Politics and Administration*

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Abstract
This paper develops a theory of the administration and effectiveness of government programs. In the model, a bureaucrat chooses a mechanism for assigning a good to clients with uncertain qualifications. The mechanism applies a costly means test to verify the client’s eligibility. A politician exercises oversight by limiting the bureaucrat’s testing resources and the number of clients to be served. The model predicts the incidence of common administrative pathologies, including inefficient and politicized distribution of resources, inflexibility, program errors, and backlogs. When the politician favors marginally qualified clients, per capita spending is low and high error rates are high. When the politician favors highly qualified clients, per capita spending is higher and error rates are lower. In the latter case the bureaucrat may also use discriminatory testing, which allows the politician to target favored clients. Such targeted programs increase budgets and reduce backlogs, but also increase error rates.

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1 Introduction

What determines the quality of public administration? In both developing and advanced
countries, the ability of the bureaucracy to deliver on stated policy goals is considered a
key component of the overall quality of governance. Accordingly, there are today many ef-
forts to measure cross-national, national and sub-national government performance. In the
United States, every recent presidential administration and several states have implemented
performance measurement initiatives. Independent interest groups and non-government or-
ganizations have also compiled numerous well-known measures of government performance.

The question raises a host of theoretical issues, as bureaucracies can produce complex
outputs and are furthermore embedded in agency relationships with politicians. Perhaps
the predominant perspective comes from an extensive family of delegation models, where a
principal trades off between policy goals and some (possibly endogenous) capability possessed
by the agency. This capability might be policy expertise (e.g., Epstein and O’Halloran 1994,
Huber and Shipan 2002), uncertainty reduction (Huber and McCarty 2004), or valence (Ting
2011, Hirsch and Shotts 2012). Other work disaggregates the bureaucracy by focusing on the
incentives and abilities of government personnel, particularly in the presence of civil service
rules (Horn 1995, Rauch 1995, Gailmard and Patty 2007). Finally, a few models address the
effects of bureaucratic structure on the distribution of Type I and Type II errors (Heimann
1997, Carpenter and Ting 2007).

This paper takes a different approach and develops a simple theory of public adminis-
tration “on the ground.” The rationale is elementary: to date, there have been few efforts
at modeling the tangible activities and outcomes of government agencies. Thus it can be
difficult to connect political variables with important administrative outputs. Beyond mod-
eling basic bureaucratic tasks, the objective is to characterize what a political principal can
achieve when it has only crude controls over the bureaucracy’s resources and authority. The
theory is potentially applicable to any environment where professional public administrators
issue politically consequential judgments. As the following examples illustrate, such settings
can be highly significant.

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1 The 1993 Government Performance and Results Act and GPRA Modernization Act of 2010 require
agencies to state objectives, develop performance metrics, and report performance in a standardized fashion.
The Bush administration developed the Performance Assessment Rating Tool in 2002 (since discontinued)
to analyze the execution of individual programs.

2 Sources include the International Country Risk Guide, Business International, the World Economic
Forum, and the World Value Survey. One measure, the World Bank’s World Governance Indicators,
([http://www.govindicators.org](http://www.govindicators.org)), combines the results of many existing data sources.
1. Disability Insurance. In the early 1980s, the Reagan administration aggressively enforced policies that required federal disability insurance recipients to undergo regular re-determinations of eligibility. These actions were motivated in part by perceptions of widespread fraud in the program, but they were accompanied by neither changes in the program’s eligibility requirements and benefits, nor improvements in the Social Security Administration’s administrative capacity. Consequently, the administrative courts that heard appeals of eligibility rulings saw untenable backlogs that ultimately led to litigation and significant program changes (Derthick 1990).

2. Tax-Exempt Organizations. Under Section 501(c)(3) of the U.S. Internal Revenue Code, charitable organizations can apply to the Internal Revenue Service’s Exempt Organizations (EO) division for tax-exempt status. According to a U.S. Government Accountability Office (2014) report, the EO division maintained a constant caseload following the Republican congressional victories in 2010, but capacity and enforcement suffered in all other respects. The number of EO employees dropped from 889 in fiscal year 2010 to 842 in 2013. Between 2011 and 2013, the examination rate of applications dropped from 0.81% to 0.71%, revocations of tax exempt status fell from 258 to 149, and the percentage of cases resulting in either no change in status or only a written advisory rose from 44% to 46%.

3. Prosecutorial Discretion. The Obama administration has made broad use of prosecutorial discretion at the U.S. Department of Justice to achieve its social policy goals. In 2012, it implemented the Deferred Action for Childhood Arrivals program, which granted temporary deportation relief to young undocumented immigrants who met certain eligibility criteria. In 2013, it announced that it would no longer bring charges that invoked mandatory minimum sentences for certain types of non-violent drug offenders. Both programs introduced considerable discretion to previously inflexible policies.

These examples are suggestive of the set of administrative issues that arise when bureaucrats make allocation choices that affect the welfare of clients in society. While the list of such phenomena may be quite long, the following items capture a few of the most widely recognized pathologies.

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3The EO division was also implicated in a 2013 controversy over the alleged targeting of ‘Tea Party’ groups. Such groups fall under Section 501(c)(4) of the Internal Revenue Code, which governs social welfare organizations. The division’s strategy was to scrutinize applications based on keyword matches with group names, which were later revealed to have triggered reviews of both liberal and conservative groups.
• Backlogs — Potential recipients are unreached.

• Inflexibility — Different types of clients are treated identically.

• Resource constraints — Inadequate resources per case.

• Politicization — Politically preferred groups receive greater program benefits.

• Errors — Goods are allocated with Type I and Type II errors.

Absent from this list is corruption. While corruption certainly ranks among the most important organizational failures, the topic has spawned an extensive literature. More importantly, the perception of bureaucratic failures persists even in wealthy democracies where corruption is considered relatively rare. The model therefore focuses on environments where efficiency and competence rather than corruption are the primary concerns about bureaucratic performance.

To address these pathologies, the model starts from four assumptions. First, there is adverse selection: bureaucratic allocation problems arise because it is not clear which clients are most deserving of some benefit. Second, the bureaucrat is a professionalized expert who can discern the appropriateness of allocations and establish procedures governing how allocations are made. She also has idiosyncratic preferences over the importance of approving deserving clients and denying underserving ones. These may reflect political preferences, external career incentives (e.g., Che 1995), or behavior patterns inherited from organizational history (e.g., Levitt and March 1988). Third, the resources for applying expertise are endogenous. This is the main source of a political principal’s control over the bureaucrat. Fourth, the principal has distributional preferences and does not maximize social welfare. A natural approach for this setting is therefore to treat the bureaucrat as a mechanism designer who faces controls over parameters of her mechanism from a political principal.

More specifically, in the model a large number of clients apply in sequence to a bureaucrat to receive a good or avoid a cost. Examples include a means tested welfare benefit, resident status from an immigration agency, avoidance of prosecution, or a research grants from a scientific funding agency. Each client has a private type, which determines both her valuation of the good and the effect of a means test. The test embodies the legal and procedural requirements for receiving the good, and thus the client receives the good if and only if she passes. Types are either qualified and marginal, depending on whether the bureaucrat

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prefers to accept or reject them. Qualified types are more likely to pass as testing increases, while marginal types are more likely to fail as testing increases. The bureaucrat administers the allocation of the good by committing to a screen that assigns announced client types to an examination level, subject to a budget constraint.

A politician oversees this procedure by setting the bureaucrat’s total testing budget and client population. These instruments are crude because they do not allow the principal to steer resources directly toward particular types. Instead, they reduce the bureaucrat’s discretion to a choice over the distribution of means testing intensity across client types. For example, a high budget and low enrollment generate high average per capita testing, but allows the bureaucrat some flexibility in meeting the average testing level. The politician cares about the welfare of one type in society, and also bears the cost of testing. In a separation of powers systems such as the U.S., the politician roughly corresponds to a legislature, which has the statutory ability to define aggregate program enrollment and budgets but must confront bureaucrats who are appointed by an independent executive.

The basic model makes several predictions about trade-offs between coverage and quality of service. A first intuition is that the bureaucrat’s ability to discriminate across types — her flexibility — will be highly limited. This follows from a crucial difference between this model and standard screening problems. In the latter, the screen designer typically adjusts transfers to each type in order to give incentives for agents to truthfully reveal their information. In the public sector context, these payments can be interpreted either as corruption or “red tape.” Here, however, the bureaucrat cannot impose transfers, and thus she cannot easily discriminate across types by focusing testing on types that are expected to produce the greatest return. Truthful revelation then requires that similar types receive identical testing levels. In particular, all qualified types must be tested at a uniform level, while all marginal types must be tested at another uniform level that is no higher. Any other configuration would give an incentive for some client type to misrepresent her type. One extension to the model illustrates how red tape can be used in limited ways to improve the bureaucrat’s performance.

The principal’s distributive preferences generate two broad categories of program implementation. First, if his preferred client type is marginal, then the simple solution is to “starve” the bureaucrat. If the under-resourced bureaucrat is sufficiently likely to approve the principal’s favored type, then the result is broad service: these clients collectively benefit from wide eligibility, maximized acceptance probabilities, and minimized testing costs. This outcome features widespread bureaucratic errors, and performance is independent of
the bureaucrat’s preferences.

Second, if the principal’s preferred client type is qualified, then he has an incentive to pay for testing. Programs will then generate high per capita spending and lower error rates, without necessarily reducing the client population. Within this implementation category, bureaucratic preferences matter. The key condition is whether the bureaucrat is acceptance biased or rejection biased, where the former prefers rewarding qualified types over depriving marginal types and the latter prefers the reverse. A rejection biased bureaucrat must test all types identically, because incentive compatibility rules out testing marginal types at a higher level than qualified types. This generates high per capita costs and limits the program’s client population, which may create backlogs. It also severely limits the politician’s ability to target his favored group.

By contrast, an acceptance biased bureaucrat can be flexible by testing qualified types at a higher level. This enables some targeting, which the principal exploits by setting a larger budget in order to reach a larger population (i.e., reduce backlogs). This discrimination reduces per capita spending and in turn causes more Type I errors, as marginal types become more likely to receive the good. Thus, two somewhat counter-intuitive implications emerge: testing discrimination is associated with higher error rates, and higher budgets are associated with higher client populations but lower per capita spending. In the language of public administration scholarship, the flexible administration induced by acceptance bias might be termed “responsive competence,” while the inflexible administration induced by rejection bias might be termed “neutral competence” (Moe 1985).

The theory shares a number of features with and is inspired in part by Banerjee’s (1997) seminal article on government corruption. In both models, a bureaucrat designs a mechanism to allocate scarce goods to a population of clients with private valuations. However, the mechanism in the Banerjee model includes the price the bureaucrat charges to each type, as well as “red tape” that is costly to both the bureaucrat and the client. The overseeing politician punishes the bureaucrat when a mechanism is found to be improper, for example due to excessively high prices (i.e., bribes). By contrast, my model does not incorporate corruption and instead focuses on the distortions created by politically motivated principals and resource-constrained agents.

This paper joins a number of literatures related to the administration of government policies. In addition to Banerjee (1997), other models of government corruption and red tape also use related mechanism design or screening approaches (e.g., Laffont and N’Guessan 1999, Guriev 2004, Banerjee, Hanna, and Mullainathan 2012). Baron (2000) and Antic and
Iaryczower (2015) develop screening models of ideological oversight, and Gailmard (2009) examines bureaucratic oversight with multiple principals. Finally, a number of other models consider errors by a bureaucrat who assesses client applications (e.g., Prendergast 2003, Leaver 2009).

More generally, theoretical and empirical studies of government quality have focused heavily on corruption (Besley and McLaren 1993, Shleifer and Vishny 1993, Rose-Ackerman 1999, Svensson 2005, Bandiera, Prat, and Valletti 2009). However, a collection of empirical studies has developed or used administrative quality measures that are not based on corruption as either dependent or independent variables (e.g., Knack and Keefer 1995, La Porta et al. 1999, Rauch and Evans 2000, Krause, Lewis, and Douglas 2006, Bertelli and John 2010). In the American context, numerous studies have examined the links between administrative quality and political control of the bureaucracy (e.g., Moe 1989, Derthick 1990, Lewis 2008, Moynihan, Herd, and Harvey 2015). Finally, the links between policy and administration have been a concern for generations of political scientists and public administration scholars (e.g., Wilson 1887, Bertelli and Lynn 2006).

The paper proceeds as follows. The next section describes the model. Section 3 derives the results and discusses their implications. Section 4 develops extensions of the model that explore one case in which red tape is especially useful, alternative objectives for the principal and bureaucrat, and minimum or maximum testing standards. A final extension on costly goods appears in the online appendix. Section 5 concludes.

2 Model

The model features a simple mechanism whereby a bureaucrat makes a binary allocation decision for each member of a set citizens or clients, under the supervision of a political principal. While the model is intended to capture a range of possible bureaucratic decisions, the following discussion will refer to the product of an affirmative bureaucratic decision as a good.

The set of potential recipients of the good is a large population of \( N \) citizens. Each has a private valuation or type drawn i.i.d. from the a finite set \( \Theta \), where \( |\Theta| \geq 2 \). Each type \( \theta_i \in \Theta \) satisfies \( \theta_i < \theta_{i+1} \) for all \( i \), and \( \theta_1 > 1 \). Let \( \pi_i \) denote the probability that a client is of type \( \theta_i \).

The bureaucrat uses a means test to determine whether each client receives the good. Denote by \( t \geq 1 \) the level of bureaucratic testing effort. Testing generates a binary result
corresponding to “fail” and “pass,” where \( \phi(t; \theta_i) \) is the probability of passage. The good is allocated if and only if the client passes. The probability of passing is either increasing and concave or decreasing and convex, as follows:

\[
\phi(t; \theta_i) = \begin{cases} 
1 - \frac{1}{c_i t^\alpha} & \text{if } \theta_i \in \Theta^h \\
\frac{1}{c_i t^\alpha} & \text{if } \theta_i \in \Theta^l.
\end{cases}
\] (1)

The sets \( \Theta^h \) and \( \Theta^l \) are non-empty and partition \( \Theta \). Set \( \Theta^h \) is referred to as \textit{qualified}, and benefits from more testing, while \( \Theta^l \) is referred to as \textit{marginal} and benefits from less testing. The parameter \( \alpha > 0 \) is a measure of bureaucratic expertise. Higher values of \( \alpha \) generate higher probabilities of acceptance (respectively, rejection) for types in \( \Theta^h \) (respectively, \( \Theta^l \)). The parameter \( c_i \in [2, \theta_i) \) is a measure of testing effectiveness, with higher values increasing the “default” probability of acceptance (respectively, rejection) at \( t = 1 \) for types in \( \Theta^h \) (respectively, \( \Theta^l \)). For example, \( c_i = 2 \) represents the most difficult testing problem, as all types pass with probability \( \frac{1}{2} \) when tested at the minimum level of 1. This functional form usefully eliminates most corner solutions. It will be convenient to denote by \( I^l = \{ i | \theta_i \in \Theta^l \} \) and \( I^h = \{ i | \theta_i \in \Theta^h \} \) the set of indices in \( \Theta^l \) and \( \Theta^h \), respectively.

The bureaucrat maximizes the program’s “quality,” or its weighted ability to deliver the proper benefit to each type. She receives \( w_i > 0 \) for any client of type \( \theta_i \in \Theta^h \) who receives the good, or any client of type \( \theta_i \in \Theta^l \) who does not receive the good. Thus \( w_i \) serves as a measure for the extent to which the bureaucrat is interested in investigating a type-\( \theta_i \) client. Combined with (1), these payoff assumptions implicitly represent the bureau’s authority and expertise in designing its testing scheme: for each type her desired outcome becomes more likely as testing increases.

The bureaucrat chooses a direct mechanism or screen \( (t(\theta_i)) \) that tests clients at level \( t(\theta_i) \) for a report of type \( \theta_i \). Clients “arrive” at the bureaucrat in i.i.d. fashion, and so the probability of a type-\( \theta_i \) client is always simply \( \pi_i \). Given truthful reporting, the bureaucrat’s objective is then:

\[
u_b(t(\theta_1), \ldots, t(\theta_{|\Theta|})) = \sum_{i \in I^h} \pi_i w_i \phi(t(\theta_i); \theta_i) + \sum_{i \in I^l} \pi_i w_i (1 - \phi(t(\theta_i); \theta_i))
\] (2)

The politician moves first by specifying a pair \( (s, b) \) for the bureaucrat prior to her mechanism choice. The parameter \( s \) \((0 \leq s \leq N)\) is the size of the population that the bureaucrat

\[\text{footnote} 5: \text{Goods do not impose a direct cost on either the bureaucrat or the politician. The appendix derives a result for the case where the politician pays a unit cost for each acceptance.}\]

\[\text{footnote} 6: \text{Notably, the bureaucrat does not care directly about the number of clients served, the budget or budgetary “slack.” Both the budget and client population are determined prior to the bureaucrat’s actions in the model, and thus the bureaucrat simply designs the best program possible within these constraints.}\]
is mandated to serve through means tests and allocation choices, where for analytical convenience I allow $s$ to take on non-integer values\footnote{The population choice might represent an explicit limit on the bureaucrat’s services, or it may represent the selection of clients based on some observable characteristic that is independent of the type distribution, such as geography.} The parameter $b \geq s$ is the bureaucrat’s budget, which constrains the ex ante number of clients she can test as follows:

\[ s \sum_i \pi_i t(\theta_i) \leq b. \] (3)

Thus, the bureaucrat’s expected service cost is linear in $s$, and each client “costs” $t(\theta_i)$. Since the bureaucrat’s objective \footnote{The model generalizes straightforwardly to a principal who cares about a set of types in $\Theta^h$, or a set of types in $\Theta^l$. This change would add more welfare terms to his objective \footnote{where $q > 0$ is a measure of the cost or difficulty of testing.} and allow him to internalize more of the consequences of testing, but would not alter the nature of his maximization problem.} is increasing in all $t(\theta_i)$, the budget constraint (3) must bind.

Clients care only about receiving the good. Testing does not impose any direct costs on clients, though Section 4.1 considers a variant of the model in which the bureaucrat can associate some type announcements with “red tape.” A client therefore only cares about his report insofar as it affects testing. A type-$\theta_i$ client who announces type $\theta_j$ receives the following expected utility:

\[ u_c(\theta_j; \theta_i) = \phi(t(\theta_j); \theta_i) \theta_i. \] (4)

Types in $\Theta^h$ benefit from greater scrutiny, while types in $\Theta^l$ are hurt by it.

Finally, the principal wishes to maximize the net surplus of citizens of some type $\theta_p$, but pays the cost of bureaucratic testing resources\footnote{7}. This favored type may be either qualified or marginal. Given truth-telling under the bureaucrat’s screen, the principal’s objective can be written:

\[ u_p(s, b; \theta_p) = \pi_p s \phi(t(\theta_p); \theta_p) \theta_p - \frac{q}{2} b^2, \] (5)

where $q > 0$ is a measure of the cost or difficulty of testing.

\section{Results}

As is standard, attention can be restricted to direct and truthful mechanisms. Thus, clients optimally report their true types, and the bureaucrat commits not to use the client’s revealed information ex post to extract her surplus. Optimal truthful reporting is captured by the client’s incentive compatibility (IC) constraints, which require that each type $\theta_i$ prefer reporting $\theta_i$ to any $\theta_j \neq \theta_i$:

\[ \phi(t(\theta_i); \theta_i) \theta_i \geq \phi(t(\theta_j); \theta_i) \theta_i. \] (6)
Additionally each type $\theta_i$ must be willing to participate in the allocation mechanism, as captured by her individual rationality (IR) constraint:

$$\phi(t(\theta_i); \theta_i) \theta_i \geq 0.$$  

(7)

The specific interpretation of the IR constraints is that any client can choose to fail the exam (e.g., by not showing up) and not receive the good. Because there are no examination costs in the basic model, IR is always satisfied and can be disregarded.

### 3.1 First Best

I begin with the standard exercise of deriving the bureaucrat’s solution under the assumption that client types are known. The bureaucrat then maximizes her objective (2) subject to her budget constraint (3), taking as given her budget $b$ and population mandate $s$. Performing the straightforward constrained optimization problem yields the first result on the relationship between testing levels for different types. For notational simplicity, I hereafter abuse notation slightly and let $t_i = t(\theta_i)$.

**Lemma 1** First Best. *Under the first best, at an interior solution the testing levels for any types $\theta_i$ and $\theta_j$ satisfy:*

$$t_j^f = \left( \frac{c_i w_j}{c_j w_i} \right)^{\frac{1}{1+\alpha}} t_i^f.$$  

(8)

**Proof.** All proofs are in the Appendix.

The lemma defines a system of equations that characterizes testing levels at an interior solution. Each $t_i^f$ is a fixed proportion of every other testing level, and is independent of $b$, $s$, and the distribution of types. This implies that there is a unique profile of testing levels that satisfies the budget constraint with equality. Corner solutions are sometimes possible.

The comparative statics on testing are mostly intuitive. Type $\theta_j$’s relative testing intensity is increasing in $w_j/c_j$. This ratio, which measures the return to testing type $\theta_j$, will be useful throughout the remainder of the paper. A high value means that the bureaucrat cares greatly about the correct result and testing has a relatively large effect in changing the

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9Adding linear testing costs for marginal types would cause the IR constraints to bind for some testing levels. However, this change would not change any of the qualitative results of the model, and would also create many cumbersome corner solutions for the optimal mechanism.
probability of acceptance. The effect of expertise ($\alpha$) depends on these expected returns: the testing level for $\theta_j$ relative to $\theta_i$ is increasing in $\alpha$ if $w_j/c_j < w_i/c_i$, and decreasing otherwise. This reflects the greater impact of expertise on types with lower expected returns.

For the principal’s decision problem under the first best, the effects of the budget and population follow directly from Lemma [1] and the fact that the budget constraint binds. Since the ratio between testing levels is independent of $b$, a change in the budget produces a proportional change in all inspection levels. The effect of $s$ is simply the inverse of the effect of $b$. A principal who favors a qualified type ($\theta_p \in \Theta^h$) would choose a large budget and client population if the returns from doing so were sufficiently high. This requires not only a bureaucrat who is willing to choose a high $t^f_p$, but also a high client valuation $\theta_p$ and a high proportion $\pi_p$ of such types in the population.

### 3.2 Main Results

When types are private information, the principal faces the problem of offering a testing profile that elicits honest reporting in an incentive compatible way. The key implication of incentive compatibility is that the bureaucrat largely loses her ability to discriminate across types. Since all qualified types prefer higher testing, they would opt for the highest offered testing level. Similarly, all marginal types would opt for the lowest available testing level. Lemma 2 establishes that there can then be at most two levels of testing across the entire client population, with all qualified types tested at one common level, and all marginal types tested at another. Moreover, these two levels can be unequal in only one way: qualified types can be tested more stringently because more testing attracts them but deters marginal types. Compared to the first-best solution, the optimal incentive compatible testing profile is relatively unresponsive to the bureaucrat’s preferences.

**Lemma 2** Testing Uniformity. $t^*_i = t^l$ for all $\theta_i \in \Theta^l$, $t^*_i = t^h$ for all $\theta_i \in \Theta^h$, and $t^l \leq t^h$.

The result is a consequence of the assumption that the bureaucrat has only one dimension – the level of means testing – to control each type’s payoff. This contrasts with other common screening problems, where the uninformed player typically has the ability to impose different side payments on different types. As Section 4.1 shows, bureaucratic side payments would enable a greater degree of discrimination across types.

To characterize testing levels, it is useful to consider a version of the first best in which the bureaucrat tests all marginal types in $\Theta^l$ at one level and all types in $\Theta^h$ at another,
but without the constraint that $t^* \leq t^h$. If the derived $\Theta^h$ testing level is greater than the $\Theta^l$ testing level, then IC (6) is satisfied. Otherwise, IC is violated and the best the bureaucrat can do is to impose a “uniform” test of $b/s$ on all types. In both cases, the screening mechanism is implementable if the testing levels are feasible (i.e., at least 1).

The bureaucrat’s desire to test qualified versus marginal types depends on $w_i/c_i$ for each type $\theta_i$. Recall that this ratio captures her expected return from testing. If these ratios are generally high for qualified types and low for marginal types, then she will be more interested in correct acceptances than in correct rejections. Accordingly, the bureaucrat will want higher testing for qualified types. The acceptance bias condition and its complement, rejection bias, capture the predilections for acceptances and rejections, respectively, and are formally defined as follows.

**Definition 1** Acceptance and Rejection Bias. The bureaucrat is acceptance biased (respectively, rejection biased) if:

$$\frac{\sum_{i \in I}^h \pi_i}{\sum_{i \in I}^l \pi_i} < \left(\frac{\sum_{i \in I}^h \frac{\pi_i w_i}{c_i}}{\sum_{i \in I}^l \frac{\pi_i w_i}{c_i}}\right)$$

The next result then characterizes the optimal implementable screening mechanisms.

**Proposition 1** Testing. (i) If the bureaucrat is rejection biased, then $t^* = t^h = b/s$.

(ii) At an interior solution, if the bureaucrat is acceptance biased, then $t^* = b/m$, $t^h = b/m$, and $t^* < t^h$, where:

$$m = 1 - \left(1 - \left[\frac{\left(\sum_{i \in I}^h \pi_i\right)\sum_{i \in I}^l \frac{\pi_i w_i}{c_i}}{\left(\sum_{i \in I}^l \pi_i\right)\sum_{i \in I}^h \frac{\pi_i w_i}{c_i}}\right]^{\frac{1}{1+\alpha}}\right)\sum_{i \in I}^l \pi_i$$

$$\bar{m} = 1 - \left(1 - \left[\frac{\left(\sum_{i \in I}^l \pi_i\right)\sum_{i \in I}^h \frac{\pi_i w_i}{c_i}}{\left(\sum_{i \in I}^h \pi_i\right)\sum_{i \in I}^l \frac{\pi_i w_i}{c_i}}\right]^{-\frac{1}{1+\alpha}}\right)\sum_{i \in I}^h \pi_i.$$

Proposition shows that acceptance and rejection biases play a central role in program implementation. Manipulating expression (9) reveals that acceptance bias implies $m < 1$ and $\bar{m} > 1$. Acceptance bias thus implies that $t^h > b/s > t^*$; that is, the bureaucrat can discriminate by testing qualified types more intensively than marginal types. By contrast, rejection bias renders discrimination and the delivery of targeted benefits impossible, as each client is treated identically. From the bureaucrat’s perspective, rejection bias maximizes allocative inefficiencies, as she is forced to over-test types in $\Theta^h$ and under-test types in $\Theta^l$. 

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The final step is to characterize the principal’s budget and population choices. The principal’s maximization problem breaks down into two main cases, depending on the kind of group he favors. Each case produces distinct styles of program implementation and also generates basic measures of program performance. These measures are directly related to all of the administrative pathologies identified in the introduction except politicization, which is addressed in the following section. The outputs are as follows:

- **Client population**, measured by $s$, where $s < N$ implies a shortage or backlog.
- **Flexibility**, measured by whether $t^s < t^h$.
- **Budget** received by the agent, measured by $b$.
- **Per capita budget**, measured by $b/s$.
- **Type I error avoidance rate**, measured by $1 - \phi(t_i; \theta_i)$ for $\theta_i \in \Theta^l$.
- **Type II error avoidance rate**, measured by $\phi(t_i; \theta_i)$ for $\theta_i \in \Theta^h$.

Note that Type I errors are defined as approvals of the good to types in $\Theta^l$ (e.g., approving a bad drug), while Type II errors are denials of the good to types in $\Theta^h$ (e.g., rejecting a good drug). The errors therefore reflect the bureaucrat’s preferences over approving different types. Both types of errors are conditional upon participation in the bureaucrat’s mechanism, and therefore do not count Type II errors arising from failure to serve eligible clients.

### 3.2.1 Favored Marginal Type

When $\theta_p \in \Theta^l$, the principal’s problem is simple: members of his preferred group benefit from low testing (i.e., low $t$), and low testing has the additional benefit of reducing per capita costs. Testing is minimized at the corner $t^* = 1$. The principal can ensure this testing level with a budget exactly equal to the client population size, and thus $b^* = s^*$. Substituting these expressions into expression (5) produces a simple objective that is concave in $s$:

$$u^*_p(s, b; \theta_p) = \pi_p s \theta_p c_p - \frac{q}{2} s^2 \quad (12)$$

Maximization of this objective produces the first main result. There exists a unique interior optimum if $N$ is sufficiently large. Low testing costs allow the politician to make the program broad, but he may stop short of testing all members of society (including some members of her preferred type) because of the cost of testing types other than $\theta_p$.

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10 An identical result obtains when all types are marginal.
Proposition 2 Principal Favors Marginal Type. If \( \theta_p \in \Theta^l \), then at an interior solution, \( b^* = s^* = \frac{\pi_\theta p}{q_p}, \) and \( t^* = 1 \).

The equilibrium is quite simple, as the principal deliberately “starves” the agency with a low per capita budget. The minimal testing of clients maximizes error rates and serves the politician’s favored group, but without any meaningful targeting by either the principal or the bureaucrat. This case may be an apt description of recent developments in the IRS EO division, where Republican supporters of politically-motivated organizations have also supported cuts in IRS funding that reduced service levels.

Comment 1 characterizes program performance. The result follows straightforwardly from Proposition 2 and is therefore stated without proof.

Comment 1 Comparative Statics with a Favored Marginal Type. (i) All performance measures are independent of all \( w_i \).

(ii) The budget and client population are weakly increasing in the proportion of type \( \theta_p \) (\( \pi_p \)), and weakly decreasing in testing costs (\( q \)) and the default rejection probability (as measured by \( c_p \)).

(iii) There is no flexibility; per capita budgets are constant in all parameters and across types. Error avoidance rates depend only on \( c_i \) and \( \alpha \).

Parts (i) and (iii) of the result imply that when a marginal group is favored, the bureaucrat’s preferences are irrelevant: any bureaucrat would subject all covered clients to the same, minimal treatment. Thus, replacing the bureaucrat can matter only when a qualified group is favored. Part (ii) states that the program is larger when the expected returns from testing the population are high. This will be the case if type \( \theta_p \) clients are common in the pool of possible clients, or if type \( \theta_p \) passes at a high rate when exposed to minimal testing due to a low \( c_p \). Similarly, a reduction in \( q \) generates a positive budget shock that boosts program size.

3.2.2 Favored Qualified Type

When \( \theta_p \in \Theta^h \), the principal wants to use higher testing levels to help favored clients, but testing is costly and reduces the population that can be served. There are two subcases, depending on whether the bureaucrat is rejection or acceptance biased.

\textsuperscript{11} A possible objection to this application is that it also implies lower approval rates for qualified types, thus offsetting the reduced revocations of marginal types. The example therefore additionally requires that qualified types are less sensitive to testing; i.e., \( c_i \) is high for qualified types, and low for marginal types.
By Proposition 1, rejection bias implies a common testing level $t^*$. Since the bureaucrat’s budget constraint binds, expression (3) implies that $t^*$ is simply the per capita budget:

$$t^* = \frac{b}{s}. \quad (13)$$

The principal then chooses $b$ and $s$ by trading off between testing costs and approvals for the favored group. Substituting $t^*$ into his objective produces the following:

$$u^*_p(s, b; \theta_p) = \pi_p s \left(1 - \frac{s^\alpha}{c_p b^\alpha}\right) \theta_p - \frac{q}{2} b^2 \quad (14)$$

Proposition 3 describes the resulting budget, program size, and testing level.

**Proposition 3** Principal Favors Qualified Type, Rejection Biased Bureaucrat. If $\theta_p \in \Theta^h$ and the bureaucrat is rejection biased, then at an interior solution, $b^* = \frac{\alpha \pi_p \theta_p c_p^{1/\alpha}}{q (\alpha+1)^{1/\alpha}}$, $s^* = \frac{\alpha \pi_p \theta_p c_p^{2/\alpha}}{q (\alpha+1)^{1/\alpha}}$, and $t^* = \left(\frac{\alpha+1}{c_p}\right)^{1/\alpha}$. ■

Two comparisons with the favored marginal type case (Proposition 2) are noteworthy. Favoring a qualified type increases testing if $\alpha + 1 > c_p$; that is, if bureaucratic expertise is high. As $c_p$ increases, the need for testing decreases and the politician responds by expanding the program. Additionally, rejection bias ensures that there is again no meaningful targeting of benefits, as all clients receive either no service or uniform treatment. This combination of expertise and non-discrimination might be described as “neutrally competent” administration.

In the second subcase, acceptance bias makes limited testing discrimination possible. Using Proposition 1(ii), the principal’s objective can be written to reflect the increased testing ($t^{hs}$) that type $\theta_p$ receives as follows.

$$u^*_p(s, b; \theta_p) = \pi_p s \left(1 - \frac{s^\alpha}{c_p b^\alpha}\right) \theta_p - \frac{q}{2} b^2 \quad (15)$$

Conveniently, this objective is simply a weighted version of the rejection biased case in (14). Maximizing (15) produces the following result.

**Proposition 4** Principal Favors Qualified Type, Acceptance Biased Bureaucrat. If $\theta_p \in \Theta^h$ and the bureaucrat is acceptance biased, then at an interior solution:

$$b^* = \frac{\alpha \pi_p \theta_p c_p^{1/\alpha}}{q m (\alpha+1)^{1/\alpha}}$$

12 An identical result obtains when all types are qualified.
Many of the comparative statics for budgets \( (b^*) \) and program size \( (s^*) \) in Propositions 3 and 4 are identical to those in Proposition 2 where the politician favored a marginal type. However two significant differences are immediately apparent. First, as the expressions for testing levels make clear, the bureaucrat’s testing ability (as measured by \( \alpha \) and \( c_p \)) matters. Second, if in addition the bureaucrat is acceptance biased, then her preferences \( (w_i) \) also affect equilibrium strategies through the parameters \( \overline{m} \) and \( m \).

Comparing Propositions 3 and 4, it is evident that acceptance bias produces implementation flexibility, a larger budget, lower per capita spending and testing, and a larger client population. Flexibility gives the principal the ability to offer a measure of favorable treatment for his preferred clients. The resulting form of administration might therefore be described as “responsive competence.” The larger client population implies fewer or smaller backlogs, or situations in which the program’s capacity falls short of the client population. Notably, qualified types are tested identically across the acceptance bias and rejection bias subcases, producing an acceptance probability of \( \alpha/(1 + \alpha) \). This implies that the cut in per capita spending under acceptance bias falls exclusively on marginal types, who benefit from the increased Type I errors due to reduced scrutiny. The main comparative statics on program outputs echo this comparison, as summarized by Comment 2.

Comment 2 Comparative Statics with a Favored Qualified Type. (i) The budget and client population are weakly increasing in \( w_p \) and \( w_i/c_i \) for qualified types besides \( \theta_p \), and weakly decreasing in \( w_i/c_i \) for marginal types. They are decreasing in \( q \).

(ii) The per capita budget is weakly decreasing in \( w_p \) and \( w_i/c_i \) for qualified types besides \( \theta_p \), and weakly increasing in \( w_i/c_i \) for marginal types. It is constant in \( q \).

(iii) Flexibility occurs only if the bureaucrat is acceptance biased.

\[ s^* = \frac{\alpha \pi \theta_p c_p^{2/\alpha}}{q m^2 (\alpha + 1)^{(\alpha+2)/\alpha}} \]
\[ t^{h*} = \left( \frac{\alpha + 1}{c_p} \right)^{1/\alpha} \]
\[ t^{l*} = \frac{\overline{m}}{m} \left( \frac{\alpha + 1}{c_p} \right)^{1/\alpha}. \]

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13 The role of \( \alpha \) in other equilibrium choices is ambiguous: the budget \( b^* \) and client population \( s^* \) are decreasing in \( \alpha \) for low values of \( \alpha \), but possibly increasing for higher values.

14 Acceptance bias is sufficient if \( t^{h*} > 1 \), which requires \( \alpha + 1 > c_p \).
(iv) Holding $\alpha$ constant, the Type I error avoidance rate is weakly lower under acceptance bias than rejection bias. The Type II error avoidance rate is increasing in $\alpha$ and constant in all other parameters.\footnote{This relationship is strict if $t^{th*} > 1$, which requires $\alpha + 1 > c_p$.}

Comment 2 has two non-obvious implications. The first lies in the contrast between parts (i) and (ii): per capita spending moves in the opposite direction from the budget and client population. Budget increases caused by changes in some $w_i/c_i$ result in larger client populations and lower average testing. By contrast, changing the principal’s favored type can cause budgets and per capita spending to move in the same direction, since the smallest budgets and most error-prone programs can result when the politician favors a marginal group. The second follows from the contrast between parts (iii) and (iv): since bureaucratic flexibility results from acceptance bias, it is associated with lower average testing levels and in turn a higher frequency of only Type I errors.\footnote{The comparative statics on the Type I error avoidance rate are ambiguous under acceptance bias.}

The comparative statics in Comment 2 are generally strict when the bureaucrat is acceptance biased, or roughly when she “agrees” with a politician who favors a qualified type on the distribution of testing effort. This produces an implication for the appointment of bureaucrats. Such a politician need not appoint a bureaucrat who cares about $\theta_p$ in particular in order to induce acceptance bias. Because of the bureaucrat’s inability to discriminate across qualified types, a high motivation for testing any qualified types is sufficient.

Figure 1 illustrates the relationships between program outputs with a qualified favored type and both acceptance and rejection bias. An example of these biases is the evolution of drug approval policies at the Food and Drug Administration (FDA) in the 1980s (Carpenter 2010). The FDA initially built a strong public reputation through an emphasis on scientific rigor, resulting most prominently from its refusal to approve thalidomide in the early 1960s. This conservatism was embedded in the FDA’s rigid approval procedures, but in response to AIDS crisis the agency formalized policies for widespread “compassionate use” of drugs in the review pipeline that were intended for AIDS and other life-threatening diseases. These policies probably increased the chances of Type I errors, but also reflected a lower payoff from avoiding such errors. Thus the agency moved from rejection bias to acceptance bias, which corresponded to an increase in bureaucratic discrimination.\footnote{The Prescription Drug User Fee Act of 1992 subsequently also provided a funding mechanism that increased the budget and throughput of the FDA’s Center for Drug Evaluation and Research.}
Figure 1: Equilibrium Budgets, Client Population, Per Capita Spending, and Acceptance Probabilities with a Qualified Favored Type. Here $\Theta^l = \{\theta_1\}$, $\Theta^h = \{\theta_2\}$, $\theta_2 = 10$, $w_1 = 1$, $\alpha = 3$, $c_1 = c_2 = 2$, $q = 0.001$, $\pi_1 = 0.8$, and $\pi_2 = 0.2$. Acceptance bias holds for $w_2 > w_1$. The figure plots the budget $b^*$ (top left), client population $s^*$ (top right), per capita spending (bottom left), and acceptance probabilities (bottom right), as a function of $w_2$. Note that the acceptance probability for type $\theta_1$ is increasing because testing levels are decreasing.
3.3 Implications of Political Control and Politicization

Many of the comparative statics results of Section 3.2 held the principal’s favored group constant. However, political transitions such as elections can generate important changes in bureaucratic behavior. The next result compares equilibrium parameters between the cases where the principal favors marginal and qualified types.

**Comment 3** Comparing Marginal and Qualified Favored Types. *When the favored type is marginal:*

(i) *The per capita budget, Type I and II error avoidance rates, and flexibility are weakly lower.*

(ii) *At an interior solution, the budget is always smaller and the client population is smaller if \( c_p \) is sufficiently high.*

The overall picture presented by Comment 3 is that oversight by a principal who favors a marginal group will generally result in an under-resourced and poorly run program. Part (i) follows simply from the fact that a principal minimizes per capita testing resources when she favors a marginal type. Part (ii) shows that such a program will always have a lower budget, and sometimes a lower client population as well. Interestingly, overall enrollments are not always lower: it is possible to derive circumstances under which such a program can serve the broadest possible population when \( c_p \) is low. Thus, the most error-prone programs can actually have less backlogs than more expert or competent programs overseen by a politician who cared about a qualified type.

A final but important contrast between politicians who favor qualified versus marginal groups is their incentive to politicize bureaucratic procedures. Politicization of the bureaucracy can take on many forms, including the appointment of particular personnel and pork barrel spending. In the model, politicization might also be interpreted as bureaucratic flexibility, since she is only able to discriminate across client types in a way that benefits a favored qualified type. Here I consider whether the bureaucrat’s testing technology \( c_p \) can be manipulated to the benefit of the politician’s favored group.

Comment (i)(ii) suggests that a politician who favors a marginal group might do better by forcing the bureaucrat to use a less effective test, or equivalently by making her classification problem “harder.” A lower value of \( c_p \) increases the probability that members of such a group pass the test, and thereby gives the politician an incentive to dictate less accurate procedures or technologies, or to use less competent personnel. As an example, since the

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18 This result also holds for at least some corner solutions.
1980s Congress has actively manipulated benefit-cost analysis to limit the Federal Aviation Administration’s ability to evaluate outsourced control tower services (Mills 2013).

By contrast, a politician who favors a qualified type has the opposite incentive. In equilibrium increasing $c_p$ secures greater benefits for the preferred group through both increased program coverage and higher passage rates. Additionally, since the bureaucrat cares primarily about the quality of her implemented tests, she generally prefers higher values of $c_p$ as well. This logic holds for both acceptance biased and rejection biased bureaucrats. Thus, any bureaucrat would welcome the intervention of a politician who favored a qualified type, and resist the intervention of a supporter of a marginal type.

Comment 4 confirms these effects of $c_p$ on the principal’s utility. I note without proof that a similar result would hold for $\alpha$.

Comment 4 Politicization of Testing Technology. At an interior solution:

(i) When $\theta_p \in \Theta^l$, $u_p(s^*, b^*; \theta_p)$ is decreasing in $c_p$.

(ii) When $\theta_p \in \Theta^h$, $u_p(s^*, b^*; \theta_p)$ is increasing in $c_p$. ■

4 Extensions

4.1 Rejection Bias and Red Tape

The preceding analysis produced a stark outcome in situations where the bureaucrat is rejection biased. Incentive compatibility forces all types to be tested at the same level, and thereby places the bureaucrat at a tremendous disadvantage when she cares greatly about rejecting marginal types. In such settings, the bureaucrat has an obvious incentive to find ways to reallocate testing resources toward the most important types. How can she better discriminate across types, and how do the constraints of feasible mechanisms affect program performance?

To improve testing flexibility, I introduce a simple form of red tape, defined as costs imposed on potential program recipients that are incidental to their performance on the means test. Substantively, these costs might represent additional paperwork or procedures that are directed toward certain types. As noted in the introduction, several models of bureaucratic allocation have used red tape as a means of differentiating among types, and this extension uses it in a similar spirit. In particular, red tape exploits differences across types in marginal disutilities from testing. Among marginal types, one that gains greatly from reduced testing will be willing to suffer some red tape, while one with less to gain will
prefer a higher testing level with no red tape. The bureaucrat then benefits from red tape if a type she wishes to test more extensively prefers high testing without red tape to low testing with red tape.

To keep the analysis simple, the extension imposes some additional structure on the basic model. Let \( \theta_1 \) be the type that is least likely to pass the test for any given testing level (i.e., \( c_1 > c_i \) for all \( \theta_i \in \Theta^l \)). Suppose additionally that \( w_1/c_1 > w_i/c_i \) for all \( \theta_i \in \Theta^l \), so that type \( \theta_1 \) is the worst of the low types from the bureaucrat’s perspective. Recall that type \( \theta_1 \) has the lowest valuation for the good: \( \theta_1 \leq \theta_i \) for all \( \theta_i \in \Theta^l \). The bureaucrat can impose a single level of red tape costing \( r \geq 0 \) on some set of announced types. Finally, red tape imposes a cost \( \tau r \) on the bureaucrat, where \( \tau > 0 \). This cost does not come out of the testing budget supplied by the politician, and therefore does not affect the aggregate level of testing.

I derive perhaps the simplest mechanism that allows the bureaucrat to redirect testing resources toward \( \theta_1 \) from other marginal types. Specifically, the bureaucrat tests type \( \theta_1 \) and all qualified types at a uniform level \( \bar{t} \). All other types incur red tape and are tested at level \( \bar{t} \), with \( \bar{t} > t \). As in the basic model, individual rationality is guaranteed by the fact that an option with no red tape exists for all types. Similarly, incentive compatibility for qualified types is straightforward, as they clearly benefit from higher testing. For the marginal types, incentive compatibility requires that \( r \) deters only type \( \theta_1 \) from choosing \( \bar{t} \). Marginal types will choose \( t \) if red tape is sufficiently low:

\[
\phi(t; \theta_i) \theta_i - r \geq \phi(\bar{t}; \theta_i) \theta_i.
\]

Finally, type \( \theta_1 \) will choose \( \bar{t} \) as the bureaucrat desires if the IC expression is satisfied with equality. Thus \( r = (\phi(t; \theta_1) - \phi(\bar{t}; \theta_1)) \theta_1 \) ensures incentive compatibility for all types and is the optimal level of red tape for the bureaucrat to impose.

In choosing testing levels, the bureaucrat faces a tradeoff similar to that of the basic model. The budget constraint allows her to increase \( \bar{t} \) for type \( \theta_1 \) and types in \( \Theta^h \) only by decreasing \( t \) for all other types. The following comment shows that even under this constraint, it is possible to target type \( \theta_1 \) with higher testing.

\[19\] This assumption can be relaxed somewhat, but in general for red tape to work the valuation of any type that the bureaucrat wishes to test at a higher level must be sufficiently low.

\[20\] As an example of such costs or the absence thereof, a bureaucrat might choose to qualify applicants for a program simply by using available administrative data, rather than requiring applicants to produce evidence of eligibility. Under the Affordable Care Act, enrollment in state Medicaid programs can (at the state’s discretion) be handled largely through pre-existing data from other public assistance programs. See [http://www.medicaid.gov/medicaid-chip-program-information/program-information/targeted-enrollment-strategies/targeted-enrollment-strategies.html](http://www.medicaid.gov/medicaid-chip-program-information/program-information/targeted-enrollment-strategies/targeted-enrollment-strategies.html)
Comment 5 Rejection Bias and Red Tape. For $w_1/c_1$ sufficiently high, there exists a mechanism with red tape that tests $\theta_1$ at a higher level than other types in $\Theta'$.

The simple result is that if the bureaucrat cares enough about preventing acceptances of type $\theta_1$, then red tape allows her to “pull” its testing level up to that of the qualified types. The bureaucrat will often prefer this scheme to one without red tape, but its benefits are offset by two kinds of costs. The first is the direct cost of red tape. The second is that the increased testing of type $\theta_1$ reduces testing of other marginal types, compared to when no red tape is used. The extension more generally shows how non-testing costs like red tape facilitate the screening of marginal types. Analogously, red tape would also be necessary to deter qualified types from choosing higher testing levels, perhaps in an effort to conserve testing resources.

The addition of red tape does not affect equilibrium outcomes when the politician favors a marginal type, as she continues to starve the bureaucrat of resources. Thus red tape can make a difference only when the politician favors a qualified type. In a more general model, the bureaucrat can potentially implement as many testing levels as there are types. This may induce a more favorable distribution of Type I and Type II errors, but it cannot generally implement the first best for any profile of client type valuations.

4.2 Error Minimization

I next consider the model’s robustness to an alternative principal objective. Suppose that instead of caring about the welfare of a group, the principal cared about a weighted sum of Type I and Type II error avoidance. This captures the idea that politicians may sometimes need to balance the relative costs of different types of error, just as bureaucrats must. The extension therefore gives the principal a policy utility function that is identical in form to the bureaucrat’s.

Let $e_i \geq 0$ denote the politician’s weighting on error avoidance for each type $\theta_i$. This objective introduces two basic changes in the his incentives. First, he now always benefits from rejecting marginal types, as he avoids a Type I error with probability $1 - \phi(\cdot)$. By contrast, for qualified types, $e_i$ plays a similar role to the type valuation $\theta_i$ used in the basic model. Conveniently, this results in an objective function that is concave in testing levels for every type. Second, the principal is now also affected by the population of $N-s$ clients.

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21 Common examples include rocket launches and pharmaceutical approvals (Heimann 1997).
who are unreached by the policy. By not receiving the good, marginal clients in this pool avoid Type I error with certainty, but qualified clients suffer Type II errors with certainty.

The bureaucrat’s problem is unchanged, and her policy choice is characterized by Proposition 1. Thus, any effect on the equilibrium operates through the principal’s choice of \( b \) and \( s \). The principal’s revised objective can be written:

\[
u_p^e(s, b) = s \left[ \sum_{i \in I^l} \pi_i e_i (1 - \phi(t(\theta_i); \theta_i)) + \sum_{i \in I^h} \pi_i e_i \phi(t(\theta_i); \theta_i) \right] + (N - s) \sum_{i \in I^l} \pi_i e_i - \frac{qb^2}{2}.
\]

(17)

One additional piece of notation will be useful. Define the following weighted measure of the politician’s incentive to test all types:

\[
\epsilon(m, m) = m^\alpha \sum_{i \in I^l} \pi_i e_i + m^\alpha \sum_{i \in I^h} \pi_i e_i.
\]

The politician’s weighting of errors across types produces a result analogous to Propositions 3 and 4 follows.²²

**Proposition 5** Error Minimization. An interior solution must satisfy:

\[
b^e = \begin{cases} 
\frac{\alpha \left( \sum_{i \in I^h} \pi_i e_i \right)^{(\alpha+1)/\alpha}}{q(\alpha+1)^{(\alpha+1)/\alpha} e m m} & \text{if bureaucrat acceptance biased} \\
\frac{\alpha \left( \sum_{i \in I^h} \pi_i e_i \right)^{(\alpha+1)/\alpha}}{q(\alpha+1)^{(\alpha+1)/\alpha} \sum_i \pi_i e_i} & \text{if bureaucrat rejection biased,}
\end{cases}
\]

\[
s^e = \begin{cases} 
\frac{\alpha \left( \sum_{i \in I^h} \pi_i e_i \right)^{(\alpha+2)/\alpha}}{q(\alpha+1)^{(\alpha+2)/\alpha} e m m} & \text{if bureaucrat acceptance biased} \\
\frac{\alpha \left( \sum_{i \in I^h} \pi_i e_i \right)^{(\alpha+2)/\alpha}}{q(\alpha+1)^{(\alpha+2)/\alpha} \sum_i \pi_i e_i} & \text{if bureaucrat rejection biased,}
\end{cases}
\]

\[
t^e(\theta_i) = \begin{cases} 
\frac{b^e}{s} \frac{m}{m} & \text{if } \theta_i \in \Theta^h \text{ and bureaucrat acceptance biased} \\
\frac{b^e}{s} \frac{m}{m} & \text{if } \theta_i \in \Theta^l \text{ and bureaucrat acceptance biased} \\
b^e & \text{if bureaucrat rejection biased.}
\end{cases}
\]

Many features of the equilibrium strategies are similar to those of the basic model, but two comparisons stand out. First, because the politician benefits directly from testing all types for which \( e_i \) is strictly positive, interior testing levels are more common and the outcome of minimal testing seen in Proposition 2 becomes difficult to achieve. Second, the desire

²²While the proof of Proposition 5 does not show concavity of the objective (17), the objective is concave for a broad range of parameter values.
the avoid Type II errors (i.e., higher values of $e_i$ for qualified types) creates an incentive to
expand a program’s budget and size. By contrast, a preference for avoiding Type I errors
unambiguously shrinks both budget and population size, while increasing average testing
levels. It therefore plays a role similar to that of removing bureaucratic acceptance bias.

4.3 Budget Maximization

Dating back at least to Niskanen (1971), observers have posited budget maximization as
a basic bureaucratic objective. Budget maximization captures the notion of bureaucratic
agency heads as “empire builders” who accumulate power and resources at the expense of
social welfare.

With a simple modification, the model can incorporate this objective. Suppose that the
bureaucracy consists of two actors. One acts as the bureaucrat in the preceding analysis,
and the other begins the game by choosing the bureaucrat’s payoff weight ($w_i$) parameters
with the objective of maximizing $b^*$. This second player might be considered an agency head
or appointing official. Let $[\bar{w}, \underline{w}]$ be the set of feasible values of $w_i$ for all types.

A first observation is that if the principal favors a marginal type, then by Proposition 2
the budget is independent of the bureaucracy’s preferences and the weights of the subordinate
bureaucrat do not matter. Focusing on the case where the principal favors a qualified type,
it is easy to see from Propositions 3 and 4 that the equilibrium budget $b^*$ is decreasing in
$\ddot{m}$, and higher when the bureaucrat is acceptance biased. The following result characterizes
the agency head’s optimal configuration of weights.

Comment 6 Budget Maximization. For $\underline{w}$ sufficiently low, a budget-maximizing agency
head chooses $w_i = \bar{w}$ for $\theta_i \in \Theta^h$, and $w_i = \underline{w}$ for $\theta_i \in \Theta^l$.

From a budget maximizer’s perspective, the optimal bureaucrat is one who is maximally
biased toward approvals. The subordinate bureaucrat in this agency aggressively investigates
declared qualified types, and is lax toward marginal types.

The result follows from the fact that a strong approval bias maximizes the bureaucrat’s
discrimination between the two type classes. This encourages a politician who cares about a
qualified type to spend more, as no resources are wasted investigating marginal types. Thus,
a budget-maximizing agency will generate high allocations of the good even when the good
itself is not costly.

23The assumption is not universally accepted; Wilson (2000) offers some important counter-examples.
4.4 Testing Standards

Suppose that, in addition to b and s, the bureaucrat was bound by a minimum or maximum testing level for all clients. In some educational settings, legislatures impose testing requirements that bureaucrats use for advancing students. In law enforcement settings, courts impose “due process” requirements on bureaucrats. One obvious trade-off is that a principal could conceivably improve the payoff of a favored group through higher mandated testing levels, but at the cost of higher testing expenditures.

Formally, suppose that testing levels for all types are constrained to satisfy \( t(\theta_i) \in [t_{\text{min}}, t_{\text{max}}] \), rather than allowing any \( t(\theta_i) \geq 1 \). When the constraint is binding, its effect is easily calculated in the cases where the bureaucrat does not discriminate across types. In these cases, it is clear that the bureaucrat will choose a uniform testing level \( b/s \), and any testing standard can only constrain the principal. For example, if the principal’s favored type is marginal and \( t_{\text{min}} > 1 \), then by the same argument as in the basic model he simply minimizes testing and chooses a budget to match: \( b^* = \frac{\pi_p \theta_p}{q_p t_{\text{min}}} \), \( s^* = \frac{\pi_p \theta_p}{q_p t_{\text{min}}} \), and \( t^* = t_{\text{min}} \).

More generally, the following comment shows that the principal cannot benefit from a testing restriction. Such restrictions are too crude of a means of inducing higher testing for a favored client type. Because of the bureaucrat’s budget constraint, for any given budget ceilings and floors can only prevent the principal from enhancing the acceptance probability of a favored group.

Comment 7 Testing Restrictions. The principal cannot benefit from a testing standard.

What might explain the presence of testing standards? One possibility is that they are useful when the principal is unable to specify other parameters of the bureaucrat’s behavior. For example, if the principal cannot feasibly designate \( s \), then a mandated maximum testing level could establish a floor on the client population. A second possibility is that testing standards can be imposed by other principals. A second principal who cared about not approving too many marginal types could use a minimum testing standard to force more scrutiny upon types that the first principal and bureaucrat would otherwise neglect. This situation might be expected in programs with an exogenous (e.g., constitutional) requirement of broad participation, which may drive down per capita testing. The additional principal might be a court, a different level of government in a federal system, or another legislator whose support is necessary for enacting the program in question.
5 Conclusions

Theories of political control of the bureaucracy have made considerable progress in recent decades, focusing in particular on informational problems between political principals and bureaucratic agents. Perhaps the canonical question in this area is the extent to which principals should delegate authority to more knowledgable bureaucrats. A central rationale for this paper is that bureaucrats face their own informational problems and design mechanisms to cope with them. In this setting, principals have great control over broad policy parameters but less control over the ways in which bureaucrats interact with potential program beneficiaries. Because the bureaucrat’s mechanism is a conduit for group-specific benefits, the politician will use crude policy tools like the budget to influence the distribution of goods. Thus, the model developed here bridges the classic divide between “policy” and “administration,” by tying political control of the bureaucracy to the concrete problems of public administration. It also identifies a central role for the bureaucracy in distributive politics (e.g., Arnold 1979).

The model begins with a foundation of incomplete information, bureaucratic expertise, resource constraints, and political control. There is a natural technological trade-off between client populations and aggregate per capita spending (or equivalently, testing), but the model provides more specific guidance as to how other distributive issues are resolved. In particular, the politician’s distributive concerns and the bureaucrat’s testing preferences both generate stark predictions about implementation flexibility, budget size, errors, and whether policy implementation tends toward breadth or depth. These results will help to guide empirical inquiry by linking political variables with both granular administrative data and recently emergent data on the quality of governance.

Three styles of programs emerge from the analysis. The largest and most error-prone programs result when the politician cares about a marginal group and the bureaucrat uses an inaccurate testing technology. In this environment, the bureaucrat has minimal resources per client, and her preferences are irrelevant to the outcome. Perversely, the principal also has an incentive to weaken the bureaucrat’s testing technology. By contrast, a principal who cares about a qualified group induces higher per capita spending and lower error rates. Here the bureaucrat’s preferences matter: depending on her testing inclinations (i.e., acceptance and rejection bias), programs are either smaller and inflexible or larger and flexible, which correspond roughly to neutrally competent and responsively competent administration, respectively. Some non-obvious implications of this environment are that budgets are positively
correlated with client populations and negatively correlated with per capita spending, and benefits can be targeted only with an acceptance biased bureaucrat.

While the model addresses a broad set of bureaucratic outputs, it omits some important technological and institutional features. Perhaps most significantly, the purely distributive setting with costless goods has non-trivial implications. In policy areas such as regulation, allocation choices can generate externalities across clients, and other environments feature natural limits on the quantity of the good. Additionally, the model considers only a minimal set of player strategies. In reality, politicians may be able to provide performance incentives to bureaucrats, or they may impose procedural constraints such as burden of proof requirements. They would also be constrained by the bureaucrat’s labor market alternatives and possible collusion with clients. Finally, the roles of other principals such as voters, courts, different levels of government, and legislative overseers deserve fuller consideration.
6 References


APPENDIX

This appendix develops an additional extension to the model, where bureaucratically provided goods have a unit cost. It also provides proofs of the paper’s results.

Extension: Costly Goods

An important assumption of the preceding analysis is that the bureaucrat’s decision to allocate the good imposes no direct costs on the principal. In many instances, this is not the case: social welfare benefits can obviously be quite costly, and building permits create opportunity costs. Alternatively, not prosecuting a case may actually reduce costs. Such costs create significant changes in the principal’s incentives to test types in \( \Theta_h \) and \( \Theta_l \).

To see the effects of costly goods, suppose that each allocated good has a unit cost \( \gamma \), where \( |\gamma| < 1 \). I focus on the case where \( \theta_p \in \Theta_h \) and the bureaucrat is rejection biased, so she does not discriminate across types. The principal’s objective can then be written:

\[
 u_p^*(s, b; \theta_p) = \pi_p s \left( 1 - \frac{s^\alpha}{c_p b^\alpha} \right) \theta_p - \gamma s \left[ \sum_{i \in I_h} \pi_i \left( 1 - \frac{s^\alpha}{c_i b^\alpha} \right) + \sum_{i \in I_l} \pi_i \frac{s^\alpha}{c_i b^\alpha} \right] - \frac{q}{2} b^2. \tag{18}
\]

The following comment generalizes the corresponding case of Proposition 3 and presents the equilibrium budget, client population, and testing level.

Comment 8 Costly Goods. If \( \theta_p \in \Theta_h \) and the bureaucrat is rejection biased, then at an interior solution:

\[
 b^* = \frac{\alpha}{q} \left( \frac{\pi_p \theta_p - \gamma \sum_{i \in I_h} \pi_i}{\alpha + 1} \right)^{\frac{\alpha + 1}{\alpha}} \left( \frac{\pi_p \theta_p}{c_p} - \gamma \left[ \sum_{i \in I_h} \frac{\pi_i}{c_p} - \sum_{i \in I_l} \frac{\pi_i}{c_p} \right] \right)^{-\frac{1}{\alpha}}
\]

\[
 s^* = \frac{\alpha}{q} \left( \frac{\pi_p \theta_p - \gamma \sum_{i \in I_h} \pi_i}{\alpha + 1} \right)^{\frac{\alpha + 2}{\alpha}} \left( \frac{\pi_p \theta_p}{c_p} - \gamma \left[ \sum_{i \in I_h} \frac{\pi_i}{c_p} - \sum_{i \in I_l} \frac{\pi_i}{c_p} \right] \right)^{-\frac{2}{\alpha}}
\]

\[
 t^* = \left( \frac{\pi_p \theta_p - \gamma \sum_{i \in I_h} \pi_i}{\alpha + 1} \right)^{-\frac{1}{\alpha}} \left( \frac{\pi_p \theta_p}{c_p} - \gamma \left[ \sum_{i \in I_h} \frac{\pi_i}{c_p} - \sum_{i \in I_l} \frac{\pi_i}{c_p} \right] \right)^{\frac{1}{\alpha}}.
\]

These expressions convey an intuitive result. Compared to the basic model, when \( \gamma > 0 \) and there is a sufficiently high probability weight on types in \( \Theta_l \), costly goods will reduce both the budget and the client population. However, per capita testing increases. The effect of positive costs therefore resembles that of moving from acceptance to rejection bias. When \( \gamma < 0 \), these effects are reversed.

I note finally that if \( \theta_p \in \Theta_l \), then costly goods might produce higher per capita testing than the minimum seen in Proposition 2. This requires that there be sufficient probability weight on types in \( \Theta_l \) besides \( \theta_p \), which from the principal’s perspective generates many wasted allocations.
Proofs

Proof of Lemma 1. As the objective is concave with respect to all \( t_i \) and the constraint is linear, first order conditions are sufficient for characterizing the optimal testing levels. The Lagrangian is:

\[
L = \sum_{i \in I^h} \pi_i w_i \phi(t_i; \theta_i) + \sum_{i \in I^l} \pi_i w_i (1 - \phi(t_i; \theta_i)) + \lambda \left( b - s \sum_i \pi_i t_i \right).
\]

Differentiation yields:

\[
\frac{\partial L}{\partial t_i} = \alpha \pi_i w_i \frac{t_i^{-\alpha-1}}{c_i} - \lambda s \pi_i = 0
\]

\[
\frac{\partial L}{\partial \lambda} = b - s \sum_i \pi_i t_i = 0.
\]

Manipulation of \( \frac{\partial L}{\partial t_i} \) produces:

\[
\lambda = \frac{\alpha w_i}{c_i s} t_i^{-\alpha-1}
\]

Substituting into \( \frac{\partial L}{\partial t_j} \) then yields:

\[
\frac{\alpha \pi_j w_j t_j^{-\alpha-1}}{c_j} = \frac{\alpha \pi_j w_i t_i^{-\alpha-1}}{c_i} \left( \frac{1}{1+\alpha} \frac{c_i t_j}{c_j t_i} \right)
\]

\[
t_j = \left( \frac{c_i t_j}{c_j t_i} \right)^{\frac{1}{1+\alpha}} t_i
\]

This solution applies for all interior \( t_i, t_j \), as claimed.

Proof of Lemma 2. To show that \( t_i^* = l^i \) for all \( \theta_i \in \Theta^l \) and \( t_i^* = t^h \) for all \( \theta_i \in \Theta^h \), suppose otherwise; i.e., there exists some \( t_j^* > t_k^* \) for types \( \theta_j \) and \( \theta_k \), where either \( \theta_j, \theta_k \in \Theta^l \) or \( \theta_j, \theta_k \in \Theta^h \). For marginal types, it is clear that \( u_c(\theta_j; \theta_j) < u_c(\theta_k; \theta_j) \). Thus, type \( \theta_j \) would strictly prefer to claim to be type \( \theta_k \), contradicting IC. For qualified types we have \( u_c(\theta_k; \theta_k) < u_c(\theta_j; \theta_k) \), and so type \( \theta_k \) would strictly prefer to claim to be type \( \theta_j \), also contradicting IC.

To show that \( l^i \leq t^h \), suppose otherwise. Then for all types \( \theta_j \in \Theta^l \) and \( \theta_k \in \Theta^h \), \( u_c(\theta_j; \theta_j) < u_c(\theta_k; \theta_j) \) and \( u_c(\theta_k; \theta_k) < u_c(\theta_j; \theta_k) \). Thus, all types in \( \Theta^l \) (respectively, \( \Theta^h \)) would strictly prefer to claim to be of a type in \( \Theta^h \) (respectively, \( \Theta^l \)), contradicting IC.

Proof of Proposition 1. I first derive characteristics of the bureaucrat’s optimal testing levels for qualified and marginal types, ignoring IC (6). As the objective is concave with
respect to all \( t_i \) and the constraint is linear, first order conditions are sufficient for characterizing the optimal testing levels. Denote by \( \bar{t} \) and \( \bar{t} \) the (uniform) optimal testing levels for types in \( \Theta^l \) and \( \Theta^h \), respectively. The Lagrangian is:

\[
\mathcal{L} = \sum_{i \in I^l} \pi_i w_i (1 - \phi(t; \theta_i)) + \sum_{i \in I^h} \pi_i w_i \phi(t; \theta_i) + \lambda \left( b - s \sum_{i \in I^l} \pi_i \bar{t} - s \sum_{i \in I^h} \pi_i \bar{t} \right).
\]

Differentiation yields:

\[
\frac{\partial \mathcal{L}}{\partial \bar{t}} = \alpha \bar{t} - \alpha - 1 \sum_{i \in I^l} \frac{\pi_i w_i c_i}{\bar{t}^\alpha} - \lambda s \sum_{i \in I^l} \pi_i - 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \bar{t}} = \alpha \bar{t} - \alpha - 1 \sum_{i \in I^h} \frac{\pi_i w_i c_i}{\bar{t}^\alpha} - \lambda s \sum_{i \in I^h} \pi_i = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = b - s \sum_{i \in I^l} \pi_i \bar{t} - s \sum_{i \in I^h} \pi_i \bar{t} = 0.
\]

Let \( P^l = \sum_{i \in I^l} \pi_i \), \( P^h = \sum_{i \in I^h} \pi_i \), \( S^l = \sum_{i \in I^l} \frac{\pi_i w_i c_i}{\bar{t}^\alpha} \), and \( S^h = \sum_{i \in I^h} \frac{\pi_i w_i c_i}{\bar{t}^\alpha} \). Then manipulation of \( \frac{\partial \mathcal{L}}{\partial \bar{t}} \) produces:

\[
\lambda = \frac{\alpha S^l}{s P^l} \bar{t}^{-\alpha - 1}
\]

Substituting into \( \frac{\partial \mathcal{L}}{\partial \bar{t}} \) then yields the following ratio (at an interior solution):

\[
S^h \bar{t}^{-\alpha - 1} = \frac{P^h S^l}{P^l \bar{t}^\alpha}
\]

\[
\bar{t} = \left( \frac{P^h S^l}{P^l S^h} \right)^{\frac{1}{\alpha + 1}} \bar{t}.
\]

(i) Rejection bias \( \Theta \) implies \( \bar{t} < \bar{t} \), and thus IC is not satisfied. Abusing notation slightly, let \( u_b(t^h, t^l) \) denote the bureaucrat’s objective when all types \( \Theta^h \) receive testing level \( t^h \) and all types in \( \Theta^l \) receive testing level \( t^l \). Note also that if \( t^h = t^l \), then the optimal testing level for all types is \( b/s \). Suppose that there exists a feasible solution \( (\bar{t}, \bar{t}) \) such that \( \bar{t} > \bar{t} \). If \( (\bar{t}, \bar{t}) \) gives the bureaucrat higher utility than \( t_i = b/s \) for all \( \theta_i \), then combining results we have:

\[
u_b(\bar{t}, \bar{t}) > u_b(b/s, b/s)
\]

\[
u_b(\bar{t}, \bar{t}) > u_b(b/s, b/s).
\]

Now observe that \( (\bar{t}, \bar{t}), (\bar{t}, \bar{t}), \) and \( (b/s, b/s) \) all lie along the bureaucrat’s budget constraint, which is linear in testing levels. Thus, any \( (\bar{t}, \bar{t}) \) such that \( \bar{t} > \bar{t} \) must violate the concavity of the bureaucrat’s objective function: contradiction. The optimal allocation satisfying IC is therefore \( (b/s, b/s) \).

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(ii) Substituting \( t \) and \( l \) into the budget constraint produces:

\[
\begin{align*}
t^{h*} &= \frac{b}{s \left[1 - P^l + P^l \left( \frac{P^h S^l}{P^l S^h} \right)^{\frac{1}{1+\alpha}} \right]} \\
t^{l*} &= \frac{b}{s \left[1 - P^h + P^h \left( \frac{P^h S^l}{P^l S^h} \right)^{-\frac{1}{1+\alpha}} \right]}
\end{align*}
\]

Note that these solutions are interior iff \( t^{h*} \geq 1 \) and \( t^{l*} \geq 1 \. Defining \( m \) and \( m \) as follows yields the expressions in the result:

\[
\begin{align*}
m &= 1 - P^l + P^l \left( \frac{P^h S^l}{P^l S^h} \right)^{\frac{1}{1+\alpha}} = 1 - \left( 1 - \left( \frac{\sum_{i \in I^h} \pi_i \sum_{i \in I^l} \frac{\pi_{il} \alpha_i}{c_i}}{\sum_{i \in I^l} \pi_i} \right)^{\frac{1}{1+\alpha}} \right) \left( \frac{\sum_{i \in I^l} \pi_i}{1} \right) \\
m &= 1 - P^h + P^h \left( \frac{P^h S^l}{P^l S^h} \right)^{-\frac{1}{1+\alpha}} = 1 - \left( 1 - \left( \frac{\sum_{i \in I^h} \pi_i \sum_{i \in I^l} \frac{\pi_{il} \alpha_i}{c_i}}{\sum_{i \in I^l} \pi_i} \right)^{-\frac{1}{1+\alpha}} \right) \left( \frac{\sum_{i \in I^l} \pi_i}{1} \right)
\end{align*}
\]

Finally, by Lemma \( \ref{lem:ic} \) IC is satisfied for all types if \( t^{h*} > t^{l*} \. Using the expressions for \( t^{h*} \) and \( t^{l*} \) above, this simplifies to:

\[
P^l \left[1 - \left( \frac{P^h S^l}{P^l S^h} \right)^{\frac{1}{1+\alpha}}\right] > P^h \left[1 - \left( \frac{P^h S^l}{P^l S^h} \right)^{-\frac{1}{1+\alpha}}\right]
\]

This expression holds iff \( P^h S^l < P^l S^h \), which is equivalent to acceptance bias (9). \( \blacksquare \)

**Proof of Proposition \( \ref{prop:proof1} \)** Since \( \theta_p \in \Theta^l \), \( u_c(\cdot) \) is decreasing in \( t_p \) and hence \( u_p(s, b; \theta_p) \) is also decreasing in \( t_p \). By Lemma \( \ref{lem:ic} \), a minimal level of testing satisfies IC and so \( t^*_p = t^{l*} = t^{h*} = 1 \. Given this, it is furthermore easily verified that \( u^*_p(s, b; \theta_p) \) (5) is increasing in \( s \) and decreasing in \( b \), thus implying \( b^* = s^* \) (since the minimum testing level is 1). Substituting \( t^*_p \) and \( b^* \) into (5) produces a concave function of \( s \) (12). The resulting first order condition is:

\[
\frac{du_p}{ds} = \frac{\pi_p \theta_p}{c_p} - q s = 0.
\]

This produces a unique interior solution \( s^* = \frac{\pi_p \theta_p}{qc_p} \). \( \blacksquare \)

**Proof of Proposition \( \ref{prop:proof2} \)** When \( \theta_p \in \Theta^h \) and the bureaucrat is rejection biased, Proposition \( \ref{prop:proof1}(i) \) implies \( t^{l*} = t^{h*} = b/s \. Substituting in \( t^{h*} \), the principal’s objective is given by (14).

I first establish concavity of the objective. Straightforward differentiation produces the Hessian:

\[
\begin{vmatrix}
-\frac{(1+\alpha)s^{-1+\alpha} \pi_p b^{-\alpha} \theta_p}{(1+\alpha)c_p} & \frac{(1+\alpha) \pi_p b^{-1-\alpha} \theta_p}{(1+\alpha)c_p} \\
\frac{(1+\alpha) \pi_p b^{-1-\alpha} \theta_p}{c_p} & -q - \frac{(1+\alpha) \pi_p b^{-2-\alpha} \theta_p}{c_p}
\end{vmatrix}
\]

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The quadratic form evaluates to \(-qb^2\), which is strictly negative, and thus the principal’s objective (14) is strictly concave. Thus conditions for an optimum are:

\[
\frac{\partial u_p}{\partial b} = \alpha \pi_p \frac{s^{\alpha+1}}{c_p b^{\alpha+1}} \theta_p - qb = 0
\]

\[
\frac{\partial u_p}{\partial s} = \pi_p \theta_p - (\alpha + 1) \pi_p \frac{s^{\alpha}}{c_p b^{\alpha}} \theta_p = 0.
\]

Solving yields:

\[
b = \frac{\alpha \pi_p \theta_p c_p^{\frac{1}{\alpha}}}{q (\alpha + 1)^{\frac{\alpha}{\alpha + 1}}} \]

\[
s = \frac{\alpha \pi_p \theta_p c_p^{\frac{2}{\alpha}}}{q (\alpha + 1)^{\frac{\alpha + 2}{\alpha}}}
\]

Testing levels \(t^*\) are derived simply by substituting into (3). 

**Proof of Proposition 4.** When \(\theta_p \in \Theta^h\) and the bureaucrat is acceptance biased, \(t^{h*}\) is given by Proposition 1(ii) and the principal’s corresponding objective is given by (15). By an analysis similar to that in the proof of Proposition 3 it is straightforward to verify that this objective is globally concave. Thus necessary conditions for an optimum are:

\[
\frac{\partial u_p}{\partial b} = \alpha \pi_p \frac{s^{\alpha+1}}{c_p b^{\alpha+1}} \theta_p - qb = 0
\]

\[
\frac{\partial u_p}{\partial s} = \pi_p \theta_p - (\alpha + 1) \pi_p \frac{s^{\alpha}}{c_p b^{\alpha}} \theta_p = 0.
\]

Solving yields:

\[
b = \frac{\alpha \pi_p \theta_p c_p^{\frac{1}{\alpha}}}{q m (\alpha + 1)^{\frac{\alpha}{\alpha + 1}}} \]

\[
s = \frac{\alpha \pi_p \theta_p c_p^{\frac{2}{\alpha}}}{q m^2 (\alpha + 1)^{\frac{\alpha + 2}{\alpha}}}
\]

The testing levels \(t^{l*} = b^*/(s^*m)\) and \(t^{h*} = b^*/(s^*m)\) follow from Proposition 1(ii). 

**Proof of Comment 2.** (i) First consider the values of \(b^*\) and \(s^*\) from Propositions 3 and 4 conditional upon whether bureaucratic acceptance bias holds. When it does not, \(b^*\) and \(s^*\) are independent of all \(w_i\) and \(c_i\), except for \(c_p\). Note that \(m\) is decreasing in \(w_i\) for \(\theta_i \in \Theta^h\) and \(c_i\) for \(\theta_i \in \Theta^l\), and increasing in \(w_i\) for \(\theta_i \in \Theta^l\) and \(c_i\) for \(\theta_i \in \Theta^h\). Thus when the bureaucrat is acceptance biased, \(b^*\) and \(s^*\) are increasing in \(w_i\) for \(\theta_i \in \Theta^h\) and \(c_i\) for \(\theta_i \in \Theta^h\) and \(\theta_i \in \Theta^l\), and decreasing in \(w_i\) for \(\theta_i \in \Theta^l\) and \(c_i\) for \(\theta_i \in \Theta^h\) and \(\theta_i \neq \theta_p\).
Next, consider the values of $b^*$ and $s^*$ when both sides of the acceptance bias condition (9) are equal. This implies $m = 1$, and hence $b^*$ and $s^*$ are continuous in all $w_i$ and $c_i$. Since acceptance bias is satisfied when $w_i$ for $\theta_i \in \Theta^h$ or $c_i$ for $\theta_i \in \Theta^l$ are sufficiently large, or $w_i$ for $\theta_i \in \Theta^l$ or $c_i$ for $\theta_i \in \Theta^h$ and $\theta_i \neq \theta_p$ are sufficiently small, the result follows.

The result for $q$ follows from the expressions for $b^*$ and $s^*$ in Propositions 3 and 4.

(ii) Using Propositions 3 and 4 per capita spending is given by:

$$\frac{b^*}{s^*} = \begin{cases} \frac{m(\alpha+1)^{1/\alpha}}{c_p^{1/\alpha}} & \text{if bureaucrat acceptance biased} \\ \frac{m(\alpha+1)^{1/\alpha}}{c_p^{1/\alpha}} & \text{if bureaucrat rejection biased} \end{cases}$$

This expression is increasing in $m$ if the bureaucrat is acceptance biased, and also dependent on $c_p$. Thus it is straightforward to show that the comparative statics for $w_i$ and $c_i$ are the reverse of those for $b^*$ and $s^*$ in part (i).

The result for $q$ follows from the expressions for $b^*$ and $s^*$ in Propositions 3 and 4.

(iii) This follows immediately from Proposition 1.

(iv) To show the result for Type I errors, note that by Propositions 3 and 4 the probability of acceptance $\phi(t; \theta_i)$ for qualified types depends only on $\alpha$. By Proposition 1 $t^* < t^{h*}$ at an interior solution when the bureaucrat is acceptance biased, and thus any combination of parameters such that the bureaucrat is acceptance biased results in a higher probability of passage for types in $\Theta^l$, thus establishing the result.

The result on Type II errors follows directly from calculating $\phi(t; \theta_i)$ using the expressions for $t^{h*}$ in Propositions 3 and 4. 

Proof of Comment 3. (i) When $\theta^p \in \Theta^l$, $b^*/s^* = t^* = 1$, which is the minimum feasible testing level. By (1), this testing level also minimizes Type I and II error avoidance rates. Finally, flexibility is minimized since all types are tested at $t^*$.

(ii) For the budget calculation I begin by showing the result for an acceptance biased bureaucrat. At an interior solution $b^*$ when $\theta^p \in \Theta^l$ is smaller than when $\theta^p \in \Theta^h$ if:

$$\frac{\pi_p \theta_p}{q c_p} \leq \frac{\alpha \pi_p \theta_p c_p^{1/\alpha}}{q m (\alpha + 1)^{(\alpha+1)/\alpha}}$$

$$1 \leq \frac{\alpha}{m} \left( \frac{c_p}{\alpha + 1} \right)^{(\alpha+1)/\alpha}.$$  \hspace{1cm} (19)

Since $c_p \geq 2$ by assumption and $c_p < \alpha + 1$ at an interior solution, it is sufficient to check that (19) is true for $\alpha \geq 1$ and $c_p \geq 2$. At $\alpha = 1$ and $c_p = 2$, the right-hand side of (19) evaluates to $1/m$; under acceptance bias, $m < 1$, and so (19) holds.

To show that (19) holds for other values of $\alpha$ and $c_p$, observe that the right-hand side is strictly increasing in $c_p$. Taking the derivative of the right-hand side of (19) with respect to $\alpha$ yields:

$$\left(1 + \alpha\right)^{-\frac{\alpha}{\alpha + 1}} c_p^{\frac{\alpha}{\alpha + 1}} \left(\ln(1 + \alpha) - \ln c_p\right)$$

$$\frac{m}{\alpha}.$$
This expression is clearly positive for all \( \alpha > c_p - 1 \), and hence (19) always holds for any feasible value of \( \alpha \) and \( c_p \).

The result under rejection bias follows by substituting in \( \bar{m} = 1 \).

For the client population calculation, I first consider an acceptance biased bureaucrat. At an interior solution \( s^* \) when \( \theta_p \in \Theta^l \) is smaller than when \( \theta_p \in \Theta^h \) if:

\[
\frac{\pi_p \theta_p}{qc_p} \leq \frac{\alpha \pi_p \theta_p c_p^{2/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(\alpha+2)/\alpha}} \cdot \frac{1}{\alpha} \left( \frac{c_p}{\alpha + 1} \right)^{(\alpha+2)/\alpha}
\]

Observe that the right-hand side of (20) is strictly increasing in \( c_p \) and \( \lim_{c_p \to \alpha + 1} \left( \frac{c_p}{\alpha + 1} \right)^{(\alpha+2)/\alpha} = 1 \). Then by the fact that \( c_p < \alpha + 1 \) at an interior solution, it is sufficient to check that (20) holds strictly for \( c_p = \alpha + 1 \). At this value, the right-hand side of (20) evaluates to \( \alpha / \bar{m}^2 \).

Since \( c_p \geq 2 \) by assumption and \( c_p < \alpha + 1 \), we have that \( \alpha > 1 \) at an interior solution. Acceptance bias implies \( \bar{m} < 1 \), and so the right-hand side of (20) is bounded away from 1.

The result under rejection bias follows by substituting in \( \bar{m} = 1 \).

**Proof of Comment 4** (i) When \( \theta_p \in \Theta^l \), \( b^*/s^* = 1 \). Substituting into the principal’s objective (12) yields:

\[
u_p(s^*, b^*; \theta_p) = \pi_p s^* \frac{\theta_p}{c_p} - \frac{q}{2} b^2 = \frac{\pi_p^2 \theta_p}{2qc_p^2}.
\]

This expression is clearly decreasing in \( c_p \).

(ii) Substituting from Proposition 4, the objective for both the acceptance biased and rejection biased cases is:

\[
u_p(s^*, b^*; \theta_p) = \pi_p s^* \left( \frac{\alpha}{\alpha + 1} \right) \theta_p - \frac{q}{2} b^2 = \pi_p s^* \left( \frac{\alpha}{\alpha + 1} \right) \theta_p - \frac{q}{2} b^2
\]

In the subcase of an acceptance biased bureaucrat, substituting \( s^* \) and \( b^* \) into (21) produces:

\[
u_p(s^*, b^*; \theta_p) = \pi_p \left( \frac{\alpha \pi_p \theta_p c_p^{2/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(\alpha+2)/\alpha}} \right) \left( \frac{\alpha}{\alpha + 1} \right) \theta_p - \frac{q}{2} \left( \frac{\alpha \pi_p \theta_p c_p^{1/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(\alpha+1)/\alpha}} \right)^2
\]

\[
= \frac{\alpha^2 \pi_p^2 \theta_p^2 c_p^{2/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(2\alpha+2)/\alpha}} - \frac{q}{2} \left( \frac{\alpha \pi_p \theta_p c_p^{1/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(\alpha+1)/\alpha}} \right)^2
\]

\[
= \frac{\alpha^2 \pi_p^2 \theta_p^2 c_p^{2/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(\alpha+2)/\alpha}} - \frac{q}{2} \left( \frac{\alpha \pi_p \theta_p c_p^{1/\alpha}}{q \bar{m}^2 (\alpha + 1)^{(\alpha+1)/\alpha}} \right)^2
\]

\[
= \frac{\alpha^2 \pi_p^2 \theta_p^2 c_p^{2/\alpha}}{2q \bar{m}^2 (\alpha + 1)^{(2\alpha+2)/\alpha}}
\]

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This expression is obviously increasing in \( c_p \).

Next, for the subcase of a rejection biased bureaucrat, note that the calculation is identical after substituting \( \overline{m} = 1 \). \qed

**Proof of Comment 5.** I construct a modified mechanism with two testing levels \( t^* \) and \( t^h \). All types in \( \Theta^l \setminus \{ \theta_1 \} \) are tested at level \( t^* \) and receive red tape causing disutility \( (\phi(t^*; \theta_1) - \phi(t^h; \theta_1))\theta_1 \). All other types are tested at level \( t^h \) and receive no red tape.

As the objective is concave with respect to all \( t_i \) and the constraint is linear, first order conditions are sufficient for characterizing the optimal testing levels. Let \( T' = T \setminus \{ 1 \} \) and \( T'h = T \cup \{ 1 \} \) denote the set of indices of types that receive low and high testing levels, respectively. Denote by \( t \) and \( \overline{t} \) the (uniform) low and high testing levels, respectively.

The Lagrangian is:

\[
L = \sum_{i \in T'} \pi_i w_i \left( 1 - \phi(t; \theta_i) \right) + \sum_{i \in T} \pi_i w_i \phi(\overline{t}; \theta_i) + \pi_1 w_1 \left( 1 - \phi(\overline{t}; \theta_1) \right) - \tau \sum_{i \in T'} \pi_i \left( \phi(t; \theta_1) - \phi(\overline{t}; \theta_1) \right) \theta_1 + \lambda \left( b - s \sum_{i \in T'} \pi_i t - s \sum_{i \in T'h} \pi_i \overline{t} \right).
\]

Differentiation yields:

\[
\frac{\partial L}{\partial \overline{t}} = \alpha \overline{t}^{\alpha-1} \left[ \sum_{i \in T'} \frac{\pi_i w_i}{c_i} - \sum_{i \in T'} \frac{\pi_i t_i}{c_i} + \frac{\pi_1 w_1}{c_1} \right] - \lambda s \sum_{i \in T'h} \pi_i \overline{t} = 0
\]

Following the proof of Proposition 1 let \( P^l = \sum_{i \in T} \pi_i, P^h = \sum_{i \in T'h} \pi_i, S^l = \sum_{i \in T'} \frac{\pi_i w_i}{c_i} \), and \( S^h = \sum_{i \in T'h} \frac{\pi_i w_i}{c_i} \). Additionally, define the analogous terms for the sets \( T' \) and \( T'h \): let \( P'^l = \sum_{i \in T'} \pi_i, P'^h = \sum_{i \in T'h} \pi_i, S'^l = \sum_{i \in T'} \frac{\pi_i w_i}{c_i} \), and \( S'^h = \sum_{i \in T'h} \frac{\pi_i w_i}{c_i} \). Finally let \( T' = \sum_{i \in T'} \pi_i t_i \). Then manipulation of \( \frac{\partial L}{\partial \overline{t}} \) produces:

\[
\lambda = \alpha \overline{t}^{\alpha-1} \left( \frac{S'^l + T'}{sP'} \right)
\]

Substituting into \( \frac{\partial C}{\partial \overline{t}} \) then yields:

\[
\overline{t}^{-\alpha-1} \left[ S^h - T' + \frac{\pi_1 w_1}{c_1} \right] = \frac{(S'^l + T')P'^h}{P'^l} \overline{t}^{-\alpha-1} \quad \frac{1}{\alpha} \overline{t}
\]

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Substituting into the budget constraint produces:

\[
b = s \left[ P^{t'} \left( \frac{(S^{t'} + T^{t'})P^{h'}}{(S^{h'} - T^{t'}) + \frac{\pi_1 w_1}{c_1}} \right)^{\frac{1}{1+\alpha}} + P^{h'} \right] \tilde{t}
\]

\[
t_{h^*} = \frac{b}{s \left[ P^{h'} + P^{t'} \left( \frac{(S^{h'} + T^{h'})P^{h'}}{(S^{h'} - T^{h'}) + \frac{\pi_1 w_1}{c_1}} \right)^{\frac{1}{1+\alpha}} \right]}
\]

\[
t_{l^*} = \frac{b}{s \left[ P^{t'} + P^{h'} \left( \frac{(S^{t'} + T^{h'})P^{h'}}{(S^{h'} - T^{t'}) + \frac{\pi_1 w_1}{c_1}} \right)^{-\frac{1}{1+\alpha}} \right]}
\]

Note that these solutions are interior iff \( t_{h^*} \geq 1 \) and \( t_{l^*} \geq 1 \).

Now consider the IC constraints. For types in \( \Theta^h \), it is obvious that IC is satisfied by declaring truthfully if \( t_{h^*} > t_{l^*} \). For types \( \theta_i \in \Theta^l \setminus \{\theta_1\} \), IC is satisfied if:

\[
\phi(t_{l^*}; \theta_i) - \phi(t_{l^*}; \theta_1) - \phi(\phi(t_{h^*}; \theta_1)) \theta_1 \geq \phi(t_{h^*}; \theta_i) \theta_i.
\] (22)

By the assumptions that \( c_1 > c_i \) and \( \theta_1 \leq \theta_i \) for all \( \theta_i \in \Theta^l \setminus \{\theta_1\} \), (22) must hold for all types in \( \Theta^l \setminus \{\theta_1\} \). Finally, by construction expression \( \phi(t_{h^*}; \theta_i) \theta_i \) holds with equality for type \( \theta_1 \), and thus type \( \theta_1 \) truthfully announces her type as well.

I finally provide conditions under which \( t_{h^*} > t_{l^*} \). The expressions for \( t_{h^*} \) and \( t_{l^*} \) imply that this is true if:

\[
(S^{t'} + T^{t'})P^{h'} < (S^{h'} - T^{h'}) + \frac{\pi_1 w_1}{c_1} P^{t'}
\]

\[
S^{t'} P^{h'} + T^{t'} < S^{h'} P^{t'}
\] (23)

From the proof of Proposition \( \text{[1]} \), the bureaucrat is rejection biased if \( P^{h'} S^l > P^l S^{h'} \). Observe that \( S^l \) and \( S^{h'} \) are increasing in \( w_1/c_1 \), and all other terms are independent of \( w_1/c_1 \). Thus for \( w_1/c_1 \) sufficiently high, the bureaucrat is rejection biased and (by (23)) uses red tape to discriminate between type \( \theta_1 \) and other types in \( \Theta^l \).

\section*{Proof of Proposition \( \text{[5]} \).} First consider an acceptance biased bureaucrat. From the derivation in Section \( \text{[3.2]} \) \( m \) and \( m_2 \) are the ratios between \( b/s \) and the bureaucrat’s testing levels for high and low types, respectively. Thus, the bureaucrat tests types in \( \Theta^h \) and \( \Theta^l \) at level \( b/(sm) \) and \( b/(sm) \), respectively.

The principal’s objective \( \text{[17]} \) can then be rewritten in terms of \( s \) and \( b \) as follows.

\[
u_p(s, b; \theta_p) = s \sum_{i \in \Theta^l} \pi_i e_i \left( 1 - \frac{s^\alpha m^\alpha}{c_i b^\alpha} \right) + \sum_{i \in \Theta^h} \pi_i e_i \left( 1 - \frac{s^\alpha m^\alpha}{c_i b^\alpha} \right) + (N - s) \sum_{i \in \Theta^l} \pi_i e_i - \frac{q b^2}{2}.
\] (24)
This function is clearly concave in \(s\) and \(b\); thus an interior solution must satisfy the following first order conditions.

\[
\frac{\partial u^e}{\partial b} = \alpha \left( m^\alpha \sum_{i \in I^P} \pi_i e_i s_i^{\alpha + 1} c_i b_i^{\alpha + 1} + m^\alpha \sum_{i \in I^B} \pi_i e_i s_i^{\alpha + 1} c_i b_i^{\alpha + 1} \right) - q b = 0
\]

\[
\frac{\partial u^e}{\partial s} = \sum_i \pi_i e_i - (\alpha + 1) \left( \sum_{i \in I^P} \pi_i e_i s_i^\alpha m_p c_p b_i^{\alpha + 1} + \sum_{i \in I^B} \pi_i e_i s_i^\alpha m_p c_p b_i^{\alpha + 1} \right) - \sum_i \pi_i e_i = 0.
\]

These simplify to:

\[
b^e = \left[ \frac{s^\alpha + \alpha}{q} \epsilon(m, m) \right]^{\frac{1}{\alpha + 2}}
\]

\[
s^e = b \left[ \frac{\sum_{i \in I^B} \pi_i e_i}{(\alpha + 1) \epsilon(m, m)} \right]^{\frac{1}{\alpha}}.
\]

Solving then produces:

\[
b^e = \frac{\alpha \left( \sum_{i \in I^B} \pi_i e_i \right)^{\frac{\alpha + 1}{\alpha}}}{q (\alpha + 1) \frac{\alpha + 1}{\alpha} \epsilon(m, m)}
\]

\[
s^e = \frac{\alpha \left( \sum_{i \in I^B} \pi_i e_i \right)^{\frac{\alpha + 2}{\alpha}}}{q (\alpha + 1) \frac{\alpha + 2}{\alpha} \epsilon(m, m)^2}.
\]

The average testing level is \(b^e/s^e = \left( \frac{(\alpha + 1) \epsilon(m, m)}{\sum_{i \in I^B} \pi_i e_i} \right)^{1/\alpha} \). The equilibrium testing levels are then \(t^B = b^e/(s^e m)\) and \(t^P = b^e/(s^e m)\).

Next, consider a rejection biased bureaucrat. By Proposition 1, all testing levels are identically \(b/s\). The principal’s solution for \(b\) and \(s\) can then be found by setting \(m = m = 1\) in the preceding derivation. This produces:

\[
b^e = \frac{\alpha \left( \sum_{i \in I^B} \pi_i e_i \right)^{\frac{\alpha + 1}{\alpha}}}{q (\alpha + 1) \frac{\alpha + 1}{\alpha} \sum_i \pi_i e_i c_i}
\]

\[
s^e = \frac{\alpha \left( \sum_{i \in I^B} \pi_i e_i \right)^{\frac{\alpha + 2}{\alpha}}}{q (\alpha + 1) \frac{\alpha + 2}{\alpha} \sum_i \pi_i e_i c_i}.
\]

**Proof of Comment 6.** From Propositions 3 and 4, the budget when \(\theta_p \in \Theta^B\) is:

\[
b^* = \begin{cases} 
\frac{\alpha \pi_p \theta_p \epsilon_p^{1/\alpha}}{q^\alpha (\alpha + 1)^{\alpha + 1}/\alpha} & \text{if bureaucrat acceptance biased} \\
\frac{\alpha \pi_p \theta_p \epsilon_p^{1/\alpha}}{q^\alpha (\alpha + 1)^{\alpha + 1}/\alpha} & \text{if bureaucrat rejection biased,}
\end{cases}
\]
where \( \overline{m} \) is given in (10). \( b^* \) is maximized by minimizing \( \overline{m} \) subject to bureaucratic acceptance bias holding (equivalently, \( \overline{m} < 1 \)). Rewriting the acceptance bias condition (9) produces:

\[
\left( \sum_{i \in I^h} \pi_i \right) \sum_{i \in I^l} \frac{\pi_i w_i}{c_i} < \left( \sum_{i \in I^l} \pi_i \right) \sum_{i \in I^h} \frac{\pi_i w_i}{c_i}.
\]

(25)

It is clear that \( \overline{m} \) is minimized by choosing \( w_i = \overline{w} \) for \( \theta_i \in \Theta^l \), and \( w_i = \overline{m} \) for \( \theta_i \in \Theta^h \). By the continuity of the left-hand side of (25) in \( w_i \), if \( \overline{w} \) sufficiently close to 0 then (25) is also satisfied at this solution, and thus \( \overline{m} < 1 \) and the bureaucrat is acceptance biased. ■

**Proof of Comment 7.** The argument for why the principal cannot benefit from any restrictions on \( t \) when the bureaucrat is rejection biased is given in the text.

When the bureaucrat is acceptance biased, suppose that under the testing restriction \( t(\theta_i) \in [t_{\min}, t_{\max}] \) the bureaucrat implements \( t' \) for types in \( \Theta^l \) and \( t'' \) for types in \( \Theta^h \), where \( t' \leq t'' \). Clearly, \( t' \) and \( t'' \) satisfy the bureaucrat’s budget constraint for some population \( s' \) and budget \( b' \). By Proposition 1 in the absence of the testing restriction, the principal could use the same \( s' \) and \( b' \) to induce the bureaucrat to implement \( t'' = \frac{b'}{\overline{m}s'} \) for types in \( \Theta^h \) and \( t' = \frac{b'}{\overline{m}s'} \) satisfying the budget constraint for types in \( \Theta^l \). Since the budget constraint binds, there are two possibilities; first, either \( t'' > t' \) and \( t' < t'' \), or second, \( t'' < t' \) and \( t' > t'' \). Under the first, the principal does better without the testing restriction if \( t' \not\in [t_{\min}, t_{\max}] \) or \( t'' \not\in [t_{\min}, t_{\max}] \). Otherwise, the testing restriction does not bind and by the concavity of the bureaucrat’s objective either \( t' \) and \( t'' \) or \( t' \) and \( t'' \) cannot be the solution to their respective problems. Since the principal cannot benefit from a testing restriction for any given \( b \) and \( s \), he cannot benefit from any testing restriction. ■

**Proof of Comment 8.** Differentiating (18), the necessary conditions for an optimum are:

\[
\frac{\partial u_p}{\partial b} = \alpha \pi_p s^{\alpha + 1} \frac{c_p b^{\alpha + 1}}{c_p b^{\alpha + 1}} \theta_p - \alpha \gamma \left[ \sum_{i \in I^h} \pi_i s^{\alpha + 1} \frac{c_p b^{\alpha + 1}}{c_p b^{\alpha + 1}} - \sum_{i \in I^l} \pi_i s^{\alpha + 1} \frac{c_p b^{\alpha + 1}}{c_p b^{\alpha + 1}} \right] - qb = 0
\]

\[
\frac{\partial u_p}{\partial s} = \pi_p \theta_p - \pi_p \frac{(\alpha + 1)s^\alpha}{c_p b^\alpha} \theta_p - \gamma \left[ \sum_{i \in I^h} \pi_i \left( 1 - \frac{(\alpha + 1)s^\alpha}{c_i b^\alpha} \right) + \sum_{i \in I^l} \pi_i \frac{(\alpha + 1)s^\alpha}{c_i b^\alpha} \right] = 0.
\]

Simplifying and solving the system yields:

\[
b = \left( \frac{\alpha \pi_p \theta_p - \alpha \gamma \sum_{i \in I^h} \pi_i c_p - \sum_{i \in I^l} \pi_i c_p}{q c_p} \right)^{\frac{1}{\alpha + 2}} \frac{s^{\alpha + 1}}{s^{\alpha + 2}}
\]

\[
s = \left( \frac{\pi_p \theta_p - \gamma \sum_{i \in I^h} \pi_i c_p}{\pi_p \frac{(\alpha + 1)s^\alpha}{c_p} - \gamma \sum_{i \in I^h} \pi_i \frac{(\alpha + 1)s^\alpha}{c_i} - \sum_{i \in I^l} \pi_i \frac{(\alpha + 1)s^\alpha}{c_i}} \right)^{\frac{1}{\alpha}} b
\]

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\[
b = \left( \frac{\pi_p \theta_p - \gamma \sum_{i \in I^h} \pi_i}{(\alpha + 1) \left[ \frac{\pi_p \theta_p}{c_p} - \gamma \left( \sum_{i \in I^h} \pi_i \frac{c_i}{c} - \sum_{i \in I^l} \pi_i \frac{c_i}{c} \right) \right]} \right)^{\frac{\alpha + 1}{\alpha}} \cdot \left( \frac{\alpha \pi_p \theta_p}{qc_p} - \frac{\alpha \gamma}{q} \left[ \sum_{i \in I^h} \pi_i c_p - \sum_{i \in I^l} \pi_i c_p \right] \right) \]

\[
s = \left( \frac{\pi_p \theta_p - \gamma \sum_{i \in I^h} \pi_i}{(\alpha + 1) \left[ \frac{\pi_p \theta_p}{c_p} - \gamma \left( \sum_{i \in I^h} \pi_i \frac{c_i}{c} - \sum_{i \in I^l} \pi_i \frac{c_i}{c} \right) \right]} \right)^{\frac{\alpha + 2}{\alpha}} \cdot \left( \frac{\alpha \pi_p \theta_p}{qc_p} - \frac{\alpha \gamma}{q} \left[ \sum_{i \in I^h} \pi_i c_p - \sum_{i \in I^l} \pi_i c_p \right] \right) \]

Further simplification yields the resulting \(b^*\) and \(s^*\). By \([3]\), the testing level is simply \(t^* = b^*/s^*\). \(\blacksquare\)