price-cap period is $T$, prices are nonincreasing, and the firm is guaranteed a competitive return on its rate base at the end of the price-cap period, then (i) there exists a $t_\tilde{1}$ such that if $T$ is less than $t_\tilde{1}$, then the firm has no incentive to retire any capacity; and (ii) for larger values of $T$, many vintages are replaced too soon relative to the optimal path, while the replacement of some later vintages is deferred inefficiently.

### 5. Discussion

In the United States, rate-of-return regulation has prevailed as a method of regulating telecommunications for much of the century. Beginning in the 1980s, price-cap regulation gradually became ascendant. Now, particularly since the passage of the 1996 Telecommunications Act, there are emerging new forms of wholesale price regulation based on long-run marginal cost concepts.\(^{27}\)

There seems to be a consensus that assets were underdepreciated under rate-of-return regulation. The conventional wisdom is that regulators felt short-term political pressures to keep rates low.\(^{28}\) Our models suggest that, as a consequence, with ongoing technological progress, historical costs, as reflected in the rate base, eventually became high relative to long-run marginal cost. This appears to be consistent with telecommunications interconnection proceedings in many states. Thus, perhaps paradoxically, a political motivation to keep prices low may ultimately have resulted in excessively high prices.

As we observed in Section 3, rate-of-return regulation provides poor incentives for plant and equipment replacement. Price-cap regulation was motivated largely by its incentive properties for cost reduction. When price is exogenous, the firm has better

\(^{27}\) New wholesale price regulation in telecommunications markets typically is based on the concept of forward-looking economic cost (FLEC). Roughly, the FLEC of a network element is the allocated cost of building a modern network at current prices, converted to a rental rate. It is reasonable to interpret FLEC methodologies for pricing network elements as attempts to approximate long-run marginal cost. See Salinger (1998).

\(^{28}\) A complementary explanation is that regulated firms had an incentive for slow depreciation schedules in order to earn above-normal rates of return on capital, as discussed by Schmalensee (1989). We thank Paul Joskow for pointing out this connection.
incentives to minimize cost by making timely capital replacement decisions. In the
wake of price-cap regulation, there does appear to have been substantial new investment
in capital equipment, as telecommunications firms adopted fiber optic and digital tech-

do
tologies (see Greenstein, McMaster, and Spiller, 1995). Our results about price-cap
regulation (Theorem 3) suggest that new investment would be “front loaded” at the
start of the price-cap period. This remains an open empirical question.

Price-cap regulation might have been an opportunity to establish efficient prices
based on long-run marginal cost. However, price-cap and related incentive regimes gen-
erally were adopted voluntarily and, therefore, were structured to recover historical costs.
In some cases, price caps were set to decline faster than the estimated rate of productivity
improvement or were initialized below prevailing prices. In these cases, it is likely that
prices turned out to be lower than they would have been under rate-of-return regulation,
but arguably the regulated firms were adequately compensated by the returns from the
cost efficiencies that resulted from price-cap incentives.

The Telecommunications Act placed important new obligations on incumbent local
exchange carriers (ILECs). In particular, ILECs are required to provide unbundled net-
work elements at cost-based rates. In a controversial regulation, the FCC established a
particular pricing methodology (called TELRIC, for total element long-run incremental
cost) and required the ILECs to provide the entire platform of network elements at
TELRIC prices—essentially enabling the resale of existing services. Subsequently, the
U.S. 8th Circuit Court determined that the Telecommunication Act did not require the
wholesale provision of the entire platform of network elements, and that the FCC lacked
jurisdiction over the pricing of unbundled elements. The Supreme Court vacated the
8th Circuit decision and restored the FCC’s authority to establish a pricing methodol-
ogy, but it qualified that ILECs were obligated to provide access only to those network
elements that were “necessary” for competition and whose withholding would “im-
pair” competition. More recently, the 8th Circuit found that the TELRIC methodology
violated the act, an issue that is likely to return to the Supreme Court.

One possible interpretation of the FCC’s wholesale pricing rule is that it was an
attempt to restore the efficient pricing of retail services. In our model, this appears to
create a stranded-costs problem. In reality, this is not so clear. In addition to placing
unbundling requirements on the ILECs, the Telecommunications Act removed certain
line-of-business restrictions on the Bell Operating Companies (BOCs). In particular,
the BOCs are able to enter long distance markets once they have opened their local
markets to competition to the satisfaction of the FCC. It is possible that the expected
profits from these new lines of business adequately compensate the ILECs for any
stranded costs. To the extent that there are remaining stranded costs in need of recovery
in the transition to a new regulatory regime, our analysis indicates that they should be
recovered by sales taxes based on standard Ramsey principles. The Supreme Court is
also likely to address the stranded-costs issue in connection with TELRIC pricing.

We conclude with some topics for future research. First, we have assumed through-
out the article that technological progress is exogenous to the regulatory process. While
this assumption can be justified by assuming that the regulated firm under considera-
tion is not a substantial part of the capital equipment market, it is clear that the form of
regulation affects the demand for new capital goods, which in turn affects upstream
incentives for innovation. A next step would be to examine an equilibrium model with

29 An interesting discussion of the implementation of the 1996 Telecommunications Act is in Harris
and Kraft (1997).

30 As of this writing, Bell Atlantic has gained FCC authority to provide in-region long distance service
in New York, and SBC has gained authority in Texas.

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endogenous technological progress. Second, we have assumed that capital is a continuous variable. In many instances, the choice variable for investment is “approximately” continuous. For example, telecommunications firms can adjust the capacity of fiber optic cable with the degree of electronics applied to the cable. In other instances, capacity investments are lumpy. For example, setting the capacity for utility poles or adding a new electricity generator are lumpy decisions. The optimal regulation of firms when capital is discrete is also an interesting and important topic for further research. Finally, an important direction for future research is to consider additional consequences of technological and demand uncertainty.

Appendix A

Proof of Theorem 1 and Corollary 1. The interior first-order conditions of the optimal investment problem with respect to X(0) and x(τ) have an identical structure:

\[
\int_r^{τ_1} e^{-r-\tau} u(t, τ) \left( 1 + \frac{λ}{e} p(t) - (1 + λ)c(t) \right) dt - e^{-r}(1 + λ)q(τ) = 0
\]

for all τ ≥ 0. Moreover, from the first-order conditions for u(τ, τ), it easy to show that an optimal utilization function takes on two values: either 0 or 1. In particular,

\[
u(t, τ) = \begin{cases} 
1 & \text{if } \left(1 + \frac{λ}{e} p(t) - (1 + λ)c(t)\right) ≥ 0 \\
0 & \text{otherwise.}
\end{cases}
\]

Assuming that p(t) is decreasing, there exists a scrapping date for capital of vintage τ such that u(t, τ) = 1 for τ ≤ t ≤ τ + Δ(t) and u(τ, t) = 0 for τ > τ + Δ(t). We can rewrite the first-order conditions for investments as

\[
\int_r^{τ_1} e^{-r-\tau} u(t, τ) \left( 1 + \frac{λ}{e} p(t) - (1 + λ)c(t) \right) dt - e^{-r}(1 + λ)q(t) = 0.
\]

Interchanging t and τ, dividing by e^{-r}, and differentiating this with respect to t gives

\[-\left(1 + \frac{λ}{e} p(t) - (1 + λ)c(t)\right) + (1 + λ)(r + δ)q(t) - (1 + λ)c(t) \int_r^{τ_1} e^{-r}(1 + λ)q(t) = 0.\]

and rearranging and substituting for μ and θ gives

\[
1 - \frac{λ}{(1 + λ)e} p(t) = c(t) + (r + δ + μ)q(t) + θc(t) \int_r^{τ_1} e^{-r}(1 + λ)q(t) = 0, \tag{A1}
\]

with Δ(t) satisfying

\[
1 - \frac{λ}{(1 + λ)e} p(t + Δ(t)) = c(t). \tag{A2}
\]

Equations (A1) and (A2) are solved simultaneously to determine p(t) and Δ(t).

Since q is declining at rate μ and e is declining at rate θ, we have, from equations (A1) and (A2),

\[
γ(t) = \left[1 + \frac{θ}{e} \int_r^{τ_1} e^{-r}(1 + λ)q(t) dt\right] γ(t)e^{-μτ} + (r + δ + μ)γ(t)e^{-θτ},
\]

where γ(τ) = [p(t)]/q(t), which gives condition (6).

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Therefore, the relevant objective function for the social planner is expected social welfare, with expectations.

The value function interpretation. We give a value function interpretation of Theorem 1 for the special case $F = 0$. Given the economic life of assets $\Delta(t)$, we define the "state space" to be the stock of each vintage of unretired capacity, i.e., the state is \( \{ y(t, \tau); t - \Delta(t) \leq \tau \leq t \} \), with

\[
y(t, \tau) = e^{-\lambda \tau} x(\tau) \quad \text{and} \quad X(t) = \int_{t-\Delta(t)}^t y(t, s) \, ds.
\]

The value function

\[
V = \int_{-\infty}^\infty e^{-\gamma t} [\Phi(X(t)) - p(t)X(t)] \, dt + \int_{-\infty}^t \int_{s-\Delta(t)}^{s+t} e^{-\gamma (s+t) - \tau} [p(\tau) - c(s)] y(s, t) \, d\tau
\]

satisfies the Bellman equation,\(^{31}\)

\[
rV = \max_i \left\{ \Phi(X_i) - cX_i - q_i x + \int_{s-\Delta(t)}^{s+t} \frac{\partial V}{\partial y_i} y_i \, ds + \frac{\partial V}{\partial t} \right\},
\]

if \( p(\tau) \) satisfies price path (8). The Bellman equation (A3) says that the rate of return on the value function \( rV \) is equal to the flow of social surplus plus the rate of change of value resulting from depreciation of the assets \( \int_{s-\Delta(t)}^{s+t} (\partial V / \partial y_i) y_i \, ds \) and exogenous technological progress \( \partial V / \partial t \). This value function itself has two components. The first component is the present discounted value of consumer surplus. The second component is the value of the state of assets defining the state, i.e., the value of each unit of capacity is the present discounted value of the cash flow it generates over its remaining economic life.

Appendix B

Technological uncertainty. We examine the model when technological progress is uncertain and technological improvements arrive according to a Poisson distribution. For simplicity we maintain the assumptions that fixed costs are zero and operating costs and capital costs decline at the same rate. Ramsey principles would apply for the recovery of fixed costs analogously to the deterministic model. That is, the percentage price markup above the long-run marginal cost is inversely proportional to the elasticity of demand. In the model below with no fixed cost, price is simply equal to long-run marginal cost, for which we derive the formula.

Consider an infinite sequence of dates \( \{ t_i \} \) determined by a Poisson arrival process with a constant hazard rate \( \lambda > 0 \). The initial date is \( t_0 = 0 \). Corresponding to these dates is a decreasing sequence of operating costs and capital equipment costs, \( (c_i, q_i) \), satisfying

\[
(c_i, q_i) = ((1 + \alpha c_{i-1}, (1 + \alpha) q_{i-1})
\]

for some constant \( \alpha > 0 \). Since both cost elements are declining at the same rate, we have \( c_i = \gamma q_i \) for a constant \( \gamma \geq 0 \). We assume that the cost structure for new capacity acquired at a date \( t_i \) is \( (c(t_i, q(t_i)) = (c_i, q_i) \). Thus, the expected rate of productivity growth in this model is

\[
\mu = \lambda \frac{\alpha}{1 + \alpha}.
\]

We refer to each \( t_i \) as an innovation date, and to the time between innovation dates as an innovation period.

The social welfare has the same structure as in the deterministic model. It is given by (3), with \( X(t) \) and \( C(t) \) defined by (1) and (2) respectively. The difference here is that \( c(t) \) and \( q(t) \) decline stochastically. Therefore, the relevant objective function for the social planner is expected social welfare, with expectations.

\(^{31}\) It is also straightforward to use Bellman’s principle to similarly characterize utilization rates, and hence the economic life of assets.
taken over the sequence of innovation dates. Moreover, optimal investment and utilization paths are conditioned on the evolving history of innovations.

**Theorem B1.** When technological progress is uncertain (having a Poisson distribution), the optimal investment path implies that \( p(t) = p_i \) for \( t \in [t_i, t_{i+1}) \), with

\[
p_{t_i} - c_i = (r + \delta + \mu)q_i + \mu c (\frac{1}{r + \delta + \lambda}) \sum_{m=0}^{n-1} \left( \frac{\lambda}{r + \delta + \lambda} \right)^m + \left( \frac{\lambda}{r + \delta + \lambda} \right)^n [p_{t_{i+1}} - c_{i+1}] \quad \text{and} \quad p_{t_{i+1}} \geq c_i \geq p_{t_{i+1}}
\]  

(B3)

for some integer \( n \). The optimal utilization path satisfies \( u(t, \tau) = 1 \) if \( p(t) - c(\tau) \geq 0 \) and \( u(t, \tau) = 0 \) otherwise.

**Proof.** Choosing an investment and capacity retirement path that maximizes expected social welfare implies a sequence of prices \( \{p_i\} \) such that \( p(t) = p_i \) for \( t \in [t_i, t_{i+1}) \), and

\[
\frac{\sum_{m=0}^{\infty} E[e^{-r\delta}e^{-\lambda}]E[1 - e^{-(r+\delta)(t_{i+1}-t_{i})}]}{r + \delta} [p_{t_{i+1}} - c_i] = q_i.
\]  

(B5)

where expectations are over the sequence of innovation dates. The first-order conditions show that price is constant within an innovation period, due to the stationarity of the Poisson process. The first-order condition for optimal utilization at each date implies that the capacity replacement cycle is determined by (B4). Capacity is retired after \( n \) innovation periods. Equation (B5) is a competitive break-even condition stating that expected discounted cash flow over the life of an asset exactly covers its acquisition cost.

Substituting

\[
E[e^{-r\delta}e^{-\lambda}] = \left( \frac{\lambda}{r + \delta + \lambda} \right)^n
\]

and

\[
E[1 - e^{-(r+\delta)(t_{i+1}-t_{i})}] = \frac{1}{r + \delta + \lambda}
\]  

(B6)

into (B5) gives

\[
\frac{1}{r + \delta + \lambda} \sum_{m=0}^{\infty} \left( \frac{\lambda}{r + \delta + \lambda} \right)^m [p_{t_{i+1}} - c_i] = q_i.
\]  

(B7)

Subtracting

\[
\sum_{m=0}^{\infty} \left( \frac{\lambda}{r + \delta + \lambda} \right)^m [p_{t_{i+1}} - c_{i+1}] = \lambda q_{i+1}
\]

from each side of (B7) and rearranging gives

\[
p_{t_i} - c_i = (r + \delta)q_i + \lambda (q_i - q_{i+1}) + (c_i - c_{i+1}) \sum_{m=0}^{\infty} \left( \frac{\lambda}{r + \delta + \lambda} \right)^m + \left( \frac{\lambda}{r + \delta + \lambda} \right)^n [p_{t_{i+1}} - c_{i+1}].
\]  

(B8)

Using (B1) and (B2) gives (B3). \( \text{Q.E.D.} \)

The theorem has a straightforward interpretation which parallels closely that for the deterministic model (Theorem 1). As in the deterministic model, capacity is scrapped when price drops below its operating cost. In the Poisson model, each vintage of capacity is scrapped after \( n \) innovation periods. Price is constant between innovations and drops discretely with the arrival of each innovation. The term \( (r + \delta + \mu)q_i \) in the pricing formula of the theorem is the counterpart of the user cost of capital for the basic deterministic model with neither operating costs nor fixed costs (\( y = 0 \) and \( F = 0 \)), except that here \( \mu \) must be interpreted as the expected rate of productivity growth.

The term

\( \mu \) represents

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the present value of forgone operating cost evaluated at the expected life of assets. It equals the expected present value of operating improvements resulting from delaying the marginal investment, and it reflects the fact that an operating improvement is an uncertain prospect that occurs only if delay results in a discrete improvement. The likelihood of such improvements is \( \lambda \), and the consequence is to reduce operating costs of new capacity by an amount \( \alpha(1 + \alpha) c_i \); therefore, the expected reduction of operating costs over an interval of size \( dt \) is \( \mu c_i \). The term \( \mu c_i \int_0^{\infty} e^{-\alpha t} dt \) recognizes that the value of the operating cost improvement accrues over a random period of length \( i_{n+1} \), which is the time it takes for the remaining \( (n - 1) \) innovation periods of the asset’s life.

The last term,

\[
\left( \frac{\lambda}{r + \delta + \lambda} \right)^n (p_{n+1} - c_{n+1}),
\]

reflected the fact that, in the event of a technological advance, a new vintage of capacity will earn a return or an additional innovation period compared to the economic life of the previous vintage. This is something of a nuisance term. In view of (B4) this term is bounded above by \( \alpha(1 + \alpha) [\lambda(r + \delta + \lambda)]^n c_i \). Clearly, this bound goes to zero as \( \alpha \) goes to zero. Holding \( \mu \) constant, a small value of \( \alpha \) corresponds to frequent small innovations. Indeed, for a given \( \mu \), the model converges to the deterministic case in the limit as \( \alpha \) goes to zero. However, even if \( \alpha \) is not small, a numerical analysis shows that \( \alpha(1 + \alpha) [\lambda(r + \delta + \lambda)]^n \) is approximately zero for reasonable parameter values. This is because, for a given rate of expected technological progress \( \mu \), the arrival rate of new technology \( \lambda \) becomes small as the size of the innovation \( \alpha \) becomes large. Thus, we conclude that, typically,

\[
\frac{p_i - c_i}{q_i} = r + \delta + \mu + \mu \gamma \frac{\lambda}{r + \delta + \lambda} \sum_{n=0}^{n-1} \left( \frac{\lambda}{r + \delta + \lambda} \right)^n.
\]

Moreover, for given values of the other parameters, it can be shown analytically that there is an interval of values for \( \gamma \) that support a given replacement cycle of size \( n \), and approximation (B10) becomes exact as \( \gamma \) goes to the upper bound of this interval. Thus, for any positive integer \( n \), there exist parameter values such that the optimal pricing formula under uncertainty is the exact analog of the pricing formula for the deterministic case.

\[33\] **Demand uncertainty.** The following analysis is adapted from Dixit and Pyndyck (1994). To introduce demand uncertainty, we specify the inverse demand curve

\[
p = YX^{-\theta},
\]

where \( \theta = 1/\epsilon \) is the inverse demand elasticity, and \( Y \) is a geometric Brownian motion with drift parameter \( \alpha \) and variance parameter \( \sigma \). We assume that the only costs are the costs of acquiring new capacity, and that the price of new capacity, \( q \), declines deterministically at rate \( \mu \). In particular, there are no additional operating costs \( (\gamma = 0) \) or fixed costs \( (F = 0) \), and no technological uncertainty. However, we do allow for positive depreciation \( (\delta > 0) \). It is assumed that the interest rate \( (r) \) is sufficiently large that the optimal investment problem is well defined.

The solution to the optimal investment problem establishes a ceiling on the price ratio \( \gamma = p/q \). Positive investment keeps price from rising above the ceiling. When price drops below the ceiling, due to negative demand shocks, investment in new capacity is zero. More precisely, the optimal investment and price path is characterized by the complementary slackness conditions

\[
p \leq \left( r + \delta + \mu + \frac{\alpha^2 \beta}{2} \right) q, \quad \dot{X} + \delta X \geq 0, \quad \text{and} \quad \left( r + \delta + \mu + \frac{\alpha^2 \beta}{2} \right) q - p \left[ \dot{X} + \delta X \right] = 0,
\]

where \( \beta \) is the positive root of the quadratic equation

\[32\] We thank Roy Radner for pointing this out to us.

\[33\] This approximation suggests that technological uncertainty tends to lower the path of prices compared to deterministic progress at the same expected rate. This is due to a version of Jensen’s inequality, which states that the expected present value of forgone operating cost improvements in the Poisson model exceeds the present value of forgone operating cost evaluated at the expected life of assets.
\[
\frac{\sigma^2}{2} \beta (\beta - 1) + (\alpha - \mu + \delta \theta) \beta - (r + \delta + \mu) = 0.
\]

In other words, optimal investment equates prices to the user cost of capacity \([r + \delta + \mu + (\sigma^2 \beta / 2)]q\). If price falls below the user cost of capacity, then it is optimal not to invest. The user cost of capacity has an upward adjustment to account for demand uncertainty. This additive adjustment is equal to \(\sigma^2 \beta / 2\). The parameter \(\beta\) is itself a function of \(\sigma\) and other parameters. However, it is straightforward to show that \(\sigma^2 \beta\) is monotonically increasing in \(\sigma\), is zero when \(\sigma = 0\), and goes to infinity as \(\sigma\) goes to infinity. Thus, increased demand uncertainty raises the path of prices under an optimal investment plan, possibly substantially.

We sketch the argument for characterizing the threshold value of \(y = p/q\) that triggers positive investment. The state of the market at any date is a triple \((Y, X, q)\). There will be a region of the state space where it is optimal for no investment to take place. The value function over this region of inaction is denoted \(V(Y, X, q)\). It makes sense that the value function is homogeneous of degree one in \(Y\) and \(q\), in which case

\[
V(Y, X, q) = qv(Z, X),
\]

where \(Z = Y/q\). Bellman’s principle and Ito’s lemma imply

\[
(r + \mu)v = \frac{ZX^{-s}v_x}{1 - \theta} + \frac{Z^2 \sigma^2}{2} v_{zz} + (\alpha - \mu) Z v_x - \delta X v_y
\]

over the interior of the zero-investment region. Now define \(f = v_x\) and observe that

\[
(r + \delta + \mu)f = ZX^{-s} + \frac{Z^2 \sigma^2}{2} f_{zz} + (\alpha - \mu) Z f_x - \delta X f_y.
\]

Suppose that

\[
f(Z, X) = g(y),
\]

where

\[
y = ZX^{-s}.
\]

Then

\[
(r + \delta + \mu)g = y + \frac{y^2 \sigma^2}{2} g'' + (\alpha - \mu + \delta \theta) yg'.
\]

The solution to this differential equation has the form

\[
g(y) = By^\alpha + \frac{y}{r - \alpha + 2\mu + (1 - \theta)\delta}
\]

where it is assumed that \(r > \alpha - 2\mu - (1 - \theta)\delta\). The value-matching and smooth-pasting conditions imply that the ceiling value of \(y\) at the boundary of the inactive region is

\[
y^* = \frac{\beta}{\beta - 1} [r - \alpha + 2\mu + (1 - \theta)\delta].
\]

The quadratic equation for \(\beta\) can be rewritten as

\[
\frac{\sigma^2}{2} \frac{\beta}{r + \delta + \mu} + 1 = \frac{r - \alpha + 2\mu + (1 - \theta)\delta}{r + \delta + \mu} \frac{\beta}{\beta - 1},
\]

whence

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Thus, the optimal investment and price path is characterized by the complementary slackness conditions

\[ p \leq y^*q, \quad X + \delta X \geq 0, \quad [y^*q - p][X + \delta X] = 0. \]

The term \( \sigma^2 \beta / 2 \) is an upward correction to the ceiling value of \( p/q \) to account for demand uncertainty.

References


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