Fall 2006, APMA E6901
Special topics in Applied Mathematics: Linear and Nonlinear Waves

Time / Place: MW, 11am-12:15pm; Room: 1106B SW Mudd

Instructor: Michael I. Weinstein
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Course description: We study the mathematical theory of linear and nonlinear dispersive waves by considering, in depth, equations of nonlinear Schrödinger (NLS) / Gross-Pitaevskii type. These partial differential equations arise in many imporant areas of Fundamental and Applied Physics (Bose-Einstein condensation, nonlinear optics, fluid dynamics) and Mathematics (integrable systems, geometry).

Topics:

(0) Review and introduction to analytical methods: Fourier analysis, spectral theory, calculus of variations

(1) Linear dispersive wave equations, (linear) *Schrödinger* equation; dispersion, phase and group velocity, time-decay estimates

(2) Rigorous derivation of NLS as a canonical envelope equation in strongly dispersive systems governing slowly varying wave envelopes.

(3) Properties of NLS: Lagrangian/Hamiltonian structure, symmetries, conservation laws

(4) Well-posedness theory for NLS

(5) Coherent structures - solitons and solitary waves, vortices, . . . – Characterizations by variational and bifurcation analysis; symmetry breaking

(6) Dynamics and coherent structures – Orbital Lyapunov stability, Asymptotic stability, Scattering, Resonant wave interactions

(7) Applications - Nonlinear optics and Bose-Einstein condensation

Prerequisites: A graduate level course in PDE.

Course texts: Selected texts and journal articles to be placed on reserved. Lecture notes on many topics to be provided.

Grading: Course grade is based on a project, *e.g* a review of an article or articles related to the theme of the course, selected by the student and instructor.