1 Introduction

This lab manual is meant to serve as a general introduction to the quantum optics setup and basics of the quantum phenomena you will be observing in the next few weeks. For these experiments, you will be working with a modern quantum optics setup, based on the “Entanglement Demonstrator” from quTools.

This setup uses a variety of components – a free running diode laser, mirrors, lenses, waveplates, and fibers – as well as one new component: a non-linear crystal (The key to entanglement!) Before you begin fiddling with the apparatus, it’s important to understand what each item does as well as its purpose.

This modern toolkit will allow you to go in-depth into a variety of topics. However, there is not enough time in this course to cover all of them comprehensively. Hence this manual will only cover a few potential experiments you can conduct, but it’s really up to you on what you are interested in investigating!

Additionally, a (non-exhaustive) list can be found here. Many other groups around the world have modified this apparatus to investigate cutting-edge physics and actually published their results! If you have any interesting ideas you would like to pursue, you are more than welcome to contact the lab instructors and see if they can lend you additional optics/equipment from Prof Will or Prof Zelevinsky’s labs.

1.1 Learning Objectives

The main learning objective is for you to gain an appreciation and understanding of fundamental quantum phenomena. This manual will only go into detail on a few of them.

- Polarization Entanglement: Violation of Bell’s Inequality
- One-photon Interference: Michelson Interferometer
- Two-photon Interference: Hong, Ou & Mandel Interferometer

It is key that you understand the theory behind the unintuitive measurements you will be observing equally as well as the experimental components.

1.2 The entangled photon source

Our entanglement source comes from the QuTools QuED package. It utilizes a blue pump laser diode tuned to wavelength $\lambda = 405$ nm and a pair of Type I non-linear BBO crystals with optical axes perpendicular to one another to generate signal and idler photons at wavelength $\lambda = 810$ nm. An illustrative diagram can be seen in Fig. 1.
Figure 1: Diagram of the parametric down conversion process. In this case, a β-barium borate (BBO) crystal is pumped with 405 nm light. Depending on its polarization relative to the BBO crystal a 405 nm photon with the same polarization can be down converted into two separate 810 nm photons with the same polarization. The λ/2 waveplate can adjust the initial input polarization of the pump laser. Entanglement occurs when the input polarization is set to 45° between linear horizontal/vertical.

The first crystal’s optical axis and the pump beam define the vertical plane. Owing to Type I phase matching, an incoming photon which is vertically polarized gets down-converted and produces two horizontally polarized photons in the first crystal, whereas a horizontally polarized photon gets similarly down-converted into two vertically polarized photons in the second crystal. Notice that the polarization entanglement of the two outgoing photons can be created by using the λ/2 waveplate to rotate the input polarization, see Table 1.

Table 1: Polarization dependent entanglement depending on the input polarization

<table>
<thead>
<tr>
<th>Input Polarization of Pump Laser</th>
<th>Post Parametric Down Conversion</th>
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<tbody>
<tr>
<td>$</td>
<td>H\rangle_p$</td>
</tr>
<tr>
<td>$</td>
<td>V\rangle_p$</td>
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<tr>
<td>$\frac{1}{\sqrt{2}}(</td>
<td>H\rangle_p +</td>
</tr>
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By definition, an entangled state cannot be expressed as the tensor product of two different single-particle states.

$$|\psi_{prod}\rangle = (a |H\rangle_1 + b |V\rangle_1) \otimes (c |H\rangle_2 + d |V\rangle_2) \neq |H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2$$

Prove it to yourself! This state is really key to all the weird behavior of quantum mechanics. Let us focus a bit more on what this state means:

- According to this, if a measurement is made and photon 1 is found to be horizontally polarized, then you can say with 100% certainty than photon 2 is also horizontally polarized and vice versa.

**Question:** If you were to look inside the laser housing, you would see pre-/post compensation crystals sandwiching the BBO crystal. What is the purpose of the compensation crystals?
2 Startup Procedure

1. Power on the quCR unit
2. Power on laser
3. Set the operating current to 33.7A
4. Verify that the single counts are within the $64,000 \pm 6,000$ count range and the coincidence counts are $\approx 5,000$.
   - If this is not the case adjust the knobs on the polarizer.
   - Should the issues persist consult the manual.

Note: Always power off the laser before powering off the quCR unit.

3 Experiment: Polarization Entanglement

3.1 History

A summary of the Bell’s Inequality experiment would not be complete without briefly going through the historical debate. In 1935 Einstein, Podalsky, and Rosen (EPR) published a paper[3] highlighting the difficulty of reconciling a quantum mechanical description of the world with general relativity. The intent was to demonstrate the existence of a more objective reality beyond the statistical superpositions described by quantum mechanics. One of the key assumptions was locality, the notion that a measurement is only affected by quantities in the immediate vicinity, which was used to show an apparent paradox in quantum mechanical descriptions of entangled two particle systems. By the introduction of physical quantities which quantum mechanics failed to consider, so-called hidden variables, the paper claimed that the paradox could be resolved.

In 1964 J.S. Bell published a paper[1] in which he derived a famous inequality (known as Bell’s Inequality) as a necessary consequence of a local hidden variable theory. He then showed that this inequality was incompatible with the predictions of quantum mechanics, which was later further backed by experimental evidence in the decades that followed. These results both disproved the principle of locality and showed inviability of any local hidden variable theory.

3.2 Theoretical Background

3.2.1 Polarization measurement in a rotated basis

If we measure the polarizations of the entangled state there are two possible outcomes: both vertical or both horizontal, each occurring half the time.

If we instead measure the polarizations using polarizers rotated by an angle $\alpha (\beta)$ for photon 1(2):

\[|+\alpha\rangle = \cos \alpha |V\rangle - \sin \alpha |H\rangle\]
\[|-\alpha\rangle = \sin \alpha |V\rangle + \cos \alpha |H\rangle\]

In this basis, the state is

\[|\psi_{DC}\rangle = \frac{1}{\sqrt{2}}(|+\alpha\rangle_1 |+\alpha\rangle_2 + |-\alpha\rangle_1 |-\alpha\rangle_2)\]

For a pair produced in $|\psi_{DC}\rangle$, the probability of coincidence detection:

\[P_{++}(\alpha, \beta) = |\langle +\alpha_1 +\beta_2 | \psi_{DC}\rangle|^2\]
The ++ subscripts on P indicate the measurement outcome \( +\alpha +\beta \), where both photons vertical in the base of their respective polarizers. More generally, for any pair of angles \( \alpha, \beta \) there are four possible outcomes: \( +\alpha +\beta, +\alpha -\beta, -\alpha +\beta, \) and \( -\alpha -\beta \).

In the rotated basis:

\[
P_{++}(\alpha, \beta) = \frac{1}{2} \left( \cos \alpha \cos \beta e^{i\delta} + \sin \alpha \sin \beta \right)^2
\]

\[
= \frac{1}{2} \left( \sin^2 \alpha \sin^2 \beta + \cos^2 \alpha \cos^2 \beta + \frac{1}{2} \sin 2\alpha \sin 2\beta \cos \delta \right)
\]

Notice that a special case occurs when \( \delta = 0 \),

\[
P_{++}(\alpha, \beta) = \frac{1}{2} \cos^2(\beta - \alpha)
\]

where the probability depends only on the relative angle \( \beta - \alpha \).

**Question:** Calculate the probabilities of \( P_{--}(\alpha, \beta) \), \( P_{+-}(\alpha, \beta) \), and \( P_{-+}(\alpha, \beta) \). Are they what you expect it to be?

### 3.2.2 CHSH Bell Inequality

Quantum mechanics implies that if either photon is measured with the polarizer set to angle \( \alpha \), the result will be \( -\alpha \) or \( +\alpha \), each occurring half the time. The Copenhagen interpretation tells us the state collapses from \( |+\alpha\rangle_1 |+\alpha\rangle_2 + |-\alpha\rangle_1 |-\alpha\rangle_2 \) to either \( |+\alpha\rangle_1 |+\alpha\rangle_2 \) or \( |-\alpha\rangle_1 |-\alpha\rangle_2 \). It is the random measurement outcome on one of the photons that determines the state of the second photon. The choice of \( \alpha \) gives the second photon a defined polarization in the rotated \( \alpha \) basis, which it previously did not have. This implies a non-local process where the quantum state changes instantaneously even though the particles can be separated by large distances.


Any local realistic interpretation of entanglement depends on a hidden variable \( \lambda \), which could be a single variable or a set of variables. The values of \( \lambda \) affect the measurements of the entangled particles and vary according to unknown rules, but are distributed according to a probability \( \rho(\lambda) \), where

\[
\rho(\lambda) \geq 0
\]

and

\[
\int d\lambda \rho(\lambda) = 1
\]

Suppose two particles are entangled in the property \( q \) and that the measurement of \( q \) only allows for two outcomes, \( +1 \) and \( -1 \). For a local theory, measurement of the first particle at some detector \( A \) must be completely determined by the settings at \( A \), \( \alpha \) and the local hidden variable \( \lambda \). It is assumed that \( \alpha \) can be set independently of \( \lambda \). The result of the measurement at \( A \) is then given by some function \( A(\lambda, \alpha) = \pm 1 \). Measurement of the second particle at detector \( B \) follows similar rules such that \( B(\lambda, \beta) = \pm 1 \) also holds true. The functions \( \rho, A \) and \( B \) can be any functions satisfying Eq. 9, 10 and \( A, B = \pm 1 \)

For this experiment, a physical interpretation could be \( \lambda \) being some polarization angle that relates the two photons and \( q \) being the polarization, with \( \pm 1 \) corresponding to orthogonal polarizations.

An inequality can be constructed using these arguments. The probability that \( A = x \), where \( x = \pm 1 \), is

\[
\langle A = x \rangle = \int \frac{1 + xA(\lambda, \alpha)}{2} \rho(\lambda) \, d\lambda
\]

Similarly, the probability that \( A = x \) and \( B = y \) is

\[
P_{xy}(\alpha, \beta) = \int \frac{1 + xA(\lambda, \alpha)}{2} \rho(\lambda) \frac{1 + yB(\lambda, \beta)}{2} \rho(\lambda) \, d\lambda
\]
Bell’s inequality constrains the measured degree of polarization correlation at different polarizer angles. The proof involves two measures of correlation.

We can first define $E$, the expectation value of the product $AB$,

$$E(\alpha, \beta) \equiv P_{++} + P_{--} - P_{-+} - P_{+-} = \int A(\lambda, \alpha)B(\lambda, \beta)\rho(\lambda) \, d\lambda$$  \hspace{1cm} (13)

$E$ spans all possible measurement outcomes and ranges $E \in \{-1, +1\}$.

The second particle correlation that we define is $S$,

$$S(\alpha, \alpha', \beta, \beta') \equiv E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta')$$  \hspace{1cm} (14)

where $\alpha, \alpha', \beta, \beta'$ are four different polarizer angles. There is no clear physical meaning for $S$. Its importance comes from the fact that $S \leq 2$ for any local theory. However, this does not hold for quantum mechanics. Substituting Eq. 7 and its other possibilities in 13 and 14, and using polarizer angles $\alpha, \alpha', \beta, \beta' = -45^\circ, 0^\circ, 22.5^\circ, 22.5^\circ$, we find a maximal violation of

$$S_{QM} = 2\sqrt{2}$$  \hspace{1cm} (15)

Thus, a measurement of $S > 2$ disproves all local theories.

For the purpose of this experiment we write $E(\alpha, \beta)$ in terms of the coincidence account rate for the corresponding polarizer settings as

$$E(\alpha, \beta) = \frac{C(\alpha, \beta) - C(\alpha, \beta_\perp) - C(\alpha_\perp, \beta) + C(\alpha_\perp, \beta_\perp)}{C(\alpha, \beta) + C(\alpha, \beta_\perp) + C(\alpha_\perp, \beta) + C(\alpha_\perp, \beta_\perp)}$$  \hspace{1cm} (16)

where $\alpha_\perp$ and $\beta_\perp$ represent orientations perpendicular to $\alpha$ and $\beta$ respectively.

### 3.3 Experimental Procedure

1. Follow the steps in Section 2 to start and calibrate the quED setup.

2. Using sufficiently long integration time begin recording coincidence counts with the given angles in the previous subsection. Four experimental runs are required for each value of $E(\alpha, \beta)$, for a total of 16 in the calculation of $S$. 

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![Figure 2: quED setup with the quCR unit](image_url)
3. After calculating $S$, interpret your results and use them to calculate the standard deviation from $S$,

$$
\Delta S = \sqrt{\sum_{a=\alpha,\alpha'} \sum_{b=\beta,\beta'} \Delta E(a, b)^2}
$$

(17)

The errors $\Delta E(a, b)$ can be obtained from Gaussian Error Propagation: where Poisson photon number statistics as well as independent measurement errors are assumed.

For a detailed experimental procedure, please refer here.

3.3.1 Additional questions to think about

- Starting with the entangled state $\frac{1}{\sqrt{2}}(|H\rangle_1 |H\rangle_2 + |V\rangle_1 |V\rangle_2)$ and using Eqs. 2, 3, show Eq. 4. Repeat for any general pair of angles $\alpha, \beta$.

- Looking at the observed value of $S$, what does it represent physically? If the value of $S$ obeyed locality or did not exceed 2 by much, speculate with regards to what could have been improved.

4 Experiment: Single-photon interference

4.1 Theoretical Background

4.1.1 Classical wave-particle picture

Figure 3: Interference between two waves, the first of which is completely in phase ($\Delta \phi = 2m\pi$), and the second completely out of phase ($\Delta \phi = (2m + \pi)$). These cases will result in maximal intensity or zero, respectively, where $m$ is an integer.

In classical physics you were taught concepts relating to waves and particles. The phenomena of interference is a strictly "wave-like" phenomena that occurs when two separate wave amplitudes either add or subtract, resulting in an intensity that can be up to four times the initial amplitude or zero.
In a classical Michelson interferometer experiment the observed interference pattern is dependent on the optical path difference, $\Delta x = d_2 - d_1$, taken between the two arms:

$$
\Delta \phi = \frac{2\pi}{\lambda} \Delta x = k \Delta x
$$

(19)

where $\lambda$ is the wavelength of the light. As a recap, the electric field amplitude is:

$$
E = E_0 e^{i(\omega t - kz - 2kd_1)} + E_{\text{out}} e^{i(\omega t - kz - 2kd_2)}
$$

(20)

$$
= E_0 e^{i(\omega t - kz)} \left(e^{-2ikd_1} + e^{-2ikd_2}\right)
$$

(21)

The classical intensity sum, $I_{\text{tot}}$ is:

$$
I_{\text{tot}} = |E|^2 = 2E_0^2 (1 + \cos(2k(d_2 - d_1)))
$$

(22)

Would this picture still hold if we zoomed down to a single photon? Recall that the classical particle picture considers the particle as well-localized at any time, and its path to be known. A particle travelling through the interferometer can only travel down one of the arms of the interferometer. In this picture, we should not expect any interference, but a flat statistical probability of the particle coming back through one of the arms to the detector.

### 4.1.2 Quantum statistics and wave-particle duality

In the quantum picture, consider a 50/50 beam splitter where a photon is incident at port 1. The two possible inputs of a photon into the beamsplitter can be represented as a two state system, where a possible input, $|\text{in}\rangle$, is represented by a 2 x 1 matrix. A photon entering port 1 and a photon entering port 2 are represented by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ respectively. We can write the action of the beamsplitter as a 2 x 2 matrix, $R$:

$$
R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}
$$

(24)

We can also approximate the effect of the path difference from the adjustable distance in path b as:

$$
\Phi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta \phi} \end{pmatrix}
$$

(25)
Thus, for the output ports of the interferometer that we measure, with an incident photon on port (a) we obtain:

\[ |\text{out}\rangle = R\Phi R \]

\[ = \frac{1}{2} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta\phi} \end{pmatrix} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]

\[ = ie^{i\Delta\phi/2} \begin{pmatrix} -\sin \frac{\Delta\phi}{2} \\ \cos \frac{\Delta\phi}{2} \end{pmatrix} \]

The probability of observation in ports c and d respectively are

\[ \langle \text{out}|\text{out}\rangle = \left(\sin^2 \frac{\Delta\phi}{2}\right) \cos^2 \frac{\Delta\phi}{2} \]

Thus, we see that interference of a single photon does occur. Note that our result here is the same as the classical picture. It is also important to mention that most of the considerations we present here are not limited to photons but are equally well applicable to massive particles. In the field of matter-wave interferometry, beam splitters have been constructed successfully for electrons, neutrons, and a number of different atoms and molecules.

### 4.2 Experimental Procedure

For a detailed procedure on conducting the experiment, please see [here](#).

#### 4.2.1 Additional questions to think about

- Why must the optical path length be close to zero before you can observe interference fringes?
- What scenario that would occur if you could tell whether the photon travelled in path c or d?
- Derive the matrix \( R \) for the action of a lossless 50/50 beamsplitter with 2 inputs and 2 outputs.
- Consider the case of two identical photons entering the two input ports at the same time. What are the possible observed results? Think carefully! It may be helpful to write it out or draw a picture. Confused? Perhaps the next experiment can offer some answers.
5 Experiment: Two-photon interference

5.1 Theoretical Background

5.1.1 Quantum Interference of indistinguishable particles

Our experimental setup is designed to introduce two photons into the apparatus at the same time. Consider the case for which the incident photons are in identical polarization states. There then result two-photon interference effects that can only be explained by quantum mechanics.

For the case of 2 photons there are two cases we will consider, and each photon will have to be treated in its own Hilbert space. (Why is this necessary?)

![Figure 6: 2 photons incident on a 50/50 beamsplitter (a) illustrates the case where 2 photons enter via the same port (b) show the case where each photon enters separately ports](image)

\[
|\psi_{in}\rangle = |1\rangle_1 |2\rangle_2
\]

(30)

where \(|1\rangle_1\) means photon 1 is in port 1 and so on.

For the case shown in Fig. 6 (a), by applying Equation 24 twice we obtain

\[
|\psi_{out}\rangle = \frac{1}{2}(|a\rangle_1 |a\rangle_2 + i |a\rangle_1 |b\rangle_2 + i |b\rangle_1 |a\rangle_2 - |b\rangle_1 |b\rangle_2)
\]

(31)

It is left to the reader to calculate the probabilities of the different cases in the output ports.

For the case shown in Fig. 6 (b), a new observation has to be made, namely with quantum statistics. If as we assume that the two particles are indistinguishable (that we cannot know which one exists which port), then we must write the quantum state as a superposition of both states. The phase factor between them would be determined by the symmetry properties of the particles, whether they are bosonic or fermionic in nature.

In the case of bosons, they are symmetric under exchange

\[
|\psi\rangle_{boson} = \frac{1}{\sqrt{2}}(|1\rangle_1 |2\rangle_2 + |2\rangle_1 |1\rangle_2)
\]

(32)

and antisymmetric under exchange for fermions

\[
|\psi\rangle_{fermions} = \frac{1}{\sqrt{2}}(|1\rangle_1 |2\rangle_2 - |2\rangle_1 |1\rangle_2)
\]

(33)
Applying Equation 24, we obtain the result for bosons

\[ |\psi_{\text{output}}\rangle = \frac{i}{\sqrt{2}} (|a\rangle_1 |a\rangle_2 + |b\rangle_1 |b\rangle_2) \]  

(34)

This implies that for bosons, we will find both particles either in port a or b, but never separately. This phenomena is the result of quantum interference, and is known as boson bunching due to the destructive or constructive interference of the particle wavefunction. You will be observing this result using the Hong-Ou-Mandel Interferometer.

**Question:** Calculate the expected result for fermions.

### 5.2 Experimental Procedure

For a detailed procedure on conducting the experiment, please see here.

### 5.2.1 Additional questions to think about

### 6 Remote Access Instructions

1. Connect using the following credentials in TeamViewer:
   - ID: 652 880 539
   - Password: qutools
2. Open VNC Viewer and connect to the following IP:
   - 169.254.52.183
3. Cameras are accessible through the camera app, with three different webcams being available via the switch camera button.

### References

