BARGAINING IN BICAMERAL LEGISLATURES: WHEN AND WHY DOES MALAPPORTIONMENT MATTER?[1]

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Abstract

Malapportionment of seats in bicameral legislatures, it is widely argued, confers disproportionate benefits to overrepresented jurisdictions. Ample empirical research has documented that unequal representation produces unequal distribution of government expenditures in bicameral legislatures. The theoretical foundations for this empirical pattern are weak. It is commonly asserted that this stems from unequal voting power per se. Using a non-cooperative bargaining game based on the closed-rule, infinite-horizon model of Baron and Ferejohn (1989), we assess the conditions under which unequal representation in a bicameral legislature may lead to unequal division of public expenditures. Two sets of results are derived. First, when bills originate in the House and the Senate consider the bill under a closed rule, the equilibrium expected payoffs of all House members are, surprisingly, *equal*. Second, we show that small-state biases can emerge when: (1) there are supermajority rules in the malapportioned chamber, (2) the Senate initiates bills, which produces maldistributed proposal probabilities, and (3) the distributive goods are "lumpy."

1. Introduction

Bicameralism is common in the legislatures of the world's democracies. It is also common for one chamber to represent population and the other to represent geographical areas, such as provinces, cantons, states, or counties. This arrangement often emerges as a compromise, perhaps unavoidable, in the formation of the nation or union to balance the representation of people and of regions. Federal systems such as Argentina, Brazil, Germany, Switzerland, and the United States often have such an arrangement, and the European Union employs a mix of representation of population and representation of states.

The result is malapportionment, with highly unequal representation of the population in at least one chamber of the legislature.[1] Malapportionment, in turn, has been found to have substantial and direct effects on public policy. An extensive empirical literature has documented a strong, positive association between a geographic area's per capita seats in the legislature and the share of public expenditures it receives.[2] Such results have stirred a growing chorus of criticism of malapportionment in bicameral legislatures; most notably, Dahl (2002) identifies the representation of states in the Senate as a fundamental flaw in the U.S. constitution.

Less certain is why. Most often, it is argued that the *voting power* of over-represented areas leads directly to their disproportionate influence over public policy. Lee (2000, p. 59) puts it is follows:[3]

[1]Lijphart (1982) documents the severity of malapportionment in 6 bicameral legislatures: Australia, Austria, Canada, Germany, Switzerland, and the United States. Samuels and Snyder (2001) document this for a wide set of countries for the current period. They find that when severe malapportionment exists it is typically in the upper chamber. The worst cases are Argentina, Brazil, Bolivia, Dominican Republic, United States, Switzerland, Russian Federation, Venezuela, Chile, Australia, Spain, Germany, Mexico, South Africa, and Poland. In all these countries, the index of malapportionment in the upper house is .20 or higher. In all cases the index of malapportionment in the lower house is less than half as large as that for the upper house, and in most cases it is less than one-fourth as large. David and Eisenberg (1961) document the situation in the U.S. states for the 50 years prior to the *Baker v. Carr* decision.

[2]See Atlas, *et al.* (1995), Lee (1998, 2000), and Lee and Oppenheimer (1999) for studies of federal spending in the U.S. states; see Ansolabehere, Gerber, and Snyder (2002) for a study of U.S. state spending prior to 1960; see Gibson, Calvo and Falleti (1999) for a study of Argentina and Brazil; see Rodden (2001) for a study of the European Union; and see Horiuchi and Saito (2001) for a study of Japan.

[3]Dahl (2002, p. 49) makes a similar argument: "With a population in 2000 of nearly 34 million, California had two senators. But so did Nevada, with only 2 million residents. Because the votes of U.S. senators are counted equally, in 2000 the vote of a Nevada resident for the U.S. Senate was, in effect, worth about seventeen times the vote of a California resident. A Californian who moved to Alaska might lose some points on climate, but she would stand to gain a vote worth about fifty-four times as much as her vote in California." See also Rodden (2001) for a similar analysis of the European Union.

> "The great variation in state population means that some states have far greater need for federal funds than others, but all senators have equal voting weight. As a result, even though all senators' votes are of equal value to the coalition builder, they are not of equal 'price.' Coalition builders can include benefits for small states at considerably less expense to program budgets than comparable benefits for more populous states."

The intuition behind these arguments comes from theoretical results derived for the unicameral legislatures. In a single legislative body, politicians with greater voting weight will receive higher shares of the division of the public dollar (Shapley and Shubik 1954; Snyder, Ting, and Ansolabehere 2002). This logic, however, may not extend to the bicameral setting. The unicameral logic would apply readily to the entire chamber if coalition formation in one chamber were independent of coalition formation in the other chamber. As Buchanan and Tullock (1962) observe, the legislators' preferences in the two chambers are not independent: Senators are more likely to support a bill if their House members are part of a coalition, because members of the lower house represent areas within the geography represented by a upper house.

What is the logic of bargaining over the division of public expenditures in bicameral legislatures? Most previous analyses employ a "power index" from cooperative game theory, such as the Shapley-Shubik index or the Banzhaf index.[4] As is well known, cooperative game theory models of voting power do not explicitly consider proposal power. Thus they cannot incorporate potential differences in proposal power of areas stemming from unequal representation. These measures also do not readily accommodate correlated preferences across chambers. Almost all of the previous work on coalition formation in bicameral legislatures assumes that the preferences of the legislators in the two chambers are independent.[5] This assumption is made in order to apply the voting power indices to the bicameral problem, but it is almost surely wrong.

[4]See, *e.g.*, Shapley and Shubik (1954), Deegan and Packel (1978), Dubey and Shapley (1979), and Brams (1989). Diermeier and Myerson (1999) is one of the few papers using a non-cooperative approach. There is also a small literature examining whether bicameralism produces unbeatable points in a multi-dimensional issue space, which can be viewed as a hybrid—*e.g.*, Hammond and Miller (1987) and Tsebelis and Money (1997).

[5]This is true for all of the papers employing power indices cited in footnote 4.

To isolate the effects of voting power, proposal power, and other institutional features, we analyze divide-the-dollar politics in a bicameral legislature using the non-cooperative legislative bargaining model developed by Baron and Ferejohn (1989). In this framework, a legislator is randomly chosen to make a proposal about how to divide a dollar among all legislators; then, voting on the proposal occurs. Each legislator's expected share of public spending equals what he or she receives when making a proposal times the probability of being proposer, plus what the legislator must be paid to join a coalition times the probability of being included in a coalition.[6]

Within this framework we can ascertain the expected division of public expenditures under a range of institutional arrangements or rules. Our basic model begins by considering the simplest case: proposals originate in the House, may not be amended by the Senate, and must be approved by majority rule in each chamber. Legislators care about the welfare of their median voter. In this situation, the opportunity to make proposals is allocated evenly across the population because House districts are assumed to have equal populations and because every House member has the same probability of being chosen as a proposer. The voting power of the areas differs because of unequal representation of population in the Senate. As a practical matter, this case emerges in many real-world legislatures. In the large majority of bicameral legislatures, the lower house initiates money bills. This is true, for example, of the U.S. House of Representatives. Tsebelis and Money (1997) document that many bicameral systems also limit upper house amendment powers. In some, such as Australia and the Netherlands, the upper house cannot amend.

Surprisingly, the basic model predicts an *equal* (expected) division of the public expenditure. The immediate implication that the argument that unequal voting power predicts maldistribution of public spending is incorrect. In a bicameral legislature, unequal voting power *per se* is not sufficient to explain maldistribution of government spending.

To produce maldistribution of public expenditures in the non-cooperative bicameral bargaining game, other institutional rules or policy arenas are required. We consider three such factors here: (1) supermajority rules, (2) Senate proposal power, and (3) the nature of public expenditure programs. All are quite common in legislative politics. The cloture rules of the

[6] The Baron-Ferejohn model is based on Rubinstein (1982). It is the most widely used model of legislative bargaining in distributive politics.

U.S. Senate are perhaps the best-known supermajority rules. The European Union provides an important example where the upper chamber (the Council of Ministers) proposes money legislation, and the lower chamber (the European Parliament) votes whether to reject these bills (Tsebelis and Money, 1997, Table 2.2B). In this case, proposal power, as well as voting power, is maldistributed. Finally, many kinds of public expenditures cannot be targeted at specific districts, thereby restricting the divide-the-dollar policy space of our basic model.

Our formalization also advances non-cooperative models of legislative bargaining and coalition formation. Over the last 15 years, an extensive literature, much of it in this *Review*, has used the Baron-Ferejohn model to explore the effects of government institutions on distributive outcomes.[7] All of this research examines a legislature with a single chamber, significantly limiting the application of theoretical research.[8] Strictly speaking, none of these models apply, for example, to the U.S. Congress or to 49 of the American states. We present a framework within which to analyze bicameral legislative bargaining, and hope it provides the basis for further analysis.

2. Basic Model

We analyze a variant of the closed-rule, divide-the-dollar game studied by Baron and Ferejohn (1989). In the bicameral setting, we must make further assumptions about the structure of the politics in order to characterize how agreements are reached across chambers. We begin with one specific formulation of the problem and consider variations on this setup in Section 4.

2.1. Assumptions

Here we lay out some of the assumptions about the structure of the bicameral setting. Readers already familiar with this type of model in the literature may wish to skip to the formal presentation below.

[7]See, *e.g.*, Winter (1996), Diermeier and Feddersen (1998), LeBlanc, Snyder and Tripathi (2000), McCarty (2000).

[8]The only other paper we have found that explicitly models the linkage between the two chambers in a bicameral legislature is Kalandrakis (n.d.). He assumes that the utility functions of the Senators and House members in a state are not just linked but are the same. This is a very restrictive assumption. For a Senator to agree to vote for a proposal requires that *all* House members in the Senator's state would also vote for that proposal.

A1. The lower chamber (House) represents districts with equal population and the upper chamber (Senate) represents states, which may containing different numbers of districts. There is one House member per district, and one Senator per state.

A2. Public expenditures are divisible to the district level. Legislators are responsive to their median voters.

A3. Both chambers vote by majority rule. The lower chamber moves first, and the other chamber votes on that proposal without amendment, *i.e.*, under a closed rule.

Assumption A1 describes the link between House members and Senators: the geographic areas of representation are nested, as in the U.S. Congress. One may imagine violations of this assumption, as occurs in some state legislatures where assembly district boundaries cut across senate district boundaries. Those lead to greater independence of coalition formation across the legislative chambers. But, most bicameral legislatures nest districts, either completely or to a high degree.

Assumption A2 helps to characterize the decision rule for House members and Senators. As we will show, it implies that in order for a legislator to support a proposal the proposal must give some money to at least half of that legislator's voters. Thus, House members must receive some money to support a proposal, and Senators must receive money in more than half of their districts. We can vary this assumption, as suggested in Section 4.3. However, the simple majority rule approximates the behavior of legislators in practice.[9]

Assumption A3 defines voting and proposal power. Importantly, we assume for the basic model that voting power is unequal but proposal power is equal. We consider the case of unequal voting power and unequal proposal power in Section 4.2. The closed rule is surely a special case, but it means that we do not have to model the resolution of differences between the chambers. Modeling that additional decision process adds a layer of complication that is not needed to gain important insights. We also relax the majority rule assumption in Section 4.1.

[9]We examined all roll call votes in the U.S. House and Senate from 1989 to 2000 involving final passage on appropriations and authorizations. When a majority of a state's House delegation supports a money proposal, that state's Senators vote for the bill 90 percent of the time. When a majority of a states' House delegations votes against a bill, the Senators vote for the bill 64 percent of the time.

2.2. Formal Description

There are two legislative chambers, a *House* and a *Senate*. Seats in the House are apportioned on a per-capita basis, while seats in the Senate are apportioned geographically. For convenience we refer to geographical units as *states*. Each state has a *type*, identified by population, and there are at least two types of states. A type-t state gets 1 seat in the Senate and t seats in the House, where $t \geq 1$ is odd. (With some modifications, the results also hold for t even.) Let $m_t \geq 0$ be the number of type-t states, and $n_t = m_t t$ be the number of House seats from type-t states. Let T denote the size of the largest state. The total number of seats in the House is then $n = \sum_{t=1}^{T} n_t = \sum_{t=1}^{T} m_t t$ and the total number of seats in the Senate is $m = \sum_{t=1}^{T} m_t$. We assume that n and m are both odd. We call the House legislators *representatives*, and we call the Senate legislators *senators*. We equate each representative with his district, and each senator with his state.

Legislators in both chambers wish to maximize the expected utility of their constituency's median voter. We assume that voters in each district have identical, quasi-linear preferences. Further, spending is *indivisible* at the level of the House district—that is, it consists of local government expenditure programs consumed by all voters in the district. Thus, representatives simply wish to maximize the funds flowing to their district. Because they may represent multiple districts, senators care not only about the quantity of goods flowing to their state, but also the distribution thereof. A type-t senator attempts to maximizes the benefit of the d_t-th highest per-district benefit that a bill promises in a type-t state, where $d_t = (t+1)/2$. The idea is that we are studying distributive spending, and any spending that goes into a district is valued both by the House member from that district and the senator from the state containing the district.

All proposals originate in the House. In period 1, Nature randomly draws one representative to be the proposer, who proposes a division of the dollar across representatives (House districts). Formally, a proposal is an n-dimensional vector from the set $X = \{\mathbf{x} \mid x_i \in [0,1], \sum_{i=1}^{n} x_i \leq 1\}$. All legislators in both chambers then simultaneously vote for or against the proposal. If the proposal receives a majority in both chambers, then the dollar is divided and the game ends. If the proposal is rejected, then a new representative is randomly drawn to be the proposer. The game has an infinite horizon, and no discounting.

To identify coalitions, we adopt the following notation. Let $\mathbf{N}$ be the set of all representatives (districts), and for each $t = 1, \dots, T$, let $\mathbf{N}_t$ be the set of all representatives (districts) from type-t states. If $\mathbf{C}$ is any coalition of representatives, let $\mathbf{N}_t(\mathbf{C})$ be the set of representatives in $\mathbf{C}$ from type-t states. Let $n_t(\mathbf{C})$ be the total number of representatives in $\mathbf{N}_t(\mathbf{C})$, and let $n(\mathbf{C}) = \sum_{t=1}^{T} n_t(\mathbf{C})$ be the total number of representatives in $\mathbf{C}$. For each state j, let $\mathbf{N}^j(\mathbf{C})$ be the set of representatives in $\mathbf{C}$ that are drawn from j, and let $n^j(\mathbf{C})$ be the number of representatives in $\mathbf{N}^j(\mathbf{C})$. Analogously, let $\mathbf{M}_t(\mathbf{C})$ be the set of type-t states such that $\mathbf{C}$ contains at least $(t+1)/2$ representatives from each of these states, and let $m_t(\mathbf{C})$ be the number of states in $\mathbf{M}_t(\mathbf{C})$. Thus, $\mathbf{M}_t(\mathbf{C})$ can be thought of as the set of senators from type-t states that are "in" $\mathbf{C}$, and $m_t(\mathbf{C})$ can be thought of as the number of senators from type-t states that are "in" $\mathbf{C}$. Let $\mathbf{M}(\mathbf{C}) = \cup_{t=1}^{T} \mathbf{M}_t(\mathbf{C})$ be the set of senators "in" $\mathbf{C}$, and let $m(\mathbf{C}) = \sum_{t=1}^{T} m_t(\mathbf{C})$ be the total number of senators "in" $\mathbf{C}$. We call a coalition $\mathbf{C}$ *winning* if and only if $n(\mathbf{C}) \geq (n+1)/2$ and $m(\mathbf{C}) \geq (m+1)/2$. Denote by $\mathcal{W} = \{\mathbf{C} \mid n(\mathbf{C}) \geq (n+1)/2 \text{ and } m(\mathbf{C}) \geq (m+1)/2\}$ the set of winning coalition.

The game can be treated as a sequence of identical subgames, where each subgame begins with nature's move to draw a proposer. We look for symmetric, stationary, subgame perfect equilibria (SSSPE's). Our definition of symmetry is that strategies treat all representatives of the same type symmetrically, although different types may be treated differently.[10] Stationarity means that each legislator uses history-independent strategies at all proposal-making stages, and voting strategies that only depend on the current proposal. This implies that we may suppress notation for time and game histories.

For all types $t = 1, \dots, T$, SSSPE strategies are then as follows. A proposal strategy for a type-t representative is $w_t \in \Delta(X)$, where $\Delta(X)$ is the set of probability distributions over X. Voting strategies for type-t representatives and senators are $y_t : X \to \{0, 1\}$ and $z_t : X \to \{0, 1\}$, respectively, mapping allocations into votes, where a 1 represents approval.

SSSPE's have the following properties, which simplify the analysis. By symmetry, for each type t, the continuation value of all type-t representatives at the beginning of each subgame will be equal. By stationarity, these values will also be the same for each subgame. Let v_t

[10]This definition of symmetry is somewhat non-standard. Along with stationarity, the assumption will constrain equilibrium payoffs within types to be equal, but it does not artificially constrain equilibrium payoffs *across* types. Without the assumption, a much wider range of payoff distributions can be sustainable.

and v_t^s be the continuation values of type-t representatives and senators at the beginning of each subgame, respectively. At an SSSPE, the proposer must offer at least v_t to a type-t representative in order to obtain that representative's support for his proposal (that is, $y_t(\mathbf{x}) = 1$ for legislator i of type t iff $x_i \geq v_t$). Likewise, the proposal must offer at least v_t^s to a type-t senator in order to gain her vote. Note that a type-t senator's allocation is effectively an "order statistic" indicating the d_t-th highest per-district benefit that a bill promises to the state. Since a proposal will pass if and only if it receives majority support in both chambers, it must offer at least v_t to $(n+1)/2$ representatives and v_t^s to $(m+1)/2$ senators.

3. Main Results

The assumptions A1, A2, and A3 describe a situation in which voting power is unequal but proposal power is equal. In equilibrium, the expected share of public expenditures is *the same* in all districts—the expected division of expenditures is *not* skewed toward the areas that are over-represented in the Senate. The intuition behind this result is as follows. To build a winning coalition, a proposer collects a majority of districts. Because the proposer keeps the surplus from any bargain, the proposer wishes to build the lowest cost minimal winning coalition. Under simple majority rule it is possible to do this without having to distribute any money solely in order to obtain votes in the Senate. As a result, the "marginal value" of any Senator to the coalition is zero. Small states therefore do not have disproportionate bargaining power even though they have disproportionately more votes.

To show this claim, we proceed in two steps. Our first result identifies a fundamental relationship between chambers.

Proposition 1. Suppose $\mathbf{C}$ is a coalition such that $m(\mathbf{C}) = (m+1)/2$, $n^j(\mathbf{C}) = (t+1)/2$ (where t is state j's type) for all $j \in \mathbf{M}(\mathbf{C})$, and $n^j(\mathbf{C}) = 0$ for all $j \notin \mathbf{M}(\mathbf{C})$. If $\mathbf{M}(\mathbf{C})$ contains all states with $t > 1$, then $n(\mathbf{C}) = (n+1)/2$. If $\mathbf{M}(\mathbf{C})$ does not contain all states with $t > 1$, then $n(\mathbf{C}) < (n+1)/2$. ∎

Proof. All proofs are in the Appendix. ∎

That is, if a coalition $\mathbf{C}$ has just enough representatives drawn from just enough states to win the Senate, then $\mathbf{C}$ either is a minimal winning coalition in the House or it loses in the House. A bare victory the Senate typically leaves the proposer short of winning the House. Intuitively, this implies that no minimum winning majority in the Senate is less desirable in the sense of requiring more than a minimum winning majority in the House. Thus, from a simple counting perspective, attracting a sufficient number of votes from the malapportioned chamber is not a binding constraint. Any small-state advantage in an SSSPE must therefore arise from variations in v_t or v_t^s across types.

To keep the analysis simple, we search for an equilibrium satisfying $v_t \geq v_t^s$ for all t. This relationship is obviously true for type $t = 1$, as $v_1 = v_1^s$. In the equilibrium that we identify, the inequality becomes strict for $t > 2$. We will show that this restriction produces a unique distribution of expected payoffs in the class of SSSPEs. We suspect that all SSSPEs satisfy this condition, but leave the question for future work.

This restriction links the chambers in the following, deterministic way: the senator from a state will support a bill if more than half of the state's representatives support it. So, to obtain the support of a state's senator, it is sufficient for proposers to pay a majority of its representatives their reservation values. The following result shows that in equilibrium, it is also *necessary*; that is, if $v_t^s < v_t$, no representative ever receives v_t^s.

Lemma. If $v_t \geq v_t^s \ \forall t$, then there is no optimal coalition in which $x_k \in (0, v_t)$ for any representative k of type t. ■

Our problem is therefore reduced to one of characterizing winning House coalitions that are drawn from the states in such a way as to include more than half of the representatives in more than half of the states. For each coalition $\mathbf{C} \subseteq \mathbf{N}$, let $v(\mathbf{C}) = \sum_{t \in \mathbf{T}} v_t n_t(\mathbf{C})$ be the total "cost" of $\mathbf{C}$. Clearly, $v(\mathbf{N}) = 1$. For each type-t representative, let $\underline{v}_t = \min_{\{\mathbf{C}|\mathbf{C}\cap\mathbf{N_t}\neq\emptyset, \mathbf{C}\in\mathcal{W}\}} v(\mathbf{C})$ be the minimum-value winning coalition for a type-t proposer (including herself). Then the minimum that a type-t proposer must pay her coalition partners is $\underline{v}_t - v_t$. Let $\mathcal{C}_t$ be the set of coalitions that solve the problem: $\min_{\{\mathbf{C}|\mathbf{C}\cap\mathbf{N_t}\neq\emptyset, \mathbf{C}\in\mathcal{W}\}} v(\mathbf{C})$. Thus, $\mathcal{C}_t$ is the set of "cheapest" coalitions for a type-t representative. At an SSSPE, each type-t proposer chooses some $\mathbf{C} \in \mathcal{C}_t$, offers v_r to all type-r representatives in $\mathbf{C}$ other than herself, offers 0 to all representatives outside $\mathbf{C}$, and keeps $1 - v(\mathbf{C}) + v_t = 1 - \underline{v}_t + v_t$ for herself.

For each type-t representative, let q_t be the average probability that the representative is chosen as a coalition partner, given that someone other than the representative is the proposer. Then the continuation value for a type-t representative satisfies

$$v_t = \frac{1}{n}(1-\underline{v}_t+v_t) + \frac{n-1}{n}q_t v_t.$$

Or,

$$v_t = \frac{1-\underline{v}_t}{(n-1)(1-q_t)}. \tag{1}$$

Proposition 1 and the Lemma provide sufficient leverage for us to identify the following, "unique" SSSPE in the bicameral bargaining game.[11]

Proposition 2. An SSSPE exists. Any SSSPE satisfies $v_t = 1/n$ for all t. ■

Thus, the House districts in large states will *not* have lower expected payoffs than the House districts in small states. Since the House districts are apportioned on a per-capita basis, voters in large states are worse-off than voters in small states if and only if the expected payoffs to the large-state House districts are smaller than the expected payoffs to the small-state House districts. Thus, the proposition says that voters in large states are *not* worse-off than voters in small states.

4. Extensions: Possible Sources of Small-State Bias

Proposition 2 above shows that when both chambers require simple majorities to pass bills and the Senate cannot propose or amend bills, over-representation in the Senate does not lead to a bias in expected allocations in purely distributive policy areas. That is, differences in voting power *per se* in one chamber do not automatically translate into differences in expected payoffs. Something else is required to explain distributive biases in favor of small states. In this section we consider three factors that can produce small-state biases: supermajoritarian requirements in the Senate, proposal power in the Senate, and "lumpy" distributive goods.

4.1. Supermajoritarian Rules

Supermajority rules, such as the cloture requirement in the U. S. Senate, can create small state biases. The intuition behind Proposition 2 is that the marginal value of a Senator

[11]Note that because of the restriction $v_t \geq v_t^s$, we cannot invoke the results of Banks and Duggan (2000) in establishing uniqueness.

is effectively zero: it is possible to build minimal winning coalitions in the House that guarantee a minimal winning coalition in the Senate. With supermajority rules in the upper chamber, the proposer may be forced to buy some small state Senators in order to clear the supermajority hurdle. The marginal value of small state Senators, then, becomes non-zero, and small states are able to extract additional payments for their legislative votes. The extreme case is when unanimity is required in the Senate. When all Senators must be in the coalition, money is divided equally among the states, but on a per capita basis this results in an unequal distribution of expenditures to people across states. Supermajority hurdles in the lower chamber, by assumption apportioned on the basis of population, lessen the small-state bias.

To simplify the analysis, we focus on a special case with two types of states, one type with a single district and the other with $k \geq 3$ districts. The total number of states is $m = m_k + m_1$, and the total number of districts is $n = km_k + m_1$.

Let $Q_S \geq (m+1)/2$ be the number of votes required to pass a bill in the Senate, and let $Q_H \geq (n+1)/2$ be the number required in the House. For simplicity, we assume $Q_H < m_1 + (km_k+1)/2$.[12] Also, let $r_k = \lfloor (km_k+1)/(k+1) \rfloor$, where $\lfloor a \rfloor$ is the greatest integer less than or equal to a. Then r_k is the maximum number of type-k senators a proposal can attract if the proposal attracts the votes of exactly $(km_k+1)/2$ type-k representatives.

Proposition 3. There is a bias in favor of small states—*i.e.*, $v_k < v_1$—if and only if $Q_S > r_k + Q_H - (km_k + 1)/2$. ■

Several comparative statics reveal the effects of supermajority hurdles on biases.

First, if Q_S is a simple majority of the Senate, then the necessary and sufficient condition for small state bias will not be met. If Q_H is a simple majority in the House, then a sufficiently high Q_S produces small state bias. At the extreme with unanimity rule in the Senate and simple majority rule in the House, the necessary and sufficient condition certainly holds.

Second, raising Q_H makes the necessary and sufficient condition more difficult to obtain. At the extreme where there is unanimity rule in both the House and the Senate then the

[12] This assumption is made strictly for convenience, to avoid the proliferation of subcases. Note that this approximates what many consider to be true of the U.S. Congress, in which only the Senate is clearly supermajoritarian because of the filibuster (*e.g.*, Krehbiel 1998).

necessary and sufficient condition cannot hold.

Third, raising r_k also makes the necessary and sufficient condition for small state bias more difficult to obtain. The term r_k is increasing in m and k. The intuition is that as m or k rises, the number of type-k states that are bought grows when buying exactly just over 1/2 of the type-k House members. In the limit, all type-k states are won. This makes small state Senators less vital to the coalition.

An example strengthens the intuition behind the result. Suppose there are four type-1 states and one type-3 state, with $Q_S = 4$ (out of 5) and $Q_H = 4$ (out of 7). Then $v_1 = 2/11$ and $v_3 = 1/11$. The strategies supporting this are: type-3 proposers always choose a coalition with one type-3 district and three type-1 districts, while type-1 proposers mix. They choose three type-1 partners with probability 1/4, and two type-1's and two type-3's with probability 3/4. In both cases, $\underline{v}_1 = 6/11$. Note that sometimes a "surplus" coalition is bought in the House.

4.2. Senate Proposal Power

In the situation studied in Proposition 2, only members of the well-apportioned chamber, the House, have proposal power. Small state bias can exist when proposals originate in the malapportioned chamber. In this case, all Senators are assumed to have equal proposal probabilities, and thus *proposal* power is maldistributed. When any Senator is chosen he or she builds a coalition of other Senators and House members and keeps the surplus to distribute among a majority of his or her own voters. Proposers will spend the same amount for a coalition as before. However, because a small state has a higher likelihood of making a proposal than if the legislation were initiated in the House, small states have higher expected returns.

Aside from Senate proposal power, we maintain all of the assumptions of the model in Sections 2 and 3. Additionally, as noted in the introduction, the analysis of Senate proposals also forces us to take a stand on how proposal power is distributed in the Senate. We will make the simplest assumption—that each senator has equal probability $(1/m)$ of being recognized to make a proposal. Also, we will assume that the House cannot make proposals or amendments, but simply passes or rejects proposals that pass the Senate. As a result, the likelihood that a small state senator is proposer is higher than that state's share of the population (House seats). The large disparity in proposal probabilities leads to a difference in expected payoffs in which small-state districts receive more than large-state districts.

Proposition 4. If proposals originate in the Senate and $n \geq t(2m+1)+2$, then $v_t > 1/n$. ■

The condition of the proposition is obviously most easily satisfied for small states, thereby implying that such states are the "first" to receive disproportionately large payoffs. This occurs because proposal power becomes more important as n increases (holding m constant), and because districts in small states capture more of the benefit of their senator being proposer. Even if small states are never included in a coalition, their proposal power alone can give their districts payoffs in excess of $1/n$, thus making the proportional equilibrium identified in Proposition 2 impossible.

The condition of the proposition also conveys an intuitive logic about the distribution of state sizes. Since $n \geq \underline{t}m$, where $\underline{t}$ is the size of the smallest state, the condition is essentially that less than half of the districts are in the smallest states. When the condition does not hold, the distribution of districts across states is relatively even. In this environment, Senate proposal power does not imply heavily disproportionate "recognition probabilities" for districts in small states, and so more proportional equilibrium payoffs are possible.

While the conditions of Proposition 4 are sufficient for maldistribution, they are not necessary. Consider a legislature with two type-1 states and one type-3 state; thus, $m = 3$, $n = 5$, and $\underline{t}(2m+1) + 3 = 10$. It is then easily demonstrated that at any SSSPE, the expected payoffs are approximately $v_1 = .270$ and $v_3 = .153$. So, the expected payoff in the small-state districts is much higher than that in the large-state districts.

4.3. Lumpy Distributive Goods

Many publicly funded distributive goods are not divisible down to the district level. Others produce benefits that spill over into other districts. Examples are inter-state highways, river navigation projects, large-scale irrigation and hydroelectric power projects, and intra-city highway, mass-transit or airport projects in large cities that contain several districts.[13]

[13] Lumpiness or spillovers may be important in practice. Ansolabehere, Snyder, and Woon (1998) study the public support for initiatives that sought to apportion that state's senate on the basis of area (county)

If the distributive goods divided by the legislature are "lumpy," then there will typically be a bias in favor of small states. An extreme case is where the distributive goods are not divisible within states. A model studied by Kalandrakis (n.d.) covers this case. In his model, legislators in the upper and lower chamber from any given state have identical utility functions. Thus, if one House district receives an amount x per capita, then the entire state containing that district must also receive x per capita.

A simple example provides the intuition about why this situation leads to a small-state bias. Suppose the distributive goods are completely divisible across states, but they are not divisible within a state.

Example. Suppose there are four type-1 states and one type-3 state; so $m = 5$ and $n = 7$. Then at any SSSPE, the expected payoffs are $v_1 = 1/6$ and $v_3 = 1/9$. So, the expected payoff in the small-state districts (type-1) is much higher than that in the large-state districts (type-3).

Because of the assumed indivisibility, there are just two sorts of minimal winning coalitions: those consisting of all four type-1 districts, and those consisting of two type-1 districts and all three type-3 districts. Note also that the indivisibility implies that $v_3 = v_3^s$.

When the type-3 senator is proposer, she always offers v_1 to two of the type-1 districts and the remainder is shared evenly by her own districts. The optimal proposals for the type-1 senators depend on the relative values of v_1 and v_3. If $3v_1 < v_1 + 3v_3$, then each type-1 proposer always offers v_1 to the other three type-1 districts, and keeps the rest for her own state (district). If $3v_1 > v_1 + 3v_3$, then each type-1 proposer always offers v_1 to one of the other type-1 districts and v_3 to each of the type-3 districts, and keeps the rest for her own state. If $3v_1 = v_1 + 3v_3$, then type-1 proposers are indifferent between offering v_1 to the other three type-1 districts, and offering v_1 to one of the other type-1 districts and v_3 to each of the type-3 districts. We show that this last condition must hold in equilibrium.

Suppose $3v_1 < v_1 + 3v_3$, that is, $2v_1 < 3v_3$. Then $v_1 = \frac{1}{5}[1 - 3v_1] + \frac{3}{5}v_1 + \frac{1}{5}(\frac{1}{2})v_1$ (the first term covers the case where the given type-1 senator is proposer, the second term covers the case where one of the other type-1 senators is proposer, and the third terms cover the case

rather than population. The patterns of voting suggest that the 10 counties around the San Francisco Bay area benefited from county-based representation in the Senate even though several of them would have lost seats. By contrast, Los Angeles County represented a similar geographic area, and had no spillovers.

where the type-3 senator is proposer); and $v_3 = \frac{1}{5}(\frac{1}{3})[1-2v_1] + \frac{4}{5}(0)$ (the first term covers the case where the type-3 senator is proposer, and the second term covers the case where a type-1 senator is proposer). Solving these two equations yields $v_1 = \frac{2}{9}$ and $v_3 = \frac{1}{27}$. But then $2v_1 = \frac{4}{9} > \frac{1}{9} = 3v_3$, contradicting the assumption that $2v_1 < 3v_3$.

Next, suppose $3v_1 > v_1 + 3v_3$, that is, $2v_1 > 3v_3$. Then $v_1 = \frac{1}{5}[1-v_1-3v_3] + \frac{3}{5}(\frac{1}{3})v_1 + \frac{1}{5}(\frac{1}{2})v_1$; and $v_3 = \frac{1}{5}(\frac{1}{3})[1-2v_1] + \frac{4}{5}v_3$. Solving these two equations yields $v_1 = 0$ and $v_3 = \frac{1}{3}$. But then $2v_1 = 0 < 1 = 3v_3$, contradicting the assumption that $2v_1 > 3v_3$.

Thus, at any SSSPE we must have $3v_1 = v_1 + 3v_3$, that is $2v_1 = 3v_3$. Let p be the probability that a type-1 proposer offers v_1 to the other three type-1 districts, and let $1-p$ be the probability a type-1 proposer offers v_1 to one of the other type-1 districts and v_3 to each of the type-3 districts. Then $v_1 = \frac{1}{5}[1-3v_1] + \frac{3}{5}[p + (1-p)(\frac{1}{3})]v_1 + \frac{1}{5}(\frac{1}{2})v_1$; and $v_3 = \frac{1}{5}(\frac{1}{3})[1-2v_1] + \frac{4}{5}(1-p)v_3$. Also, since none of the dollar is ever wasted, $4v_1 + 3v_3 = 1$. Solving these three equations yields $v_1 = 1/6$, $v_3 = 1/9$, and $p = 1/4$. (Note that $2v_1 = 3v_3$, as required.) Since $v_1 > v_3$, there is a bias in favor of small-state districts. ■

Lumpy public expenditure programs violate a key feature of Proposition 1. When goods are divisible, it is possible to build a minimal winning coalition in the lower house that guarantees a coalition in the upper house, so the marginal cost of a Senator to a coalition is zero. That is no longer the case with lumpy goods. The lumpy expenditure assumption makes the marginal cost of the large state Senator higher than the marginal cost of a small-state Senator. To buy a Senator from a state with, say, 3 House members, a proposer must pay the price of all 3 House members. If all members cost the same, then for the same cost, a proposer could buy 3 House members from small states and get 3 Senators from the small states. The cost of large states, then, must be higher than the cost of small states. Recognizing this, small states can command higher (per capita) prices for their membership in a coalition.

The "lumpy goods" and supermajority results offer insight about relaxing the assumption that Senators are responsive to their median voters (A2). So far we have assumed that a simple majority of districts (a threshold of 50 percent) is needed to gain a Senator's support for a bill. The lumpy good argument suggests that thresholds above 50 percent imply small state biases. The highest threshold occurs when a Senator will join a coalition only if a proposal distributes funds to all districts; that is a completely indivisible good. A threshold below 50 percent will weaken the pressures toward small state biases. Thinking about Proposition 2, if a Senator votes for a proposal that gives money to less than half of a state's House delegation, then it becomes even easier to build minimal winning coalitions in the House that guarantee a majority in the Senate.

5. Discussion

Geographic linkages across chambers in bicameral legislatures complicate distributive politics. Unlike unicameral politics, unequal representation in a bicameral legislature does not lead inexorably to unequal distributions of public expenditures. The need to win in both chambers tempers the importance of raw voting power in each chamber separately. When only voting power is unequal and when the lower house districts are nested within the geography represented by the upper house, then it is possible to form minimum winning coalitions entirely within the House without having to "pay extra" to get the Senate. Other inequities in political power must exist in a bicameral legislature – such as proposal power or supermajority requirements – in order to generate maldistribution of government expenditures.

Several interesting empirical predictions follow from our analysis. We consider three briefly.

First, the effects of malapportionment in bicameral legislatures on the distribution of public funds should depend on the extent to which each of the two chambers deviates from equal representation. The U.S. state legislatures prior to 1964 provide empirical support for this general pattern. Many state legislatures paralleled the federal system of representation, with counties being the analog of the states, but many were also malapportioned in both chambers. Ansolabehere, Gerber, and Snyder (2002) show that malapportionment of the state legislatures strongly affected the distribution of public spending. We analyzed the data they considered with an eye to the specific claim here. We regressed the share of state transfers received by counties on the counties' representation in the legislature (called the RRI Index) and other factors, all variables in logarithms. We tested for a differential effect of the RRI Index in states where both chambers were badly apportioned and found that there is a statistically significant interaction.[14]

Second, the U. S. Congress is an interesting test case not of the effects of malapportionment *per se*, as is sometimes argued, but of the effects of supermajority rules and unequal proposal power in the face of malapportionment. Consistent with the results in Section 4, Atlas, *et al.*, (1995) and Lee and Oppenheimer (1999) document that inequitable divisions of federal expenditures are a persistent and striking feature of American public finance.

Third, our results have implications for preferences over the choice of constitutions and legislative rules in federal systems. The filibuster in the U.S. Senate provides one important example. The U.S. Senate determines its own rules about the number of votes required to end debate. The number of votes required for cloture has varied over time, from two-thirds of the entire Senate to two-thirds of those present to three-fifths. Proposition 3 suggests that Senators from smaller states would favor more stringent requirements for cloture. In fact, this seems to be the case. Senator Harry Reid (D,NV) put it as follows:

> "Checks and balances has nothing to do with protecting a small state. Vetoes have nothing to do with it, unless you have the ear of the Chief Executive of this country. The filibuster is uniquely situated to protect a small state in population like Nevada." (Binder and Smith, 1997, p. 98)

Dozens of roll call votes have been taken on this issue over the years. Binder and Smith (1997) study these votes and find that, even after controlling for party, ideology, region, and other factors, there is a tendency for Senators from smaller states to favor more stringent requirements for cloture.[15]

[14]The badly apportioned states were the states with the lowest percent of the population required to elect a majority in both chambers. The coefficient on RRI (in logarithms) is .15 (se = .013) and the coefficient on the interaction term is .06 (se = .016), meaning the slope on representation in the malapportioned states is substantially larger (i.e., .21 = .15+.06). Other factors in the model are state fixed effects, county population, income, poverty, percent black, percent old, percent school aged, percent unemployed, percent Democratic.

[15]Actually, Binder and Smith argue that their evidence shows *weak* support for the hypothesis that small states favor the filibuster. In particular, they find a statistically significant positive effect of the "small-state" dummy in only 2 out of the 12 roll calls they examined. A closer look, however, shows that the effect is quite strong. First, in 10 out of 12 cases the effect is in the right direction—small state senators supporting a more stringent cloture rule. Second, they examine only a subset of all the roll calls on cloture reform. We pooled all of the relevant data for the post-World War II period—69 roll calls in all—and find a large and highly significant "small-state Senator" effect. (The details of this analysis are available upon request.)

These empirical patterns support the basic idea forwarded here – that malapportionment in bicameral legislatures depends not only on the constitution of the representative body but on the rules of the chambers. To make our basic point, we have focused on distributive politics under some restrictive assumptions, and a more extensive analysis will yield further insights.

One important class of extensions would examine situations where bills may be amended. Bills may be considered under an open rule, or one chamber may be allowed to make a counterproposal. The latter situation is considerably more complicated than our basic model, as it requires further assumptions about how the chambers resolve differences between them. Our intuition is that any (randomized) amendment power will tend to reduce the rents from being proposer, and spread benefits more equally across districts. However, the logic of different proposal powers across chambers might still result in maldistribution. Drawing on our analysis above, Senate amendment powers might work like a supermajority requirement, and therefore increase small-state bias. Likewise, House amendment powers would decrease small-state bias. One empirical implication would then be that small states prefer open rules in the Senate, while large states prefer open rules in the House.

A related model might also explore the role of conference committees, which are involved in most major U.S. legislation. Our results are suggestive of the likely outcomes under a conference committee procedure similar to that used in the U.S. Within such committees, proposals must "pass" *both* the House and Senate delegations on the committee separately. The resulting proposal must then pass each chamber. Modelling this situation would require non-trivial assumptions about the distribution of proposal power. However, conference committees on large money bills often have a preponderance of House members, so our basic assumption that proposal probabilities are proportional to population may be a reasonable approximation.[16]

A second type of extension would change the policy space. One alternative is to incorporate taxation schemes and project size into legislators' strategies. This would allow us to examine policy characteristics beyond the distribution of expenditures. Another alternative

[16]For example, the 1984 budget reconciliation bill conference committee was composed of 90 representatives and 32 senators, and the 1987 omnibus trade bill conference committee had 155 representatives and 44 senators.

is to add an ideological dimension to the simple division of the public dollar. Politicians may use public expenditures to “buy” votes in the ideological domain. Standard results suggest that moderates on the ideological dimension may command higher prices. That intuition might be altered in a bicameral setting.

A final generalization is to allow legislatures to determine the rules under which divide-the-dollar politics will occur. Diermeier and Myerson (1999) argue that the two chambers in a bicameral legislature will erect countervailing hurdles, thereby equalizing the institutional power of the chambers. In their model, legislators compete for interest group contributions, and roadblocks increase a chamber’s expected contributions. In our setting, hurdles have different effects across chambers and state sizes. Small-state senators and large-state representatives prefer supermajority requirements in their respective chambers, but large-state senators and small-state representatives do not. Thus, our model would not predict a continual escalation of legislative hurdles.

Appendix

Proof of Proposition 1. Expanding, $n(\mathbf{C}) = \sum_{t=1}^{T} m_t(\mathbf{C})(t+1)/2 = \frac{1}{2}\sum_{t=1}^{T} m_t(\mathbf{C})t + \frac{1}{2}\sum_{t=1}^{T} m_t(\mathbf{C}) = \frac{1}{2}\sum_{t=1}^{T} m_t(\mathbf{C})t + (m+1)/4$. Now, $(n+1)/2 = \frac{1}{2}\sum_{t=1}^{T} m_t t + \frac{1}{2}$, so $n(\mathbf{C}) \leq (n+1)/2$ if and only if $\frac{1}{2}\sum_{t=1}^{T} m_t(\mathbf{C})t + (m+1)/4 \leq \frac{1}{2}\sum_{t=1}^{T} m_t t + \frac{1}{2}$, or $(m-1)/2 \leq \sum_{t=1}^{T}[m_t - m_t(\mathbf{C})]t$. Now, $\sum_{t=1}^{T}[m_t - m_t(\mathbf{C})] = m - (m+1)/2 = (m-1)/2$, and $t \geq 1$ for all t, so the desired inequality holds. Moreover, the inequality is strict unless $m_t(\mathbf{C}) = m_t$ for all t with $t > 1$. Note that for each t, the term $m_t - m_t(\mathbf{C})$ is the number of type-t states that are *not* in $\mathbf{M}(\mathbf{C})$. So, the inequality is strict unless $\mathbf{M}(\mathbf{C})$ contains *all* states with $t > 1$. ■

Proof of Lemma. Suppose otherwise. This clearly implies the existence of a type t' such that representatives receive $v_{t'}^s < v_{t'}$ with positive probability. Let $\mathbf{W} \in \mathcal{W}$ represent an optimal winning coalition in which a type-t' representative, k', receives $x_{k'} = v_{t'}^s$. This implies three facts. First, $\mathbf{W}$ contains no "surplus" legislators of type t' (*i.e.*, all type t' states have either 0 or $d_{t'}$ representatives receiving $x_k > 0$). Second, $m(\mathbf{W}) = \frac{M+1}{2}$, for otherwise $\mathbf{W} \setminus \{k'\} \in \mathcal{W}$. Third, $m(\mathbf{W}) = \frac{M+1}{2}$ and Proposition 1 imply the existence of a type $t'' \neq t'$ that contains surplus legislators receiving v_t. In order for $\mathbf{W}$ to be optimal, all surplus legislators must be of the least expensive type, and so $t'' \in \{t | v_t = \min\{v_t\}\}$.

The existence of a type t'' surplus legislator implies that $v_{t''} + v_{t'}^s < v_{t'}$. Thus the proposer would replace as many type t' legislators with type t'' (or identically inexpensive) legislators as possible. There are two cases. First, some type t' legislators receive $v_{t'}$ and all type t'' legislators receive $v_{t''}$ (*i.e.*, $q_{t''} = 1$). This generates an obvious contradiction of (1). Second, no type t' legislators receive $v_{t'}$ (*i.e.*, $q_{t'} = 0$). Let ρ_t^s and ρ_t represent the equilibrium probabilities that a type-t senator receives v_t^s and v_t, respectively. Then $v_t^s = \rho_t^s v_t^s + \rho_t v_t$, and $q_{t'} = 0$ implies $\rho_{t'} = 0$, so $\rho_{t'}^s = 1$. Thus, every type-t' state has $d_{t'}$ representatives receiving $v_{t'}^s$ and $t' - d_{t'}$ representatives receiving 0. An identical argument applies for all types in $\mathbf{W}$ containing a representative who receives $v_t^s < v_t$.

Denote by $\mathbf{A}$ the set of all representatives receiving $v_t^s < v_t$. Then $\mathbf{W} = \mathbf{A} \cup \mathbf{B}$, where $n(\mathbf{A}) = 0$, $n(\mathbf{B}) \geq \frac{N+1}{2}$, $m(\mathbf{A}) > 0$, and $m(\mathbf{B}) < \frac{M+1}{2}$. Note that the number of surplus representatives must satisfy $\hat{s} < d_{t''}$, for otherwise a type-t'' senator could be bought and a senator from $\mathbf{A}$ dropped. So, other than these $\hat{s}$ representatives, all representatives in $\mathbf{B}$

must be among the d_t necessary to buy their state's senator. But since $m(\mathbf{B}) < \frac{M+1}{2}$, by Proposition 1 $n(\mathbf{B}) < \frac{N+1}{2}$: contradiction. Thus no coalition containing a payment of $v_t^s < v_t$ can be optimal. ■

Proof of Proposition 2. (Existence) For generality, we present a result that allows both odd and even types. To show existence, note that in such an equilibrium, Proposition 1 implies that exactly $(n-1)/2$ legislators receive v_t. Thus, $\underline{v}_t$ is constant across t, and expression (1) implies that q_t is also constant across t. Thus $q_t = 1/2$ for all types. It is sufficient to identify a proposal strategy for any proposer i such that, *ex ante*, $\Pr\{x_k = v_t | k \neq i\} = \Pr\{x_k = 0 | k \neq i\} = 1/2$, subject to the constraint that at least $(m+1)/2$ states receive at least v_t in more than half of their districts. Let i belong to state j of type $\tilde{t}$, and let $\mathcal{S}^j$ denote the set of states not including j. Consider any partition $\{\mathcal{S}_1, \mathcal{S}_2\}$ of $\mathcal{S}^j$ with the following properties: (i) $\mathcal{S}_1$ consists of one state from each type $t \neq \tilde{t}$ such that m_t is odd, and one from type $\tilde{t}$ if $m_{\tilde{t}}-1$ is odd. (ii) $\mathcal{S}_2 = \mathcal{S}^j \setminus \mathcal{S}_1$ (that is, a largest subset of $\mathcal{S}^j$ such that the number of states of each type is even). Note that $|\mathcal{S}_1|$ and $|\mathcal{S}_2|$ must both be even, and that $\mathcal{S}_1$ or $\mathcal{S}_2$ may be empty. We assign $x_k = v_t$ across districts as follows (all other districts receive $x_k = 0$):

1. In $\mathcal{S}_2$: for each type $t \neq \tilde{t}$, choose $m_t/2$ states if t is even, and $(m_t-1)/2$ states if t is odd, at random with equal probability. For type $\tilde{t}$, choose $m_{\tilde{t}}/2-1$ states if $\tilde{t}$ is even, and $(m_{\tilde{t}}-1)/2$ states if $\tilde{t}$ is odd, at random with equal probability. Assign $x_k = v_t$ to all representatives in these states.

2a. If $\tilde{t}$ is odd, in $\mathcal{S}_1$: the number of districts in $\mathcal{S}_1$ is even. Thus, there exists a partition $\{\mathcal{S}_1^l\}_{l=1,\ldots,|\mathcal{S}_1|/2}$ of $\mathcal{S}_1$ such that for each l, $\mathcal{S}_1^l$ contains a pair of states, both with either odd or even numbers of districts. Choose any such partition. For each $\mathcal{S}_1^l$, label the member states $j^{l'}$ and $j^{l''}$, of types $t^{l'}$ and $t^{l''}$ respectively, where $t^{l'} \leq t^{l''}$. With probability $1/2$, assign $x_k = v_t$ randomly to $(t^{l'}+t^{l''})/2$ representatives in state $j^{l''}$ with equal probability. With probability $1/2$, assign $x_k = v_t$ to all representatives in state $j^{l'}$ and randomly to $(t^{l''}-t^{l'})/2$ representatives in state $j^{l''}$ with equal probability.

   In state j: assign $x_k = v_t$ randomly to $(\tilde{t}-1)/2$ representatives with equal probability.

2b. If $\tilde{t}$ is even, in $\mathcal{S}_1$ and state j: the number of districts in $\mathcal{S}_1$ is odd. Thus, there exists a state $\hat{j}$ of type $\hat{t}$, where $\hat{t}$ is odd. Suppose that $\tilde{t}-1 \leq \hat{t}$. Then with probability

1/2, assign $x_k = v_t$ randomly to $(\hat{t}+\tilde{t}-1)/2$ representatives in state $\hat{j}$, with equal probability. With probability 1/2, assign $x_k = v_t$ to all representatives except i in state j and randomly to $(\hat{t}-\tilde{t}+1)/2$ representatives in state $\hat{j}$, with equal probability. A symmetrical result assignment is used for $\tilde{t}-1 > \hat{t}$. For the set of states in $\mathcal{S}_1 \setminus \{\hat{j}\}$, follow the procedure in step 2a for $\mathcal{S}_1$, replacing $\mathcal{S}_1$ with $\mathcal{S}_1 \setminus \{\hat{j}\}$.

Given this proposal strategy, and letting $v_t = 1/n$ for all t, it is easily verified that $q_t = 1/2$ and (1) holds for all types. Further, exactly $(m+1)/2$ states receive at least v_t in more than half of their districts. Finally, since $v_1^s \leq v_1$ trivially and only one district can receive strictly more than v_t, $v_t^s \leq v_t$ for all types, as required.

(Uniqueness of Expected Payoffs) To prove that $v_t = 1/n$ for all t, suppose to the contrary that there is an SSSPE with some type t such that $v_t \neq 1/n$. Without loss of generality, let $v_c = \min_t\{v_t\}$ and $v_e = \max_t\{v_t\}$. Clearly, $v_c < 1/n < v_e$. We show this leads to a contradiction.

Consider the set of representatives from states with $v_t = v_e$, and let $\mathbf{A}$ denote the representatives from the largest type in this set. Note two facts that follow from equation (1). First, if $q_t = 0$, then $v_t = (1 - \underline{v}_t)/n < 1/n$. Second, if $q_t = 1$, then $v_t = (1-\underline{v}_t) > 1/n$. There are two cases.

Case 1: $m_1 < (m-1)/2$. There are two subcases: (i) $m(\mathbf{A}) \leq (m-1)/2$ and (ii) $m(\mathbf{A}) \geq (m+1)/2$. In (i), a cheapest coalition always includes $(m+1)/2$ states in $\mathbf{N} \setminus A$ and none from $\mathbf{A}$. If $n(\mathbf{A}) < (n+1)/2$, then $q_t = 0$ and $v_e < 1/n$ for all representatives in $\mathbf{A}$: contradiction. If $n(\mathbf{A}) \geq (n+1)/2$, then $q_t = 1$ and $v_t > 1/n > v_c$ for all representatives in $\mathbf{N}\setminus A$: contradiction. So (ii) must hold. In this subcase a cheapest coalition always includes a minimum winning majority in all states with representatives in $\mathbf{N}\setminus A$, plus $(m+1)/2 - m(\mathbf{N}\setminus A)$ states from $\mathbf{A}$. Since cheapest coalitions include one or more representatives of each type, there exists a cheapest coalition for each proposer that includes herself; thus, $\underline{v}_t = \underline{v}$ for all t. Substituting into equation (1),

$$v_t = \frac{1-\underline{v}}{(n-1)(1-q_t)}.$$

Obviously, v_t is strictly increasing in q_t. The fact that a cheapest coalition always includes a minimum winning majority in all states with representatives in $\mathbf{N} \setminus A$ means that $q_t > 1/2$

for all representatives in $\mathbf{N} \setminus A$. Since cheapest minimum winning coalitions include exactly $(n+1)/2$ representatives, they never include more than half of representatives in $\mathbf{A}$; thus, $q_t < 1/2$ for all representatives in $\mathbf{A}$. This implies v_t is larger for representatives in $\mathbf{N} \setminus A$ than for those in $\mathbf{A}$: contradiction.

Case 2: $m_1 \geq (m-1)/2$. There are two subcases: (i) $\mathbf{A}$ consists of type 1 representatives, and (ii) $\mathbf{A}$ does not. In (i), if $m_1 = (m\text{–}1)/2$, then a winning coalition can be drawn from types $t \neq 1$. This implies $q_1 = 0$ and $v_1 < 1/n$: contradiction. If $m_1 > (m-1)/2$, then subcase (ii) of Case 1 applies. In (ii), if $m(\mathbf{A}) \leq (m-1)/2$ then $q_t = 0$ and $v_e < 1/n$ for representatives in $\mathbf{A}$: contradiction. Otherwise, $m(\mathbf{A}) = (m+1)/2$ so $q_1 = 1$ and $v_t > 1/n > v_c$ for all representatives in $\mathbf{N} \setminus A$: contradiction. ∎

Proof of Proposition 3. Suppose $Q_S > r_k + Q_H - (km_k+1)/2$ and $v_k \geq v_1$. We show this leads to a contradiction. If $v_k > v_1$, then $v_k > 1/n > v_1$. In this case, if $m_1 \geq Q_H$ and $m_1 \geq Q_S$, then $q_k = 0$, and so $v_k < 1/n$: contradiction. Otherwise, $q_1 = 1$ and thus $v_1 > 1/n$: contradiction.

Thus $v_k = v_1 = 1/n$. In this case, since there are only two types, there exists a least-cost coalition that includes both type-1 and type-k representatives, which implies that $\underline{v}_k = \underline{v}_1 = \underline{v}$. Since $(n+1)/2 \leq Q_H < n$, this implies $1/2 < q_k = q_1 < 1$. So, there exist optimal winning coalitions that do not contain all type-1 representatives. Any such coalition must have exactly Q_H representatives, and therefore costs $\underline{v} = Q_H/n$. Thus, *all* optimal winning coalitions must have exactly Q_H representatives (and cost $\underline{v} = Q_H/n$). Consider a coalition with Q_H representatives, at least $(km_k+1)/2$ of which are of type-k. This coalition can win no more than $r_k + Q_H - (km_k+1)/2$ senators, where $r_k = \lfloor (km_k+1)/(k+1) \rfloor$. (This is achieved by distributing $(km_k+1)/2$ payouts of v_k to $(k+1)/2$ districts in as many type-k states as possible. The remaining $Q_H - (km_k+1)/2$ districts are allocated to type-1 states.) Note that the assumption $Q_H < m_1 + (km_k+1)/2$ (made in the text) ensures that there are enough type-1 districts to make this distribution feasible. Thus, since $Q_S > r_k + Q_H - (km_k+1)/2$ by assumption, it is impossible to construct a coalition that contains exactly Q_H representatives, has at least $(km_k+1)/2$ members in type-k districts, and wins the Senate. So, $q_k < 1/2 < q_1$: contradiction. Thus, if $Q_S > r_k + Q_H - (km_k+1)/2$, then $v_k < v_1$.

Finally, when $Q_S \leq r_k + Q_H - (km_k+1)/2$, an argument analogous to Proposition 2 shows that $v_1 = v_k = 1/N$ (*i.e.*, no small state bias exists). ■

Proof of Proposition 4. Suppose otherwise (*i.e.*, $v_t \leq 1/n$). By the lemma, no representative receives an allocation in $(0, v_t)$. This implies that under an optimal allocation for any senator:

$$\frac{1}{n} \geq \frac{1}{m}\frac{d_t}{t}\frac{1}{d_t}\left[1 - \left(\frac{n+1}{2} - d_t\right)\frac{1}{n}\right] + \frac{m-1}{m}q_t\frac{1}{n}.$$

where q_t represents the average probability of being included in a coalition, conditional on not being the proposer. Collecting terms, this implies:

$$m \geq \frac{1}{t}\left[n - \left(\frac{n+1}{2} - d_t\right)\right] + (m-1)q_t,$$

and thus:

$$q_t \leq \frac{tm - (n-1)/2 + d_t}{t(m-1)}.$$

This implies $q_t < 0$, generating a contradiction, if $tm < (n-1)/2 - d_t$, or: $n > 2(tm + d_t) + 1$. Since $2d_t + 1 = t + 2$, the condition is satisfied if $n > t(2m+1) + 2$. If $m_1 \geq 1$, $v_1 > 1/n$ if $n > 2m+3$.

Note that by substituting equalities for the inequalities, the expressions above also imply that an equilibrium in which $v_t = 1/n$ for all t is impossible if $q_t > 1$. This occurs if $t > (n-1)/2 - d_t$, or: $n < 2(t + d_t) + 1$, which is satisfied if $n < 3t + 2$ for some t. ■

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