Direct and Indirect Representation
ONLINE APPENDIX

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This online appendix contains the proofs of theoretical results and supplementary tables for "Direct and Indirect Representation," published in the *British Journal of Political Science*.

1 Theoretical Results and Proofs

The first proof makes use of the following expression from the main text for the expected payoff of an incumbent legislator in party P_1 with tide cutpoint τ^n:

$$\frac{1}{L} + \frac{r(m_1^1)}{m_1^1} + \frac{1 - F(\tau^n)}{L} + \sum_{k=n}^{L} \left[F(\tau^{k+1}) - F(\tau^k)\right] \frac{r(k)}{k}. \tag{2}$$

Proposition 1 *(Multi-member districts) For any party p and district i, if $m_p^1 < 2(\theta - 1) - J$, then y^*_{pij} is weakly decreasing in j for incapacitated legislator C_{pij}.* ∎

Proof. As noted in the text, groups without incapacitated legislators have a weakly dominant strategy of supporting their candidate. We therefore focus without loss of generality on party P_1 and assume that a party P_1 legislator occupying the $\tilde{j}$-th seat in district $\tilde{i}$ is incapacitated. Thus the group backing this legislator is $G_{1\tilde{i}\tilde{j}}$.

Notationally, let the set of national cutpoints when all P_1 groups support their candidates, given the support strategies of P_2 groups, be denoted by $\{\tau^n\}$. Likewise, let the set of national tide cutpoints under the same support strategies, with the exception that $g_{1ij} = 0$, be denoted $\{\tau^{n'}\}$. Note that $\{\tau^n\} \cap \{\tau^{n'}\} \supseteq \cup_{i \neq \tilde{i}, j} \{\tau_i^j\}$; i.e., the incapacitation affects only tide cutpoints in district $\tilde{i}$.

For $G_{1\tilde{i}\tilde{j}}$, we derive the voting contract that induces the maximum payment from P_1 in $\mathbf{y}_1$. To do this, we find the maximum set-aside allocation $\mathbf{y}_1$ that a majority of non-incapacitated P_1 legislators would support.

We first claim that P_1 legislators not in district $\tilde{i}$ with tide cutpoints no higher than τ^θ are a majority of surviving P_1 legislators. This obviously holds if $m_1^1 \leq \theta$. For larger values of m_1^1, this holds if $\theta - J - 1 > m_1^1 - \theta + 1$, or $m_1^1 < 2(\theta - 1) - J$, as assumed.

Next, we derive the expected utilities of such legislators when $G_{1\tilde{i}\tilde{j}}$ withholds support. Consider a P_1 legislator not in district $\tilde{i}$ with tide cutpoint $\tau^n < \tau^\theta$. His/her expected utility is:

$$\frac{1}{m_1^1 + m_2^1} + \frac{r(m_1^1)}{m_1^1} + \frac{1 - F(\tau^n)}{L} + \sum_{k=1}^{L} \left[F(\tau^{k+1'}) - F(\tau^{k'})\right] \frac{r(k)}{k}. \tag{6}$$

Likewise, if P_1 proposes a set-aside allocation of y to $G_{1\tilde{i}\tilde{j}}$ and all P_1 groups provide support, then we can modify (2) to express this surviving legislator's expected utility as:

$$\frac{1}{m_1^1 + m_2^1} + \frac{r(m_1^1)}{m_1^1} - \frac{y}{m_1^1} + \frac{1 - F(\tau^n)}{L} + \sum_{k=1}^{L} \left[F(\tau^{k+1}) - F(\tau^k)\right] \frac{r(k)}{k}. \tag{7}$$

This legislator is then indifferent between the two expected payoffs if the set-aside y allocated to $G_{1\tilde{i}\tilde{j}}$ in $\mathbf{y}_1$ satisfies:

$$\frac{y}{m_1^1} = \sum_{k=1}^{L} \left[F(\tau^{k+1}) - F(\tau^k)\right] \frac{r(k)}{k} - \sum_{k=1}^{L} \left[F(\tau^{k+1'}) - F(\tau^{k'})\right] \frac{r(k)}{k} \tag{8}$$

$$
\begin{aligned}
&= \sum_{k=1}^{L}\Big[F(\tau^{k+1}) - F(\tau^{k}) - F(\tau^{k+1'}) + F(\tau^{k'})\Big]\,\frac{r(k)}{k} \\
&= \Big[F(\tau^{L+1}) - F(\tau^{L+1'})\Big]\,\frac{r(L)}{L} + \sum_{k=2}^{L}\Big[F(\tau^{k'}) - F(\tau^{k})\Big]\left(\frac{r(k)}{k} - \frac{r(k-1)}{k-1}\right) \\
&\quad + \Big[F(\tau^{1'}) - F(\tau^{1})\Big]\,r(1) \\
&= \sum_{k=\theta}^{L}\Big[F(\tau^{k'}) - F(\tau^{k})\Big]\left(\frac{r(k)}{k} - \frac{r(k-1)}{k-1}\right). \qquad (9)
\end{aligned}
$$

Note that the last line follows from the fact that $r(k) = 0$ for $k < \theta$.

Since P_1 legislators not in district $\tilde{i}$ with tide cutpoints below τ^{θ} are a majority of surviving P_1 legislators, the per capita set-aside that would induce indifference among surviving P_1 legislators is given by (9). We denote the total set-aside value y^{med}.

To establish the equilibrium set-aside and voting strategies, observe that a majority of non-incapacitated party P_1 legislators will vote in favor of any set-aside proposal satisfying $y_{1\tilde{i}\tilde{j}} \leq \min\{y^{med}, 1{+}r(m_1^1)/m_1^1\}$, if that proposal assures that $g_{1\tilde{i}\tilde{j}} = 1$. $G_{1\tilde{i}\tilde{j}}$ therefore optimally offers the contract: $g_{1\tilde{i}\tilde{j}} = 1$ iff $y_{1\tilde{i}\tilde{j}} \geq \min\{y^{med}, 1{+}r(m_1^1)/m_1^1\}$.

P_1 therefore proposes $\mathbf{y}_1^*$ with: $y^*_{1\tilde{i}\tilde{j}} = \min\{y^{med}, 1{+}r(m_1^1)/m_1^1\}$ and $y^*_{1ij} = 0$ for all $(i,j) \neq (\tilde{i},\tilde{j})$. Expression (9) implies that a majority of surviving P_1 legislators prefer $\mathbf{y}_1^*$ to $\mathbf{y}_1 = \mathbf{0}$. As a result, $\mathbf{y}_1^*$ passes and all P_1 groups support their candidates in equilibrium. By symmetry, all P_2 groups support their candidates when all P_1 groups support their candidates. Thus there is a unique subgame perfect equilibrium in which all $g_{pij} = 1$ for all groups.

We now examine comparative statics on $y^*_{1\tilde{i}\tilde{j}}$. For each possible $\tilde{j}$, let $\tau^{n'}(\tilde{j})$ denote the n-th tide cutpoint given $g_{1\tilde{i}\tilde{j}} = 0$ and $g_{1ij} = 1$ for all $(i,j) \neq (\tilde{i},\tilde{j})$. Also, let $\mathcal{T}(\tilde{j}) \equiv \{\tau^{n'}(\tilde{j}) \mid \tau^{n'}(\tilde{j}) \neq \tau^{n}\}$ denote the set of tide cutpoints for electing n legislators that change if $G_{1\tilde{i}\tilde{j}}$ does not support its candidate.

We claim that $\mathcal{T}(\tilde{j}) \subseteq \mathcal{T}(\tilde{j}{-}1)$. To show this, observe first that tide cutpoints for electing each legislator not in district $\tilde{i}$ are independent of $\tilde{j}$. Next, in district $\tilde{i}$, the tide required to achieve office for candidate k is $\tau_{\tilde{i}}^{k-1'}$ for all $k > \tilde{j}$, and remains at $\tau_{\tilde{i}}^{k}$ for all $k < \tilde{j}$. Thus for any $\tilde{j}$, $\mathcal{T}(\tilde{j})$ contains only tide cutpoints satisfying $\tau^{n'}(\tilde{j}) \geq \tau_{\tilde{i}}^{\tilde{j}-1'}$. Furthermore, the set of induced tide cutpoints in $\mathcal{T}(\tilde{j})$ with a value of at least $\tau_{\tilde{i}}^{\tilde{j}-1'}$ is independent of whether the group associated with candidate $1,\ldots,\tilde{j}$ withholds support. Since exactly one candidate does not have its group's support, the rank ordering of all such tide cutpoints is also independent of $\tilde{j}$, and hence for all $\tau^{n'}(\tilde{j}) \geq \tau_{\tilde{i}}^{\tilde{j}-1'}$, we have $\tau^{n'}(\tilde{j}) = \tau^{n'}(\tilde{j}{-}1) = \cdots = \tau^{n'}(1)$, which implies the claim.

Now since $\tau^{n'} \geq \tau^{n}$ for all n, with the inequality strict for some n, the claim expression (9) then implies that y^{med} is weakly decreasing in $\tilde{j}$. We conclude that $y^*_{1\tilde{i}\tilde{j}'}$ is weakly decreasing in $\tilde{j}$ for incapacitated legislator $C_{1\tilde{i}\tilde{j}}$ within district $\tilde{i}$. ■

Proposition 2 *(Single-member districts) For $J = 1$, any party p and district i, if $m_p^1 < 2\theta - 3$, then y^*_{pi1} is increasing in ρ_{pi1} for incapacitated legislator C_{pi1}.* ■

Proof. As in the proof of Proposition 1, we focus without loss of generality on an incapacitated party P_1 legislator in district $\tilde{i}$. We show that $G_{1\tilde{i}1}$'s demand, given by expression (8), is increasing in $\rho_{p\tilde{i}1}$, holding $\rho_{p\tilde{i}1} + \rho^0_{\tilde{i}}$ constant. As in the proof of Proposition 1, the condition $m^1_1 < 2\theta - 3$ ensures that a majority of surviving P_1 members have tide cutpoints no higher than τ^θ. We consider two values of $\rho_{1\tilde{i}1}$, where $\hat{\rho}_{1\tilde{i}1} > \overline{\rho}_{1\tilde{i}1}$, inducing corresponding tide cutpoints $\{\hat{\tau}^k\}$ and $\{\overline{\tau}^k\}$ when $G_{1\tilde{i}1}$ supports $C_{1\tilde{i}1}$, and $\{\hat{\tau}^{k'}\}$ and $\{\overline{\tau}^{k'}\}$ otherwise.

First we show that $\sum_{k=1}^L \left[F(\tau^{k+1'}) - F(\tau^{k'})\right] r(k)/k$ is decreasing in $\rho_{p\tilde{i}1}$. We begin by showing that if $\hat{\tau}^{k'} \geq \overline{\tau}^{k'}$ for all k, then:

$$\sum_{k=1}^{L} \left[F(\hat{\tau}^{k+1'}) - F(\hat{\tau}^{k'})\right] \frac{r(k)}{k} < \sum_{k=1}^{L} \left[F(\overline{\tau}^{k+1'}) - F(\overline{\tau}^{k'})\right] \frac{r(k)}{k}. \tag{10}$$

Rearranging terms, this expression can be rewritten as:

$$\begin{aligned} 0 \quad > \quad & \sum_{k=1}^{L} \left[(F(\overline{\tau}^{k'}) - F(\hat{\tau}^{k'})) - (F(\overline{\tau}^{k+1'}) - F(\hat{\tau}^{k+1'}))\right] \frac{r(k)}{k} \\ = \quad & \left[F(\overline{\tau}^{1'}) - F(\hat{\tau}^{1'})\right] r(1) + \sum_{k=2}^{L} \left[F(\overline{\tau}^{k'}) - F(\hat{\tau}^{k'})\right] \left(\frac{r(k)}{k} - \frac{r(k-1)}{k-1}\right) \\ & - \left[F(\overline{\tau}^{L+1'}) - F(\hat{\tau}^{L+1'})\right] \frac{r(L)}{L}. \end{aligned}$$

It is clear that since $r(k)/k$ is increasing in k and $F(\overline{\tau}^{L+1'}) = F(\hat{\tau}^{L+1'}) = 1$, the right-hand side of the above expression is negative when $\hat{\tau}^{k'} \geq \overline{\tau}^{k'}$ for all k. Thus, (10) holds.

Now it is sufficient to show that $\hat{\rho}_{1\tilde{i}1} > \overline{\rho}_{1\tilde{i}1}$ implies $\hat{\tau}^{k'} \geq \overline{\tau}^{k'}$ for all k. Note first that for all districts besides $\tilde{i}$, groups support their candidates in equilibrium. Thus the only tide cutpoint affected by a change in $\rho_{1\tilde{i}1}$ is $\tau^{1'}_{\tilde{i}}$. This implies that it is sufficient to show that $\hat{\tau}^{1'}_{\tilde{i}} \geq \overline{\tau}^{1'}_{\tilde{i}}$. The condition for the P_1 candidate to win when its group withholds support is $\tau\rho^0_{\tilde{i}} > \rho_{2\tilde{i}1} + (1-\tau)\rho^0_{\tilde{i}}$, or equivalently $\tau > 1/2 + \rho_{2\tilde{i}1}/(2\rho^0_{\tilde{i}})$. Since the right-hand side of this expression is decreasing in $\rho^0_{\tilde{i}}$, $\tau^{1'}_{\tilde{i}}$ is increasing in $\rho_{1\tilde{i}1}$.

By an analogous argument, $\sum_{k=1}^L \left[F(\tau^{k+1}) - F(\tau^k)\right] r(k)/k$ is increasing in $\rho_{p\tilde{i}1}$. (This follows from the observation that $\hat{\tau}^k \leq \overline{\tau}^k$ for all k.) Thus $G_{1\tilde{i}1}$'s demand is increasing in $\rho_{p\tilde{i}1}$. ∎

2 Supplementary Tables

In Table A1, we present the results when we estimate equation (5) without including the three additional covariates. The results are substantively similar to the main results in Table 4.

Table A1
Transfers to Municipality Governments
Excluding Additional Covariates
1977 to 1992

	National Treasury Disbursements			Local Allocation Tax		
LDP Died	−0.00	−0.00	−0.00	−0.01	−0.01	−0.01
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
LDP Died *Jiban*	−0.03	0.02	0.02	0.01	−0.00	−0.02
	(0.04)	(0.05)	(0.06)	(0.03)	(0.03)	(0.04)
LDP Died *"Weak" Jiban*		−0.11†	−0.11†		0.02	0.03
		(0.06)	(0.06)		(0.03)	(0.04)
LDP *Jiban* Not Passed On			−0.00			0.06
			(0.10)			(0.04)
Non-LDP Died	−0.01	−0.01	−0.01	0.00	0.00	0.00
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Non-LDP Died *Jiban*	−0.08	−0.08	−0.08	−0.00	−0.00	−0.00
	(0.12)	(0.12)	(0.12)	(0.05)	(0.05)	(0.05)
Observations	50051			50008		

Municipality×legislative session fixed-effects and year fixed-effects are included in all regressions. Standard errors are clustered by district×legislative session.

† indicates statistical significance at the 10% level.

In Table A2, we present the results from focusing only on those districts with a deceased representative. Again the substantive findings do not appear to be significantly affected. Excluding year fixed effects does affect the statistical significance, but we know that the total amount of national treasury disbursements does appear to trend over the period we are examining.

Table A2
Transfers to Municipality Governments
Restricting the Sample to Districts with Deceased Legislators
1977 to 1992

	National Treasury Disbursements			Local Allocation Tax		
LDP Died	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
	(0.04)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)
LDP Died *Jiban*	−0.03	0.02	0.02	0.02	0.01	0.00
	(0.04)	(0.05)	(0.07)	(0.03)	(0.04)	(0.05)
LDP Died *"Weak" Jiban*		−0.12†	−0.11		0.01	0.02
		(0.06)	(0.07)		(0.04)	(0.05)
LDP *Jiban* Not Passed On			0.01			0.03
			(0.10)			(0.05)
Non-LDP Died	−0.00	−0.00	−0.00	0.04	0.04	0.04
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
Non-LDP Died *Jiban*	−0.08	−0.08	−0.08	0.00	0.00	0.00
	(0.11)	(0.11)	(0.11)	(0.05)	(0.05)	(0.05)
Observations	3499			3498		

Municipality×legislative session fixed-effects and year fixed-effects are included in all regressions. Standard errors are clustered by district×legislative session.

† indicates statistical significance at the 10% level.

In Table A3, we examine three alternative measures of *jiban* strength. In the first column, *jiban* strength is determined by the last election. If a representative came in last place in the election, then he/she may be considered to have a "weak" jiban during the next legislative session. The measure used in the second column satisfies the criteria described in Subsection 4.1.1 with the exception that the first election after a representative passes away is not dropped. Finally, the measure in the last column uses the same criteria described in Subsection 4.1.1 but only includes the elections in the fifteen years prior to when the representative passes away. The results in Table A3 suggest that the substantive findings remain statistically significant for each of these measures.

Table A3
National Treasury Disbursements to Municipality Governments
Varying the Measure of "Weak" *Jiban*
1977 to 1992

	Last Place Previous Election	+/− 10 Yrs Include Next Election	+/− 10 Yrs Exclude Next Election	Previous 15 Yrs
LDP Died	−0.01	−0.01	−0.01	−0.01
	(0.02)	(0.02)	(0.02)	(0.02)
LDP Died *Jiban*	0.01	0.09	0.04	0.02
	(0.04)	(0.06)	(0.06)	(0.06)
LDP Died *"Weak" Jiban*	−0.14*	−0.17*	−0.12*	−0.09
	(0.06)	(0.06)	(0.06)	(0.07)
LDP *Jiban* Not Passed On		−0.08	−0.02	−0.01
		(0.11)	(0.11)	(0.12)
Non-LDP Died	−0.02	−0.02	−0.02	−0.02
	(0.03)	(0.03)	(0.03)	(0.03)
Non-LDP Died *Jiban*	−0.07	−0.07	−0.07	−0.07
	(0.12)	(0.12)	(0.12)	(0.12)
ln (Dep Pop / Pop)	0.79*	0.78*	0.78*	0.78*
	(0.31)	(0.31)	(0.31)	(0.31)
ln (Prim Ind Workers/Workers)	0.07	0.07	0.07	0.07
	(0.10)	(0.10)	(0.10)	(0.10)
ln (fiscal strength)	−0.33*	−0.33*	−0.33*	−0.33*
	(0.04)	(0.04)	(0.04)	(0.04)
Observations	48273			

Municipality×legislative session fixed-effects and year fixed-effects are included in all regressions. Standard errors are clustered by district×legislative session.

* indicates statistical significance at the 5% level.

The results are somewhat sensitive to the vote thresholds used to identify the *jiban*. In Table A4, we present the results varying the vote share thresholds used to classify a municipality as being part of a *jiban*. In first column, municipalities in the five- and six-member districts are considered to be part of a candidate's *jiban* if that candidate receives more than 15% of the vote. In the three- and four-member districts, the threshold for being part of a *jiban* are 25% and 20% respectively. The threshold increases by 2% in the subsequent columns. The estimates in the table demonstrate how the significance of the main finding presented Table 4 declines as the vote share thresholds used to identify the *jiban* are increased or decreased by 5 or more percentage points.

Table A4
National Treasury Disbursements to Municipality Governments
Varying the Threshold for *Jiban* Identification
1977 to 1992

	15-20 -25	17-22 -27	19-24 -29	21-26 -31	23-28 -33	25-30 -35
LDP Died	−0.02	−0.01	−0.01	−0.01	−0.01	−0.01
	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)
LDP Died *Jiban*	0.06	0.03	0.04	0.04	0.05	0.02
	(0.04)	(0.05)	(0.05)	(0.05)	(0.06)	(0.07)
LDP Died *"Weak" Jiban*	−0.11*	−0.11*	−0.13*	−0.16*	−0.17*	−0.13
	(0.04)	(0.05)	(0.05)	(0.07)	(0.07)	(0.09)
LDP *Jiban* Not Passed On	−0.03	0.02	−0.02	−0.06	−0.09	−0.08
	(0.06)	(0.10)	(0.12)	(0.11)	(0.13)	(0.15)
ln (Dep Pop / Pop)	0.78*	0.77*	0.78*	0.78*	0.78*	0.77*
	(0.31)	(0.31)	(0.31)	(0.31)	(0.31)	(0.31)
ln (Prim Ind Work/Work)	0.07	0.07	0.07	0.07	0.07	0.07
	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
ln (fiscal strength)	−0.33*	−0.33*	−0.33*	−0.33*	−0.33*	−0.33*
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
Observations	48273					

Municipality×legislative session fixed-effects and year fixed-effects are included in all regressions. Standard errors are clustered by district×legislative session.

* indicates statistical significance at the 5% level.