Fall 2006, APMA E6901
Special topics in Applied Mathematics: Linear and Nonlinear Waves

Time / Place: MW, 11am-12:15pm; Room: 1106B SW Mudd

Instructor: Michael I. Weinstein
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Course description: We study the mathematical theory of linear and nonlinear dispersive waves by considering, in depth, equations of nonlinear Schrödinger (NLS) / Gross-Pitaevskii type. These partial differential equations arise in many important areas of Fundamental and Applied Physics (Bose-Einstein condensation, nonlinear optics, fluid dynamics) and Mathematics (integrable systems, geometry).

Topics:

1. Review and introduction to analytical methods: Fourier analysis, spectral theory, calculus of variations
2. Linear dispersive wave equations, (linear) Schrödinger equation; dispersion, phase and group velocity, time-decay estimates
3. Rigorous derivation of NLS as a canonical envelope equation in strongly dispersive systems governing slowly varying wave envelopes.
4. Properties of NLS: Lagrangian/Hamiltonian structure, symmetries, conservation laws
5. Well-posedness theory for NLS
6. Coherent structures - solitons and solitary waves, vortices, ... - Characterizations by variational and bifurcation analysis; symmetry breaking
7. Dynamics and coherent structures - Orbital Lyapunov stability, Asymptotic stability, Scattering, Resonant wave interactions
8. Applications - Nonlinear optics and Bose-Einstein condensation

Prerequisites: A graduate level course in PDE.

Course texts: Selected texts and journal articles to be placed on reserved. Lecture notes on many topics to be provided.

Grading: Course grade is based on a project, e.g. a review of an article or articles related to the theme of the course, selected by the student and instructor.