Primary Elections and the Provision of Public Goods*

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Abstract

We develop a model of electoral competition in which candidates from two parties compete in primary and general elections. There are three groups of voters, two representing “core” supporters for each party and one “swing” group. In the primary elections, each party’s core voters choose a candidate to run in the general election. Candidates within a party share a fixed ideology and can promise to distribute a unit of public spending across private goods for each group and a public good. Without primaries, candidates offer only public goods when they are very valuable, and only private goods to the swing group otherwise. By appealing to both types of voters, primaries can encourage the provision of public goods when the swing group is small and core groups are moderate, but not in general. The level of public good provision is non-monotonic in ideological polarization. Additionally, primary elections shift benefits toward core voters at the expense of swing voters, but less so as core voters become more ideologically extreme.

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1 Introduction

How do political institutions shape public policy? This is a fundamental question in political economy. One important sub-question is: What features of the political system provide incentives for politicians or parties to spend government funds on public goods that benefit the vast majority of citizens, rather than goods targeted at narrow groups? This has been the subject of a number of recent theoretical and empirical studies, including Lancaster (1986), Persson and Tabellini (1999, 2003, 2004a, 2004b), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002), Persson, Roland and Tabellini (2000, 2007), and Blume et al. (2009).

These studies focus on key features of electoral systems and the separation of powers, such as plurality rule vs. proportional representation systems, district magnitude, and parliamentary vs. presidential systems. One of the main arguments is that the “winner-take-all” structure of electoral outcomes under plurality rule with single-member-districts implies that the minimum winning coalition of voters to gain a majority in the legislature is smaller than under proportional representation. Thus, plurality rule induces politicians to target small but pivotal constituencies in individual electoral districts with local public goods and specific transfers. In contrast, under proportional representation “every vote counts” no matter where it is cast, and additional votes always translate directly into additional seats, providing incentives for politicians to seek the support of voters across the country. Proportional representation therefore induces politicians to favor policies benefiting large groups of voters such as general public goods and broad-based transfer programs that affect voters in many electoral districts.\footnote{Persson and Tabellini (1999, 2004b), Milesi-Ferretti, Perotti, and Rostagno (2002), Gagliarducci, Nannicini and Naticchioni (2011) find empirical support for these predictions. See also Lancaster and Patterson (1990), Stratmann and Baur (2002), and Kunicova and Remington (2005) for other evidence that is broadly consistent with the underlying incentives facing politicians. Many other features of the political system have also been studied, including bicameralism, vetoes, confidence procedures, party organization, and federalism. See, for example, Inman and Fitts (1990), Diermeier and Feddersen (1998), McCarty (2000a, 2000b), Bradbury and Crain (2002), Lockwood (2002), Ansolabehere, Snyder, and Ting (2003), Kalandrakis (2004), Cutrone and McCarty (2006), Berry (2008, 2009), Berry and Gersen (2009), Primo and Snyder (2010), Tergiman (2013), and Parameswaran (2012).}

One important political factor omitted from this analysis is internal party structure. In particular, there is no treatment of the various ways candidates are nominated. A variety of nomination methods are used around the world, including national, state, and local party conventions, caucuses, meetings restricted to small party elites, and direct primary elections. Primary elections are the dominant system for nominations in the U.S., are used widely in
many Latin American countries. They also appear to be increasingly popular, having been
used in recent years by parties in Italy, Spain, South Korea and elsewhere. Hirano, Snyder
and Ting (2009) show that nomination systems may have significant effects on the allocation
of distributive government spending. In particular, when electoral outcomes are uncertain,
direct primaries may provide strong incentives for politicians to offer transfers to “core
supporters” in addition to “swing voters.”

This paper shows that direct primaries may also provide incentives for politicians to sup-
ply public goods that benefit all voters, rather than distributive goods or narrowly targeted
transfers that only benefit specific constituencies. The logic is straightforward. If there are
no primary elections and candidates simply maximize their probability of winning in the gen-
eral election, then they are driven to compete mainly for swing voters. Thus, when deciding
between public goods and targeted goods, candidates are biased toward choosing targeted
goods — and targeting them at swing voters. They will only choose public goods if public
goods have an extremely high ratio of social benefits to costs. Under primaries, however,
candidates may offer a more even distribution of targeted goods, aimed both at swing voters
and core voters, since they must win both swing voters in the general election, and core
voters in the primary election. Thus, they have incentives to offer public goods even when
public goods have a relatively modest (but still favorable) ratio of benefits to costs.

Our theory is based on the models developed by Lindbeck and Weibull (1987) and Dixit
and Londregan (1995, 1996). In their work, office-minded candidates from two parties com-
pete for votes by promising distributions of particularistic spending across a large number of
ideologically heterogeneous groups. Parties have fixed ideological positions, and are therefore
advantaged in certain districts. We simplify this framework by considering only three groups
of voters, corresponding to a swing group and core groups for each party. We further assume
that core groups are “off limits” to the opposition in the sense that there is no incentive
for the opposing party to offer particularistic goods in the hope of gaining votes. We add
two important features. First, there are simultaneous primary elections in each party that
determine which of its candidates will proceed to the general election. The pivotal voter in

2For background on U.S. primary elections, see Merriam and Overacker (1928) and Ware (2002). For
more details on primaries in Latin America see Carey and Polga-Hecimovich (2006) and Kemahliliglu, Weitz-
Shaprio, and Hirano (2009).

3There is an extensive literature on the policy consequences of primaries or the incentives for adopting
primary systems, including Aronson and Ordeshook (1972), Coleman (1972), Owen and Grofman (2006),
Caillaud and Tirole (1999), Jackson, Mathevet, and Mattes (2007), Adams and Merrill (2008), Castanheira,
Crutzen, and Sahuguet (2010), Serra (2011), Crutzen (2014), and Negri (2014). Most of these models are
concerned with outcomes in a spatial or valence framework.
each primary is a member of the corresponding core group. Second, in addition to promising allocations of particularistic spending at the group level, candidates can commit to spending some portion of the budget on a public good that benefits all voters evenly.

As intuition might suggest, public goods will be attractive as platforms only when the value of the public good is high. Candidates from both parties choose private goods when that value is sufficiently low, and the public good when that value is sufficiently high. For intermediate values, parties will typically adopt platforms that combine the public good and private goods for either the swing group or their core group. Thus, equilibrium platforms never promise the public good and private goods to both groups. It is also possible in some cases for only one party to choose a positive level of public goods.

It is useful to illustrate the logic of platform selection in the model by considering the incentives of core groups when the value of the public good is relatively high. In choosing who wins their primary election, core groups must trade off between the probability of winning and the benefits received conditional upon victory. Extreme core voters are more inclined to give up private goods than moderates, because they are more concerned about ideological payoffs and private goods for swing voters raise a party’s probability of election. Consequently, private goods will go to the swing group when a party’s core group is extreme, and to the core group when it is moderate. In between these ideological extremes, public goods are a useful compromise between the two types of private goods. While the equilibrium of the general model can be quite complex, these relationships are examined in more detail in a variant of the model where the factions are symmetric with respect to core group size and ideology. Under these conditions, the level of public goods provision is non-monotonic: it increases initially in ideological polarization, and then abruptly drops to zero as candidates use only private goods to chase the swing group.

The model generates a number of predictions about the level of provision of private goods. Somewhat surprisingly, compared to a world without primary elections, public goods are not always better provided under a primary system. The reason for this is that when a core group is small, the per capita value of private goods can be so high that candidates will have to offer some in order to win the primary election. Compared to a non-primary system, the threshold social benefit at which public goods will be offered exclusively by all candidates is lower under a primary system when the swing group is the smallest group. Thus primaries can be said to encourage the exclusive production of public goods when core groups in society are both large and not too extreme. On another metric, however, primaries can do well in encouraging public goods. The threshold social benefit for candidates to promise some positive level of
public goods is lower than the threshold for offering only public goods. Thus the range of parameter values under which some public goods are offered will typically be larger under a primary system. But despite this, the model also predicts that this threshold is strictly above the level at which public goods are socially efficient.

The paper proceeds as follows. The next section describes the model. Section 3 derives the results for the model with and without primary elections, as well as for the special case of symmetric ideologies. Section 4 develops an extension that relaxes the assumption of core groups forming the majority of a primary electorate. Section 5 concludes.

2 Model

Our model considers electoral competition between two parties, $X$ and $Y$. There are two main variants of the model. In the first, there are no primary elections, and in the second, we introduce primaries within both parties. All elections are decided by plurality rule.

Voters are divided into three groups, indexed $i = 1, 2, 3$. The relative size of each group is $n_i$, with $\sum_{i=1}^{3} n_i = 1$. No group is an outright majority, so $n_i < 1/2$ for $i = 1, 2, 3$. Group membership is important to the model because candidates are able to offer transfers that are targeted specifically toward a group. Within each group, members enjoy the benefits of a targeted transfer equally.

Candidates in each party are denoted $a$ and $b$. Then $\mathbf{x}^j = (x_0^j, x_1^j, x_2^j, x_3^j)$ is the platform offer by candidate $j \in \{a, b\}$ in party $X$, where $x_0^j \geq 0$ is the amount allocated to a public good that is enjoyed by all citizens and for $i > 0$, $x_i^j \geq 0$ is targeted toward group $i$. Similarly, $\mathbf{y}^k = (y_0^k, y_1^k, y_2^k, y_3^k)$ is the platform offered by candidate $k \in \{a, b\}$ in party $Y$. Offers must satisfy the budget constraints $x_0^j + \sum_i n_i x_i^j = 1$ and $y_0^k + \sum_i n_i y_i^k = 1$. The transfers of private goods are “per capita,” while for the public good they are multiplied by a parameter $s > 0$ that measures the efficiency of the public good. Platforms are binding policy commitments and cannot be changed.

Candidates care only about winning office. Voters care about a “fixed” policy issue, private good transfers, and public goods. All voters in each group have the same preference on the fixed issue. For each group $i = 1, 2, 3$, let $\gamma_i$ denote the members’ relative preference for party $X$’s position on the fixed issue. Groups 1 and 3 are the “extremist” or core groups for party $X$ and $Y$, respectively, and group 2 consists of “moderate” or swing voters. We

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4The results of the model also hold for any larger number of candidates in each party. The important assumption is that candidates are ex ante identical to voters.
assume $\gamma_1 > K$ and $\gamma_3 < -K$, where $K = \max\{1/n_1, 1/n_2, 1/n_3, s\}$. Among other things, this guarantees that party $X$ can never buy the support of group 3 voters, and party $Y$ can never buy the support of group 1 voters. Primary voters are forward-looking when voting in the primary, taking into account the expected outcome in the general election.

The preferences of group 2 voters on the fixed issue are stochastic. This could represent a utility shock from the general election campaign that only group 2 voters cared about. For simplicity, we assume that $\gamma_2$ is distributed uniformly on the interval $[-\theta/2, \theta/2]$. So, the density of $\gamma_2$ is $1/\theta$ for $\gamma_2 \in [-\theta/2, \theta/2]$ and 0 otherwise, and the c.d.f. is $F(\gamma_2) = \gamma_2/\theta + 1/2$ for $\gamma_2 \in [-\theta/2, \theta/2]$. We also allow party $X$ to have a party-specific electoral advantage, by giving group 2 voters $\alpha \in [0, \theta/2]$ in valence from either party $X$ candidate.

Voter utility is linear in income. If candidate $k$ from party $Y$ wins, a group $i$ voter receives a payoff of $s y_k^i + y_k^i$. If candidate $j$ from party $X$ wins the general election, then a voter from group $i = 1, 3$ receives a payoff of $s x_0^i + x_j^i + \gamma_i$, and therefore votes for party $X$’s candidate in the general election if $\gamma_i > y_k^i - x_j^i + s(y_0^i - x_0^i)$. From our assumptions on $\gamma_1$ and $\gamma_3$, this always results in a vote for the ideologically proximate party. Similarly, a voter from group 2 receives $s x_0^2 + x_2^2 + \gamma_2 + \alpha$ and votes for party $X$’s candidate in the general election if

$$\gamma_2 > y_k^2 - x_2^2 + s(y_0^2 - x_0^2) - \alpha.$$

In the game without primaries, any selection process for general election candidates produces the same result, since candidates are ex ante identical. We let each candidate be chosen by Nature with probability $1/2$. In the game with primaries, we require some relatively mild assumptions about the division of voters across primary elections. Assume that $n_1 > n_2/2$ and $n_3 > n_2/2$. Let the electorate in the party $X$ primary be group 1 and half of group 2, and likewise let the electorate in the party $Y$ primary be group 3 and the other half of group 2. This ensures that groups 1 and 3 are the majorities in the party $X$ and $Y$ primaries, respectively. Our results also hold under any assumption about the distribution of group 2 voters’ participation in the primaries (such as complete abstention), as long as they are a minority in both primaries.

5Respondents in the presidential primary exit poll surveys claim to value electability when deciding how to vote. For example in the 2004 Democratic primaries, more exit poll respondents cited the ability to “defeat George W. Bush” than any other response to the question “Which ONE candidate quality mattered most in deciding how you voted today?”

6The logic and qualitative results of the model hold for a large class of symmetric, unimodal distributions.

7The linearity assumption is made for simplicity. The logic and qualitative results of the model hold for any strictly increasing, concave utility function.
All actions in the game are perfectly observable. The sequence of play for both games is as follows.

1. Candidates simultaneously offer transfer vectors $\pi^a$, $\pi^b$, $\gamma^a$, and $\gamma^b$.

2. Without primaries, Nature chooses each party’s general election candidate. With primaries, primary voters for each party vote for one of the party’s two candidates.

3. Nature reveals $\gamma_2$.

4. All voters vote for one of the general election candidates.

We derive subgame perfect equilibria in undominated voting strategies. An equilibrium consists of transfer announcements for each candidate and voting strategies for each voter at each election. Voting strategies map the set of platforms to votes for their party’s candidates in the primary elections, and map the sets of platforms, primary election votes (if any), general election candidates, and values of $\gamma_2$ to a vote for one of the parties in the general elections.

3 Results

We begin by deriving an expression for party $X$’s probability of winning the general election, which will occur frequently in what follows. For any platforms $(\pi^j, \gamma^k)$, all voters in group 1 vote for the party $X$ candidate and all voters in group 3 vote for the party $Y$ candidate. Since group 2 voters are pivotal, the party $X$ candidate wins if $\gamma_2 > y^k_2 - x^j_2 + s(y^k_0 - x^j_0) - \alpha$. Thus, at an interior solution, the probability that the party $X$ candidate wins is:

$$1 - F(y^k_2 - x^j_2 + s(y^k_0 - x^j_0) - \alpha) = \frac{x^j_2 - y^k_2 - s(y^k_0 - x^j_0) + \alpha}{\theta} + \frac{1}{2}. \quad (1)$$

3.1 No Primaries

In an environment where general election candidates are selected randomly, each party’s candidates must maximize the probability of winning the general election. It follows from (1) that the uniquely optimal strategy for each candidate is to maximize transfers to group 2. The first remark summarizes the resulting allocation and voting strategies.
Remark 1 Transfers and Voting Without Primaries. Without primaries, all candidates offer the transfer vector

\[ \mathbf{x}^a = \mathbf{x}^b = \mathbf{y}^a = \mathbf{y}^b = \begin{cases} (0, 0, \frac{1}{n_2}, 0) & \text{if } s < \frac{1}{n_2} \\ (1, 0, 0, 0) & \text{otherwise} \end{cases}. \]  

Group 1 and 3 members vote for the party X and Y candidates, respectively. Group 2 members vote for party X’s candidate if \( \gamma_2 > -\alpha \) and for party Y’s candidate if \( \gamma_2 < -\alpha \).

It is straightforward to calculate that these strategies imply that party X’s probability of victory is \( 1/2 + \alpha/\theta \).

3.2 Primaries

Now suppose that there are primary elections in each party. Candidates running in party X’s primary offer to maximize expected utility of group-1 voters, who are decisive in the primary election. This implies trading off optimally (from a group-1 voter’s point of view) between the ideological benefits of winning the general election and transfers conditional upon victory. Similarly, candidates in party Y’s primary maximize the expected utility of a group-3 voter.

We characterize a pure strategy equilibrium by finding the optimal platform strategy within each party, given an expected winning platform from the opposing party. Let \( \mathbf{x} \) and \( \mathbf{y} \) denote arbitrary platforms from parties X and Y. The expected utilities of group-1 and group-3 voters are then:

\[ E_1(\mathbf{x}, \mathbf{y}) = \left[ \frac{x_2 - y_2 - s(y_0 - x_0) + \alpha}{\theta} \right] + \frac{1}{2} \left( x_1 - y_1 + s(x_0 - y_0) + \gamma_1 \right) + y_1 + sy_0 \]  

\[ E_3(\mathbf{x}, \mathbf{y}) = \left[ \frac{x_2 - y_2 - s(y_0 - x_0) + \alpha}{\theta} \right] + \frac{1}{2} \left( x_3 - y_3 + s(x_0 - y_0) + \gamma_3 \right) + y_3 + sy_0. \]

Our first pair of lemmas establish some important properties of these functions. Lemma 1 shows that the efficiency of the public good plays a role in determining the kinds of goods that are offered. As ensured by our assumptions on \( \gamma_i \), allocating toward the opposing party’s core group is dominated for any \( s \). Next, as intuition would suggest, for \( s \) sufficiently low, any best response must include only private goods. At any such \( s \), there is no incentive for candidates to offer public goods because any public good allocation could improved upon by an allocation of private goods. For \( s \) sufficiently high, any best response must have only
the public good. Finally, the problem becomes more interesting for intermediate values of s. When s is higher than $1/(n_1 + n_2)$, a candidate offers private goods to at most one group. A symmetric result holds for party Y.

**Lemma 1** Efficiency of Public Goods and Optimal Platforms. *Party X candidates’ best responses satisfy the following:*

(i) $x^*_3 = 0$.

(ii) If $s \leq \frac{1}{n_1 + n_2}$, then $x^*_0 = 0$.

(iii) If $s > \frac{1}{n_i}$ (i.e., $i \in \{1, 2\}$), then $x^*_i = 0$; if $s > \max\{\frac{1}{n_1}, \frac{1}{n_2}\}$, then $x^*_0 = 1$.

(iv) If $s \in \left(\frac{1}{n_1 + n_2}, \max\{\frac{1}{n_1}, \frac{1}{n_2}\}\right]$, then either $x^*_1 = 0$ or $x^*_2 = 0$.  

Intuitively, group 1 members would accept zero benefits if they care greatly about the ideological benefits (i.e., $\gamma_1$ is high) and if group 1 is large relative to group 2. Extremism increases the importance of victory, and hence increases payments to group 2, while a large size dilutes the benefit of private goods. The opposite would happen when group 1 is moderate and relatively small.

The next important property of (3) and (4) affects our approach toward characterizing best responses. From the candidates’ perspectives, these objective functions are never locally concave when $s$ is high enough for public goods to be undominated. This also implies that (as Lemma 1 establishes) at any best response, solutions are at a corner for at least one private good.

**Lemma 2** Nonconcavity of Party Objectives. *Party X and Y candidates’ objective functions are never locally concave for $s > \frac{1}{n_1 + n_2}$ and $s > \frac{1}{n_3 + n_2}$, respectively.*

We use these results to derive platforms that satisfy necessary conditions for an optimum. For party X candidates, since at most one of $x_1$ and $x_2$ can be strictly positive at an optimal platform, we have $x_0 + n_i x_i = 1$ for $i \in \{1, 2\}$. We can therefore rewrite the party X objective (3) in terms of $x_0$ by substituting this constraint for each group $i$. There are two cases, corresponding to whether group 1 or 2 receives private goods. In the first, $x_1 > 0$ and $x_2 = 0$; using the fact that $y^*_i = 0$, the objective can be rewritten:

$$E_1(\bar{x}, \bar{y}) = \left[ \frac{-y_2 - s(y_0 - x_0) + \alpha}{\theta} + \frac{1}{2} \right] \left( \frac{1 - x_0}{n_1} + s(x_0 - y_0) + \gamma_1 \right) + sy_0$$

This is concave in $x_1$ when $n_1 s < 1$. By Lemma 1(iii), this condition is necessary for an interior solution for $x_1$, as the public good would clearly be preferable otherwise. The interior
solutions for $x_1$ and $x_0$ are then:
\[
\begin{align*}
\tilde{x}_1 &= \frac{n_1 \gamma_1 + (2n_1s - 1)(1 - y_0)}{2n_1(n_1s - 1)} + \frac{\alpha + \theta/2 - y_2}{2n_1s} \\
\tilde{x}_{01} &= \frac{(2n_1s - 1)y_0 - n_1 \gamma_1 - 1}{2(n_1s - 1)} - \frac{\alpha + \theta/2 - y_2}{2s}.
\end{align*}
\]

In the second case, $x_1 = 0$ and $x_2 > 0$, and the objective is concave if $n_2s < 1$. Again, this condition is necessary for an interior value of $x_2$ to be chosen, for otherwise party $X$ candidates would prefer the public good. Straightforward maximization yields the following interior solutions for $x_2$ and $x_0$:
\[
\begin{align*}
\tilde{x}_2 &= \frac{(n_2s - 1)\gamma_1 - s(2n_2s - 1)(y_0 - 1)}{2n_2s(n_2s - 1)} + \frac{\alpha + \theta/2 - y_2}{2(n_2s - 1)} \\
\tilde{x}_{02} &= \frac{-(n_2s - 1)\gamma_1 + s(2n_2s - 1)(y_0 - 1)}{2s(n_2s - 1)} - \frac{n_2(\alpha + \theta/2 - y_2)}{2(n_2s - 1)}.
\end{align*}
\]

Similarly, for party $Y$ candidates the necessity of a corner solution implies $y_0 + n_3y_i = 1$ for $i \in \{2, 3\}$. This produces a simplified version of party $Y$ objective (4) that is concave in $y_0$ for $n_3s < 1$. There are again two cases, corresponding to whether group 3 or 2 receives private goods. In the first, $y_3 > 0$ and $y_2 = 0$; noting that $x_3^* = 0$, we have the following interior solutions for $y_3$ and $y_0$:
\[
\begin{align*}
\tilde{y}_3 &= \frac{(1 - 2n_3s)(x_0 - 1) - n_3 \gamma_3}{2n_3(n_3s - 1)} - \frac{\alpha - \theta/2 + x_2}{2n_3s} \\
\tilde{y}_{01} &= \frac{(2n_3s - 1)x_0 + n_3 \gamma_3 - 1}{2(n_3s - 1)} + \frac{\alpha - \theta/2 + x_2}{2s}.
\end{align*}
\]

And finally when $y_3 = 0$ and $y_2 > 0$, we have for $y_2$ and $y_0$:
\[
\begin{align*}
\tilde{y}_2 &= \frac{(1 - n_2s)\gamma_3 - s(2n_2s - 1)(x_0 - 1)}{2n_2s(n_2s - 1)} - \frac{\alpha - \theta/2 + x_2}{2(n_2s - 1)} \\
\tilde{y}_{02} &= \frac{(n_2s - 1)\gamma_3 + s(2n_2sx_0 - x_0 - 1)}{2s(n_2s - 1)} + \frac{n_2(2\alpha - \theta + 2x_2)}{4(n_2s - 1)}.
\end{align*}
\]

For high values of $s$, the choice between core or swing voters is obvious: if $s > 1/n_2$ then only core groups can possibly receive private goods, while if $s > 1/n_1$ or $s > 1/n_3$ then only the swing group can receive private goods from party $X$ or $Y$ candidates, respectively. Otherwise, these expressions do not obviously determine whether candidates promise private goods to swing or core voters. However, for moderate values of $s$ it is straightforward to show which of $\tilde{x}_1$ and $\tilde{x}_2$ (analogously, $\tilde{y}_3$ and $\tilde{y}_2$) can be interior, and that any such interior solution must be optimal. The following result establishes these conditions for private good allocations toward each group.
Lemma 3 Private Good Allocations Under Intermediate $s$. (i) For $s \in \left(\frac{1}{n_1+n_2}, \min\left\{\frac{1}{n_1}, \frac{1}{n_2}\right\}\right)$, $x_1^* = \min\left\{\frac{1}{n_1}, \tilde{x}_1\right\} > 0$ and $x_2^* = 0$ if and only if:

$$(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \alpha - \theta/2) + n_1s\gamma_1 < 0,$$

and $x_1^* = 0$ and $x_2^* = \min\left\{\frac{1}{n_2}, \tilde{x}_2\right\} > 0$ if and only if:

$$(2n_2s^2 - s)(1 - y_0) - n_2s(y_2 - \alpha - \theta/2) - (1 - n_2s)\gamma_1 < 0.$$

Otherwise, $x_0^* = 1$.

(ii) For $s \in \left(\frac{1}{n_2+n_3}, \min\left\{\frac{1}{n_3}, \frac{1}{n_2}\right\}\right)$, $y_3^* = \min\left\{\frac{1}{n_3}, \tilde{y}_3\right\} > 0$ and $y_2^* = 0$ if and only if:

$$(2n_3s^2 - s)(1 - x_0) + (1 - n_3s)(x_2 + \alpha - \theta/2) - n_3s\gamma_3 < 0,$$

and $y_3^* = 0$ and $y_2^* = \min\left\{\frac{1}{n_2}, \tilde{y}_2\right\} > 0$ if and only if:

$$(2n_2s^2 - s)(1 - x_0) - n_2s(x_2 + \alpha - \theta/2) + (1 - n_2s)\gamma_3 < 0.$$

Otherwise, $y_0^* = 1$. □

We use this result and other features of the candidates’ best responses to establish generally the existence of a pure strategy equilibrium. As we will show, there is a unique equilibrium under a broad range of parameter values, but uniqueness is not guaranteed in general.

Proposition 1 Equilibrium Existence. There exists a pure strategy equilibrium. □

3.2.1 Private Goods Equilibrium

We first consider the case where $s \leq \min\{1/(n_1 + n_2), 1/(n_2 + n_3)\}$. By Lemma [ii], this implies that candidates offer only private goods to voters. The main feature of the private goods equilibrium is that in contrast to the no-primaries private goods equilibrium (see Remark [i]), candidates will usually offer positive allocations to their core groups. This is because citizens in groups 1 and 3 would prefer a small allocation and a slightly reduced probability of winning to a zero allocation that maximized their party’s probability of winning. In fact, allocations to core voters are strategic complements: a higher payment to core voters in one party raises the expected return to core allocations in the other. As noted
previously, more extreme core voters demand less private goods, as they are relatively more interested in winning in order to achieve ideological goals.

Proposition 2 characterizes the private goods equilibrium and establishes a simple condition on $s$ for its existence and uniqueness. The characterization is simplified by the fact that each candidate’s objective functions is strictly concave, and $|\gamma_1|$ and $|\gamma_3|$ are large enough to prevent candidates from offering anything to the opposing party’s core group. This generates a unique platform that maximizes the utility of core voters, and as a result both candidates will adopt it in equilibrium.

**Proposition 2** Private Goods Equilibrium. At an interior solution of an equilibrium with only private goods:

$$(x_1^P, y_3^P) = \left( \frac{3n_2\theta + 2n_2\alpha + 2n_3\gamma_3}{6n_1}, \frac{2\gamma_1}{3}, \frac{3n_2\theta - 2n_2\alpha - 2n_1\gamma_1}{6n_3} + \frac{2\gamma_3}{3} \right).$$

When $(x_1^P, y_3^P)$ is not interior, the following corner solutions arise:

$$(x_1^P, y_3^P) = \begin{cases} 
(0,0) & \text{if } n_2(\theta/2 + \alpha) \leq n_1\gamma_1 \text{ and } n_2(\theta/2 - \alpha) \leq -n_3\gamma_3 \\
\left(0, \frac{n_3\gamma_1 + n_2(\theta/2 - \alpha)}{2n_3}\right) & \text{if } n_2(3\theta/2 + \alpha) \leq 2n_1\gamma_1 - n_3\gamma_3 \text{ and } \\
n_2(\theta/2 - \alpha) \in (-n_3\gamma_3, 2 - n_3\gamma_3) \\
(0, \frac{1}{n_3}) & \text{if } n_2(\theta/2 + \alpha) \leq n_1\gamma_1 - 1 \text{ and } \\
n_2(\theta/2 - \alpha) \geq 2 - n_3\gamma_3 \\
\left(-\frac{n_1\gamma_1 + n_2(\theta/2 + \alpha)}{2n_1}, 0\right) & \text{if } n_2(\theta/2 + \alpha) \in (n_1\gamma_1, 2 + n_1\gamma_1) \text{ and } \\
n_2(3\theta/2 - \alpha) \leq n_1\gamma_1 - 2n_3\gamma_3 \\
\left(\frac{1}{n_1}, 0\right) & \text{if } n_2(\theta/2 + \alpha) \geq 2 + n_1\gamma_1 \text{ and } \\
n_2(\theta/2 - \alpha) \leq -1 - n_3\gamma_3 \\
\left(\frac{1}{n_1}, 1 + n_3\gamma_1 + n_2(\theta/2 - \alpha)\right) & \text{if } n_2(3\theta/2 + \alpha) \geq 3 + 2n_1\gamma_1 - n_3\gamma_3 \text{ and } \\
n_2(\theta/2 - \alpha) \in (-1 - n_3\gamma_3, 1 - n_3\gamma_3) \\
\left(\frac{1-n_1\gamma_1 + n_2(\theta/2 + \alpha)}{2n_1}, \frac{1}{n_3}\right) & \text{if } n_2(\theta/2 + \alpha) \in (n_1\gamma_1 - 1, 1 + n_1\gamma_1) \text{ and } \\
n_2(3\theta/2 - \alpha) \geq 3 + n_1\gamma_1 - 2n_3\gamma_3 \\
\left(\frac{1}{n_1}, \frac{1}{n_3}\right) & \text{if } n_2(\theta/2 + \alpha) \geq 1 + n_1\gamma_1 \text{ and } \\
n_2(\theta/2 - \alpha) \geq 1 - n_3\gamma_3.
\end{cases}$$

This is the unique equilibrium for $s < \min\{1/(n_1+n_2), 1/(n_2+n_3)\}$.

With one exception, which occurs when both core groups are large and extreme, some core voters receive positive allocations in equilibrium. Primaries also tend to benefit both parties’ core groups indirectly because their party’s probability of victory increases when the opposition party reduces its allocation to group 2. By contrast, primaries typically hurt
group 2 citizens in a private goods equilibrium, relative to a world in which there are no primaries.

Elsewhere (Hirano, Snyder, and Ting 2009), we show that each party gives more (per capita) to its core group when it shrinks in size relative to the swing group, when it becomes more moderate, when electoral uncertainty ($\theta$) increases, and when its relative valence advantage increases. We also show that each party’s probability of victory is increasing in the size and ideological extremism of its core group, as well as in the size of its relative valence advantage.

### 3.2.2 Public Goods Equilibrium

We now turn to the case of high values of $s$, so that public goods are efficient enough to be offered exclusively by at least one party in equilibrium. A central question of the paper is when primaries encourage the provision of public goods, compared to a world with no primaries. In particular, can public goods be provided when $s < 1/n_2$? The threshold of $s = 1/n_2$ for provision without primaries generates an inefficiency because public goods provide benefit society whenever $s > 1$. Lemma 1(ii) implies that some inefficiencies must persist in the presence of primaries: party $X$ and $Y$ candidate candidates never offer public goods when $s$ is below $1/(n_1 + n_2)$ and $1/(n_3 + n_2)$, respectively.

Proposition 3(i) shows that when the swing group is the smallest group, primaries are strictly better for producing public goods in the sense of reducing the threshold value $s$ needed to induce all candidates to offer only public goods. Group 2 be the smallest in order to reduce the per capita value of private goods for core groups. As the proof of this result makes clear, a similar sufficient condition for candidates in just one party to offer only public goods is more easily met: each party’s candidates will do so if the swing group is smaller than their core group.

Parts (ii)-(iv) of Proposition 3 characterize the other cases where at least one party offers only public goods. They follow immediately from Lemma 1 and manipulation of (5) and (9), and are stated without proof.

**Proposition 3** Public Goods Equilibrium. (i) If $n_2 < \min\{n_1, n_3\}$, then $x_0^* = y_0^* = 1$ for all $s > \tilde{s}$, where $\tilde{s} < \frac{1}{n_2}$.

(ii) If $s \geq \max\{\frac{1}{n_1}, \frac{1}{n_2}, \frac{1}{n_3}\}$, then $x_0^* = y_0^* = 1$.

(iii) If $s \geq \max\{\frac{1}{n_1}, \frac{1}{n_2}\}$ and $s < \frac{1}{n_3}$, then $x_0^* = 1$ and $y_3^* = \max\{0, \min\{\tilde{y}_3, \frac{1}{n_3}\}\}$, where:

$$\tilde{y}_3 = \frac{\gamma_3}{2(1 - n_3s)} + \frac{\theta/2 - \alpha}{2n_3s}.$$
(iv) If \( s \geq \max\{\frac{1}{n_2}, \frac{1}{n_3}\} \) and \( s < \frac{1}{n_1} \), then \( y_0^* = 1 \) and \( x_1^* = \max\{0, \min\{\tilde{x}_1, \frac{1}{n_1}\}\} \), where:

\[
\tilde{x}_1 = -\frac{\gamma_1}{2(1-n_1 s)} + \frac{\theta/2 + \alpha}{2n_1 s}.
\]

Notably, parts (iii) and (iv) of Proposition 3 imply that there are conditions (e.g., \(|\gamma_i|\) small, \(\theta\) large) under which public goods are not promised by all candidates even though \( s > 1/n_2 \). Thus, a small core group can result in a party providing a lower level of public goods under a primary system. These results illustrate the general point that smaller groups will tend to receive private goods, due to the higher per capita value of a given amount of transfers.

### 3.2.3 Mixed Goods Equilibria

Now consider the cases where \( s \) is intermediate, so that candidates will offer combinations of public and private goods. A first question is when candidates offer some public goods in their platform. Proposition 4 provides a simple lower bound on \( s \) for when party \( X \) candidates will do so. A symmetric result holds for party \( Y \).

**Proposition 4** Some public goods under primaries. If \( s > \frac{1}{n_1 + n_2} \), then \( x_0^* > 0 \) if:

\[
s > \max\left\{\frac{\alpha + \theta/2}{n_1(\gamma_1 + \alpha + \theta/2)}, \frac{\gamma_1}{n_2(\gamma_1 + \alpha + \theta/2) - 1}\right\}.
\]

The bound on \( s \) in Proposition 4 is for many parameter values quite low. In particular, for sufficiently large values of \( \alpha \), \( \theta \), and \( \gamma_1 \), the threshold for partial adoption of public goods under primaries can be much lower than the \( 1/n_2 \) threshold for adoption without primaries. As an example, suppose \( n_1 = 0.4 \), \( n_2 = 0.3 \), \( \theta = 10 \), \( \alpha = 2 \), and \( \gamma_1 = 4 \). By Lemma 1, party \( X \) offers no public goods if \( s < 1/(n_1 + n_2) \approx 1.43 \), and \( X \) offers only public goods if \( s > 3.33 \). The threshold from the result is about 1.74, and so candidates offer some public goods even when they are relatively close to being dominated by private goods.

Unfortunately, when \( s \) is moderate general closed-form solutions for platform choices are quite cumbersome. We can, however, derive solutions for a broad set of cases that suffice to illustrate the general logic of the model. Under some simple conditions on group ideology, a party’s candidates will never allocate private goods to either its core or swing group. When the core group is sufficiently moderate, platforms never include private goods for the swing group, and when the core group is sufficiently extreme, platforms never include the core group. These conditions simplify equilibrium characterization because when only one group
receives private goods, each party’s maximization problem reduces to a univariate choice over the level of public good to provide.

Lemma 4 summarizes these conditions and uses the following bounds, defined as follows:

\[
\gamma_1 \equiv \max\{-2n_1 s^2 + s, 0\} + (1 - n_1 s)(\alpha + \theta/2) \\
(17)
\]

\[
\check{\gamma}_1 \equiv n_2 s(\alpha + \theta/2) - 2s \\
(18)
\]

\[
\gamma_3 \equiv \min\{2n_3 s^2 - s, 0\} + (1 - n_3 s)(\alpha - \theta/2) \\
(19)
\]

\[
\check{\gamma}_3 \equiv n_2 s(\alpha - \theta/2) + 2s. \\
(20)
\]

Importantly, these bounds typically exclude only a relatively “small” set of values of \(\gamma_1\) and \(\gamma_3\), and in many cases exclude none at all.

**Lemma 4** Independence of Private Goods Recipient.

(i) For \(s \in \left(\frac{1}{n_1 + n_2}, \min\left\{\frac{1}{n_1}, \frac{1}{n_2}\right\}\right)\), \(x_1^\# = 0\) if \(\gamma_1 \geq \check{\gamma}_1\), and \(x_2^\# = 0\) if \(\gamma_1 \leq \check{\gamma}_1\).

(ii) For \(s \in \left(\frac{1}{n_2 + n_3}, \min\left\{\frac{1}{n_3}, \frac{1}{n_2}\right\}\right)\), \(y_3^\# = 0\) if \(\gamma_3 \leq \check{\gamma}_3\), and \(y_2^\# = 0\) if \(\gamma_3 \geq \check{\gamma}_3\).

The mild parameter restrictions from this result allow us to characterize a unique equilibrium for a broad range of parameters. Under these conditions, the decision to target one group with private goods is “dominant” in the sense that it is independent of the other party’s platform. Note also that the result complements Lemma 4(iii), which established that a groups would not receive private goods when \(s\) is sufficiently large.

Proposition 5 derives the interior platforms for values of \(s\) lower than those in Proposition 3. For each case, the platforms are the solutions of the appropriate system of equations chosen from expressions (6) through (12). For convenience, it only states the allocations that are made to the group receiving private goods; all of the remaining candidate budgets are allocated toward the public good.

**Proposition 5** Mixed Goods Equilibrium. Let \(s \geq \max\{\frac{1}{n_1 + n_2}, \frac{1}{n_1}, \frac{1}{n_2}\}, s \leq \max\{\frac{1}{n_1}, \frac{1}{n_2}\}, \) and \(s \leq \max\{\frac{1}{n_2}, \frac{1}{n_3}\}.\)

\(^8\)For example, when \(n_1 = n_2 = n_3 = 1/3\), Lemma 4 implies that there are no restrictions on \(\gamma_1\) if \(\alpha + \theta/2 > 2.5\) and \(s > 2.033\). There are no restrictions on \(\gamma_3\) if \(\alpha - \theta/2 < -0.5\) and \(s > 2.753\).

\(^9\)Taken together, Propositions 2, 3, and 5 cover all possible values of \(s\), with the exception of \(s \in (\min\{1/(n_1 + n_2), 1/(n_3 + n_2)\}, \max\{1/(n_1 + n_2), 1/(n_3 + n_2)\})\). This case is relatively straightforward, combining features of the private goods equilibrium for one party and mixed goods for the other. Section 3.2.4 develops this case for ideologically symmetric voters.
(i) If $\gamma_1 \leq \gamma_1$ and $\gamma_3 \geq \gamma_3$, the non-zero private good allocations at an interior equilibrium are:

$$
x_1^* = \frac{(1-n_3s)(2n_2s(\theta+\gamma_1) - 2\gamma_1 + \gamma_3) - n_3s\gamma_3(1-2n_2s)}{n_1s(2n_1s + 2n_3s - 3)}
$$

$$
y_3^* = \frac{(1-n_1s)(2n_3s(\theta-\gamma_3) + \alpha - 3\theta/2) + n_1s\gamma_1(1-2n_3s)}{n_3s(2n_1s + 2n_3s - 3)}
$$

(ii) If $\gamma_1 \geq \gamma_1$ and $\gamma_3 \geq \gamma_3$, the non-zero private good allocations at an interior equilibrium are:

$$
x_2^* = \frac{(1-n_3s)(2n_2s(\theta+\gamma_1) - 2\gamma_1 + \alpha - \theta/2) - n_3s\gamma_3(1-2n_2s)}{2n_2s^2 - n_2s + n_3s - 1}
$$

$$
y_3^* = \frac{(1-n_3s - n_2s)(2n_2s(\alpha - 3\theta/2) + (1-n_2s)\gamma_1) + 2n_2n_3s^2((1-n_2s)(\gamma_1 - \gamma_3) - n_2s\theta)}{n_3s(2n_2s^2 - n_2s + n_3s - 1)}
$$

(iii) If $\gamma_1 \leq \gamma_1$ and $\gamma_3 \leq \gamma_3$, the non-zero private good allocations at an interior equilibrium are:

$$
x_1^* = \frac{(1-n_1s - n_2s)(2n_2s(-\alpha - 3\theta/2) - (1-n_2s)\gamma_3) + 2n_2n_1s^2((1-n_2s)(\gamma_1 - \gamma_3) - n_2s\theta)}{n_1s(2n_2s^2 - n_2s + n_1s - 1)}
$$

$$
y_2^* = \frac{(1-n_1s)(2n_2s(\theta - \gamma_3) + 2\gamma_3 - \alpha - \theta/2) + n_1s\gamma_1(1-2n_2s)}{2n_2s^2 - n_2s + n_1s - 1}
$$

(iv) If $\gamma_1 \geq \gamma_1$ and $\gamma_3 \leq \gamma_3$, there is no interior equilibrium. $x_2^* = \frac{1}{n_2}$ and $y_2^* = \frac{1}{n_2}$ for $\gamma_1$ and $|\gamma_3|$ sufficiently large. 

One implication of this result is that, similar to the private good equilibrium, more extreme core groups receive no private goods. For such voters, the ideological payoff from victory generates an incentive to maximize the probability of victory, thereby shifting all private good provision to the swing group. In part (iv), where all core voters are extreme, all candidates allocate their entire budget to private good for group 2. In the other cases, public goods soften the choice between core and swing groups, and allow extreme core groups to benefit from non-ideological payoffs. Moderate core voters are most willing to trade off between private goods and the probability of victory, and therefore generally receive private goods in equilibrium.

### 3.2.4 Ideological Symmetry

While Proposition 5 is suggestive of the role of ideology in party strategies, it does not provide comparative statics over the full range of ideological polarization. We can obtain
sharper predictions by considering a special case of the game with symmetric ideological parameters. In particular, suppose core groups have the same ideological motivation (i.e., $\gamma \equiv \gamma_1 = -\gamma_3$), and that the swing group is not biased toward either party (i.e., $\alpha = 0$). There are two cases of interest; in the first, the core groups are also of the same size, while in the second, one core group is larger than the other groups.

The first case allows us to isolate the effect of ideological polarization. Proposition 6 confirms the general intuition about the effects of extremism in Proposition 5. For the interesting values of $s$ (i.e., where $s$ is intermediate), candidates shift benefits from core groups to the swing group as ideological extremism $\gamma$ increases. Public goods then help to “smooth” the transition between private goods for the two groups.

**Proposition 6 Symmetric Core Groups.** Suppose $s \in (\max\{\frac{1}{2n_1}, \frac{1}{2n_2}\}, \min\{\frac{1}{n_1}, \frac{1}{n_2}\})$, $n_1 = n_3$, $\alpha = 0$, and $\gamma_1 = -\gamma_3 = \gamma$. In the unique equilibrium:

$$
\begin{align*}
    x_1^* &= y_3^* = \min\left\{\frac{1}{n_1}, \frac{(1-n_1s)\theta}{2n_1s} - \gamma\right\}, \\
    x_0^* &= y_0^* = 1 - n_1x_1^* & \text{if } \gamma \leq \frac{(1-n_1s)\theta}{2n_1s} \\
    x_2^* &= y_2^* = 1 & \text{if } \gamma \in \left(\frac{(1-n_1s)\theta}{2n_1s}, \frac{n_2s\theta}{2(1-n_2s)}\right) \\
    x_2^* &= y_2^* = \frac{1}{n_1} & \text{if } \gamma \geq \frac{n_2s\theta}{2(1-n_2s)}. 
\end{align*}
$$

The symmetry of core group ideologies here implies that equilibrium platforms are also symmetric. For the lowest values of $\gamma$, candidates promise a mix of private goods for the core group and public goods. These private goods are linearly decreasing in $\gamma$ and increasing in $\theta$. For higher values of $\gamma$ the allocation hits a corner of entirely public goods. Finally, for extreme values of $\gamma$ candidates promise only private goods to the swing group. Interestingly, as suggested by Proposition 5(iv), the transition from public goods to private goods for the swing group is discontinuous at $\gamma = n_2s\theta/(2-2n_2s)$: there is no “interior” solution for private goods to the swing group. Overall, then, the provision of public goods is non-monotonic in $\gamma$. Figure 1 depicts the allocations to each group as a function of $\gamma$ for the special case where all group sizes are equal. Although not part of the result, the equilibrium is similar when $s$ is large enough to dominate the provision of some private good (i.e., $s > 1/n_1$ or $s > 1/n_2$); in these cases the public good is simply substituted for that private good.

Proposition 6 allows us to calculate simple solutions for the key question of when public goods are provided as a function of the efficiency of public goods. It is obvious that increasing $s$ strictly increases the appeal of offering public goods. By manipulating the conditions on $\gamma$ in the proposition, we derive the following bounds when public goods are provided exclusively, as well as when public goods are provided at all. The result is stated without proof.
Remark 2 Public Goods Thresholds Under a Symmetric Equilibrium. Under the conditions of Proposition 6, $x_0^* = 1$ if:

$$s > \max \left\{ \frac{2\gamma}{n_2(2\gamma + \theta)}, \frac{\theta}{n_1(2\gamma + \theta)} \right\}.$$ 

And $x_0^* > 0$ if:

$$s > \frac{2\gamma}{n_2(2\gamma + \theta)}.$$ 

This result implies that if $\theta > 2\gamma$, all candidates will offer some public goods whenever $s$ is in the “intermediate” range identified in Proposition 6. Notably if $\theta = 2\gamma$ and all groups are of equal size, then the threshold $s$ for offering only public goods is the same as the threshold for public goods to be undominated by private goods (i.e., $s = 1/(n_1 + n_2)$). Values of $\theta$ below $2\gamma$ will raise the threshold for offering public goods, while higher values of $\theta$ will raise the threshold for offering only public goods.

Figure 2 plots an example of these thresholds under equal group sizes. It is immediately evident that primaries can sometimes greatly facilitate public good provision. Under the parameters of the example, when there are no primaries public goods are not provided for
\( s \in (3/2, 3) \). When \( \gamma \) is not too high, primaries fill much of this public good provision gap. Of course, no public goods are provided when they are socially efficient but dominated by private goods \( (s \in (1, 3/2)) \).

Figure 2: Efficiency Thresholds in a Symmetric Game. Here \( n_1 = n_2 = n_3 = 1/3 \), \( \gamma = \gamma_1 = -\gamma_3 \), \( \alpha = 0 \), and \( \theta = 20 \). This figure plots public good provision levels as a function of \( s \) and \( \gamma \). Below \( s = 1.5 \), public goods are a dominated platform strategy, while above \( s = 3 \) public goods are exclusively provided when there are no primaries. At \( \gamma = 10 \), public goods are exclusively provided by all candidates when they are undominated by private goods.

Several other comparative statics on the range of \( \gamma \) for which public goods are exclusively offered by all candidates follow immediately from Proposition 6. The size of this range is increasing in \( \theta \), which measures electoral uncertainty. It also shifts “upwards” in \( \gamma \) as \( \theta \) increases, which implies that as elections become more uncertain, the value of contributions to the swing group decreases while the value of contributions to the core group increases. Finally, as \( n_1 \) increases (implying that \( n_2 \) decreases), this range shifts “downwards” in \( \gamma \). This reflects the dilution of the value of core contributions as \( n_1 \) increases along with the concentration of swing contributions as \( n_2 \) decreases.

The second case examines the effect of asymmetry in core group sizes. Suppose that \( n_1 = n_2 \), and \( n_3 > n_1 \), so that party \( Y \) faces a larger constituency. In addition, suppose that

\[ \text{\textsuperscript{10}} \text{The range of } \gamma \text{ for which some public goods are offered is also increasing in } \theta. \]
$s < 1/(n_1 + n_2)$ and $s > 1/(n_2 + n_3)$ (which also implies that $s < 1/n_i$ for all $i$). Thus party $X$ offers only private goods, while party $Y$ may offer some public goods.

Proposition 7 shows that the logic of offering increasingly valuable allocations to swing voters as extremism increases remains when the opposite party uses only private goods. Figure 3 plots the party $Y$ candidates’ allocation strategies as a function of $\gamma$. The most notable difference between this figure and Figure 1 is the smoother transition from public goods to private goods for the swing group. By contrast, party $X$ candidates simply trade core allocations for swing allocations as $\gamma$ increases.

**Proposition 7** Asymmetric Core Groups. Suppose $s < \frac{1}{n_1 + n_2}$, $s > \frac{1}{n_3 + n_2}$, $n_1 = n_2$, $\alpha = 0$, and $\gamma_1 = -\gamma_3 = \gamma$. In equilibrium there exist $\gamma'$ and $\gamma''$ such that:

(i) If $\gamma < \gamma'$, then $y_2^* = 0$ and $y_3^*$ is piecewise linear and weakly decreasing in $\gamma$.
(ii) If $\gamma > \gamma''$, then $y_3^* = 0$.
(iii) If $\gamma \in [\gamma', \gamma'']$, then $y_0^* = 1$.

At an interior solution, $\gamma' = \frac{(1-n_3s)(2s-1/n_2+3\theta/2)-2n_3s^2}{1+n_3s}$ and $\gamma'' = \frac{s(3-4n_2s-3n_3\theta/2)}{n_2s-2}$.

Figure 3: Equilibrium Allocations in a Symmetric Equilibrium. Here $n_1 = n_2 = 0.32$, $n_3 = .36$, $\gamma = \gamma_1 = -\gamma_3$, $\alpha = 0$, $s = 1.5$, and $\theta = 10$. This figure plots the per capita allocations for each platform component for party $Y$ candidates, as a function of $\gamma$.  

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The cutpoints $\gamma'$ and $\gamma''$ correspond to the endpoints of the interval where only public goods are offered. (This interval also appears in Proposition 0.) It can be shown that this interval is nonempty whenever $\gamma' > 1/n_2$, which holds for $\theta$ sufficiently large. Since $\gamma > 1/n_2$ by assumption, the condition $\gamma' > 1/n_2$ is sufficient for the existence of a region where only party Y provides public goods.

4 Extension: Pivotal Swing Voters

An important assumption in the previous results was that the pivotal voter in each party’s primary election belonged to a core group. However, a broad-based party might have more swing than core voters. If instead the pivotal voter belonged to the the swing group, then that party’s candidates could focus exclusively on the general election. In this section, we examine the case where party Y’s pivotal primary voter belongs to group 2.

It is clear that party Y candidates will choose to maximize the expected payoffs of group 2 voters. Thus their platform strategies will be identical to those in the no-primaries world:

$$y^a = y^b = \begin{cases} (0, 0, \frac{1}{n_2}, 0) & \text{if } s < \frac{1}{n_2} \\ (1, 0, 0, 0) & \text{otherwise} \end{cases}.$$

The party Y strategies generate two cases. When public goods are highly valuable (i.e., $s > 1/n_2$), party Y candidates offer only public goods, and when public goods are less valuable, they focus exclusively on private goods for group 2. The following result characterizes the equilibrium platforms for party X in both cases. We restrict attention here to values of $s$ that are high enough to ensure that public goods are undominated.

**Remark 3 Pivotal Swing Voters.** Suppose $s > \frac{1}{n_1+n_2}$ and group 2 voters are a majority of the party Y primary electorate.

(i) If $s > \frac{1}{n_2}$, then $x^*_0 = 1 - n_1 x^*_1$ and

$$x^*_1 = \begin{cases} 0 & \text{if } n_1 \geq \frac{\alpha+\theta/2}{s(\gamma_1+\alpha+\theta/2)} \\ \max \left\{ \frac{1}{n_1}, \frac{\gamma_1}{2(n_1s-1)} + \frac{\alpha+\theta/2}{2n_1s} \right\} & \text{otherwise.} \end{cases}$$

(ii) If $s < \frac{1}{n_2}$, then $x^*_0 = 1 - n_1 x^*_1 - n_2 x^*_2$ and

$$x^*_1 = \begin{cases} 0 & \text{if } n_1 \geq \frac{\alpha+\theta/2-1/n_2}{s(2s+\gamma_1+\alpha+\theta/2-1/n_2)} \\ \max \left\{ \frac{1}{n_1}, \frac{\gamma_1+\theta}{2(n_1s-1)} + \frac{1}{2n_1} \left( 1 - \frac{1}{n_2s} \right) + \frac{\alpha+\theta/2}{2n_1s} \right\} & \text{otherwise,} \end{cases}$$

$$x^*_2 = \begin{cases} 0 & \text{if } n_2 \geq \frac{2s+\gamma_1}{s(2s+\gamma_1+\alpha+\theta/2)} \\ \max \left\{ \frac{1}{n_2}, \frac{\gamma_1}{2n_2s} + \frac{n_2(2s+\alpha+\theta/2)-2}{2n_2(n_2s-1)} \right\} & \text{otherwise.} \end{cases}$$
Party $X$’s equilibrium platforms are derived simply from Lemma 1 and equations (5) and (7). The basic properties of the original game continue to hold here. For example, while it is possible for either group 1 or group 2 to benefit from private goods in part (ii), Lemma 3 continues to hold, and thus at most one of $x_1$ and $x_2$ can be positive. Additionally, consistent with the logic of Proposition 3(i), part (i) implies that $x_1^* > 0$ only if $n_1 < n_2$. Again, small group sizes are conducive to offering private goods, even when the opposition can be considerably more appealing to swing voters.

One clear implication of this environment is that it helps party $Y$ to win. A more interesting question is how party $X$ candidates respond. In a world without public goods, pivotal swing voters in party $Y$ generally induce party $X$ candidates to allocate more to the swing group in order to compensate for their reduced probability of victory (Hirano, Snyder, and Ting 2009). Public goods can muddle this result by reducing the stakes of victory. For example, suppose that the equilibrium core group allocations are interior regardless of which group controls the party $Y$ primary (implying $s < 1/n_1$), and compare swing and core voter control of the party $Y$ primary. When $s > 1/n_2$, $y_2^* = 0$ and swing voter control (weakly) raises $y_0$ to 1. It can be easily shown that if $s > 1/(2n_1)$, then shifting to swing voter control increases $x_1^*$, while if $s < 1/(2n_1)$, the relationship is reversed. The increase in core allocations in party $X$ is due to the high value of the public good: with increased payoffs from losing, group 1 members are willing to accept a lower probability of victory and higher payoffs conditional upon victory. By contrast, when party $Y$ does not provide public goods, party $X$ candidates would respond by improving their offers to swing voters.

5 Conclusions

This paper investigates the effect of primary elections on the distribution of public spending. The main intuition is that primary elections provide an incentive for candidates to increase the provision of public as opposed to particularistic goods. The incentive is generated by the simple observation that public goods simultaneously benefit both core and swing voters. Thus they can present candidates with a more efficient way of maximizing the utility of both core voters in a primary election as well as swing voters in the general election.

The model produces a number of non-obvious predictions. First, despite the preceding intuition, primaries do not necessarily increase the provision of public goods. Second, large core groups encourage the provision of public goods. A sufficient condition for primaries to make the provision of public goods “easier” is for the swing group to be the smallest
group, thus diluting the value of private goods for core groups. Finally, public goods will be appealing when core voters are not too extreme. In this case, an equilibrium where only public goods are offered might arise when the public goods are not efficient enough to be offered in the absence of primary elections. By contrast, the most extreme voters will wish to maximize the probability of receiving ideological benefits, and this often implies targeting the swing group with private goods.

The model suggests that primaries produce mixed distributional consequences. To the extent that they increase in the use of public goods, primaries increases aggregate social utility and equalize payoffs across society. Primaries also tend to draw private good allocations away from swing voters and toward core voters. Whether this produces more egalitarian outcomes depends on the size of the core and swing groups. Primaries have no effect on distributions when both core groups are sufficiently extreme and $s$ is “intermediate,” as candidates simply maximize their offer of private goods to the swing group in both games.

Our model is simple and may be extended in several ways. For example, what if one candidate had an incumbency advantage, modeled as a candidate-specific valence term? How would the model be affected by endogenous decisions to adopt primaries? The robustness of our results is also worth exploration. The corner solutions in the model are driven in part by the linearity of voter utility, and interior solutions might follow from non-linear utility over public goods. While our results are robust to multiple candidates in each party, the effects of multiple parties, more groups, or alternative nomination systems are not clear.

Finally, the results suggest some avenues for empirical research. The main challenge lies in classifying government spending as public versus particularistic. Previous research has grappled with this problem, but there is no clear consensus regarding classification schemes. U.S. states and localities spend on a variety of goods and services — education, health, transportation, police, fire departments, courts, sewerage and trash pickup, etc. — and these are all partially public and partially excludable and targetable goods.

An alternative measure, which we plan to explore in future work, is based on “project size.” Within a relatively narrow category of spending, projects that larger in scale are “more public” than smaller projects. Compare for example a hospital with a 1,000 beds centrally located in a county to a 10 hospitals scattered throughout the county each with 100 beds. The former is closer to the theoretical ideal of a public good than the latter. One way to measure project size is from data on local or state government bond issues.
APPENDIX

Proof of Lemma 1. Throughout, suppose that a party $X$ candidate allocates some $\pi$, where (by weak dominance) $\pi$ satisfies the budget constraint $x_0 + n_1x_1 + n_2x_2 + n_3x_3 = 1$.

(i) It is clear from inspection of (3) that any positive allocation to group 3 is dominated by a reallocation toward either group 1 or 2, or the public good. Thus, $x_3 = 0$.

(ii) We derive the condition on $s$ for the party $X$ candidate to reallocate all of $x_0$ to private goods of equal value for groups 1 and 2 at lower cost. A platform giving $sx_0 + x_1$ to group 1 voters and $sx_0 + x_2$ to group 2 voters is feasible if $n_1(sx_0 + x_1) + n_2(sx_0 + x_2) \leq 1$.

Rearranging and applying the budget constraint, we obtain:

$$n_1sx_0 + n_2sx_0 \leq x_0$$

$$s \leq \frac{1}{n_1 + n_2}.$$

(iii) To show the result for $s > 1/n_i$, observe that a party $X$ candidate could replace $x_i$ with $n_ix_i$ units of $x_0$. This revised allocation strictly benefits all voters.

To show the result for $s > \max\{1/n_1, 1/n_2\}$, we derive the condition on $s$ for the party $X$ candidate to reallocate all of $x_1$ and $x_2$ to the public goods and benefit groups 1 and 2 at lower cost. The platform $x'_0 = 1$ is feasible and gives $s$ to all voters. It provides greater utility to voters in group $i$ if $s > sx_0 + x_i$. Since the right-hand side is maximized either at $x_i = 1/n_i$ (implying $x_0 = 0$), or 0 (implying $x_0 = 1$), the condition holds for any $s > 1/n_i$.

(iv) Suppose that $x_i > x_j > 0$ for $i, j \in \{1, 2\}$ and $j \neq i$. We provide conditions under which a party $X$ candidate would do strictly better by offering a different platform $\pi'$ where $x'_j = 0$, $x'_i = x_i - x_j$, and $x'_0 = x_0 + x_j/s$. This platform clearly provides all voters in groups 1 and 2 identical utility as $\pi$. Thus we need only verify the feasibility of $\pi'$, which, given part (ii), is assured if:

$$n_i(x_i - x_j) + x_0 + x_j/s < 1$$

$$s > \frac{1}{n_1 + n_2}.$$

Therefore for any such $s$, an optimal platform must have $x_1 = 0$ or $x_2 = 0$. By part (iii), for $s > \max\{1/n_1, 1/n_2\}$, this is strengthened to $x_1 = 0$ and $x_2 = 0$.

Proof of Lemma 2. The budget constraints and weak domination imply that $x_2 = (1 - n_1x_1 - n_3x_3 - x_0)/n_2$ and $y_2 = (1 - n_1y_1 - n_3y_3 - y_0)/n_2$. Further, Lemma (i) implies $x'_3 = y'_1 = 0$. Substituting these into (3) and (4) yields:

$$E_1(\pi, y) = \left[ \frac{n_3y_3 - n_1x_1 - (n_2s - 1)(y_0 - x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (x_1 + s(x_0 - y_0) + \gamma_1) + sy_0 \quad (21)$$

$$E_3(\pi, y) = \left[ \frac{n_3y_3 - n_1x_1 - (n_2s - 1)(y_0 - x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (-y_3 + s(x_0 - y_0) + \gamma_3) + y_3 + sy_0. \quad (22)$$
There are two cases. First, if \( n_2s > 1 \), then it is clear that (21) and (22) are strictly convex in \( x_0 \) and \( y_0 \), respectively. Second, if \( n_2s < 1 \), then we may write the first order conditions as follows.

\[
\begin{align*}
\frac{\partial E_1(x, y)}{\partial x_1} &= -\frac{2n_1}{\theta n_2}x_1 + \frac{n_3y_3 - n_1\gamma_1 - (n_2s - n_1s - 1)(y_0 - x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \\
\frac{\partial E_1(x, y)}{\partial x_0} &= \frac{2(n_2s - 1)s}{\theta n_2}x_0 + \frac{(n_2s - 1)(x_1 - sy_0 + \gamma_1) + s(n_3y_3 - n_1x_1 - (n_2s - 1)y_0)}{\theta n_2} \\
&\quad + \left(\frac{\alpha}{\theta} + \frac{1}{2}\right)s \\
\frac{\partial E_3(x, y)}{\partial y_3} &= -\frac{2n_3}{\theta n_2}y_3 + \frac{n_1x_1 + n_3\gamma_3 + (n_2s - n_3s - 1)(y_0 - x_0)}{\theta n_2} - \frac{\alpha}{\theta} + \frac{1}{2} \\
\frac{\partial E_3(x, y)}{\partial y_0} &= \frac{2(n_2s - 1)s}{\theta n_2}y_0 - \frac{(n_2s - 1)(-y_3 + sx_0 + \gamma_3) + s(n_3y_3 - n_1x_1 + (n_2s - 1)x_0)}{\theta n_2} \\
&\quad - \left(\frac{\alpha}{\theta} - \frac{1}{2}\right)s.
\end{align*}
\]

Now consider whether the conditions for local concavity are possible. For party \( X \), the Hessian is:

\[
\begin{bmatrix}
-\frac{2n_1}{\theta n_2} & \frac{n_2s - n_1s - 1}{\theta n_2} \\
\frac{n_2s - n_1s - 1}{\theta n_2} & \frac{2(n_2s - 1)s}{\theta n_2}
\end{bmatrix}
\]

The diagonal elements are clearly negative, and the determinant is non-negative if:

\[
\begin{align*}
-4n_1(n_2s - 1)s - (n_2s - n_1s - 1)^2 &\geq 0 \\
1 - 2n_1n_2s^2 - (n_1s - 1)^2 - (n_2s - 1)^2 &\geq 0
\end{align*}
\]

It is straightforward to show that this expression is never positive, and can be satisfied with equality if and only if \( n_1s + n_2s = 1 \). But when \( s = 1/(n_1 + n_2) \), party \( X \) does just as well by giving private goods \( x_1 = x_2 = 1/(n_1 + n_2) \). Thus the objective has no local maxima whenever \( s \) is such that public goods might be optimal. The analysis for party \( Y \) is symmetrical and therefore omitted. ■

**Proof of Lemma 3.** (i) Let \( E_{1i}(x_0, y) \) denote group 1 utility \( E_1(x, y) \) when \( x_i = (1 - x_0)/n_i \) and all other private good allocations are zero.

We first show that \( \frac{dE_{1i}}{dx_0}(1, y) \geq 0 \) for either \( i = 1 \) or 2. Substituting \( x_i \) into (3) and differentiating \( E_{1i}(x_0, y) \) gives the following expressions:

\[
\begin{align*}
\frac{dE_{11}}{dx_0} &= 2n_1s^2(x_0 - y_0) + (1 - n_1s)(y_2 - \alpha - \theta/2) + s(1 - 2x_0 + y_0) + n_1s(\gamma_1 - y_1) \\
\frac{dE_{12}}{dx_0} &= 2n_2s^2(x_0 - y_0) - n_2s(y_2 - \alpha - \theta/2) + s(1 - 2x_0 + y_0) - (1 - n_2s)(\gamma_1 - y_1)
\end{align*}
\]

Suppose that \( \frac{dE_{1i}}{dx_0}(1, y) < 0 \) and \( \frac{dE_{1j}}{dx_0}(1, y) < 0 \). Clearly, for each \( i \), \( \frac{dE_{1i}}{dx_0}(1, y) < 0 \) iff the corresponding numerator in the above expressions is negative. This implies that the sum of
the numerators evaluated at \( x_0 = 1 \) must also be negative. Substituting and simplifying, we obtain:

\[
2n_1 s^2 (1 - y_0) + (1 - n_1 s) (y_2 - \alpha - \theta/2) + s(y_0 - 1 + n_1 (\gamma_1 - y_1)) + \\
2n_2 s^2 (1 - y_0) - n_2 s(y_2 - \alpha - \theta/2) + s(y_0 - 1 + n_2 (\gamma_1 - y_1)) - \gamma_1 + y_1 \\
= (1 - (n_1 + n_2)s)[2s(y_0 - 1) + y_2 - \alpha - \theta/2 - \gamma_1 + y_1].
\]

By assumption, \( s > 1/(n_1 + n_2) \), and thus \( 1 - (n_1 + n_2)s < 0 \). The above expression is then positive if \( y_1 + y_2 < \gamma_1 \), which always holds since \( y_1 + y_2 \leq \max\{1/n_1, 1/n_2\} < \gamma_1 \): contradiction. Thus either \( dE_{11} \) is \( 0 \) or \( dE_{12} \), or both.

It is easily verified from the expressions for \( \frac{dE_i}{dx_0} \) that \( E_{1i}(x_0, \overline{y}) \) is concave in \( x_0 \) for \( s < 1/n_i \). By concavity, if \( \frac{dE_i}{dx_0} (1, \overline{y}) > 0 \) then the optimal platform that excludes group \( j \neq i \) is \( x_0 = 1 \) and \( x_i = 0 \). If both \( \frac{dE_{11}}{dx_0} (1, \overline{y}) > 0 \) and \( \frac{dE_{12}}{dx_0} (1, \overline{y}) > 0 \), then the optimal platform is \( x_0^* = 1 \). And if \( \frac{dE_{1i}}{dx_0} (1, \overline{y}) < 0 \) for some \( i \), then the optimal platform that excludes group \( j \neq i \) must have \( x_0^* < 1 \) and \( x_i^* > 0 \). It is therefore either a corner at \( x_0 = 0 \) and \( x_i = 1/n_i \), or the interior solution given by \( \tilde{x}_i \) and \( \tilde{x}_0 \) from the appropriate expression in (5)-(8). This platform must be the unique optimal platform since \( \frac{dE_{1i}}{dx_0} (1, \overline{y}) > 0 \) \( (j \neq i) \) implies that the optimal platform that excludes \( i \) is \( x_0 = 1 \).

Substituting \( y_i^* = 0 \) and \( x_0^* = 1 \) into the numerators of \( \frac{dE}{dy_0} (1, \overline{y}) \) produces the result.

(ii) Let \( E_{3i}(\overline{x}, y_0) \) denote group 3 utility \( E_3(\overline{x}, y) \) when \( y_i = (1 - y_0)/n_i \) and all other private good allocations are zero.

We first show that \( \frac{dE_{3i}}{dy_0} \geq 0 \) for either \( i = 2 \) or 3. Substituting \( y_i \) into (4) and differentiating \( E_{3i}(\overline{x}, y_0) \) gives the following expressions:

\[
\frac{dE_{33}}{dy_0} = \frac{2n_3 s^2 (y_0 - x_0) + (1 - n_3 s)(x_2 + \alpha - \theta/2) + s(1 - 2y_0 + x_0) - n_3 s(\gamma_3 + x_3)}{\theta n_3} \\
\frac{dE_{32}}{dy_0} = \frac{2n_2 s^2 (y_0 - x_0) - n_2 s(x_2 + \alpha - \theta/2) + s(1 - 2y_0 + x_0) + (1 - n_2 s)(\gamma_3 + x_3)}{\theta n_2}
\]

Suppose that \( \frac{dE_{3i}}{dy_0} (\overline{x}, 1) < 0 \) and \( \frac{dE_{3i}}{dy_0} (\overline{x}, 1) < 0 \). Clearly, for each \( i \), \( \frac{dE_{3i}}{dy_0} (1, \overline{y}) < 0 \) iff the corresponding numerator in the above expressions is negative. This implies that the sum of the numerators evaluated at \( y_0 = 1 \) must also be negative. Substituting and simplifying, we obtain:

\[
2n_3 s^2 (1 - x_0) + (1 - n_3 s)(x_2 + \alpha - \theta/2) + s(x_0 - 1 - n_3 (\gamma_3 + x_3)) + \\
2n_2 s^2 (1 - x_0) - n_2 s(x_2 + \alpha - \theta/2) + s(x_0 - 1 - n_2 (\gamma_3 + x_3)) + \gamma_3 + x_3 \\
= (1 - (n_2 + n_3)s)(2s(x_0 - 1) + x_2 - \alpha - \theta/2 + \gamma_3 + x_3).
\]

By assumption, \( s > 1/(n_2 + n_3) \), and thus \( 1 - (n_2 + n_3)s < 0 \). The above expression is then positive if \( x_2 + x_3 < |\gamma_3| \), which always holds since \( x_2 + x_3 \leq \max\{1/n_2, 1/n_3\} < |\gamma_3| \): contradiction. Thus either \( \frac{dE_{33}}{dy_0} (\overline{x}, 1) > 0 \) or \( \frac{dE_{32}}{dy_0} (\overline{x}, 1) > 0 \), or both.
It is easily verified from the expressions for \( \frac{dE_{x_1}}{dy_0} \) that \( E_{x_1}(\pi, y_0) \) is concave in \( y_0 \) for \( s < 1/n_i \). By concavity, if \( \frac{dE_{x_1}}{dy_0}(\pi, 1) > 0 \) then the optimal platform that excludes group \( j \neq i \) is \( y_0 = 1 \) and \( y_i = 0 \). If both \( \frac{dE_{x_1}}{dy_0}(\pi, 1) > 0 \) and \( \frac{dE_{x_1}}{dy_0}(\pi, 1) > 0 \), then the optimal platform is \( y_0^* = 1 \) and \( y_i^* = 0 > 0 \). It is therefore either a corner at \( y_0 = 0 \) and \( y_i = 1/n_i \), or the interior solution given by \( \tilde{y}_i \) and \( \tilde{y}_0i \) from the appropriate expression in (9)-(12). This must be the unique optimal platform since \( \frac{dE_{x_1}}{dy_0}(\pi, 1) > 0 (j \neq i) \) implies that optimal platform that excludes \( i \) is \( y_0 = 1 \).

Substituting \( y_0^* = 1 \) and \( x_3^* = 0 \) into the numerators of \( \frac{dE_{x_1}}{dy_0}(\pi, 1) \) produces the result. □

**Proof of Proposition 1.** There are three cases. First, for \( s < \min\{1/(n_1+n_2), 1/(n_2+n_3)\} \), existence is demonstrated by Proposition 2.

Second, suppose \( s \geq \max\{1/(n_1+n_2), 1/(n_2+n_3)\} \). Since candidates from both parties adopt the platform that maximizes the expected utility of their core voters, it suffices to establish a fixed point in the best response functions for each party’s common platform. We show that the pure strategy best responses satisfy the conditions of Brouwer’s fixed point theorem. Observe that each party’s set of feasible pure strategies is a simplex, and thus the set of strategy profiles is obviously compact, convex, and non-empty.

Next, we show that party X’s best response is single-valued. Let \( E_{x_1}(x_0, \vec{y}) \) denote group 1 utility \( E_{x_1}(\pi, \vec{y}) \) when \( x_i = (1-x_0)/n_i \) and all other private good allocations are zero. Note that \( E_{x_1}(x_0, \vec{y}) \) is concave in \( x_0 \) for \( s < 1/n_i \) and that \( E_{x_1}(\cdot) \) is maximized at \( x_0 = 1 \) for \( s \geq 1/n_i \). Thus, when restricted to a choice between group \( i \) and the public good, there unique solutions \( \tilde{x}_{0i} \) and \( \tilde{x}_{02} \).

Party X’s best response is \( \tilde{x}_{0i} \) if \( E_{x_1}(\tilde{x}_{0i}, \vec{y}) > E_{x_1}(\tilde{x}_{02}, \vec{y}) \) \((j \neq i)\). By the argument in the proof of Lemma 3, \( E_{x_1}(\tilde{x}_{0i}, \vec{y}) = E_{x_1}(\tilde{x}_{02}, \vec{y}) \) if and only if \( \tilde{x}_{0i} = \tilde{x}_{02} = 1 \). Thus, party X’s best response is single-valued. An identical argument establishes that party Y’s best response is also single-valued. The best response to each strategy profile is therefore single-valued.

Finally, we show that the best response function is continuous. By the concavity of \( E_{x_1}(x_0, \vec{y}), E_{x_2}(x_0, \vec{y}) \), the solutions \( \tilde{x}_{0i} \) and \( \tilde{x}_{02} \) are continuous in \( \vec{y} \). By the argument in the proof of Lemma 3, either \( \tilde{x}_{0i} = 1 \) or \( \tilde{x}_{02} = 1 \), and the platform implied by \( \tilde{x}_{0i} \) (i.e., \( x_0 = x_{0i}, x_i = (1-x_0)/n_i \)) is optimal if \( \tilde{x}_{0i} < 1 \). Thus for any \( \vec{y} \) the best response is either \( \tilde{x}_{0i} < 1 \) for some \( i \in \{1, 2\}, \) or \( x_0 = 1 \), which occurs when \( \tilde{x}_{0i} = \tilde{x}_{02} = 1 \). The resulting best response for party X is then:

\[
\begin{cases}
\tilde{x}_{0i} \text{ if } \tilde{x}_{0i} < 1, \tilde{x}_{02} = 1 \\
1 \text{ if } \tilde{x}_{0i} = \tilde{x}_{02} = 1 \\
\tilde{x}_{02} \text{ if } \tilde{x}_{02} < 1, \tilde{x}_{01} = 1.
\end{cases}
\]

This function inherits continuity in \( \vec{y} \) from the continuity of \( \tilde{x}_{0i} \) and \( \tilde{x}_{02} \). An identical argument holds for party Y’s best response.

Third, suppose \( s \in \min\{1/(n_1+n_2), 1/(n_2+n_3)\}, \max\{1/(n_1+n_2), 1/(n_2+n_3)\} \). Without loss of generality, assume that \( 1/(n_1+n_2) < 1/(n_2+n_3) \). We again show that the pure strategy
best responses satisfy the conditions of Brouwer’s fixed point theorem. For party X’s best responses the analysis is identical to the second case. For party Y, we show that the best response is single-valued and continuous.

Since $s \leq 1/(n_2+n_3)$, it is clear that $y_0 = 0$ in any best response. Noting that $y_1 = 0$ and $y_2 = (1-n_3y_3)/n_2$ in any best response, we rewrite (4) as the party Y objective as follows:

$$E_3(\bar{x}, \bar{y}) = \left[ x_2 - \frac{(1-n_3y_3)/n_2 + sx_0 + \alpha}{\theta} + \frac{1}{2} \right] (x_3 - y_3 + sx_0 + \gamma_3 + y_3 + sy_0$$

This expression is obviously concave in $y_3$. Straightforward maximization yields the solution $y_3^* = 1 + n_3x_3 + (n_3s + n_2s)x_0 + n_3\gamma_3 + n_2(\theta/2 - \alpha - x_2)/2n_3$, which is obviously single-valued and continuous. ■

**Proof of Proposition 2.** By Lemma 1(ii), the assumptions on $s$ allow us to restrict attention to strategies where $x_0 = y_0 = 0$.

We now characterize the unique equilibrium platforms. The budget constraints and weak domination imply that $x_2 = (1-n_1x_1-n_3x_3)/n_2$ and $y_2 = (1-n_1y_1-n_3y_3)/n_2$. Substituting these into (3) and (4) yields:

$$E_1(\bar{x}, \bar{y}) = \left[ \frac{\alpha + (n_1y_1 + n_3y_3 - n_1x_1 - n_3x_3)/n_2}{\theta} + \frac{1}{2} \right] (x_1 - y_1 + \gamma_1) + y_1$$

$$E_3(\bar{x}, \bar{y}) = \left[ \frac{\alpha + (n_1y_1 + n_3y_3 - n_1x_1 - n_3x_3)/n_2}{\theta} + \frac{1}{2} \right] (x_3 - y_3 + \gamma_3) + y_3.$$ 

Clearly, $\frac{\partial E_1}{\partial x_3}(\bar{x}, \bar{y}) < 0$ and $\frac{\partial E_3}{\partial y_1}(\bar{x}, \bar{y}) < 0$ for all $(\bar{x}, \bar{y})$, so $x_3^* = y_1^* = 0$. The expected utilities of group-1 and group-3 voters can then be written:

$$E_1(\bar{x}, \bar{y}) = \left[ \frac{n_3y_3 - n_1x_1}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (x_1 + \gamma_1)$$

$$E_3(\bar{x}, \bar{y}) = \left[ \frac{n_3y_3 - n_1x_1}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (-y_3 + \gamma_3) + y_3.$$ 

Expressions (23) and (24) are concave and univariate objectives in $x_1 \in [0, 1/n_1]$ and $y_3 \in [0, 1/n_3]$, respectively. Thus for any $\bar{x}$ (respectively, $\bar{y}$), there is a unique platform for party Y (respectively, X) that maximizes the utility of the pivotal voter in group 3 (respectively, 1). Each party’s candidates must therefore choose the same platform in equilibrium.

Denoting the equilibrium transfer vectors $\bar{x}^p$ and $\bar{y}^p$, the first-order conditions on (23) and (24) produce:

$$x_1^p = \frac{n_3y_3^p - n_1\gamma_1 + n_2(\theta/2 + \alpha)}{2n_1}$$

$$y_3^p = \frac{n_1x_1^p + n_3\gamma_3 + n_2(\theta/2 - \alpha)}{2n_3}.$$ 

Solving these yields the stated unique equilibrium allocations. ■
Proof of Proposition 3. (i) Note that by Lemma 1(iii), the best responses by candidates in both parties under the stated condition are to offer only public goods when \( s > 1/n_2 \).

Consider the platform choices of party \( X \) candidates when \( s \leq 1/n_2 \). By Lemma 1(iv), for any optimal platform either \( x_1^* = 0 \) or \( x_2^* = 0 \). Observe that the left-hand side of expressions (13)-(14) in Lemma 3(i) are the numerators of the derivatives with respect to \( x_0 \) of the party \( X \) objectives \( E_{11}(x_0, \bar{y}) \) and \( E_{12}(x_0, \bar{y}) \) evaluated at \( x_0 = 1 \), where the objectives are restricted to \( x_2 = 0 \) and \( x_1 = 0 \), respectively. Since the denominators of the derivatives are strictly positive, the sign of each expression is sufficient for signing the derivative.

Evaluating (14) at \( s = 1/n_2 \), \( \frac{dE_{12}}{dx_0}(1, \bar{y}) > 0 \) if:

\[
\frac{1 - y_0}{n_2} > y_2 - \alpha - \theta/2.
\]

Since \( \theta \geq \alpha \geq 0 \) and the budget constraint implies \( y_2 \leq (1 - y_0)/n_2 \), this expression always holds. Thus by the concavity of \( E_{12}(x_0, \bar{y}) \), \( x_0 = 1 \) is the optimal strategy for party \( X \) candidates when \( x_1 = 0 \) and \( s = 1/n_2 \). By the continuity of \( E_{12}(x_0, \bar{y}) \) in \( s \), there exists a nonempty set \( S \equiv [s, 1/n_2) \) such that for all \( s \in S \), \( \frac{dE_{12}}{dx_0}(1, \bar{y}) > 0 \).

Now consider \( E_{11}(x_0, \bar{y}) \). Clearly, for all \( s > 1/n_1 \), the optimal strategy for party \( X \) candidates when \( x_2 = 0 \) is \( x_0 = 1 \). Since \( n_1 > n_2 \), the region \([1/n_1, 1/n_2] \cap S \) is non-empty. Party \( X \) candidates will then choose \( x_0 = 1 \) regardless of whether their best response is to maximize \( E_{11}(x_0, \bar{y}) \) or \( E_{12}(x_0, \bar{y}) \) when \( s \geq \min\{[1/n_1, 1/n_2] \cap S\} \). Thus, for \( n_1 > n_2 \), party \( X \) candidates will offer only public goods for some \( s \) strictly less than \( 1/n_2 \).

The analysis for party \( Y \) candidates is symmetric and therefore omitted. Combining the statements for both parties yields a threshold \( \bar{s} \).

(ii)-(iv) These expressions follow immediately from Lemma 1 and the derivations of (5) and (9). □

Proof of Proposition 4. By Lemma 1(ii), we require \( s > \frac{1}{n_1 + n_2} \) for \( x_0 > 0 \) in equilibrium. Since only \( x_1 \) or \( x_2 \) can be strictly positive at an optimal platform, it is sufficient to derive conditions under which the possible private good allocations for \( x_1 \) and \( x_2 \), \( \bar{x}_1 \) and \( \bar{x}_2 \), are not maximized. Observe that for \( s > 1/n_i \), \( x_i = 0 \), and so we restrict attention to \( s \leq 1/n_i \) for \( i = 1, 2 \). Using expression (6) and the fact that \( y_2 \geq 0 \), \( \bar{x}_1 < 1/n_1 \) if:

\[
\frac{n_1\gamma_1 + (2n_1s - 1)(1 - y_0)}{2n_1(n_1s - 1)} + \frac{\alpha + \theta/2 - y_2}{2n_1s} < \frac{1}{n_1}
\]

\[
sn_1\gamma_1 + s - s(2n_1s - 1)y_0 + (\alpha + \theta/2)(n_1s - 1) > 0
\]

Noting that \((2n_1s - 1)y_0\) is bounded from above by 1, this expression simplifies to:

\[
s > \frac{\alpha + \theta/2}{n_1(\gamma_1 + \alpha + \theta/2)}.
\]

Likewise, using expression (7) and the fact that \( y_2 \leq 1/n_2 \), \( \bar{x}_2 < 1/n_2 \) if:

\[
(n_2s - 1)\gamma_1 - s(2n_2s - 1)y_0 + s + n_2s(\alpha + \theta/2 - 1/n_2) > 0
\]
Noting that \((2n_2s - 1)y_0\) is bounded from above by 1, this expression simplifies to:

\[
    s > \frac{\gamma_1}{n_2(\gamma_1 + \alpha + \theta/2) - 1}.
\]

Combining the two expressions for \(s\) yields the result. ■

**Proof of Lemma 4.** We use the results from Lemma 3.

(i) Reversing the inequality in (13), we obtain the following condition under which \(x^*_1 = 0\) must obtain: \(\gamma_1 \geq \frac{(2n_1s-s_2)(1-y_0)}{1-n_1s}(y_2-\alpha-\theta/2)\). To establish the upper bound on the right-hand side of this expression, let \(y_2 = 0\) and \(y_0 = 1\) (0) if \(2n_1s > (<) 1\). This yields \(\gamma_1\), or (17).

Next, reversing the inequality in (14), we obtain the following condition under which \(x^*_2 = 0\) must obtain: \(\gamma_1 \leq \frac{(2n_2s-s_2)(1-y_0)-n_2s(y_3-\alpha-\theta/2)}{1-n_2s}\). To establish the lower bound on the right-hand side of this expression, let \(y_2 = 1/n_2\) and \(y_0 = 1 - n_2y_2 = 0\). This yields \(\gamma_1 = \frac{n_2s(\alpha+\theta/2)}{1-n_2s} - 2s\), or (18).

(ii) Reversing the inequality in (15), we obtain the following condition under which \(y^*_3 = 0\) must obtain: \(\gamma_3 \leq \frac{(2n_3s-s_1)(1-x_0)-(1-n_3s)(x_3+\alpha-\theta/2)}{1-n_3s}\). To establish the lower bound on the right-hand side of this expression, let \(x_2 = 0\) and \(x_0 = 1\) (0) if \(2n_3s > (<) 1\). This yields \(\gamma_3\), or (19).

Next, reversing the inequality in (16), we obtain the following condition under which \(y^*_2 = 0\) must obtain: \(\gamma_3 \geq \frac{(2n_2s-s_2)(1-x_0)-(1-n_2s)(x_3+\alpha-\theta/2)}{1-n_2s}\). To establish the upper bound on the right-hand side of this expression, let \(x_2 = 1/n_2\) and \(x_0 = 1 - n_2x_2 = 0\). This yields \(\gamma_3 = \frac{n_2s(\alpha-\theta/2)}{1-n_2s} + 2s\), or (20). ■

**Proof of Proposition 5.** Combining Lemmas 1, 3, and 4 and expressions (5)-(12), we have the following choices for party \(X\) candidates. For \(s \leq \min\{1/n_1, 1/n_2\}\), \(x^*_2 = \min\{\max\{0, x_2\}, 1/n_2\}\) if \(\gamma_1 \geq \gamma_1\), and \(x^*_1 = \min\{\max\{0, x_1\}, 1/n_1\}\) if \(\gamma_1 \leq \gamma_1\). Likewise, we have the following choices for party \(Y\) candidates. For \(s \leq \min\{1/n_3, 1/n_2\}\), \(y^*_2 = \min\{\max\{0, y_2\}, 1/n_2\}\) if \(\gamma_3 \leq \gamma_3\), and \(y^*_3 = \min\{\max\{0, y_3\}, 1/n_3\}\) if \(\gamma_3 \geq \gamma_3\).

For parts (i)-(iii), the interior platforms are derived from straightforward solutions of the linear systems implied by these equilibrium best response platforms. For part (iv), the system yields:

\[
    x_2 = y_2 + \frac{\gamma_1}{2n_2s} - \frac{\theta + 2\alpha}{4(1 - n_2s)} \tag{27}
\]

\[
    y_2 = x_2 - \frac{\gamma_3}{2n_2s} + \frac{\theta - 2\alpha}{4(1 - n_2s)} \tag{28}
\]

There is clearly no generic interior solution for this system. For \(\gamma_1\) and \(\gamma_3\) sufficiently large, the unique solution is the corner at \(x_2 = 1/n_2\) and \(y_2 = 1/n_2\). ■

**Proof of Proposition 6.** We first establish party \(X\) candidates’ optimal platforms for \(\gamma > \frac{(1-n_1s)\theta}{2n_1s}\). By Lemma 3, the solution for party \(X\) candidates is \(x_1 = \max\{1/n_1, x_1\}\) if
(13) holds, or:

\[(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \theta/2) + n_1s\gamma < 0.\]

The left-hand side of this expression is increasing in \(\gamma\). Substituting in \(\gamma = \frac{(1-n_1s)\theta}{2n_1s}\), this expression reduces to:

\[(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)y_2 < 0.\]

This expression cannot hold for any \(s \in \left(\frac{1}{2n_1}, \frac{1}{n_1}\right)\), and thus there is no optimal platform where \(x_1 > 0\) when \(\gamma \geq \frac{(1-n_1s)\theta}{2n_1s}\). By Lemma (11), this implies that at any party \(X\) best response, the entire budget is used on \(x_0\) and \(x_2\), or equivalently, \(n_2x_2 + x_0 = 1\).

To derive the value of \(x_2\), we substitute \(n_2x_2 + x_0 = 1\), \(n_2y_2 + y_0 = 1\), and the assumed parameter restrictions on \(\gamma_1, \gamma_3,\) and \(\alpha\) into the expressions for \(\tilde{x}_2\) (7) and \(\tilde{y}_2\) (11), which yields the following:

\[
x_2 = y_2 + \frac{\gamma}{2n_2s} - \frac{\theta}{4(1-n_2s)} \quad (29)
\]

\[
y_2 = x_2 + \frac{\gamma}{2n_2s} - \frac{\theta}{4(1-n_2s)}. \quad (30)
\]

There is clearly no generic interior solution for this system. Since \(\frac{\gamma}{2n_2s} - \frac{\theta}{4(1-n_2s)} > (\gamma < 0)\) for \(\gamma < (\gamma < \frac{n_2s\theta}{2(1-n_2s)})\), the unique solution is \(x_2 = y_2 = 1/n_2\) (\(= 0\) for \(\gamma > (\gamma > \frac{n_2s\theta}{2(1-n_2s)})\)).

Now consider the party \(X\) candidates’ optimal platforms for \(\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}\). Observe that \(\frac{(1-n_1s)\theta}{2n_1s} < \frac{n_2s\theta}{2(1-n_2s)}\) for all \(s > \frac{1}{n_1+n_2}\), which follows from the assumption that \(s > \max\{\frac{1}{2n_1}, \frac{1}{2n_2}\}\). We first establish that \(x_2 = 0\) for any best response. To see this, note first that the system (29)-(30) implies that for \(\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}\), there can be no solution where \(x_2 > 0\) and \(y_2 > 0\); thus, \(x_2 > 0\) requires \(y_2 = 0\). By Lemma 3, party \(X\) chooses \(x_2 > 0\) if and only if (14) holds, or:

\[(2n_2s^2 - s)(1 - y_0) - n_2s(y_2 - \theta/2) - (1 - n_2s)\gamma < 0. \quad (31)\]

The left-hand side of this expression is decreasing in \(\gamma\). To show that (31) cannot hold for \(\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}\), it will be convenient to substitute in \(y_2 = 0\) and \(\gamma = \frac{n_2s\theta}{2(1-n_2s)}\). Then (31) can be satisfied for all \(\gamma < \frac{n_2s\theta}{2(1-n_2s)}\) only if:

\[(2n_2s^2 - s)(1 - y_0) < 0.\]

This expression cannot hold for any \(s > \frac{1}{2n_2}\), and thus all best responses must satisfy \(x_0 + n_1x_1 = 1\). A symmetric analysis holds for party \(Y\) candidates.

To derive the value of \(x_1\), observe that an interior solution must be given by \(\tilde{x}_1\) and \(\tilde{y}_1\), as defined by (5) and (9). The unique solution of this system is \(x_1^* = y_3^* = \min\left\{\frac{1}{n_1}, \frac{(1-n_1s)\theta}{2n_1s} - \gamma\right\}\); these are clearly non-negative for all \(\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}\). \(\blacksquare\)
Proof of Proposition 7. By Lemma 1(ii), since \( s < 1/(n_1 + n_2) \), \( x_0^* = 0 \). Substituting into (3) and maximizing then gives the following expression for \( x_1^* \) and \( x_2^* \) in terms of \( y_2 \):

\[
x_1^* = \frac{1 - n_1\gamma - n_2(y_2 - \theta/2)}{2n_1}
\]

\[
x_2^* = \frac{1 + n_1\gamma + n_2(y_2 - \theta/2)}{2n_2}
\]

By the concavity of (3), the obvious corner solutions are \( x_1^* = 0 \) and \( x_2^* = 1/n_2 \), and \( x_1^* = 1/n_1 \) and \( x_2^* = 0 \). We derive features of the party \( Y \) platforms by applying Lemma 3(ii).

(i) Simplifying from (15), \( y_3^* = \min\{1/n_3, \tilde{y}_3\} \) and \( y_2^* = 0 \) if:

\[
2n_3s^2 - s + (1 - n_3s)(x_2 - \theta/2) + n_3s\gamma < 0
\]

\[
\gamma < \frac{s - 2n_3s^2 - (1 - n_3s)(x_2 - \theta/2)}{n_3s}
\]

Since \( y_2^* = 0 \) in this region, we have \( x_2^* = \max\{0, \min\{\frac{1}{n_2}, \frac{1}{2n_2} + \frac{\gamma}{\theta} - \frac{\theta}{4}\}\} \). The upper bound implied by (34) is linear in \( x_2 \), and \( x_2^* \) is continuous, bounded, and piecewise linear in \( \gamma \). Thus a solution \( \gamma' \) for (34) in terms of \( \gamma \) exists and is weakly decreasing and piecewise linear. It is straightforward to calculate that at an interior solution this bound is:

\[
\gamma' = \frac{(1 - n_3s)(2s - 1/n_2 + 3\theta/2) - 2n_3s^2}{1 + n_3s}
\]

Finally, substituting appropriately into (9), we have the following expression for \( \tilde{y}_3 \):

\[
\tilde{y}_3 = \frac{2n_3s - 1 + n_3\gamma}{2n_3(n_3s - 1)} - \frac{x_2 - \theta/2}{2n_3s}.
\]

This expression is decreasing in \( \gamma \) for \( s < 1/n_3 \), and \( y_3^* = 0 \) for \( s \geq 1/n_3 \) when (15) holds. Thus, \( y_3^* \) is weakly decreasing in this region.

(ii) Simplifying from (16), \( y_3^* = 0 \) and \( y_2^* = \min\{1/n_2, \tilde{y}_2\} \) if:

\[
2n_2s^2 - s - n_2s(x_2 - \theta/2) - (1 - n_2s)\gamma < 0
\]

\[
\gamma > \frac{2n_2s^2 - s - n_2s(x_2 - \theta/2)}{1 - n_2s}
\]

Simplifying (12) yields the following expression for \( \tilde{y}_2 \):

\[
\tilde{y}_2 = \frac{s(2n_2s - 1) - (1 - n_2s)\gamma}{2n_2s(n_2s - 1)} - \frac{x_2 - \theta/2}{2(n_2s - 1)}.
\]

The expressions for \( y_2^* \) and \( x_2^* \) are continuous and piecewise linear in \( y_2 \) and \( x_2 \), respectively, and bounded. Thus there exists a solution to the system. At an interior solution we have:

\[
x_2^* = \frac{1}{n_2} + \frac{(2n_2s^2 - n_2s - 1)\gamma}{n_2s(4n_2s - 3)} + \frac{(3/2 - n_2s)\theta}{4n_2s - 3}
\]

\[
y_2^* = \frac{1}{n_2} + \frac{(2 - n_2s)\gamma}{3n_2s - 4n_2s^2} + \frac{3\theta}{6 - 8n_2s}
\]
To characterize \( \gamma'' \), note that since \( x^*_2 \geq 0 \), the lower bound on \( \gamma'' \) can be derived by substituting \( x^*_2 = 1/n_2 \) into (36), yielding \( \gamma'' \geq \frac{s(2n_2s^2-2n_2\theta/2)}{1-n_2s} \). Substituting the interior value of \( x^*_2 \) into (36), at an interior solution the minimum value of \( \gamma \) for this solution to obtain is:

\[
\gamma'' = \frac{s(3 - 4n_2s - 3n_2\theta/2)}{n_2s - 2}
\]

It is straightforward to verify that \( \gamma'' > \gamma' \) whenever \( \gamma' > 1/n_2 \).

(iii) By Lemma 3(ii), for all \( \gamma \) not satisfying the conditions of parts (i) and (ii), \( y^*_0 = 1 \).

Proof of Remark 3. (i) Since \( s > 1/n_2 \), Lemma 3(iii) implies that \( x^*_2 = 0 \). We use Lemma 3(i) to establish the condition under which party \( X \) candidates can offer private goods to group 1. Substituting into equation (13) yields:

\[
n_1 < \frac{\alpha + \theta/2}{s(\gamma_1 + \alpha + \theta/2)}. \tag{37}
\]

When (37) is not satisfied, \( x^*_0 = 1 \). When (37) is satisfied, \( x^*_1 = \min\{1/n_1, \bar{x}_1\} \), as given by substituting \( y_0 = 1 \) and \( y_1 = y_2 = 0 \) into (5):

\[
x^*_1 = \frac{\gamma_1}{2(n_1s - 1)} + \frac{\alpha + \theta/2}{2n_1s}.
\]

(ii) Now suppose that \( s < 1/n_2 \), which implies \( y^*_2 = 1/n_2 \). We again apply Lemma 3(i). Substituting \( y_0 = y_1 = 0 \) and \( y_2 = 1/n_2 \) into equation (13) yields the following condition:

\[
2n_1s^2 - s + (1 - n_1s) \left( \frac{1}{n_2} - \alpha - \frac{\theta}{2} \right) + n_1s\gamma_1 < 0
\]

\[
n_1 < \frac{s + \alpha + \theta/2 - 1/n_2}{s(2s + \gamma_1 + \alpha + \theta/2 - 1/n_2)}. \tag{38}
\]

When (38) is satisfied, \( x^*_1 = \min\{1/n_1, \bar{x}_1\} \), as given by substituting appropriately into (5):

\[
x^*_1 = \frac{\gamma_1 + s}{2(n_1s - 1)} + \frac{1}{2n_1} \left( 1 - \frac{1}{n_2s} \right) + \frac{\alpha + \theta/2}{2n_1s}.
\]

Similarly, party \( X \) candidates may offer private goods to group 2. Substituting into equation (14) yields the following condition:

\[
2n_2s^2 - s - n_2\left( \frac{1}{n_2} - \alpha - \frac{\theta}{2} \right) - (1 - n_2s)\gamma_1 < 0
\]

\[
n_2 < \frac{2s + \gamma_1}{s(2s + \gamma_1 + \alpha + \theta/2)}. \tag{39}
\]

When (39) is satisfied, \( x^*_2 = \min\{1/n_2, \bar{x}_2\} \), as given by substituting appropriately into (7):

\[
x^*_2 = \frac{\gamma_1}{2n_2s} + \frac{n_2(2s + \alpha + \theta/2) - 2}{2n_2(n_2s - 1)}.
\]

When neither (38) nor (39) are satisfied, party \( X \) provides only public goods.
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