E. K. A. ADVANCED PHYSICS LABORATORY
PHYSICS 3081, 4051

COINCIDENCE COUNTING

References for Coincidence Counting

Instruction manuals for the model 465 coincidence unit and for the 2037A single channel analyzer (SCA).

Melissinos A., Experiments in Modern Physics, Academic Press, 1966, pages 340-374. Theory and Measurements.

1. INTRODUCTION

The requirement that the electronic signals from two or more particle detectors occur simultaneously, *i.e.* in temporal "coincidence", is a common criterion for the detection of radiation and particles in many nuclear and elementary particle physics experiments. This experiment has two separate parts which use the coincidence technique in different ways:

1.1 $\gamma\gamma$ Correlations

The angular correlation is measured between two gamma rays, which are emitted almost simultaneously in the cascade from the decay of a radioactive nucleus. Each gamma ray is absorbed in and detected by a NaI scintillation counter, which produces an electronic pulse of approximately 1 μs duration with a height proportional to the gamma-ray energy.

1.2 Cosmic Rays

The flux and direction of penetrating cosmic rays are measured as each charged particle passes through a "telescope", an array of three, large-area, plastic scintillation counters. Each of these counters produces an electronic pulse of several nanoseconds duration. This part of the experiment is for the more advanced students. See the instructor for details.

2. ANGULAR CORRELATIONS

2.1 True vs. Random Coincidences (in the absence angular correlation)

Consider the decay of a radioactive nucleus in which two gamma rays, γ_1 and γ_2 of different energies, are emitted in almost simultaneous cascade. The gamma rays are detected using two NaI scintillation counters in which the height of the electronic output pulses is proportional to the incident gamma ray energies. By pulse height selection, in "single channel analyzers", one counter will be used to record γ_1 and the other γ_2. The counting rate, in s^{-1}, of each counter for detecting its selected gamma ray is given by:

$$R_i = N_0 \epsilon_i \frac{\Omega_i}{4\pi} \quad i = 1, 2$$

where N_0 is the number of decays per second in the radioactive source, ϵ_i is the efficiency of the detector for recording the selected gamma ray when incident on its face, and Ω_i is the geometrical "solid angle" intercepted by the face of the detector *i.e.*, $\Omega_i/4\pi$ is the fraction of the sphere centered about the source which is subtended by the face (See Fig. 1).

In the absence of angular correlations, the "true" rate of detecting gamma ray coincidences is:

$$R_{\text{true}} = R_1 \epsilon_2 \frac{\Omega_2}{4\pi} = N_0 \epsilon_1 \frac{\Omega_1}{4\pi} \epsilon_2 \frac{\Omega_2}{4\pi}$$

since, for each γ detected in counter 1, the probability of detecting the other γ in counter 2 is $\epsilon_2 \Omega_2/(4\pi)$.

The accidental coincidence rate between a γ coming from the decay of a nucleus and a γ from a different nucleus is:

$$\begin{aligned} R_{\text{random}} =& R_1 R_2 \Delta t \\ =& N_0^2 \epsilon_1 \frac{\Omega_1}{4\pi} \epsilon_2 \frac{\Omega_2}{4\pi} \Delta t \end{aligned}$$

where Δt is the resolving time of the circuit used to establish a "time coincidence" between the two detectors. If both detectors are set to respond to both gamma rays, the counting rate in each detector is the sum of the counting rates for the different gamma rays; the true coincidence rate is twice that of the former case, and the accidental rate increases by a factor of 4.

$$\begin{aligned} R_i = N_0 \frac{\Omega_i}{4\pi} (\epsilon_1 + \epsilon_2) =& 2N_0 \frac{\Omega_i}{4\pi} \epsilon \\ R_{\text{true}} = 2N_0 \epsilon_1 \frac{\Omega_1}{4\pi} \epsilon_2 \frac{\Omega_2}{4\pi} =& 2N_0 \frac{\Omega_1}{4\pi} \frac{\Omega_2}{4\pi} \epsilon^2 \qquad \text{for } \epsilon_1 \sim \epsilon_2 = \epsilon \\ R_{\text{random}} = R_1 R_2 \Delta t =& 4N_0^2 \frac{\Omega_1}{4\pi} \frac{\Omega_2}{4\pi} \epsilon^2 \Delta t \end{aligned}$$

2.2 Angular Correlation

For many radioactive sources there is a correlation in the direction of emission of pairs of gamma rays. The two sources to be used in this experiment are:

2.2.1 ^{22}Na: a positron emitter.

When the decay positron annihilates with an electron in the source itself, two gamma-rays are usually emitted. In order to conserve momentum and energy in the two-body decay of a system initially at rest, the two gamma rays must be emitted with equal and opposite momenta. Each gamma ray then has an energy equal to 0.511 MeV (one half of the combined rest mass of a positron and an electron) and the angle between the gamma rays is 180 degrees.

2.2.2 ^{60}Co: A Gamma Ray Emitter following beta decay.

There is an angular correlation between the direction of the 1.17 and 1.33 MeV gamma rays emitted in the cascade of ^{60}Co, which has the decay scheme shown in Fig. 2. The true coincidence rates must be multiplied by an angular correlation function $W(\theta)$, which is chosen to average to one over all relative directions. For ^{60}Co, $W(\theta)$ is given by:

$$W(\theta) = 1 + 0.125\cos^2\theta + 0.042\cos^4\theta$$

In particular:

$$W(180^\circ)/W(0^\circ) = 1.0$$

$$W(180^\circ)/W(90^\circ) = 1.167$$

A discussion is given by Melissinos (pp. 417-429) for the cases of ^{22}Na and ^{60}Co, and for the general theory of which ^{60}Co is an example.

3. EXPERIMENTAL APPARATUS

A block diagram of the apparatus is shown in Fig. 3. The components are as follows:

3.1 Sodium Iodide Detectors

The experiment uses two sodium iodide detectors, each consisting of a crystal of NaI, 2 inches in diameter by 2 inches in length, optically coupled to a photomultiplier tube. The tube base supplies the high voltages required along the dynode chain, and also contains a preamplifier. The two bases are connected to a single high voltage power supply through individual, variable series resistors which allow each tube gain to be independently adjusted. The detectors are mounted on the circumference of a horizontal circle at the center of which is a source holder. One detector can be rotated about the source position.

3.2 Single Channel Analyzer

The signal from each detector goes first to an amplifier which also shapes the pulse and then to a single channel analyzer (SCA) module.

The SCA produces a standard output pulse whenever the height of the amplified signal falls within a selected range. The SCA selects signals within a variable "window" above the threshold. This additional electronic function limits the timing resolution that can be obtained using the output pulse. The output pulse from each SCA can be delayed by an adjustable time, D. If the delay is set too close to zero, problems occur, so avoid very small values of delay; this is easily done, since only the relative delay of the two signals is important.

3.3 Coincidence Circuit

The 465 coincidence unit has contains 3 coincidence circuits. Each can accept from 1 to 4 inputs. The output pulses from the SCAs are then each fed to a coincidence circuit used in single-fold mode. With only a single input active, the coincidence circuit just reshapes the pulse to an adjustable width, W. The width adjustment is made using a small screwdriver. It is easy to damage the unit while making this adjustment so ask the instructor to help. The reshaped pulses go to inputs of the third coincidence circuit used as a two-fold coincidence.

The coincidence unit provides an output pulse during the time of overlap of the two shaped pulses (W_1 and W_2 in Fig. 4.) The resolving time of the coincidence is therefore, approximately W_1+W_2. The resolving time can be experimentally determined by plotting the number of coincidences versus a varied relative delay time between simultaneous input pulses (delay curve). Intrinsic resolving time of the detector and electronics can make the resolving time longer than W_1+W_2. Compare the width of the delay curve to W_1+W_2.

3.4 Scaling Circuits

The three scalers, used to record the output of the 1-2 coincidences as well as the single counts from detectors 1 and 2, are contained in the TC535P timer/multiscaler unit.

3.5 Radioactive Sources

The radioactive sources to be used in this experiment are ^{22}Na and ^{60}Co. ^{22}Na, with a half-life of 2.60 years, decays with positron emission 90.5% of the time, (electron capture the other 9.5%). The source had an activity of 122.7 microcuries (μCi) on September 1,

1983. ^{60}Co with a half-life of 5.27 years decays into 1.17 and 1.33 MeV gamma rays (see Fig. 2). There are three sources with activities of 22.05 μCi on September 1, 1983, 40 μCi on January 5, 1976, and 10 μCi on January 5, 1976. One Curie is defined as 3.7×10^{10} decays per second; approximately the decay rate of one gram of radium.

The sources have been deposited on thin 1 inch diameter aluminum discs, which are to be mounted on a holder which can be rotated between the two detectors. In order to maintain a constant, effective source area facing the detectors, at 90 and 180 degrees, it is advisable to mount the ^{60}Co source at 45 degrees.

4 MEASUREMENT PROCEDURE

4.1 Statistical Error

If the average count expected in a time interval is $\overline{N}$, the actual observed count N in each measurement fluctuates around $\overline{N}$. For small $\overline{N}$ the fluctuation of n is described by the Poisson distribution. For $N \rightarrow \infty$ the distribution becomes a gaussian of r.m.s. $\sigma = \sqrt{N}$. This fluctuation should be considered when selecting the counting time interval required to achieve a desired accuracy.

4.2 Setting Up the Electronics

Use the ^{22}Na source, with detectors at 180 degrees, to set up the electronics and to become familiar with the operation of the system.

4.2.1 Setting the Gains

Use the oscilloscope, triggered internally, to observe the amplifier output from each SCA. The maximum output obtainable from the amplifiers is approximately 10 volts, and since the amplifier is non-linear near this saturation value, it is advisable to set the total gain so that all pulses to be measured are significantly below this level. The total gain depends on a combination of the phototube high voltage and amplifier gain setting.

The ^{22}Na source will produce a strong band of pulses from the .511 MeV annihilation gamma rays and a weaker band of pulses from direct 1.2 MeV gamma rays. Adjust the high voltages (controls on the variable resistors box) and the amplifier gains so that the outputs from both counters are the same, with the 1.2 MeV gamma rays well below saturation. (This will insure that the gamma rays from ^{60}Co are also below saturation.)

4.2.2 Setting the thresholds and windows

In order to observe which pulses fall within the SCA window, use the SCA output as an external sweep trigger for the oscilloscope and observe the amplifier output. (The delay line in the amplifier output insures that the pulses are seen on the scope after the SCA output has triggered the sweep.) Observe the effect of the threshold and window controls, and set the threshold and window levels to admit most of the pulses arising from the 0.511 MeV gamma rays.

4.2.3 Observing Coincidences

If you place counter 1 far from the ^{22}Na source and counter 2 opposite to 1 and closer to the source, then nearly every 0.511 MeV gamma ray which is counted in 1 will be accompanied by its "partner" in 2. Verify this triggering the scope sweep "externally", branching with a "tee" off the output of the SCA–1 and observe, alternately, both SCA output pulses. (You can use the "Alt" mode of the scope pre–amp to display both signals at once on separate traces.) Both sets of pulses should appear at the same position on the horizontal scale and be of equal intensity (but the band of pulses from 2 should disappear when counter 2 is rotated from the 180 degree position).

4.4.4 Setting Windows and Delays

Trigger the scope sweep "externally" with one of the two SCA outputs and connect the two coincidence monitor outputs to the two scope vertical inputs and view the two signals simultaneously, using the "alternate" display feature.

Set the widths, W_1 and W_2, to approximately 0.1 μs and adjust the two delays D_1, D_2 to about 1 μs and well aligned in time on the two scope traces.

5 MEASUREMENTS WITH ^{22}Na

5.1 Delay Curve

The proper timing of the signals must be verified by observing the counting rate vs changes in the relative delays of pulses. Record the coincidence and the two single counts on the three scalers as you change the delay of one signal, keeping the other constant. The Calibration of the delay knobs can be checked on the scope. Graph the curve and make sure you clearly observe a flat region of constant coincidence rate and a fall of the rate on both sides, to the approximately constant accidental rate. Set the delay you have varied to the central value from the plot.

5.2 Angular Correlation

Plot the coincidence rate vs angle of the moveable counter. Use small changes around 180° but also take counts near 90°. Compare the results obtained with the expected correlation as smeared by the finite size of the counters. At angles far from 180°, compare the coincidence rate with the accidental rate at 180°. Also compare with the computed accidental rate, using the width settings and the single rates.

5.3 Measuring the Source Strength

The source strength, N_0, can be calculated from a measurement of the true coincidence rate for γ–rays which are time correlated but have no angular correlation (see Section I). After emitting a positron which gives rise to the two 511 keV γ's studied above, the ^{22}Ne nucleus is left in an excited state.

The ^{22}Ne ground state is reached by emission of a 1.27 MeV γ which has no angular correlation with the annihilation γ's. Adjust the single channel analyzers so that one responds to a window around 511 keV and the other to a window around 1.27 MeV. Care must be taken not to make the windows too narrow or unstable counting rates will be observed. Set the detectors at an angle far from 180° (to avoid detecting both annihilation γ's). Measure the counting rate in each detector, the coincidence rate and the random coincidence rate. Calculate the resolving time of the coincidence and the source strength using the equations on page 2. Compare the measured source strength with the activity indicated on the source, corrected for the decay since the listed date.

6 MEASUREMENTS WITH ^{60}Co

6.1 Angular Correlation

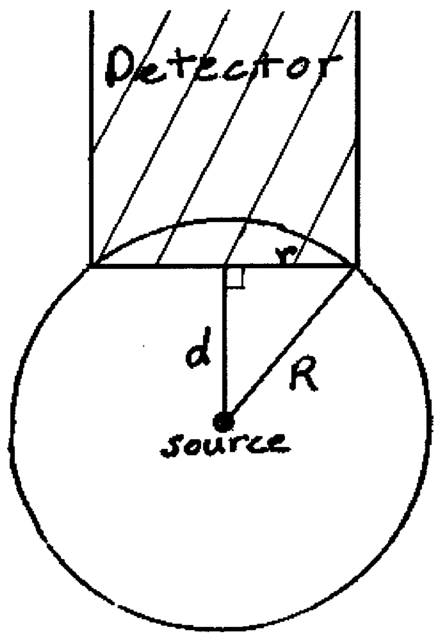
Adjust the single channel analyzers so that each responds to both the 1.17 and the 1.33 MeV γ's of ^{60}Co. Set the detectors at 180° with the source at 45° to the line joining the detectors. Measure the counting rate of each detector, the coincidence rate and the random rate. Repeat with the detectors at 90°.

Calculate:

$$A = \frac{\text{(coincidence rate} - \text{random rate) at } 180^\circ}{\text{(coincidence rate} - \text{random rate) at } 90^\circ}$$

and compare with the expected angular correlation. Note that R_1 and R_2 should not vary appreciably with angle. Count long enough to obtain a measurement of A with a 2%

statistical error.



$$\frac{\Omega}{4\pi} \approx \frac{\pi r^2}{4\pi R^2} = \frac{r^2}{4(d^2+R^2)}$$

Fig. 1 - Solid Angle, Ω

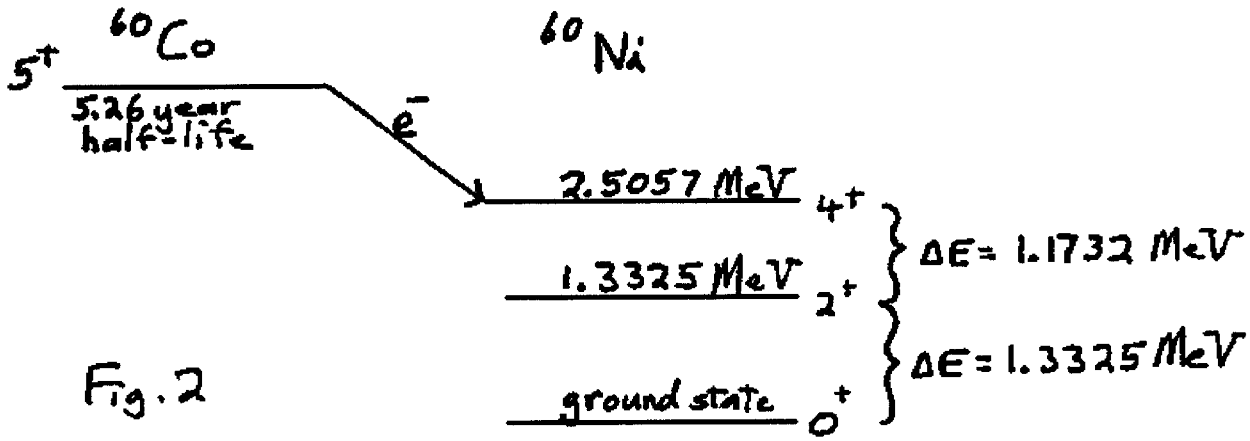


Fig. 2

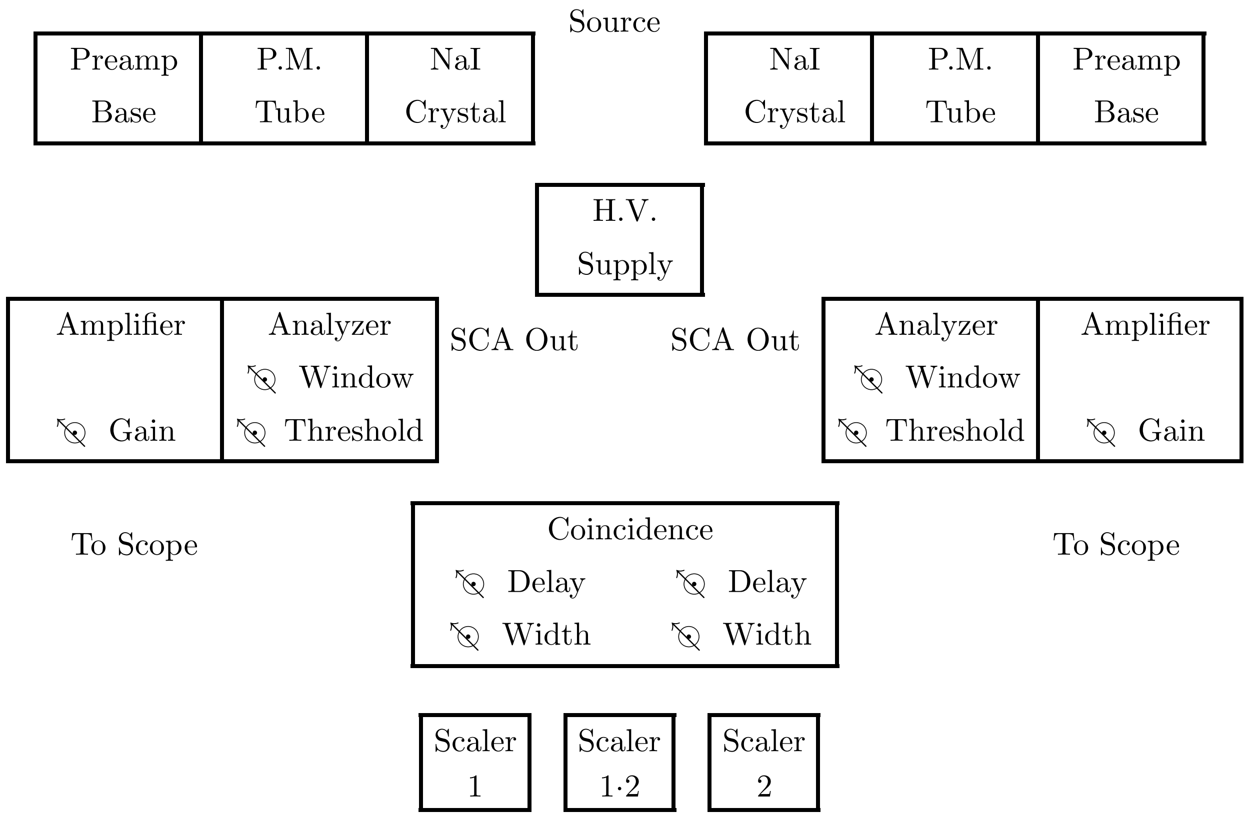


Figure 3. Block Diagram of Coincidence Apparatus

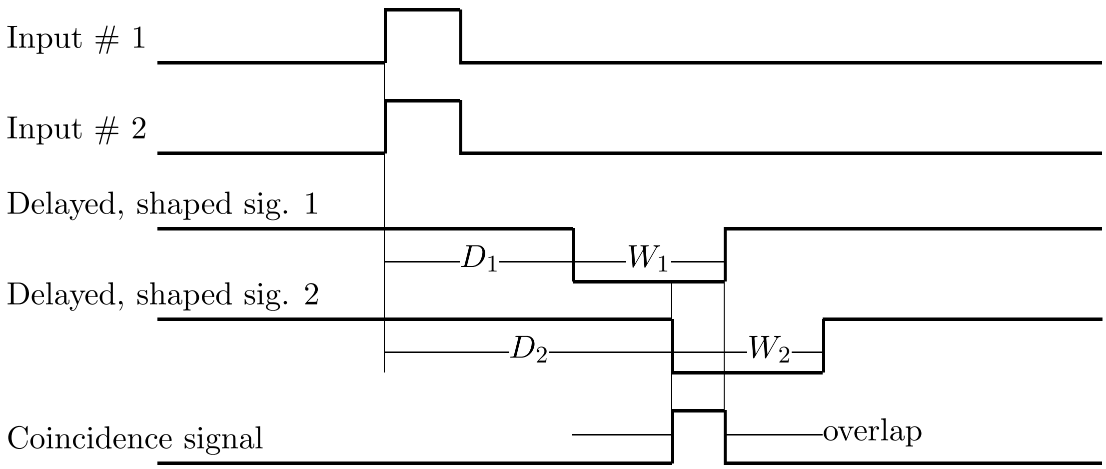


Fig. 4. Coincidence timing