Primary Elections and the Provision of Public Goods*

Running Title: Primary Elections and Public Goods

Michael M. Ting†
Columbia University

James M. Snyder, Jr.‡
Harvard University and NBER

Shigeo Hirano§
Columbia University

March 5, 2018

*We thank Deborah Beim, Mattias Polborn, Edoardo Teso, and seminar attendees at the University of North Carolina, New York University, and the 2013 Yale CSAP Conference for useful comments and suggestions.

†Professor of Political Science and International and Public Affairs, Columbia University; IAB Floor 7, 420 West 118 Street, New York NY 10027, mmt2033@columbia.edu

‡Leroy B. Williams Professor of History and Political Science, Harvard University; 1737 Cambridge Street, Cambridge, MA 02138, jsnyder@gov.harvard.edu

§Associate Professor of Political Science, Columbia University; IAB Floor 7, 420 West 118 Street, New York NY 10027, sh145@columbia.edu

Abstract

We develop a theory of primary elections and the provision of public and private goods. In our model, candidates from two parties compete for the support of "core" party voters in their respective primary elections and "swing" voters in a general election. Candidates within a party share a fixed ideology and offer platforms that distribute a unit of public spending across group-specific private goods and public goods. Without primaries, candidates offer only public goods when they are very valuable, and only private goods to the swing group otherwise. Because public goods appeal to both types of voters, primary elections increase their provision under a broad set of conditions. The level of public good provision is non-monotonic in ideological polarization. The prediction of increased public goods spending following the adoption of primaries matches empirical patterns in capital expenditures across U.S. states.

Keywords: primary elections, distributive politics, public goods

Supplementary material for this article is available in the appendix in the online edition. Replication files are available in the JOP Data Archive on Dataverse (http://thedata.harvard.edu/dvn/dv/jop).

1 Introduction

How do political institutions shape public policy? This is a fundamental question in political economy. One important sub-question is: What features of the political system provide incentives for politicians or parties to spend government funds on public goods that benefit the vast majority of citizens, rather than goods targeted at narrow groups? This has been the subject of a number of recent theoretical and empirical studies, including Lancaster (1986), Persson and Tabellini (1999, 2003, 2004a), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002), Persson, Roland and Tabellini (2000), and Blume et al. (2009).

These studies focus on key features of electoral systems and the separation of powers, such as plurality rule vs. proportional representation systems, district magnitude, and parliamentary vs. presidential systems. One of the main arguments is that the “winner-take-all” structure of electoral outcomes under plurality rule with single-member-districts implies that the minimum winning coalition of voters to gain a majority in the legislature is smaller than under proportional representation. Thus, plurality rule induces politicians to target small but pivotal constituencies in individual electoral districts with local public goods and specific transfers. In contrast, under proportional representation, “every vote counts” no matter where it is cast, and additional votes always translate directly into additional seats, providing incentives for politicians to seek the support of voters across the country. Proportional representation therefore induces politicians to favor policies benefiting large groups of voters such as general public goods and broad-based transfer programs that affect voters in many electoral districts.[1]

[1] Persson and Tabellini (2004b), Milesi-Ferretti, Perotti, and Rostagno (2002), Gagliarducci, Nannicini and Naticchioni (2011) find empirical support for these predictions. See also Lancaster and Patterson (1990) and Stratmann and Baur (2002) for other evidence that is broadly consistent with the underlying incentives facing politicians. Many other features of the political system have also been studied, including bicameralism, vetoes, confidence procedures, party organization, and federalism. See, for example, Inman and Fitts (1990), Diermeier and Feddersen (1998), McCarty (2000a, 2000b), Bradbury and Crain (2002), Lockwood (2002), Ansolabehere, Snyder and Ting (2003), Kalandrakis (2004), Cutrone and McCarty (2006), Berry (2008, 2009), Berry and Gersen (2009), Primo and Snyder (2010), Tergiman (2015), and Parameswaran (2012).

One important political factor omitted from this analysis is internal party structure. In particular, there is no treatment of the various ways candidates are nominated. A variety of nomination methods are used around the world, including national, state, and local party conventions, caucuses, meetings restricted to small party elites, direct vote by dues-paying party members, and direct primary elections. Primary elections are the dominant system for nominations in the U.S., are used widely in many Latin American countries. They also appear to be increasingly popular, having been used in recent years by parties in Italy, Spain, South Korea and elsewhere.[2] Hirano, Snyder and Ting (2009) show that nomination systems may have significant effects on the allocation of distributive government spending. In particular, when electoral outcomes are uncertain, direct primaries may provide strong incentives for politicians to offer transfers to "core supporters" in addition to "swing voters."[3]

This paper shows that direct primaries may also provide incentives for politicians to supply public goods that benefit all voters, rather than distributive goods or narrowly targeted transfers that only benefit specific constituencies. The basic logic is straightforward. If there are no primary elections and candidates simply maximize their probability of winning in the general election, then they are driven to compete mainly for swing voters. Thus, when deciding between public goods and targeted goods, candidates are biased toward choosing targeted goods — and targeting them at swing voters. They will only choose public goods if public goods have an extremely high ratio of social benefits to costs. Under primaries, however, candidates must spread benefits more evenly, since they must win both swing voters in the general election and core voters in the primary election. Thus, they have incentives to offer public goods even when public goods have a relatively modest (but still favorable)

[2]For background on U.S. primary elections, see Merriam and Overacker (1928) and Ware (2002). For more details on primaries in Latin America see Carey and Polga-Hecimovich (2006) and Kemahlioglu, Weitz-Shapiro and Hirano (2009).

[3]There is an extensive literature on the policy consequences of primaries or the incentives for adopting primary systems, including Aronson and Ordeshook (1972), Coleman (1972), Owen and Grofman (2006), Caillaud and Tirole (1999), Jackson, Mathevet and Mattes (2007), Adams and Merrill (2008), Castanheira, Crutzen and Sahuguet (2010), Serra (2011), Crutzen (2014), and Negri (2014). Most of these models are concerned with outcomes in a spatial or valence framework.

ratio of benefits to costs.

Our theory is based on distributive politics models developed by Lindbeck and Weibull (1987) and Dixit and Londregan (1995, 1996). In their work, office-minded candidates from two parties compete for votes by promising distributions of particularistic spending across a large number of ideologically heterogeneous groups. Parties have fixed ideological positions, and are therefore advantaged in certain districts. We simplify this framework by considering only three groups of voters, corresponding to a swing group and core groups for each party. Additionally, core groups are "off limits" to the opposition in the sense that there is no incentive for the opposing party to offer particularistic goods in the hope of gaining votes. We also add two important features to the framework. First, in addition to promising allocations of particularistic spending, candidates can commit to spending some portion of the budget on a public good that benefits all voters evenly. Second, following Hirano, Snyder, and Ting (2009), there are simultaneous primary elections in each party that determine which of its candidates will proceed to the general election. The pivotal voter in each primary is a member of the corresponding core group. Platforms are fixed across the two election stages, and voters are strategic in assessing platforms.[4]

As intuition might suggest, equilibrium platforms depend on the relative value of public goods. Candidates offer only public goods when that value is sufficiently high, and only private goods when that value is sufficiently low. For intermediate values, parties will adopt platforms that combine public goods and private goods for either the swing group or their core group. Optimal platforms never offer public goods and private goods to both core and swing groups, since (depending on their relative value) a unit of public goods can always either replace or be replaced by private goods for both groups.

It is also useful to consider the role of ideology in platform selection when the value of

[4]Respondents in the presidential primary exit poll surveys claim to value electability when deciding how to vote. For example in the 2004 Democratic primaries, more exit poll respondents cited the ability to "defeat George W. Bush" than any other response to the question "Which ONE candidate quality mattered most in deciding how you voted today?"

the public good is intermediate. In their primary election, core group voters must trade off between the probability of winning and the benefits received conditional upon victory. Extreme core voters are more inclined to give up private goods than moderates, because they are more concerned about ideological payoffs and private goods for swing voters may be the most effective way to raise their party's probability of election. Consequently, private goods will go to the swing group when a party's core group is extreme, and to the core group when it is moderate. In between these ideological extremes, public goods are a useful compromise for delivering benefits to both groups. While the equilibrium of the general model can be quite complex, these relationships are examined in more detail in a variant of the model where the factions are symmetric with respect to core group size and ideology. Under these conditions, the level of public goods provision is non-monotonic: it increases initially in ideological polarization, and then abruptly drops to zero as candidates use only private goods to chase the swing group.

The model makes several predictions about the effect of primaries on public goods provision. Without primaries, the stark result is that candidates offer exclusively public goods if their value to swing voters exceeds that of private goods. Otherwise, candidates offer only private goods to swing voters and core voters receive nothing. When public goods are of intermediate value to swing voters, primaries broadly shift platforms from private to public goods. The resulting allocation is more socially efficient, but primaries alone do not fully ensure socially efficient provision of public goods. One somewhat surprising exception to the increase in public goods occurs in the special case where public goods are very valuable to swing voters but core groups are both small and ideologically moderate. In this case, core voters have high per capita valuations of private goods and face low ideological stakes in the election, and thus demand private goods when there are primaries.

We finally provide some evidence on the relationship between primaries and public good spending. While public goods are difficult to measure, capital outlays such as infrastructure projects can serve as one plausible measure. The introduction of direct primaries in American states corresponded with sizable increases in such outlays. In the earliest part of the 20th century, we estimate that capital outlays as a share of total state and local government expenditures increased by about five percentage points following the introduction of direct primaries. These estimates must be taken with some caution and cannot be taken as causal, but they are nonetheless consistent with the prediction that primaries increase public goods.

The paper proceeds as follows. The next section describes the model. Section 3 derives the results for the model with and without primary elections, as well as for the special case of symmetric ideologies. Section 4 presents our data on capital expenditures. Section 5 concludes.

2 Model

Our model considers electoral competition between two parties, labeled X and Y. There are two main variants of the model. In the first, there are no primary elections, and in the second, we introduce primaries within both parties. All elections are decided by plurality rule.

Voters are divided into three groups, indexed $i = 1, 2, 3$. The relative size of each group is n_i, with $\sum_{i=1}^{3} n_i = 1$. No group is an outright majority, so $n_i < 1/2$ for $i = 1, 2, 3$. Group membership is important to the model because candidates are able to offer transfers that are targeted specifically toward a group. Within each group, members enjoy the benefits of a targeted transfer equally.

There are two identical candidates in each party, denoted a and b. The platform offer by candidate $j \in \{a, b\}$ in party X is $\overline{x}^j = (x_0^j, x_1^j, x_2^j, x_3^j)$, where $x_0^j \geq 0$ is the amount allocated to public goods that are enjoyed by all citizens and for $i > 0$, $x_i^j \geq 0$ is targeted toward group i. Similarly, $\overline{y}^k = (y_0^k, y_1^k, y_2^k, y_3^k)$ is the platform offered by candidate $k \in \{a, b\}$ in party Y. Offers must satisfy the budget constraints $x_0^j + \sum_i n_i x_i^j = 1$ and $y_0^k + \sum_i n_i y_i^k = 1$. Platforms are binding policy commitments and cannot be changed.

Candidates care only about winning office. Voters care about a "fixed" policy issue, private good transfers, and public goods. All voters in each group have the same preference on the fixed issue. For each group $i = 1, 2, 3$, let γ_i denote the members' relative preference for party X's position on the fixed issue. Groups 1 and 3 are the "extremist" or core groups for party X and Y, respectively, and group 2 consists of "moderate" or swing voters. We assume $\gamma_1 > K$ and $\gamma_3 < -K$, where $K = \max\{1/n_1, 1/n_2, 1/n_3, s\}$. The parameter $s > 0$ is a measure of the efficiency of the public good, as described below. Among other things, this guarantees that party X can never buy the support of group 3 voters, and party Y can never buy the support of group 1 voters. Primary voters are forward-looking when voting in the primary, taking into account the expected outcome in the general election.

The preferences of group 2 voters on the fixed issue are stochastic and revealed immediately before the general election. This could represent a utility shock from the general election campaign that only group 2 voters cared about. For simplicity, we assume that γ_2 is distributed uniformly on the interval $[-\theta/2, \theta/2]$, where $\theta > 0$ is a measure of electoral uncertainty. So, the density of γ_2 is $1/\theta$ for $\gamma_2 \in [-\theta/2, \theta/2]$ and 0 otherwise, and the c.d.f. is $F(\gamma_2) = \gamma_2/\theta + 1/2$ for $\gamma_2 \in [-\theta/2, \theta/2]$.[5] Party X additionally has a party-specific electoral advantage, by giving group 2 voters $\alpha \in [0, \theta/2]$ in valence from either party X candidate. For one of our results (Proposition 4) we restrict attention to the somewhat simpler case where ideologies are symmetric ($\gamma_1 = -\gamma_3$) and there is no electoral advantage ($\alpha = 0$).

Voter utility is linear in transfers, where private goods are "per capita" and public goods are multiplied by the efficiency parameter s.[6] Thus if candidate k from party Y wins, a group i voter receives a payoff of $sy_0^k + y_i^k$. If candidate j from party X wins the general election, then a voter from group $i = 1, 3$ receives a payoff of $sx_0^j + x_i^j + \gamma_i$, and therefore votes for party X's candidate in the general election if $\gamma_i > y_i^k - x_i^j + s(y_0^k - x_0^j)$. From our assumptions on γ_1 and γ_3, this always results in a vote for the ideologically proximate party. Similarly, a voter from group 2 receives $sx_0^j + x_2^j + \gamma_2 + \alpha$ and votes for party X's candidate

[5] The logic of the model holds for a large class of symmetric, unimodal distributions.

[6] The linearity of voter utility allows us to derive closed-form solutions in many cases, but our results qualitatively hold for any strictly increasing, concave utility function.

in the general election if:

$$\gamma_2 > y_2^k - x_2^j + s(y_0^k - x_0^j) - \alpha.$$

In the game without primaries, any selection process for general election candidates produces the same result, since candidates are identical. We let each candidate be chosen by Nature with probability 1/2. In the game with primaries, we require some relatively mild assumptions about the division of voters across primary elections. Assume that $n_1 > n_2/2$ and $n_3 > n_2/2$. Let the electorate in the party X primary be group 1 and half of group 2, and likewise let the electorate in the party Y primary be group 3 and the other half of group 2. This ensures that groups 1 and 3 are the majorities in the party X and Y primaries, respectively. Since core groups are decisive in primary elections, it is possible to interpret their size as the extent of intra-party enfranchisement; for example, a large n_1 might correspond to broad suffrage in the party X primary, rather than a small caucus. Our results will hold under any assumption about the distribution of group 2 voters' participation in the primaries (such as complete abstention), as long as they are a minority in both primaries. In the Supplementary Appendix, we also consider what happens when swing voters control one party's primary electorate.[7]

All actions in the game are perfectly observable. The sequence of play for both games is as follows.

1. Candidates simultaneously offer transfer vectors $\overline{x}^a$, $\overline{x}^b$, $\overline{y}^a$, and $\overline{y}^b$.

2. Without primaries, Nature chooses each party's general election candidate. With primaries, primary voters for each party vote for one of the party's two candidates.

3. Nature reveals γ_2.

4. All voters vote for one of the general election candidates.

[7]When swing voters vote in the party Y primary, party Y benefits electorally. Party X candidates respond by shifting resources to core voters to increase their probability of victory, but less so as party Y uses public goods as these reduce the stakes of victory.

We derive subgame perfect equilibria in undominated voting strategies. An equilibrium consists of transfer announcements for each candidate and voting strategies for each voter at each election. Voting strategies map the set of platforms to votes for their party's candidates in the primary elections, and map the sets of platforms, primary election votes (if any), general election candidates, and values of γ_2 to a vote for one of the parties in the general election.

3 Results

We begin by deriving an expression for party X's probability of winning the general election, which will appear frequently in what follows. For any platforms $(\overline{x}^j, \overline{y}^k)$, all voters in group 1 vote for the party X candidate and all voters in group 3 vote for the party Y candidate. Since group 2 voters are pivotal, the party X candidate wins if $\gamma_2 > y_2^k - x_2^j + s(y_0^k - x_0^j) - \alpha$. Thus, at an interior solution, the probability that the party X candidate wins is:

$$1 - F(y_2^k - x_2^j + s(y_0^k - x_0^j) - \alpha) = \frac{x_2^j - y_2^k - s(y_0^k - x_0^j) + \alpha}{\theta} + \frac{1}{2}. \tag{1}$$

3.1 No Primaries

In an environment where general election candidates are selected randomly, each party's candidates must maximize the probability of winning the general election. It follows from (1) that the uniquely optimal strategy for each candidate is to maximize transfers to group 2. The first remark summarizes the resulting allocation and voting strategies.

Remark 1 Transfers and Voting Without Primaries. *Without primaries, all candidates offer the transfer vector*

$$\overline{x}^a = \overline{x}^b = \overline{y}^a = \overline{y}^b = \begin{cases} (0, 0, \frac{1}{n_2}, 0) & \text{if } s < \frac{1}{n_2} \\ (1, 0, 0, 0) & \text{otherwise.} \end{cases} \tag{2}$$

Group 1 and 3 members vote for the party X and Y candidates, respectively. Group 2 members vote for party X's candidate if $\gamma_2 > -\alpha$ and for party Y's candidate if $\gamma_2 < -\alpha$. ∎

It is straightforward to calculate that these strategies imply that party X's probability of victory is $1/2 + \alpha/\theta$.

3.2 Primaries

Now suppose that there are primary elections in each party. As the group-2 ideology γ_2 is determined after the primary election, candidates running in each party primary offer to maximize the expected utility of its core voters, who are decisive in the primary election.[8] Thus platforms must optimally trade off between the relative ideological benefits of winning the general election and transfers conditional upon victory. As candidates all converge on an optimal platform, the addition of more candidates within a party would not change our results.[9]

We characterize a pure strategy equilibrium by finding the optimal platform within each party, given an expected winning platform from the opposing party. Let $\overline{x}$ and $\overline{y}$ denote arbitrary platforms from parties X and Y. The expected utility of group-i ($i = 1, 3$) voters is then:

$$E_i(\overline{x}, \overline{y}) = \left[\frac{x_2 - y_2 - s(y_0 - x_0) + \alpha}{\theta} + \frac{1}{2}\right](x_i - y_i + s(x_0 - y_0) + \gamma_i) + y_i + sy_0. \quad (3)$$

An immediate implication of this expression is that our assumption that candidates spend the entire budget is without loss of generality. A candidate who could claim uncommitted funds as rents would be defeated by a primary opponent who promised more to either the core or swing group.[10]

[8]The assumption that γ_2 is revealed after the primary election has some consequences for the candidates' calculations. If instead this revelation occurred before the primary election, then candidates may have an incentive to diverge, as this increases the range of realized values of γ_2 for which a party would win.

[9]The addition of voting costs would undermine convergence on the core voter's optimal platform. If primary voters are unwilling to vote over "close" platforms, then candidates would have an incentive to shade their platforms toward the swing group in order to win the general election.

[10]This conclusion would change if we relaxed the assumption of intra-party candidate heterogeneity. If candidate a were strictly advantaged for core group voters in the sense of being strictly more appealing to swing voters or better able to provide public or private goods, then she would be able to defeat candidate b by matching b's platform and retaining any unspent funds.

Our first two results establish some important implications of these objectives. Lemma 1 shows that the efficiency of the public good (s) plays a central role in determining the kinds of goods that are offered. As ensured by our assumptions on γ_i, allocating toward the opposing party's core group is dominated. Next, as intuition would suggest, for s sufficiently low, any best response must include only private goods. Candidates would then have no incentive to offer public goods because any public good allocation could improved upon by an allocation of private goods. By contrast, very high values of s rule out private goods for large groups. If $s > 1/n_i$ for any group i, then a candidate could do strictly better by offering that group public instead of private goods. Finally, the allocation problem becomes more complex for intermediate values of s. When s is higher than $1/(n_1 + n_2)$, a candidate offers private goods to at most one group. A symmetric result holds for party Y.

Lemma 1 Efficiency of Public Goods and Optimal Platforms. *Party X candidates' best responses satisfy the following:*

(i) $x_3^* = 0$.

(ii) If $s \leq \frac{1}{n_1+n_2}$, then $x_0^* = 0$.

(iii) If $s > \frac{1}{n_i}$ $(i \in \{1,2\})$, then $x_i^* = 0$; if $s > \max\{\frac{1}{n_1}, \frac{1}{n_2}\}$, then $x_0^* = 1$.

(iv) If $s \in (\frac{1}{n_1+n_2}, \max\{\frac{1}{n_1}, \frac{1}{n_2}\}]$, then either $x_1^* = 0$ or $x_2^* = 0$. ■

Holding the efficiency of public goods fixed, Lemma 1 provides an important intuition about the relationship between private goods, ideology, and group size. Group 1 members want no private goods if they care greatly about ideological benefits (i.e., γ_1 is high), or if group 1 is large. Extremism increases the importance of victory, and hence increases payments to group 2, while a large size dilutes the benefit of private goods. The opposite would happen when group 1 is moderate and relatively small.

The most interesting part of Lemma 1 is part (iv), which implies that candidates will provide private goods to at most one group if s is simply high enough for public goods to be undominated. This occurs because the candidate objectives are never locally concave, and hence solutions must be at a corner for at least one private good.[11] This fact can be used to narrow down the platforms that satisfy necessary conditions for an optimum. For party X candidates, since at most one of x_1 and x_2 can be strictly positive at an optimal platform, we have $x_0 + n_i x_i = 1$ for $i \in \{1, 2\}$. We therefore rewrite the party X objective (3) in terms of x_0 by substituting this constraint for each group i. There are two cases, corresponding to whether group 1 or 2 receives private goods. In the first, $x_1 > 0$ and $x_2 = 0$; using the fact that $y_1^* = 0$, the objective can be rewritten:

$$E_1(\overline{x}, \overline{y}) = \left[\frac{-y_2 - s(y_0 - x_0) + \alpha}{\theta} + \frac{1}{2}\right]\left(\frac{1 - x_0}{n_1} + s(x_0 - y_0) + \gamma_1\right) + sy_0$$

This is concave in x_1 when $n_1 s < 1$. By Lemma 1(iii), this condition is necessary for an interior solution for x_1, as the public good would clearly be preferable otherwise. The interior solutions for x_1 and x_0 are then:

$$\begin{aligned}
\tilde{x}_1 &= \frac{n_1\gamma_1 + (2n_1 s - 1)(1 - y_0)}{2n_1(n_1 s - 1)} + \frac{\alpha + \theta/2 - y_2}{2n_1 s} && (4)\\
\tilde{x}_{01} &= \frac{-n_1\gamma_1 + (2n_1 s - 1)y_0 - 1}{2(n_1 s - 1)} - \frac{\alpha + \theta/2 - y_2}{2s}. && (5)
\end{aligned}$$

In the second case, $x_1 = 0$ and $x_2 > 0$, and the objective is concave if $n_2 s < 1$. Again, this condition is necessary for an interior value of x_2 to be chosen, for otherwise party X candidates would prefer the public good. Straightforward maximization yields the following interior solutions for x_2 and x_0:

$$\begin{aligned}
\tilde{x}_2 &= \frac{(n_2 s - 1)\gamma_1 - s(2n_2 s - 1)(y_0 - 1)}{2n_2 s(n_2 s - 1)} + \frac{\alpha + \theta/2 - y_2}{2(n_2 s - 1)} && (6)\\
\tilde{x}_{02} &= \frac{-(n_2 s - 1)\gamma_1 + s(2n_2 s y_0 - y_0 - 1)}{2s(n_2 s - 1)} - \frac{n_2(\alpha + \theta/2 - y_2)}{2(n_2 s - 1)}. && (7)
\end{aligned}$$

It is straightforward to derive analogous expressions for $\tilde{y}_3$, $\tilde{y}_{01}$, $\tilde{y}_2$ and $\tilde{y}_{02}$. These are presented in the Appendix.

The following lemma establishes the unique best response platform for intermediate values of s. In particular, it adds to the preceding discussion sufficient conditions for party X

[11] This property is shown in Lemma 3 in the Supplementary Appendix.

candidates to choose $\tilde{x}_1$, $\tilde{x}_2$, or a pure public goods platform. The optimal platform for party Y candidates is analogous and presented in the Appendix.

Lemma 2 Private Good Allocations Under Intermediate s. *For* $s \in \left(\frac{1}{n_1+n_2}, \min\{\frac{1}{n_1}, \frac{1}{n_2}\}\right)$, $x_1^* = \min\{\frac{1}{n_1}, \tilde{x}_1\} > 0$ *and* $x_2^* = 0$ *if and only if:*

$$(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \alpha - \theta/2) + n_1s\gamma_1 < 0, \tag{8}$$

and $x_1^* = 0$ *and* $x_2^* = \min\{\frac{1}{n_2}, \tilde{x}_2\} > 0$ *if and only if:*

$$(2n_2s^2 - s)(1 - y_0) - n_2s(y_2 - \alpha - \theta/2) - (1 - n_2s)\gamma_1 < 0. \tag{9}$$

Otherwise, $x_0^* = 1$. ■

Proposition 1 uses this lemma and other features of the candidates' best responses to establish generally the existence of a pure strategy equilibrium.

Proposition 1 Equilibrium Existence. *There exists a pure strategy equilibrium.* ■

The following analysis will show that there is a unique equilibrium for a broad range of parameter values, but uniqueness is not guaranteed in general. We proceed by first examining the relatively straightforward cases of very low and very high values of s, where equilibrium platforms offer only private and public goods, respectively. We then consider intermediate values of s, where primaries induce increased levels of public goods spending.

3.2.1 Private Goods Equilibrium

For low-value public goods, where $s \leq \min\{1/(n_1 + n_2), 1/(n_2 + n_3)\}$, Lemma 1(ii) implies that candidates offer only private goods to voters. The main feature of the private goods equilibrium is that in contrast to the no-primaries private goods equilibrium (see Remark 1), candidates will usually offer positive allocations to their core groups. This is because citizens in groups 1 and 3 would prefer a small allocation and a slightly reduced probability of winning to a zero allocation that maximized their party's probability of winning. In fact, allocations to core voters are strategic complements: a higher payment to core voters in one party raises the expected return to core allocations in the other. As noted previously, more extreme core voters demand less private goods, as they are relatively more interested in winning in order to achieve ideological goals.

Proposition 2 characterizes the private goods equilibrium and establishes a simple condition on s for its existence and uniqueness. The characterization is simplified by the fact that candidate objective functions are strictly concave, and $|\gamma_1|$ and $|\gamma_3|$ are large enough to prevent candidates from offering anything to the opposing party's core group. This generates a unique platform that maximizes the utility of core voters, and as a result both candidates will adopt it in equilibrium.

Proposition 2 Private Goods Equilibrium. *At an interior solution of an equilibrium with only private goods:*

$$(x_1^P, y_3^P) = \left(\frac{3n_2\theta + 2n_2\alpha + 2n_3\gamma_3}{6n_1} - \frac{2\gamma_1}{3}, \frac{3n_2\theta - 2n_2\alpha - 2n_1\gamma_1}{6n_3} + \frac{2\gamma_3}{3} \right).$$

When (x_1^P, y_3^P) is not interior, the following corner solutions arise:

$$(x_1^P, y_3^P) = \begin{cases} (0,0) & \text{if } n_2(\theta/2+\alpha) \leq n_1\gamma_1 \text{ and } n_2(\theta/2-\alpha) \leq -n_3\gamma_3 \\ (0, \frac{n_3\gamma_3+n_2(\theta/2-\alpha)}{2n_3}) & \text{if } n_2(3\theta/2+\alpha) \leq 2n_1\gamma_1 - n_3\gamma_3 \text{ and} \\ & \quad n_2(\theta/2-\alpha) \in (-n_3\gamma_3, 2-n_3\gamma_3) \\ (0, \frac{1}{n_3}) & \text{if } n_2(\theta/2+\alpha) \leq n_1\gamma_1 - 1 \text{ and} \\ & \quad n_2(\theta/2-\alpha) \geq 2 - n_3\gamma_3 \\ (\frac{-n_1\gamma_1+n_2(\theta/2+\alpha)}{2n_1}, 0) & \text{if } n_2(\theta/2+\alpha) \in (n_1\gamma_1, 2+n_1\gamma_1) \text{ and} \\ & \quad n_2(3\theta/2-\alpha) \leq n_1\gamma_1 - 2n_3\gamma_3 \\ (\frac{1}{n_1}, 0) & \text{if } n_2(\theta/2+\alpha) \geq 2 + n_1\gamma_1 \text{ and} \\ & \quad n_2(\theta/2-\alpha) \leq -1 - n_3\gamma_3 \\ (\frac{1}{n_1}, \frac{1+n_3\gamma_3+n_2(\theta/2-\alpha)}{2n_3}) & \text{if } n_2(3\theta/2+\alpha) \geq 3 + 2n_1\gamma_1 - n_3\gamma_3 \text{ and} \\ & \quad n_2(\theta/2-\alpha) \in (-1-n_3\gamma_3, 1-n_3\gamma_3) \\ (\frac{1-n_1\gamma_1+n_2(\theta/2+\alpha)}{2n_1}, \frac{1}{n_3}) & \text{if } n_2(\theta/2+\alpha) \in (n_1\gamma_1 - 1, 1+n_1\gamma_1) \text{ and} \\ & \quad n_2(3\theta/2-\alpha) \geq 3 + n_1\gamma_1 - 2n_3\gamma_3 \\ (\frac{1}{n_1}, \frac{1}{n_3}) & \text{if } n_2(\theta/2+\alpha) \geq 1 + n_1\gamma_1 \text{ and} \\ & \quad n_2(\theta/2-\alpha) \geq 1 - n_3\gamma_3. \end{cases}$$

This is the unique equilibrium for $s < \min\{1/(n_1+n_2), 1/(n_2+n_3)\}$. ∎

With one exception, which occurs when both core groups are large and extreme, some core voters receive positive allocations in equilibrium. Primaries also tend to benefit both parties' core groups indirectly because their party's probability of victory increases when the opposition party reduces its allocation to group 2. By contrast, primaries typically hurt group 2 citizens in a private goods equilibrium.[12]

3.2.2 Public Goods Equilibrium

Next, consider the case of high values of s, so that public goods are efficient enough to be offered exclusively by at least one party in equilibrium. A central question of the paper is when primaries encourage the provision of public goods. As Remark 1 indicates, in a world with no primaries public goods are exclusively provided when $s > 1/n_2$, but this threshold is inefficient because public goods provide benefit society whenever $s > 1$. Lemma 1(ii) implies that some inefficiencies must persist in the presence of primaries: party X and Y candidates never offer public goods when s is below $1/(n_1 + n_2)$ and $1/(n_3 + n_2)$, respectively.

Proposition 3 provides the conditions under which candidates offer public goods exclusively. Part (i) shows that when the swing group is the smallest group, primaries reduce the threshold efficiency level s needed for all candidates to offer only public goods, and are thus strictly better for producing only public goods. A small group 2 implies large core groups, which derive lower per capita value from private goods. As the proof of this result makes clear, this logic works within each party: each party's candidates will offer only public goods if the swing group is smaller than their core group. Parts (ii)-(iv) characterize other cases where at least one party offers only public goods. They follow immediately from Lemma 1 and manipulation of (4) and its analog for party Y, and are stated without proof.

Proposition 3 Public Goods Equilibrium. *(i) If $n_2 < \min\{n_1, n_3\}$, then $x_0^* = y_0^* = 1$ for*

[12] Elsewhere, Hirano, Snyder, and Ting (2009) show that a core group receives more per capita when it shrinks in size relative to the swing group, when it becomes more moderate, when electoral uncertainty (θ) increases, and when its relative valence advantage increases. Each party's probability of victory is increasing in the size and ideological extremism of its core group, as well as in the size of its relative valence advantage.

all $s > \tilde{s}$*, where* $\tilde{s} < \frac{1}{n_2}$.

(ii) If $s \geq \max\{\frac{1}{n_1}, \frac{1}{n_2}, \frac{1}{n_3}\}$*, then* $x_0^* = y_0^* = 1$.

(iii) If $s \geq \max\{\frac{1}{n_1}, \frac{1}{n_2}\}$ *and* $s < \frac{1}{n_3}$*, then* $x_0^* = 1$ *and* $y_3^* = \max\{0, \min\{\tilde{y}_3, \frac{1}{n_3}\}\}$*, where:*

$$\tilde{y}_3 = \frac{\gamma_3}{2(1 - n_3 s)} + \frac{\theta/2 - \alpha}{2 n_3 s}.$$

(iv) If $s \geq \max\{\frac{1}{n_2}, \frac{1}{n_3}\}$ *and* $s < \frac{1}{n_1}$*, then* $y_0^* = 1$ *and* $x_1^* = \max\{0, \min\{\tilde{x}_1, \frac{1}{n_1}\}\}$*, where:*

$$\tilde{x}_1 = -\frac{\gamma_1}{2(1 - n_1 s)} + \frac{\theta/2 + \alpha}{2 n_1 s}. \quad \blacksquare$$

Notably, parts (iii) and (iv) of Proposition 3 imply that there are conditions under which public goods are *not* promised by all candidates even though $s > 1/n_2$. This might occur, for example, if core groups are moderate (i.e., $|\gamma_i|$ is small), or elections are uncertain (i.e., θ is large). Thus, a small and moderate core group can result in its party providing a lower level of public goods under a primary system. These results illustrate the general point that smaller groups will tend to attract private goods, due to the higher per capita value of a given amount of transfers.

3.2.3 Mixed Goods with Ideological Symmetry

Now consider the cases where s is intermediate, so that candidates will offer combinations of public and private goods. In this subsection we focus on a special case of the game with symmetric ideological parameters: groups have the same ideological motivation (i.e., $\gamma \equiv \gamma_1 = -\gamma_3$), and the swing group is not biased toward either party (i.e., $\alpha = 0$). Furthermore, the core groups are of the same size. This allows us to isolate the effect of ideological polarization and obtain convenient closed form solutions.

Proposition 4 characterizes equilibrium platforms in this setting. It shows that candidates shift benefits from core groups to the swing group as ideological extremism γ increases. Crucially, public goods are used to "smooth" the transition between private goods for the two groups, even though they are not valuable enough to be offered in the absence of primaries.

Proposition 4 Symmetric Core Groups. *Suppose* $s \in (\max\{\frac{1}{2n_1}, \frac{1}{2n_2}\}, \min\{\frac{1}{n_1}, \frac{1}{n_2}\})$*,* $n_1 = n_3$*,* $\alpha = 0$*, and* $\gamma_1 = -\gamma_3 = \gamma$*. In the unique equilibrium:*

$$\begin{cases} x_1^* = y_3^* = \min\left\{\frac{1}{n_1}, \frac{(1-n_1 s)\theta}{2n_1 s} - \gamma\right\}, & \\ \quad x_0^* = y_0^* = 1 - n_1 x_1^* & \text{if } \gamma \leq \frac{(1-n_1 s)\theta}{2n_1 s} \\ x_0^* = y_0^* = 1 & \text{if } \gamma \in \left(\frac{(1-n_1 s)\theta}{2n_1 s}, \frac{n_2 s\theta}{2(1-n_2 s)}\right) \\ x_2^* = y_2^* = \frac{1}{n_2} & \text{if } \gamma \geq \frac{n_2 s\theta}{2(1-n_2 s)}. \quad \blacksquare \end{cases}$$

The symmetry of core groups here implies that equilibrium platforms are symmetric. For the lowest values of γ, candidates promise a mix of private goods for the core group and public goods. These private goods are linearly decreasing in γ and increasing in θ, while public goods move in the opposite direction. For higher values of γ the allocation hits a corner of entirely public goods. Finally, for extreme values of γ candidates promise only private goods to the swing group. Interestingly, the transition from public goods to private goods for the swing group is discontinuous at $\gamma = n_2 s\theta/(2 - 2n_2 s)$: there is no "interior" solution for private goods to the swing group. Overall, then, the provision of public goods is non-monotonic in γ. Figure 1 depicts the allocations of each good as a function of γ. Although not part of the result, the equilibrium is similar when s is large enough to dominate the provision of one private good (i.e., $s > 1/n_1$ or $s > 1/n_2$); in these cases the public good is simply substituted for that private good.

Proposition 4 allows us to calculate simple solutions for the key question of when public goods are provided as a function of the efficiency parameter s. It is obvious that increasing s strictly increases the appeal of offering public goods. By manipulating the conditions on γ in the proposition, we derive the following bounds on when public goods are provided exclusively, as well as when public goods are provided at all. The result is stated without proof.

Remark 2 Public Goods Thresholds Under a Symmetric Equilibrium. *Under the conditions of Proposition 4,* $x_0^* = 1$ *if:*

$$s > \max\left\{\frac{2\gamma}{n_2(2\gamma+\theta)}, \frac{\theta}{n_1(2\gamma+\theta)}\right\}.$$

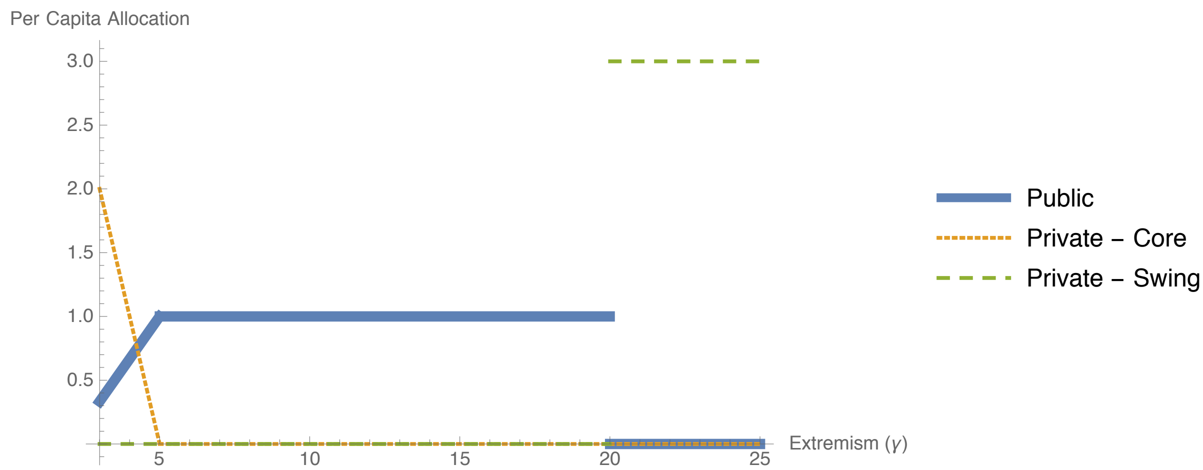

Figure 1: Candidate Platforms and Ideology. This figure plots the per capita allocations for each platform component for candidates of both parties, as a function of core group extremism (γ). Group ideologies are symmetric and group sizes are identical. Parameters are $n_1 = n_2 = n_3 = 1/3$, $\gamma = \gamma_1 = -\gamma_3$, $\alpha = 0$, $s = 2$, and $\theta = 20$.

And $x_0^* > 0$ *if:*

$$s > \frac{2\gamma}{n_2(2\gamma + \theta)}. \quad \blacksquare$$

Remark 2 provides a simple characterization of the contribution of primaries to public goods provision. In the "intermediate" range of s identified in Proposition 4, there is no public goods provision without primaries. With primaries, the set of parameters under which public goods are provided is quite broad. In particular, if $\theta > 2\gamma$ (i.e., electoral uncertainty is high), then for *all* such intermediate values of s, all candidate platforms will offer a positive level of public goods. When groups are of equal size, public goods provision is maximized at $\theta = 2\gamma$, as the threshold s for offering exclusively public goods is exactly the threshold for public goods to be undominated by private goods (i.e., $s = 1/(n_1 + n_2)$). Values of θ below 2γ raise the threshold for offering public goods, while higher values of θ raise the threshold for offering exclusively public goods.

Figure 2 plots an example of the thresholds on s for public good provision. Under the parameters of the example, when there are no primaries public goods are not provided for $s \in (3/2, 3)$. When γ is relatively low, primaries fill much of this public good provision gap.

Of course, no public goods are provided when they are socially efficient but dominated by private goods, which occurs when $s \in (1, 3/2)$.

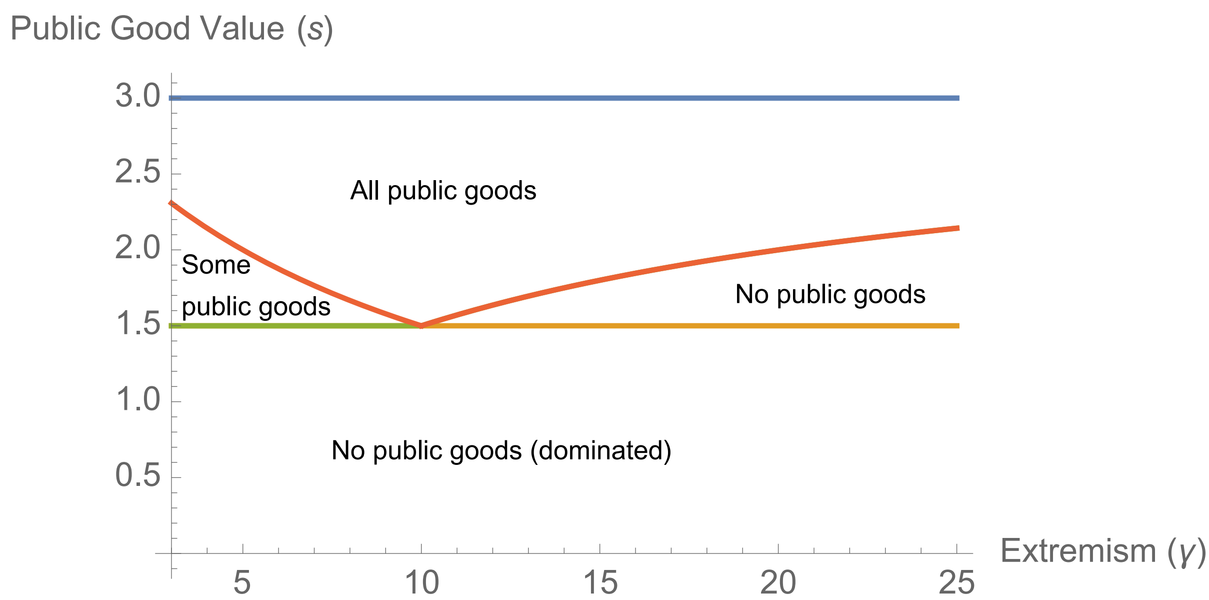

Figure 2: Public Goods and Ideology. This figure plots public good provision as a function of the efficiency of the public good (s) and and core group extremism (γ). Group ideologies are symmetric. Below $s = 1.5$, public goods are a dominated platform strategy, while above $s = 3$ public goods are exclusively provided when there are no primaries. At $\gamma = 10$, public goods are exclusively provided by all candidates when they are undominated by private goods. Parameters are $n_1 = n_2 = n_3 = 1/3$, $\gamma = \gamma_1 = -\gamma_3$, $\alpha = 0$, and $\theta = 20$.

Several other comparative statics on the range of γ for which public goods are exclusively offered by all candidates follow immediately from Proposition 4. The size of this range is increasing in electoral uncertainty (θ).[13] It also shifts "upwards" in γ as θ increases, which implies that as elections become more uncertain, the value of contributions to the swing group decreases while the value of contributions to the core group increases. Finally, as n_1 increases (implying that n_2 decreases), this range shifts "downwards" in γ. This reflects the dilution of the value of core contributions as n_1 increases along with the concentration of swing contributions as n_2 decreases. If we interpret the core group as being the party's effective primary electorate, then for low levels of ideological polarization the positive relationship

[13]The range of γ for which *some* public goods are offered is also increasing in θ.

between core group size and public goods resembles that of Lizzeri and Persico (2004).

What happens when we relax the assumption of symmetry in core group sizes? Figure 3 plots one example of party Y candidates' allocation strategies as a function of γ. In this example, $n_1 = n_2$, and $n_3 > n_1$, so group 3 is the largest group. Furthermore, $s < 1/(n_1+n_2)$, so party X candidates offer only private goods. The figure clearly shows that platform strategies are qualitatively similar. The most notable difference with Figure 1 is the smoother transition from public goods to private goods for the swing group. By contrast, party X candidates simply trade core allocations for swing allocations as γ increases. Proposition 7 in the Supplementary Appendix generalizes this example.

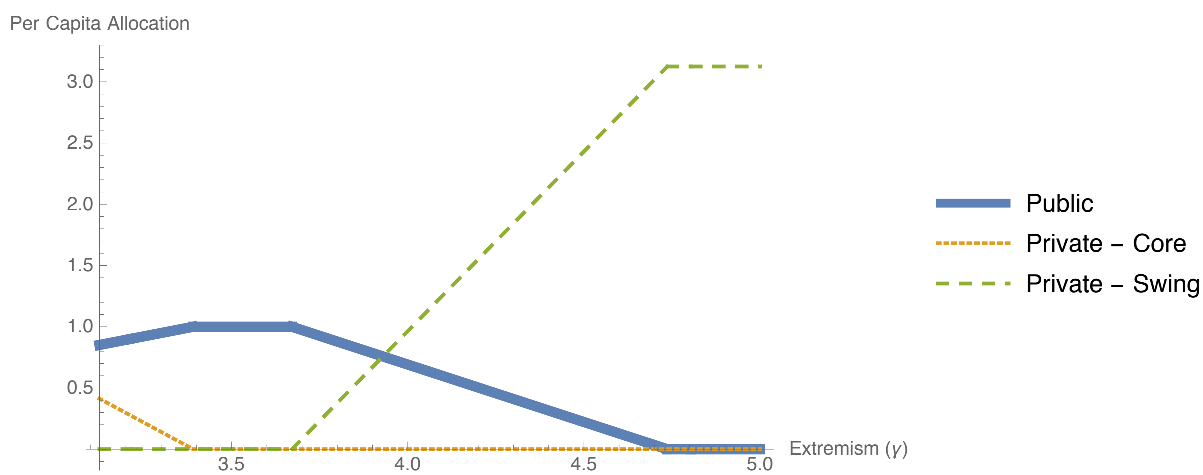

Figure 3: Candidate Platforms and Ideology with a Large Core Group. This figure plots the per capita allocations for each platform component for party Y candidates, as a function of core group extremism (γ). Group ideologies are symmetric but group 3 (party Y's core) is larger than the others. Parameters are $n_1 = n_2 = 0.32$, $n_3 = .36$, $\gamma = \gamma_1 = -\gamma_3$, $\alpha = 0$, $s = 1.5$, and $\theta = 10$.

3.2.4 General Mixed Goods Equilibria

We finally turn to the general case of mixed goods with asymmetric ideologies. While closed-form solutions for platform choices in this case are quite cumbersome, it is possible to derive results on public goods provision and platforms that illustrate the general logic of the model.

A first question is when candidates offer *some* public goods in their platform. Public goods obviously become more attractive as their efficiency increases, but primaries may not be especially helpful for their provision if the threshold for their adoption is close to the $s = 1/n_2$ threshold for adoption without primaries. Proposition 5 provides a simple lower bound on s for when party X candidates offer some public goods. A symmetric result holds for party Y.

Proposition 5 Some public goods under primaries. *If* $s > \frac{1}{n_1+n_2}$, *then* $x_0^* > 0$ *if:*

$$s > \max\left\{\frac{\alpha + \theta/2}{n_1(\gamma_1 + \alpha + \theta/2)}, \frac{\gamma_1}{n_2(\gamma_1 + \alpha + \theta/2) - 1}\right\}. \quad \blacksquare$$

The bound on s in Proposition 5 is for many parameter values quite low. In particular, for sufficiently large values of α, θ, and γ_1 (i.e., bias in favor of party X, electoral uncertainty, and party X extremism, respectively), the threshold for partial adoption of public goods under primaries can be much lower than $1/n_2$. As an example, suppose $n_1 = 0.4$, $n_2 = 0.3$, $\theta = 10$, $\alpha = 2$, and $\gamma_1 = 4$. By Lemma 1, party X offers no public goods if $s < 1/(n_1 + n_2) \approx 1.43$, and X offers only public goods if $s > 3.33$. The threshold from Proposition 5 is about 1.74, and so candidates offer some public goods even when they are relatively close to being dominated by private goods.

To characterize equilibrium platforms, recall that when a party's candidates offer some public goods, they will never allocate private goods to both core and swing groups. We can show that when a core group is sufficiently moderate, platforms *never* include private goods for the swing group, regardless of the opponent's allocation. Likewise, when the core group is sufficiently extreme, platforms never include the core group. Thus when core groups are either very moderate or very extreme, each party's problem reduces to a univariate choice over the level of public good to provide, with the remainder going to one private good.

Proposition 6 uses this logic to derive the unique platforms when core group ideologies are bounded such that $\gamma_1 \notin (\underline{\gamma}_1, \overline{\gamma}_1)$ and $\gamma_3 \notin (\underline{\gamma}_3, \overline{\gamma}_3)$.[14] These bounds are defined as follows:

$$\overline{\gamma}_1 \equiv \frac{\max\{-2n_1s^2+s,0\}+(1-n_1s)(\alpha+\theta/2)}{n_1s} \tag{10}$$

$$\underline{\gamma}_1 \equiv \frac{n_2s(\alpha+\theta/2)}{1-n_2s}-2s \tag{11}$$

$$\underline{\gamma}_3 \equiv \frac{\min\{2n_3s^2-s,0\}+(1-n_3s)(\alpha-\theta/2)}{n_3s} \tag{12}$$

$$\overline{\gamma}_3 \equiv \frac{n_2s(\alpha-\theta/2)}{1-n_2s}+2s. \tag{13}$$

Importantly, these bounds typically exclude only a relatively "small" set of values of γ_1 and γ_3, and in many cases exclude none at all.[15] For convenience, the proposition states only private good allocations; all remaining resources are allocated toward the public good.

Proposition 6 Mixed Goods Equilibrium. *Let* $s \geq \max\{\frac{1}{n_1+n_2}, \frac{1}{n_3+n_2}\}$ *and* $s \leq \max\{\frac{1}{n_1}, \frac{1}{n_2}, \frac{1}{n_3}\}$.

(i) If $\gamma_1 \leq \underline{\gamma}_1$ *and* $\gamma_3 \geq \overline{\gamma}_3$*,* $x_2^* = y_2^* = 0$ *and at an interior solution:*

$$x_1^* = \frac{(1-n_3s)(2n_1s(\theta+\gamma_1)-\alpha-3\theta/2)-n_3s\gamma_3(1-2n_1s)}{n_1s(2n_1s+2n_3s-3)}$$

$$y_3^* = \frac{(1-n_1s)(2n_3s(\theta-\gamma_3)+\alpha-3\theta/2)+n_1s\gamma_1(1-2n_3s)}{n_3s(2n_1s+2n_3s-3)}$$

(ii) If $\gamma_1 \geq \overline{\gamma}_1$ *and* $\gamma_3 \geq \overline{\gamma}_3$*,* $x_1^* = y_2^* = 0$ *and at an interior solution:*

$$x_2^* = \frac{(1-n_3s)(2n_2s(\theta+\gamma_1)-2\gamma_1+\alpha-\theta/2)-n_3s\gamma_3(1-2n_2s)}{2n_2^2s^2-n_2s+n_3s-1}$$

$$y_3^* = \frac{(1-n_3s-n_2s)(n_2s(\alpha-3\theta/2)+(1-n_2s)\gamma_1)+2n_2n_3s^2((1-n_2s)(\gamma_1-\gamma_3)-n_2s\theta)}{n_3s\,(2n_2^2s^2-n_2s+n_3s-1)}$$

(iii) If $\gamma_1 \leq \underline{\gamma}_1$ *and* $\gamma_3 \leq \underline{\gamma}_3$*,* $x_2^* = y_3^* = 0$ *and at an interior solution:*

$$x_1^* = \frac{(1-n_1s-n_2s)(n_2s(-\alpha-3\theta/2)-(1-n_2s)\gamma_3)+2n_2n_1s^2((1-n_2s)(\gamma_1-\gamma_3)-n_2s\theta)}{n_1s\,(2n_2^2s^2-n_2s+n_1s-1)}$$

$$y_2^* = \frac{(1-n_1s)(2n_2s(\theta-\gamma_3)+2\gamma_3-\alpha-\theta/2)+n_1s\gamma_1(1-2n_2s)}{2n_2^2s^2-n_2s+n_1s-1}$$

[14] Taken together, Propositions 2, 3, and 6 cover all possible values of s, with the exception of $s \in (\min\{1/(n_1+n_2), 1/(n_3+n_2)\}, \max\{1/(n_1+n_2), 1/(n_3+n_2)\})$. This case is relatively straightforward, combining features of the private goods equilibrium for one party and mixed goods for the other. Section 3.2.3 develops this case for ideologically symmetric voters.

[15] For example, when $n_1 = n_2 = n_3 = 1/3$, there are no restrictions on γ_1 if $\alpha + \theta/2 > 2.5$ and $s > 2.033$. There are no restrictions on γ_3 if $\alpha - \theta/2 < -0.5$ and $s > 2.753$.

(iv) If $\gamma_1 \geq \overline{\gamma}_1$ and $\gamma_3 \leq \underline{\gamma}_3$, there is no interior equilibrium. $x_2^* = \frac{1}{n_2}$ and $y_2^* = \frac{1}{n_2}$ for γ_1 and $|\gamma_3|$ sufficiently large. ■

In each case of Proposition 6, the platforms are the solutions of the appropriate system of equations chosen from expressions such as (5) and (7). Despite the complexity of the expressions, the proposition generalizes the main features of the symmetric equilibrium characterized in Proposition 4. Moderate core voters are most willing to trade off between private goods and the probability of victory, and therefore generally receive private goods in equilibrium. More extreme core voters receive no private goods, as ideological payoffs generate an incentive to shift private goods to the swing group to maximize the probability of victory. Public goods soften the choice between core and swing groups, and allow extreme core groups to benefit from non-ideological payoffs. Finally, in part (iv), where all core voters are extreme, all candidates allocate their entire budget to private goods for group 2.

4 Evidence

The model broadly predicts that the incentives to provide public goods increase following the introduction of primaries.[16] To examine whether this may be the case empirically, we study changes in capital outlays in states that introduced direct primaries in the early part of the 20th century. For each state we consider total state and local government capital outlays. These outlays were largely to fund public infrastructure projects – e.g., highways, roads and bridges, public sanitation works, water supply facilities, parks and recreation facilities – and public buildings – e.g., court houses, police and fire stations, libraries and schools. These are generally considered public goods at the local level, and many provide benefits much more widely across the state.[17]

[16] As Proposition 3 establishes, one caveat is that the introduction of primaries will not increase the supply of public goods when the core group is moderate and small.

[17] Some studies provide evidence that this type of spending may have a positive impact on economic growth (e.g., Aschauer 1989, Munnell 1990).

Figure 4 provides simple box plots of the change in capital outlays as a share of total state and local government expenditures by state between 1902 and 1913.[18] States that introduced primaries during this period are plotted separately from those that maintained the same nomination process. In 1900 only three of the 48 states in our sample had adopted primaries. By 1911 this number had jumped to 35. The total amount of capital outlay expenditures in a state is from ICPSR 06304 *State and Local Government [United States]: Sources and Uses of Funds, Census Statistics, Twentieth Century [Through 1982]*, which provides expenditures by both the state and local governments. The dates of primary introduction are from Ansolabehere, Hirano and Snyder (2007), and reflect when state mandatory primary election laws went into effect or when parties began regularly using primaries.[19] Since the expenditure data is for the fiscal year, we consider primaries that are enacted in year t to have an effect on fiscal expenditures reported for year $t+2$.[20] The figure shows that states that introduced primaries between 1900 and 1911 had bigger increases in their share of capital outlays than those that did not introduce primaries during this period.

We further examine the relationship between capital outlays and primary elections using the following simple specification:

$$\Delta \mathit{Capital\ Outlays}_i = \alpha + \beta \Delta Primary_i + \gamma \Delta X_i + \epsilon_i \tag{14}$$

where i indexes state. $\Delta Primary_i$ is an indicator variable for whether state i introduced primary elections between 1900 and 1911. $\Delta \mathit{Capital\ Outlays}_i$ is the change in capital outlays as a share of total government expenditures. ΔX_i is the change in other state level variables

[18]The 1913 measure excludes expenditures by localities with populations less than 2,500. This will be discussed below.

[19]The dates differ slightly for a few states. This is due to our focus on state and local offices as well as some new information about when states adopted primaries. Ansolabehere, Hirano and Snyder (2007) focus on U.S. House primaries. While the primary election laws we study generally apply to county and municipal offices as well, in some states the laws did not explicitly mandate primaries for small local governments. For example, in California by 1908 primaries were mandatory for cities with populations greater than 7,500. Thus our primary indicator variable has a degree of measurement error regarding coverage of local governments.

[20]Our findings are stronger if we use fiscal expenditures reported for year $t+3$.

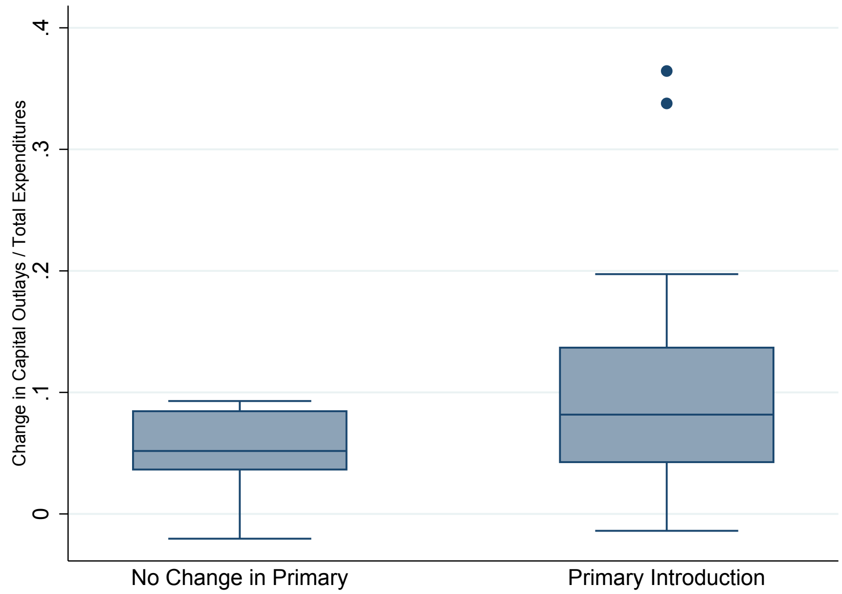

Figure 4: Change in Capital Outlays as a Share of Total Government Expenditures and the Introduction of Primary Elections

that may influence expenditures on capital outlays.

One measurement concern is that the localities used to calculate total government expenditures differed between the 1902 and 1913 U.S. Census reports. The U.S. Census report on government finances for 1902 includes government expenditures by states, counties, cities and minor civil divisions. The U.S. Census report for 1913 includes expenditures by states, counties and incorporated places with populations greater than 2,500.[21] Because of this concern, we present results including a control variable for the percentage of the state population in localities with populations less than 2,500 in 1913. We also present results where we focus on government expenditures excluding localities with populations less than 8,000 for both 1902 and 1913.[22]

[21] Another potential outcome variable of interest is capital outlays per capita. Since the populations of the municipalities included differ between 1902 and 1913, we do not present these coefficient estimates.

[22] For 1902 we calculate this expenditure by subtracting expenditures by Other Minor Civil Divisions from total government expenditures. Other Minor Civil Divisions includes localities other than counties or cities with populations greater than 8,000. For 1913 we calculated this expenditure by summing expenditures by states, counties and incorporated places over 2,500 and then subtracting expenditures by incorporated places

We consider two additional demographic control variables – change in population density and change in manufacturing output per worker.[23] Both of these variables are included as they may influence the demand for public goods. Manufacturing output per worker might also proxy for state income, which could be related to the state's ability to supply public goods. When these covariates are included, the coefficient estimates should be interpreted with caution as control variables may introduce some bias in the estimates.

Table 1: **Change in Capital Outlays as a Share of Total Expenditures and the Introduction of Primaries**

	(1)	(2)	(3)	(4)	(5)	(6)
Δ Primary Use	0.051	0.047	0.052	0.049	0.038	0.051
	(0.022)	(0.024)	(0.022)	(0.023)	(0.025)	(0.023)
Δ Pop Density				0.024	0.115	0.014
				(0.079)	(0.083)	(0.079)
Δ Mfg Out/Work				-0.028	-0.010	-0.031
				(0.094)	(0.100)	(0.094)
% Pop < 2,500			0.060			0.058
			(0.049)			(0.050)
Number Obs	48	48	48	48	48	48

Columns 2 and 5 include government expenditures excluding localities with populations less than 8,000. The other columns the share of total government expenditures is used for 1902 and the share of government expenditures excluding expenditures by incorporated places with populations less than 2,500 is used for 1913.

The coefficient estimates on change in primary use in Table 1 suggest that states that introduced primaries between 1900 and 1911 also had a roughly 5 percentage point increase

with populations between 2,500 and 8,000. One concern with this measure is that other minor civil divisions may include expenditures by entities that are not incorporated and incorporated places with populations greater than 8,000 that are not designated as cities.

[23]Population density is measured by population divided by area of the state. Manufacturing output per worker is measured by total state manufacturing output divided by the number of workers in manufacturing. Because of the skew in these variables, we take the natural log of both of these variables. These data come from ICPSR 3 *Historical, Demographic, Economic and Social Data: The United States, 1790-1970.*

in capital outlay expenditures as a share of total government expenditures.[24] Columns 2 and 5 exclude expenditures by localities with populations less than 8,000. The other columns use total government expenditures in 1902 and government expenditures excluding incorporated places with populations less than 2,500 in 1913. When the change in population density and manufacturing output per worker are included, only the results in column 5 drop in magnitude and statistical significance.

While these results are consistent with the main predictions from the model, the findings are merely suggestive. Aside from the measurement issues discussed above, further work would be required to demonstrate a causal relationship between the introduction of primaries and expenditures on capital outlays. For example, we cannot rule out the possibility that voter preferences were moving in a "progressive" direction during this period and changing at different rates in different states, and that as voters became more progressive, they demanded (and received) both primary election reforms and higher capital outlays. However, none of the existing accounts for the adoption of primaries explicitly link the reforms to expectations about public goods provision, or even more generally to issues of public finance (e.g., Merriam and Overacker 1928, Ware 2002, Reynolds 2006).

The model makes other predictions that are more subtle. Consider for example the relationship between public goods and polarization (see Proposition 4, Remark 2 and Figure 2). When the ideological gap between the core groups is small, an increase in polarization creates stronger incentives for candidates to provide public goods. After some point, however,

[24]The coefficient is smaller and no longer statistically significance in certain specifications when we exclude the two states, Oregon and Nebraska, which had substantially larger shifts in capital outlays / total expenditures during this period (see Figure 4). In addition, if we extend the analysis to include 1932 and 1942, the coefficient estimates are smaller and no longer significant in most specifications. With the Great Depression and the sizeable increase in intergovernmental transfers from the federal government following the Great Depression, there were significant changes in state and local government expenditures (see Wallis, Fishback and Kantor 2006, Wallis and Oates 1989). Thus, the weaker findings when focusing on this long time period are perhaps not unexpected. Finally, we also examined capital outlays by state governments and found little evidence that changes in these expenditures are related to the introduction of primaries. This is also probably not surprising since a large portion of government expenditures at this time was done at the local level and most state governments reported no spending on capital expenditures in 1902.

further increases in polarization will tend to result in lower public good provision. Another prediction concerns the allocation of resources to core and swing groups. When the ideological gap between the core groups is small, candidate spending targeted at swing voters should be relatively low. This spending is predicted to increase when polarization passes some threshold. Spending targeted at core voters should follow the opposite pattern – i.e., positive but decreasing in polarization for low levels of polarization and then low at some roughly constant level beyond some point. These predictions are challenging to test because the relationships are non-monotonic and we need accurate measures of the ideological preferences of core groups across and within states and localities over time. We leave this for future work.

5 Conclusions

This paper investigates the effect of primary elections on the distribution of public spending. The main intuition is that primary elections provide an incentive for candidates to increase the provision of public as opposed to particularistic goods. The incentive is generated by the simple observation that public goods simultaneously benefit both core and swing voters. Thus they can present candidates with a more efficient way of maximizing the utility of both core voters in a primary election as well as swing voters in the general election.

The model shows that primary elections cause public goods to be offered when they are socially efficient, but not efficient enough to be offered in their absence. Under these conditions, public goods are most appealing when core voters are not too extreme. Extreme voters will wish to maximize the probability of receiving ideological benefits, and this may result in targeting the swing group with private goods. We also show, however, that primaries can actually reduce the provision of public goods under a specific set of circumstances; i.e., when public goods are highly efficient but a core group is both small and moderate. A sufficient condition for primaries to increase always the provision of public goods is for the swing group to be the smallest group, thus diluting the value of private goods for core groups.

The model suggests that primaries have mixed distributional consequences. To the extent that they increase public goods, primaries increase aggregate social utility and equalize payoffs across society. Primaries also tend to draw private good allocations away from swing voters and toward core voters. Whether this produces more egalitarian outcomes depends on the size of the core and swing groups. Primaries have no effect on distributions when both core groups are sufficiently extreme and s is "intermediate," as candidates simply maximize their offer of private goods to the swing group in both games.

Our evidence on state-level capital spending in the early 20th century is consistent with the prediction of increased public goods spending following the introduction of primaries. While these results are merely suggestive, we hope that they may stimulate further empirical work. The main challenge lies in classifying government spending as public versus particularistic. Previous research has grappled with this problem, but there is no clear consensus regarding classification schemes. U.S. states and localities spend on a variety of goods and services — education, health, transportation, police, fire departments, courts, sewerage and trash pickup, etc. — that are partially public and partially excludable and targetable goods. An alternative measure may be based on "project size." Within a relatively narrow category of spending, larger scale projects tend to be "more public" than smaller projects. Compare for example a hospital with a 1,000 beds centrally located in a county to 10 hospitals with 100 beds each scattered throughout the county. The former may be closer to the theoretical ideal of a public good than the latter. One way to measure project size is from data on local or state government bond issues.

Finally, our model is simple and may be extended in several ways. For example, what if one candidate had an incumbency advantage, modeled as a candidate-specific valence term? How do public goods affect the decision to adopt primaries? The robustness of our results to the introduction of different institutional settings and other players is also worth exploration. While our results are unchanged by multiple candidates in each party, the effects of multiple parties, more groups, or alternative nomination systems remain open questions.

REFERENCES

Adams, James, and Samuel Merrill, III. 2008. "Candidate and Party Strategies in Two-Stage Elections Beginning with a Primary." *American Journal of Political Science* 52(2): 344-359.

Aschauer, David A. 1989. "Is Public Expenditure Productive?" *Journal of Monetary Economics* 23: 177-200.

Ansolabehere, Stephen, Shigeo Hirano, and James M. Snyder, Jr. 2007. "What Did the Direct Primary Do to Party Loyalty in Congress?" In David Brady and Mathew D. McCubbins, eds., *Process, Party and Policy Making: Further New Perspectives on the History of Congress, Volume 2.* Palo Alto, CA: Stanford University Press.

Ansolabehere, Stephen, James M. Snyder, Jr., and Michael M. Ting. 2003. "Bargaining in Bicameral Legislatures: When and Why Does Malapportionment Matter?" *American Political Science Review* 97: 471-481.

Aranson, Peter, and Peter Ordeshook. 1972. "Spatial Strategies for Sequential Elections." In Richard Niemi and Herbert Weisberg, eds., *Probability Models of Collective Decision Making.* Columbus, Ohio: Charles E. Merrill, 298-331.

Berry, Christopher R. 2008. "Piling On: Multilevel Government and the Fiscal Common-Pool." *American Journal of Political Science* 52: 802-820.

Berry, Christopher R. 2009. *Imperfect Union: Representation and Taxation in Multilevel Governments.* Cambridge: Cambridge University Press.

Berry, Christopher R., and Jacob Gersen. 2009. "Fiscal Consequences of Electoral Institutions." *Journal of Law and Economics* 52: 469-495.

Blume, Lorenz, Jens Muller, Stefan Voigt, and Carsten Wolf. 2009. "The Economic Effects of Constitutions: Replicating – And Extending – Persson and Tabellini." *Public Choice* 139(1/2): 197-225.

Bradbury, John C., and W. Mark Crain. 2002. "Bicameral Legislatures and Fiscal Policy." *Southern Economic Journal* 68: 646-659.

Carey, John M., and John Polga-Hecimovich. 2006. "Primary Elections and Candidate Strength in Latin America." *Journal of Politics* 68: 530-543.

Caillaud, Bernard, and Jean Tirole. 1999. "Party Governance and Ideological Bias." *European Economic Review* 43 (4-6): 779-789.

Castanheira, Micael, Benoît Crutzen, and Nicolas Sahuguet. 2010. "Party Governance and Electoral Competition." *Journal of Law, Economics, and Organization* 26(2): 212-242.

Coleman, James S. 1972. "The Positions of Political Parties in Elections." In Richard Niemi and Herbert Weisberg, eds., *Probability Models of Collective Decision Making.* Columbus, Ohio: Charles E. Merrill.

Crutzen, Benoît S. Y. 2014. "Keeping Politicians on Their Toes: The Role of the Candidate Selection Procedure and of the Power of Electoral Incentives." Unpublished manuscript, Erasmus University Rotterdam.

Cutrone, Michael, and Nolan McCarty. 2006. "Does Bicameralism Matter?" *Oxford Handbook of Political Economy.* Oxford: Oxford University Press.

Diermeier, Daniel, and Timothy J. Feddersen. 1998. "Cohesion in Legislatures and the Vote of Confidence Procedure." *American Political Science Review* 92: 611-621.

Dixit, Avinash, and John Londregan. 1995. "Redistributive Politics and Economic Efficiency." *American Political Science Review* 89: 856-866.

Dixit, Avinash, and John Londregan. 1996. "The Determinants of Success of Special Interests in Redistributive Politics." *Journal of Politics* 58: 1132-1155.

Gagliarducci, Stefano, Tommaso Nannicini, and Paolo Naticchioni. 2011. "Electoral Rules and Politicians' Behavior: A Micro Test." *American Economic Journal: Economic Policy* 3(3): 144-174.

Hirano, Shigeo, James M. Snyder, Jr., and Michael M. Ting. 2009. "Distributive Politics with Primary Elections." *Journal of Politics* 71: 1467-1480.

Inman, Robert P., and Michael A. Fitts. 1990. "Political Institutions and Fiscal Policy: Evidence from the U.S. Historical Record." *Journal of Law, Economics, and Organization* 6: 79-132.

Jackson, Matthew O., Laurent Mathevet, and Kyle Mattes. 2007. "Nomination Processes and Policy Outcomes." *Quarterly Journal of Political Science* 2(1): 67-94.

Kalandrakis, Tasos. 2004. "Bicameral Winning Coalitions and Equilibrium Federal Legislatures." *Legislative Studies Quarterly* 29: 49-79.

Kemahlioglu, Ozge, Rebecca Weitz-Shapiro, and Shigeo Hirano. 2009. "Why Primaries in Latin American Presidential Elections?" *Journal of Politics* 71(1): 339-352.

Lancaster, Thomas D. 1986. "Electoral Structures and Pork-Barrel Politics." *International Political Science Review* 7: 67-81.

Lancaster, Thomas D., and W. David Patterson. 1990. "Comparative Pork Barrel Politics: Perceptions from the West German Bundestag." *Comparative Political Studies* 22: 458-477.

Lizzeri, Alessandro, and Nicola Persico. 2001. "The Provision of Public Goods under Alternative Electoral Incentives." *American Economic Review* 91: 225-239.

Lizzeri, Alessandro, and Nicola Persico. 2004. "Why Did the Elites Extend the Suffrage? Democracy and the Scope of Government, With an Application to Britain?s 'Age of Reform'." *Quarterly Journal of Economics* 119(2): 707-765.

Lizzeri, Alessandro, and Nicola Persico. 2005. "A Drawback of Electoral Competition." *Journal of the European Economic Association* 3: 1318-1348.

Lindbeck, Assar, and Jorgen W. Weibull. 1987. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice* 52: 273-297.

Lockwood, Ben. 2002. "Distributive Politics and the Costs of Decentralization." *Review of Economic Studies* 69: 313-337.

McCarty, Nolan. 2000a. "Proposal Rights, Veto Rights, and Political Bargaining." *American Journal of Political Science* 44: 506-522.

McCarty, Nolan. 2000b. "Presidential Pork: Executive Politics and Distributive Politicss." *American Political Science Review* 94: 117-129.

Merriam, Charles E., and Louise Overacker. 1928. *Primary Elections.* Chicago: University of Chicago Press.

Milesi-Ferretti, Gian Maria, Roberto Perotti, and Massimo Rostagno. 2002. "Electoral Systems and Public Spending." *Quarterly Journal of Economics* 117: 649-657.

Munnell, Alicia H. 1990. "Why Has Productivity Growth Declined? Productivity and Public Investment." *New England Economic Review, Federal Reserve Bank of Boston.* January/February, pp. 3-22.

Negri, Margherita. 2014. "Minority Representation in Proportional Representation Systems." Unpublished manuscript, University of St. Andrews.

Owen, Guillermo, and Bernard Grofman. 2006. "Two-Stage Electoral Competition in Two-Party Contests: Persistent Divergence of Party Positions." *Social Choice and Welfare* 26: 547-569.

Parameswaran, Giri. 2012. "Less Representation is Better: How Large States Benefit from Bicameralism." Unpublished manuscript, Princeton University.

Persson, Torsten, Gerard Roland, and Guido Tabellini. 2000. "Comparative Politics and Public Finance." *Journal of Political Economy* 108: 1121-1161.

Persson, Torsten, and Guido Tabellini. 1999. "The Size and Scope of Government: Comparative Politics with Rational Politicians." *European Economic Review* 43: 699-735.

Persson, Torsten, and Guido Tabellini. 2003. *The Economic Effects of Constitutions.* Cambridge, MA: MIT Press.

Persson, Torsten, and Guido Tabellini. 2004a. "Constitutions and Economic Policy." *Journal of Economic Perspectives* 18: 75-98.

Persson, Torsten, and Guido Tabellini. 2004b. "Constitutional Rules and Economic Policy Outcomes." *American Economic Review* 94: 25-46.

Primo, David, and James M. Snyder, Jr. 2010. "Party Strength and State Government Spending." *American Journal of Political Science* 54: 354-370.

Reynolds, John F. 2006. *The Demise of the American Convention System, 1880-1911* New York: Cambridge University Press.

Serra, Gilles. 2011. "Why Primaries? The Party's Tradeoff between Policy and Valence." *Journal of Theoretical Politics* 23(1): 21-51.

Stratmann, Thomas, and Martin Baur. 2002. "Plurality Rule, Proportional Representation, and the German Bundestag: How Incentives to Pork-Barrel Differ across Electoral Systems." *American Journal of Political Science* 46: 506-514.

Tergiman, Chloe. 2015. "Institution Design and Public Good Provision: an Experimental Study of the Vote of Confidence Procedure." *Experimental Economics* 18(4): 697-717.

Wallis, John J., Price V. Fishback, and Shawn E. Kantor. 2006. "Politics, Relief, and Reform. Roosevelt's Efforts to Control Corruption and Political Manipulation during the New Deal." In Edward Glaeser and Claudia Goldin, eds., *Corruption and Reform: Lessons from America's Economic History.* Chicago: University of Chicago Press.

Wallis, John J., and Wallace Oates. 1989. "The Impact of the New Deal on American Federalism" in Bordo, Michael, Claudia Goldin and Eugene N. White, ed., *The Defining Moment: The Great Depression and the American Economy in the Twentieth Century.* Chicago: University of Chicago Press.

Ware, Alan. 2002. *American Direct Primary.* Cambridge, U.K.: Cambridge University Press.

Primary Elections and the Provision of Public Goods — Online Appendix

This appendix begins with a statement of the possible interior solutions for party Y. Next, it presents two additional theoretical results. The first characterizes equilibria in an environment where groups are ideologically symmetric but group sizes are not. The second is an extension of the model that relaxes the assumption of core groups forming the majority of a primary electorate. Proofs of the main results of the paper follow.

Interior Solutions for Party Y

For party Y candidates the necessity of a corner solution implies $y_0 + n_i y_i = 1$ for $i \in \{2, 3\}$. This produces a simplified version of party Y objective (3) that is concave in y_0 for $n_3 s < 1$. There are two cases, corresponding to whether group 3 or 2 receives private goods. In the first, $y_3 > 0$ and $y_2 = 0$; noting that $x_3^* = 0$ Performing the straightforward maximization, we have the following interior solutions for y_3 and y_0:

$$\tilde{y}_3 = \frac{(1 - 2n_3 s)(x_0 - 1) - n_3\gamma_3}{2n_3(n_3 s - 1)} - \frac{\alpha - \theta/2 + x_2}{2n_3 s} \tag{15}$$

$$\tilde{y}_{01} = \frac{(2n_3 s - 1)x_0 + n_3\gamma_3 - 1}{2(n_3 s - 1)} + \frac{\alpha - \theta/2 + x_2}{2s}. \tag{16}$$

And when $y_3 = 0$ and $y_2 > 0$, we have for y_2 and y_0:

$$\tilde{y}_2 = \frac{(1 - n_2 s)\gamma_3 - s(2n_2 s - 1)(x_0 - 1)}{2n_2 s(n_2 s - 1)} - \frac{\alpha - \theta/2 + x_2}{2(n_2 s - 1)} \tag{17}$$

$$\tilde{y}_{02} = \frac{(n_2 s - 1)\gamma_3 + s(2n_2 s x_0 - x_0 - 1)}{2s(n_2 s - 1)} + \frac{n_2(2\alpha - \theta + 2x_2)}{4(n_2 s - 1)}. \tag{18}$$

These expressions are analogous to the party X solutions in equations (4)-(7).

Symmetric Ideology, Asymmetric Group Sizes

Proposition 4 characterizes the equilibrium of the case where groups are ideologically symmetric and equal in size. Here we examine the effect of asymmetry in core group sizes. Suppose that $n_1 = n_2$, and $n_3 > n_1$, so that party Y faces a larger constituency. In addition, suppose that $s < 1/(n_1 + n_2)$ and $s > 1/(n_2 + n_3)$ (which also implies that $s < 1/n_i$ for all i). Thus party X offers only private goods, while party Y may offer some public goods.

We focus on a simple case where one party (X) uses only private goods. Proposition 7 shows that the logic of offering increasingly valuable allocations to swing voters as extremism increases remains when groups are asymmetric in size. The cutpoints γ' and γ'' correspond to the endpoints of the interval where only public goods are offered. (This interval also appears in Proposition 4.) It can be shown that this interval is nonempty whenever $\gamma' > 1/n_2$, which holds for θ sufficiently large. Since $\gamma > 1/n_2$ by assumption, the condition $\gamma' > 1/n_2$ is sufficient for the existence of a region where only party Y provides public goods.

Proposition 7 Asymmetric Core Groups. *Suppose* $s < \frac{1}{n_1+n_2}$, $s > \frac{1}{n_3+n_2}$, $n_1 = n_2$, $\alpha = 0$, *and* $\gamma_1 = -\gamma_3 = \gamma$. *In equilibrium there exist* γ' *and* γ'' *such that:*

(i) If $\gamma < \gamma'$, *then* $y_2^* = 0$ *and* y_3^* *is piecewise linear and weakly decreasing in* γ.

(ii) If $\gamma > \gamma''$, *then* $y_3^* = 0$.

(iii) If $\gamma \in [\gamma', \gamma'']$, *then* $y_0^* = 1$.

At an interior solution, $\gamma' = \frac{(1-n_3 s)(2s-1/n_2+3\theta/2)-2n_3 s^2}{1+n_3 s}$ *and* $\gamma'' = \frac{s(3-4n_2 s-3n_2\theta/2)}{n_2 s-2}$. ■

Proof of Proposition 7. By Lemma 1(ii), since $s < 1/(n_1 + n_2)$, $x_0^* = 0$. Substituting into (3) and maximizing then gives the interior solutions for x_1 and x_2 in terms of y_2:

$$x_1^* = \frac{1 - n_1\gamma - n_2(y_2 - \theta/2)}{2n_1} \tag{19}$$

$$x_2^* = \frac{1 + n_1\gamma + n_2(y_2 - \theta/2)}{2n_2} \tag{20}$$

By the concavity of (3), the obvious corner solutions are $x_1^* = 0$ and $x_2^* = 1/n_2$, and $x_1^* = 1/n_1$ and $x_2^* = 0$. We derive features of the party Y platforms by applying Lemma 2′(ii).

(i) Simplifying from (29), $y_3^* = \min\{1/n_3, \tilde{y}_3\}$ and $y_2^* = 0$ if:

$$\begin{aligned} 2n_3 s^2 - s + (1 - n_3 s)(x_2 - \theta/2) + n_3 s\gamma &< 0 \\ \gamma &< \frac{s - 2n_3 s^2 - (1 - n_3 s)(x_2 - \theta/2)}{n_3 s} \end{aligned} \tag{21}$$

Since $y_2^* = 0$ in this region, we have $x_2^* = \max\{0, \min\{\frac{1}{n_2}, \frac{1}{2n_2} + \frac{\gamma}{2} - \frac{\theta}{4}\}\}$. The upper bound implied by (21) is linear in x_2, and x_2^* is continuous, bounded, and piecewise linear in γ. Thus a solution γ' for (21) in terms of γ exists and is weakly decreasing and piecewise linear. It is straightforward to calculate that at an interior solution this bound is:

$$\gamma' = \frac{(1 - n_3 s)(2s - 1/n_2 + 3\theta/2) - 2n_3 s^2}{1 + n_3 s} \tag{22}$$

Finally, substituting appropriately into (15), we have the following expression for $\tilde{y}_3$:

$$\tilde{y}_3 = \frac{2n_3 s - 1 + n_3\gamma}{2n_3(n_3 s - 1)} - \frac{x_2 - \theta/2}{2n_3 s}.$$

This expression is decreasing in γ for $s < 1/n_3$, and $y_3^* = 0$ for $s \geq 1/n_3$ when (29) holds. Thus, y_3^* is weakly decreasing in this region.

(ii) Simplifying from (30), $y_3^* = 0$ and $y_2^* = \min\{1/n_2, \tilde{y}_2\}$ if:

$$\begin{aligned} 2n_2 s^2 - s - n_2 s(x_2 - \theta/2) - (1 - n_2 s)\gamma &< 0 \\ \gamma &> \frac{2n_2 s^2 - s - n_2 s(x_2 - \theta/2)}{1 - n_2 s} \end{aligned} \tag{23}$$

Simplifying (18) yields the following expression for $\tilde{y}_2$:

$$\tilde{y}_2 = \frac{s(2n_2 s - 1) - (1 - n_2 s)\gamma}{2n_2 s(n_2 s - 1)} - \frac{x_2 - \theta/2}{2(n_2 s - 1)}$$

The expressions for y_2^* and x_2^* are continuous and piecewise linear in y_2 and x_2, respectively, and bounded. Thus there exists a solution to the system. At an interior solution we have:

$$\begin{aligned} x_2^* &= \frac{1}{n_2} + \frac{(2n_2^2 s^2 - n_2 s - 1)\gamma}{n_2 s(4n_2 s - 3)} + \frac{(3/2 - n_2 s)\theta}{4n_2 s - 3} \\ y_2^* &= \frac{1}{n_2} + \frac{(2 - n_2 s)\gamma}{3n_2 s - 4n_2^2 s^2} - \frac{3\theta}{6 - 8n_2 s} \end{aligned}$$

To characterize γ'', note that since $x_2^* \geq 0$, the lower bound on γ'' can be derived by subustiting $x_2 = 1/n_2$ into (23), yielding $\gamma'' \geq \frac{s(2n_2 s - 2 + n_2\theta/2)}{1 - n_2 s}$. Substituting the interior value of x_2^* into (23), at an interior solution the minimum value of γ for this solution to obtain is:

$$\gamma'' = \frac{s(3 - 4n_2 s - 3n_2\theta/2)}{n_2 s - 2}$$

It is straightforward to verify that $\gamma'' > \gamma'$ whenever $\gamma' > 1/n_2$.

(iii) By Lemma 2′(ii), for all γ not satisfying the conditions of parts (i) and (ii), $y_0^* = 1$.

■

Extension: Pivotal Swing Voters

An important assumption in the previous results was that the pivotal voter in each party's primary election belonged to a core group. However, a broad-based party might have more swing than core voters. If instead the pivotal voter belonged to the swing group, then that party's candidates could focus exclusively on the general election. In this section, we examine the case where party Y's pivotal primary voter belongs to group 2.

It is clear that party Y candidates will choose to maximize the expected payoffs of group 2 voters. Thus their platform strategies will be identical to those in the no-primaries world:

$$\overline{y}^a = \overline{y}^b = \begin{cases} (0, 0, \frac{1}{n_2}, 0) & \text{if } s < \frac{1}{n_2} \\ (1, 0, 0, 0) & \text{otherwise.} \end{cases}$$

The party Y strategies generate two cases. When public goods are highly valuable (i.e., $s > 1/n_2$), party Y candidates offer only public goods, and when public goods are less valuable, they focus exclusively on private goods for group 2. The following result characterizes the equilibrium platforms for party X in both cases. We restrict attention here to values of s that are high enough to ensure that public goods are undominated.

Remark 3 Pivotal Swing Voters. *Suppose $s > \frac{1}{n_1+n_2}$ and group 2 voters are a majority of the party Y primary electorate.*

(i) If $s > \frac{1}{n_2}$, then $x_0^* = 1 - n_1 x_1^*$ and

$$x_1^* = \begin{cases} 0 & \text{if } n_1 \geq \frac{\alpha+\theta/2}{s(\gamma_1+\alpha+\theta/2)} \\ \max\left\{\frac{1}{n_1}, \frac{\gamma_1}{2(n_1 s-1)} + \frac{\alpha+\theta/2}{2n_1 s}\right\} & \text{otherwise.} \end{cases}$$

(ii) If $s < \frac{1}{n_2}$, then $x_0^* = 1 - n_1 x_1^* - n_2 x_2^*$ and

$$x_1^* = \begin{cases} 0 & \text{if } n_1 \geq \frac{s+\alpha+\theta/2-1/n_2}{s(2s+\gamma_1+\alpha+\theta/2-1/n_2)} \\ \max\left\{\frac{1}{n_1}, \frac{\gamma_1+s}{2(n_1 s-1)} + \frac{1}{2n_1}\left(1-\frac{1}{n_2 s}\right) + \frac{\alpha+\theta/2}{2n_1 s}\right\} & \text{otherwise,} \end{cases}$$

$$x_2^* = \begin{cases} 0 & \text{if } n_2 \geq \frac{2s+\gamma_1}{s(2s+\gamma_1+\alpha+\theta/2)} \\ \max\left\{\frac{1}{n_2}, \frac{\gamma_1}{2n_2 s} + \frac{n_2(2s+\alpha+\theta/2)-2}{2n_2(n_2 s-1)}\right\} & \text{otherwise.} \quad ■ \end{cases}$$

Proof of Remark 3. (i) Since $s > 1/n_2$, Lemma 1(iii) implies that $x_2^* = 0$. We use Lemma $2'$(i) to establish the condition under which party X candidates can offer private goods to group 1. Substituting into equation (8) yields:

$$n_1 < \frac{\alpha + \theta/2}{s(\gamma_1 + \alpha + \theta/2)}. \tag{24}$$

When (24) is not satisfied, $x_0^* = 1$. When (24) is satisfied, $x_1^* = \min\{1/n_1, \tilde{x}_1\}$, as given by substituting $y_0 = 1$ and $y_1 = y_2 = 0$ into (4):

$$x_1^* = \frac{\gamma_1}{2(n_1 s - 1)} + \frac{\alpha + \theta/2}{2n_1 s}.$$

(ii) Now suppose that $s < 1/n_2$, which implies $y_2^* = 1/n_2$. We again apply Lemma $2'$(i). Substituting $y_0 = y_1 = 0$ and $y_2 = 1/n_2$ into equation (8) yields the following condition:

$$\begin{aligned} 2n_1 s^2 - s + (1 - n_1 s)\left(\frac{1}{n_2} - \alpha - \frac{\theta}{2}\right) + n_1 s \gamma_1 &< 0 \\ n_1 &< \frac{s + \alpha + \theta/2 - 1/n_2}{s(2s + \gamma_1 + \alpha + \theta/2 - 1/n_2)}. \end{aligned} \tag{25}$$

When (25) is satisfied, $x_1^* = \min\{1/n_1, \tilde{x}_1\}$, as given by substituting appropriately into (4):

$$x_1^* = \frac{\gamma_1 + s}{2(n_1 s - 1)} + \frac{1}{2n_1}\left(1 - \frac{1}{n_2 s}\right) + \frac{\alpha + \theta/2}{2n_1 s}.$$

Similarly, party X candidates may offer private goods to group 2. Substituting into equation (9) yields the following condition:

$$\begin{aligned} 2n_2 s^2 - s - n_2 s\left(\frac{1}{n_2} - \alpha - \frac{\theta}{2}\right) - (1 - n_2 s)\gamma_1 &< 0 \\ n_2 &< \frac{2s + \gamma_1}{s(2s + \gamma_1 + \alpha + \theta/2)}. \end{aligned} \tag{26}$$

When (26) is satisfied, $x_2^* = \min\{1/n_2, \tilde{x}_2\}$, as given by substituting appropriately into (6):

$$x_2^* = \frac{\gamma_1}{2n_2 s} + \frac{n_2(2s + \alpha + \theta/2) - 2}{2n_2(n_2 s - 1)}.$$

When neither (25) nor (26) are satisfied, party X provides only public goods. ∎

Party X's equilibrium platforms are derived simply from Lemma 1 and equations (4) and (6). The basic properties of the original game continue to hold here. For example, while it is possible for either group 1 or group 2 to benefit from private goods in part (ii), Lemma 2 continues to hold, and thus at most one of x_1 and x_2 can be positive. Additionally, consistent with the logic of Proposition 3(i), part (i) implies that $x_1^* > 0$ only if $n_1 < n_2$. Again, small group sizes are conducive to offering private goods, even when the opposition can be considerably more appealing to swing voters.

One clear implication of this environment is that it helps party Y to win. A more interesting question is how party X candidates respond. In a world without public goods, pivotal swing voters in party Y generally induce party X candidates to allocate more to the swing group in order to compensate for their reduced probability of victory (Hirano, Snyder, and Ting 2009). Public goods can muddle this result by reducing the stakes of victory. For example, suppose that the equilibrium core group allocations are interior regardless of which group controls the party Y primary (implying $s < 1/n_1$), and compare swing and core voter control of the party Y primary. When $s > 1/n_2$, $y_2^* = 0$ and swing voter control (weakly) raises y_0 to 1. It can be easily shown that if $s > 1/(2n_1)$, then shifting to swing voter control *increases* x_1^*, while if $s < 1/(2n_1)$, the relationship is reversed. The increase in core allocations in party X is due to the high value of the public good: with increased payoffs from losing, group 1 members are willing to accept a lower probability of victory and higher payoffs conditional upon victory. By contrast, when party Y does not provide public goods, party X candidates would respond by improving their offers to swing voters.

Proofs of Main Results

We begin with two useful lemmas. The first shows that the objectives for each party's candidates is non-concave.

Lemma 3 Nonconcavity of Party Objectives. *Party X and Y candidates' objective functions are never locally concave for $s > \frac{1}{n_1+n_2}$ and $s > \frac{1}{n_3+n_2}$, respectively.* ■

Proof of Lemma 3. The budget constraints and weak domination imply that $x_2 = (1 - n_1x_1 - n_3x_3 - x_0)/n_2$ and $y_2 = (1 - n_1y_1 - n_3y_3 - y_0)/n_2$. Further, Lemma (i) implies

$x_3^P = y_1^P = 0$. Substituting these into (3) yields:

$$E_1(\overline{x},\overline{y}) = \left[\frac{n_3y_3 - n_1x_1 - (n_2s-1)(y_0-x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2}\right](x_1+s(x_0-y_0)+\gamma_1) + sy_0 \quad (27)$$

$$E_3(\overline{x},\overline{y}) = \left[\frac{n_3y_3 - n_1x_1 - (n_2s-1)(y_0-x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2}\right](-y_3 + s(x_0-y_0) + \gamma_3) + y_3 + sy_0. \quad (28)$$

There are two cases. First, if $n_2s > 1$, then it is clear that (27) and (28) are strictly convex in x_0 and y_0, respectively. Second, if $n_2s < 1$, then we may write the first order conditions as follows.

$$\begin{aligned}
\frac{\partial E_1(\overline{x},\overline{y})}{\partial x_1} &= -\frac{2n_1}{\theta n_2}x_1 + \frac{n_3y_3 - n_1\gamma_1 - (n_2s - n_1s - 1)(y_0 - x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \\
\frac{\partial E_1(\overline{x},\overline{y})}{\partial x_0} &= \frac{2(n_2s-1)s}{\theta n_2}x_0 + \frac{(n_2s-1)(x_1 - sy_0 + \gamma_1) + s(n_3y_3 - n_1x_1 - (n_2s-1)y_0)}{\theta n_2} \\
&\quad + \left(\frac{\alpha}{\theta} + \frac{1}{2}\right)s \\
\frac{\partial E_3(\overline{x},\overline{y})}{\partial y_3} &= -\frac{2n_3}{\theta n_2}y_3 + \frac{n_1x_1 + n_3\gamma_3 + (n_2s - n_3s - 1)(y_0 - x_0)}{\theta n_2} - \frac{\alpha}{\theta} + \frac{1}{2} \\
\frac{\partial E_3(\overline{x},\overline{y})}{\partial y_0} &= \frac{2(n_2s-1)s}{\theta n_2}y_0 - \frac{(n_2s-1)(-y_3 + sx_0 + \gamma_3) + s(n_3y_3 - n_1x_1 + (n_2s-1)x_0)}{\theta n_2} \\
&\quad - \left(\frac{\alpha}{\theta} - \frac{1}{2}\right)s.
\end{aligned}$$

Now consider whether the conditions for local concavity are possible. For party X, the Hessian is:

$$\begin{vmatrix} -\frac{2n_1}{\theta n_2} & \frac{n_2s-n_1s-1}{\theta n_2} \\ \frac{n_2s-n_1s-1}{\theta n_2} & \frac{2(n_2s-1)s}{\theta n_2} \end{vmatrix}$$

The diagonal elements are clearly negative, and the determinant is non-negative if:

$$\begin{aligned}
-4n_1(n_2s-1)s - (n_2s - n_1s - 1)^2 &\geq 0 \\
1 - 2n_1n_2s^2 - (n_1s-1)^2 - (n_2s-1)^2 &\geq 0
\end{aligned}$$

It is straightforward to show that this expression is never positive, and can be satisfied with equality if and only if $n_1s + n_2s = 1$. But when $s = 1/(n_1 + n_2)$, party X does just as well by giving private goods $x_1 = x_2 = 1/(n_1 + n_2)$. Thus the objective has no local maxima whenever s is such that public goods might be optimal. The analysis for party Y is symmetrical and therefore omitted. ∎

Second, Lemma 4 provides sufficient conditions on γ for zero allocations of private goods, using the cutpoints in (10)-(13). The result complements Lemma 1(iii), which established that a groups would not receive private goods when s is sufficiently large.

Lemma 4 Independence of Private Goods Recipient.

(i) For $s \in \left(\frac{1}{n_1+n_2}, \min\{\frac{1}{n_1}, \frac{1}{n_2}\}\right)$, $x_1^* = 0$ *if* $\gamma_1 \geq \overline{\gamma}_1$, *and* $x_2^* = 0$ *if* $\gamma_1 \leq \underline{\gamma}_1$.

(ii) For $s \in \left(\frac{1}{n_2+n_3}, \min\{\frac{1}{n_3}, \frac{1}{n_2}\}\right)$, $y_3^* = 0$ *if* $\gamma_3 \leq \underline{\gamma}_3$, *and* $y_2^* = 0$ *if* $\gamma_3 \geq \overline{\gamma}_3$. ∎

Proof of Lemma 4. We use the results from Lemma 2′.

(i) Reversing the inequality in (8), we obtain the following condition under which $x_1^* = 0$ must obtain: $\gamma_1 \geq \frac{-(2n_1s^2-s)(1-y_0)-(1-n_1s)(y_2-\alpha-\theta/2)}{n_1s}$. To establish the upper bound on the right-hand side of this expression, let $y_2 = 0$ and $y_0 = 1$ (0) if $2n_1s >$ (<) 1. This yields $\overline{\gamma}_1$, or (10).

Next, reversing the inequality in (9), we obtain the following condition under which $x_2^* = 0$ must obtain: $\gamma_1 \leq \frac{(2n_2s^2-s)(1-y_0)-n_2s(y_2-\alpha-\theta/2)}{1-n_2s}$. To establish the lower bound on the right-hand side of this expression, let $y_2 = 1/n_2$ and $y_0 = 1 - n_2y_2 = 0$. This yields $\underline{\gamma}_1 = \frac{n_2s(\alpha+\theta/2)}{1-n_2s} - 2s$, or (11).

(ii) Reversing the inequality in (29), we obtain the following condition under which $y_3^* = 0$ must obtain: $\gamma_3 \leq \frac{(2n_3s^2-s)(1-x_0)+(1-n_3s)(x_2+\alpha-\theta/2)}{n_3s}$. To establish the lower bound on the right-hand side of this expression, let $x_2 = 0$ and $x_0 = 1$ (0) if $2n_3s >$ (<) 1. This yields $\underline{\gamma}_3$, or (12).

Next, reversing the inequality in (30), we obtain the following condition under which $y_2^* = 0$ must obtain: $\gamma_3 \geq \frac{-(2n_2s^2-s)(1-x_0)+n_2s(x_2+\alpha-\theta/2)}{1-n_2s}$. To establish the upper bound on the right-hand side of this expression, let $x_2 = 1/n_2$ and $x_0 = 1 - n_2x_2 = 0$. This yields $\overline{\gamma}_3 = \frac{n_2s(\alpha-\theta/2)}{1-n_2s} + 2s$, or (13). ∎

We now prove the results in the text of the paper.

Proof of Lemma 1. Throughout, suppose that a party X candidate allocates some $\overline{x}$, where (by weak dominance) $\overline{x}$ satisfies the budget constraint $x_0 + n_1x_1 + n_2x_2 + n_3x_3 = 1$.

(i) It is clear from inspection of (3) that any positive allocation to group 3 is dominated by a reallocation toward either group 1 or 2, or the public good. Thus, $x_3^* = 0$.

(ii) We derive the condition on s for the party X candidate to reallocate all of x_0 to private goods of equal value for groups 1 and 2 at lower cost. A platform giving $sx_0 + x_1$ to group 1 voters and $sx_0 + x_2$ to group 2 voters is feasible if $n_1(sx_0 + x_1) + n_2(sx_0 + x_2) \leq 1$. Rearranging and applying the budget constraint, we obtain:

$$\begin{aligned} n_1sx_0 + n_2sx_0 &\leq x_0 \\ s &\leq \frac{1}{n_1 + n_2}. \end{aligned}$$

(iii) To show the result for $s > 1/n_i$, observe that a party X candidate could replace x_i with n_ix_i units of x_0. This revised allocation strictly benefits all voters.

To show the result for $s > \max\{1/n_1, 1/n_2\}$, we derive the condition on s for the party X candidate to reallocate all of x_1 and x_2 to the public goods and benefit groups 1 and 2 at lower cost. The platform $x_0' = 1$ is feasible and gives s to all voters. It provides greater utility to voters in group i if $s > sx_0 + x_i$. Since the right-hand side is maximized either at $x_i = 1/n_i$ (implying $x_0 = 0$), or 0 (implying $x_0 = 1$), the condition holds for any $s > 1/n_i$.

(iv) Suppose that $x_i > x_j > 0$ for $i, j \in \{1, 2\}$ and $j \neq i$. We provide conditions under which a party X candidate would do strictly better by offering a different platform $\overline{x}'$ where $x_j' = 0$, $x_i' = x_i - x_j$, and $x_0' = x_0 + x_j/s$. This platform clearly provides all voters in groups 1 and 2 identical utility as $\overline{x}$. Thus we need only verify the feasibility of $\overline{x}'$, which, given part (ii), is assured if:

$$\begin{aligned} n_i(x_i - x_j) + x_0 + x_j/s &< 1 \\ s &> \frac{1}{n_1 + n_2}. \end{aligned}$$

Therefore for any such s, an optimal platform must have $x_1 = 0$ or $x_2 = 0$. By part (iii), for $s > \max\{1/n_1, 1/n_2\}$, this is strengthened to $x_1 = 0$ and $x_2 = 0$. ∎

We present and prove an extended version of Lemma 2 that addresses both party X (part (i), as shown in the main text), and party Y (part (ii)).

Lemma 2′ *Private Good Allocations Under Intermediate s.* (i) For $s \in \left(\frac{1}{n_1+n_2}, \min\{\frac{1}{n_1}, \frac{1}{n_2}\}\right)$, $x_1^* = \min\{\frac{1}{n_1}, \tilde{x}_1\} > 0$ and $x_2^* = 0$ if and only if:

$$(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \alpha - \theta/2) + n_1s\gamma_1 < 0,$$

and $x_1^* = 0$ and $x_2^* = \min\{\frac{1}{n_2}, \tilde{x}_2\} > 0$ if and only if:

$$(2n_2s^2 - s)(1 - y_0) - n_2s(y_2 - \alpha - \theta/2) - (1 - n_2s)\gamma_1 < 0.$$

Otherwise, $x_0^* = 1$.

(ii) For $s \in \left(\frac{1}{n_2+n_3}, \min\{\frac{1}{n_3}, \frac{1}{n_2}\}\right)$, $y_3^* = \min\{\frac{1}{n_3}, \tilde{y}_3\} > 0$ and $y_2^* = 0$ if and only if:

$$(2n_3s^2 - s)(1 - x_0) + (1 - n_3s)(x_2 + \alpha - \theta/2) - n_3s\gamma_3 < 0, \tag{29}$$

and $y_3^* = 0$ and $y_2^* = \min\{\frac{1}{n_2}, \tilde{y}_2\} > 0$ if and only if:

$$(2n_2s^2 - s)(1 - x_0) - n_2s(x_2 + \alpha - \theta/2) + (1 - n_2s)\gamma_3 < 0. \tag{30}$$

Otherwise, $y_0^* = 1$. ∎

Proof of Lemma 2′. (i) Let $E_{1i}(x_0, \overline{y})$ denote group 1 utility $E_1(\overline{x}, \overline{y})$ when $x_i = (1-x_0)/n_i$ and all other private good allocations are zero.

We first show that $\frac{dE_{1i}}{dx_0}(1, \overline{y}) \geq 0$ for either $i = 1$ or 2. Substituting x_i into (3) and differentiating $E_{1i}(x_0, \overline{y})$ gives the following expressions:

$$\begin{aligned}
\frac{dE_{11}}{dx_0} &= \frac{2n_1s^2(x_0 - y_0) + (1 - n_1s)(y_2 - \alpha - \theta/2) + s(1 - 2x_0 + y_0) + n_1s(\gamma_1 - y_1)}{n_1\theta} \\
\frac{dE_{12}}{dx_0} &= \frac{2n_2s^2(x_0 - y_0) - n_2s(y_2 - \alpha - \theta/2) + s(1 - 2x_0 + y_0) - (1 - n_2s)(\gamma_1 - y_1)}{n_2\theta}.
\end{aligned}$$

Suppose that $\frac{dE_{11}}{dx_0}(1,\overline{y}) < 0$ and $\frac{dE_{12}}{dx_0}(1,\overline{y}) < 0$. Clearly, for each i, $\frac{dE_{1i}}{dx_0}(1,\overline{y}) < 0$ iff the corresponding numerator in the above expressions is negative. This implies that the sum of the numerators evaluated at $x_0 = 1$ must also be negative. Substituting and simplifying, we obtain:

$$\begin{aligned} & 2n_1s^2(1-y_0) + (1-n_1s)(y_2-\alpha-\theta/2) + s(y_0-1+n_1(\gamma_1-y_1)) + \\ & 2n_2s^2(1-y_0) - n_2s(y_2-\alpha-\theta/2) + s(y_0-1+n_2(\gamma_1-y_1)) - \gamma_1 + y_1 \\ = \;& (1-(n_1+n_2)s)[2s(y_0-1)+y_2-\alpha-\theta/2-\gamma_1+y_1]. \end{aligned}$$

By assumption, $s > 1/(n_1+n_2)$, and thus $1-(n_1+n_2)s < 0$. The above expression is then positive if $y_1 + y_2 < \gamma_1$, which always holds since $y_1 + y_2 \leq \max\{1/n_1, 1/n_2\} < \gamma_1$: contradiction. Thus either $\frac{dE_{11}}{dx_0}(1,\overline{y}) > 0$ or $\frac{dE_{12}}{dx_0}(1,\overline{y}) > 0$, or both.

It is easily verified from the expressions for $\frac{dE_{1i}}{dx_0}$ that $E_{1i}(x_0,\overline{y})$ is concave in x_0 for $s < 1/n_i$. By concavity, if $\frac{dE_{1i}}{dx_0}(1,\overline{y}) > 0$ then the optimal platform that excludes group $j \neq i$ is $x_0 = 1$ and $x_i = 0$. If both $\frac{dE_{11}}{dx_0}(1,\overline{y}) > 0$ and $\frac{dE_{12}}{dx_0}(1,\overline{y}) > 0$, then the optimal platform is $x_0^* = 1$. And if $\frac{dE_{1i}}{dx_0}(1,\overline{y}) < 0$ for some i, then the optimal platform that excludes group $j \neq i$ must have $x_0^* < 1$ and $x_i^* > 0$. It is therefore either a corner at $x_0 = 0$ and $x_i = 1/n_i$, or the interior solution given by $\tilde{x}_i$ and $\tilde{x}_{0i}$ from the appropriate expression in (4)-(7). This platform must be the unique optimal platform since $\frac{dE_{1j}}{dx_0}(1,\overline{y}) > 0$ ($j \neq i$) implies that the optimal platform that excludes i is $x_0 = 1$.

Substituting $y_1^* = 0$ and $x_0^* = 1$ into the numerators of $\frac{dE_{1i}}{dx_0}(1,\overline{y})$ produces the result.

(ii) Let $E_{3i}(\overline{x}, y_0)$ denote group 3 utility $E_3(\overline{x},\overline{y})$ when $y_i = (1-y_0)/n_i$ and all other private good allocations are zero.

We first show that $\frac{dE_{3i}}{dy_0} \geq 0$ for either $i = 2$ or 3. Substituting y_i into (3) and differentiating $E_{3i}(\overline{x}, y_0)$ gives the following expressions:

$$\begin{aligned} \frac{dE_{33}}{dy_0} &= \frac{2n_3s^2(y_0-x_0) + (1-n_3s)(x_2+\alpha-\theta/2) + s(1-2y_0+x_0) - n_3s(\gamma_3+x_3)}{\theta n_3} \\ \frac{dE_{32}}{dy_0} &= \frac{2n_2s^2(y_0-x_0) - n_2s(x_2+\alpha-\theta/2) + s(1-2y_0+x_0) + (1-n_2s)(\gamma_3+x_3)}{\theta n_2} \end{aligned}$$

Suppose that $\frac{dE_{33}}{dy_0}(\overline{x},1) < 0$ and $\frac{dE_{32}}{dy_0}(\overline{x},1) < 0$. Clearly, for each i, $\frac{dE_{3i}}{dy_0}(1,\overline{y}) < 0$ iff the corresponding numerator in the above expressions is negative. This implies that the sum of the numerators evaluated at $y_0 = 1$ must also be negative. Substituting and simplifying, we obtain:

$$\begin{aligned} & 2n_3s^2(1-x_0) + (1-n_3s)(x_2+\alpha-\theta/2) + s(x_0-1-n_3(\gamma_3+x_3)) + \\ & \quad 2n_2s^2(1-x_0) - n_2s(x_2+\alpha-\theta/2) + s(x_0-1-n_2(\gamma_3+x_3)) + \gamma_3 + x_3 \\ = \; & (1-(n_2+n_3)s)(2s(x_0-1)+x_2-\alpha-\theta/2+\gamma_3+x_3). \end{aligned}$$

By assumption, $s > 1/(n_2+n_3)$, and thus $1-(n_2+n_3)s < 0$. The above expression is then positive if $x_2+x_3 < |\gamma_3|$, which always holds since $x_2+x_3 \leq \max\{1/n_2, 1/n_3\} < |\gamma_3|$: contradiction. Thus either $\frac{dE_{33}}{dy_0}(\overline{x},1) > 0$ or $\frac{dE_{32}}{dy_0}(\overline{x},1) > 0$, or both.

It is easily verified from the expressions for $\frac{dE_{3i}}{dy_0}$ that $E_{3i}(\overline{x},y_0)$ is concave in y_0 for $s < 1/n_i$. By concavity, if $\frac{dE_{3i}}{dy_0}(\overline{x},1) > 0$ then the optimal platform that excludes group $j \neq i$ is $y_0 = 1$ and $y_i = 0$. If both $\frac{dE_{33}}{dy_0}(\overline{x},1) > 0$ and $\frac{dE_{32}}{dy_0}(\overline{x},1) > 0$, then the optimal platform is $y_0^* = 1$. And if $\frac{dE_{3i}}{dy_0}(\overline{x},1) < 0$ for some i, then the optimal platform that excludes group $j \neq i$ must have $y_0^* < 1$ and $y_i^* > 0$. It is therefore either a corner at $y_0 = 0$ and $y_i = 1/n_i$, or the interior solution given by $\tilde{y}_i$ and $\tilde{y}_{0i}$ from the appropriate expression in (15)-(18). This must be the unique optimal platform since $\frac{dE_{3j}}{dy_0}(\overline{x},1) > 0$ ($j \neq i$) implies that optimal platform that excludes i is $y_0 = 1$.

Substituting $y_0^* = 1$ and $x_3^* = 0$ into the numerators of $\frac{dE_{3i}}{dy_0}(\overline{x},1)$ produces the result. ∎

Proof of Proposition 1. There are three cases. First, for $s < \min\{1/(n_1+n_2), 1/(n_2+n_3)\}$, existence is demonstrated by Proposition 2.

Second, suppose $s \geq \max\{1/(n_1+n_2), 1/(n_2+n_3)\}$. Since candidates from both parties adopt the platform that maximizes the expected utility of their core voters, it suffices to establish a fixed point in the best response functions for each party's common platform. We show that the pure strategy best responses satisfy the conditions of Brouwer's fixed point theorem. Observe that each party's set of feasible pure strategies is a simplex, and thus the set of strategy profiles is obviously compact, convex, and non-empty.

Next, we show that party X's best response is single-valued. Let $E_{1i}(x_0, \overline{y})$ denote group 1 utility $E_1(\overline{x}, \overline{y})$ when $x_i = (1-x_0)/n_i$ and all other private good allocations are zero. Note that $E_{1i}(x_0, \overline{y})$ is concave in x_0 for $s < 1/n_i$ and that $E_{1i}(\cdot)$ is maximized at $x_0 = 1$ for $s \geq 1/n_i$. Thus, when restricted to a choice between group i and the public good, there unique solutions $\tilde{x}_{01}$ (5) and $\tilde{x}_{02}$ (7). Party X's best response is $\tilde{x}_{0i}$ if $E_{1i}(\tilde{x}_{0i}, \overline{y}) > E_{1j}(\tilde{x}_{0i}, \overline{y})$ $(j \neq i)$. By the argument in the proof of Lemma 2, $E_{11}(\tilde{x}_{01}, \overline{y}) = E_{12}(\tilde{x}_{02}, \overline{y})$ if and only if $\tilde{x}_{01} = \tilde{x}_{02} = 1$. Thus, party X's best response is single-valued. An identical argument establishes that party Y's best response is also single-valued. The best response to each strategy profile is therefore single-valued.

Finally, we show that the best response function is continuous. By the concavity of $E_{11}(x_0, \overline{y})$, $E_{12}(x_0, \overline{y})$, the solutions $\tilde{x}_{01}$ and $\tilde{x}_{02}$ are continuous in $\overline{y}$. By the argument in the proof of Lemma 2, either $\tilde{x}_{01} = 1$ or $\tilde{x}_{02} = 1$, and the platform implied by $\tilde{x}_{0i}$ (i.e., $x_0 = x_{0i}$, $x_i = (1 - x_{0i})/n_i$) is optimal if $\tilde{x}_{0i} < 1$ $(j \neq i)$. Thus for any $\overline{y}$ the best response is either $\tilde{x}_{0i} < 1$ for some $i \in \{1, 2\}$, or $x_0 = 1$, which occurs when $\tilde{x}_{01} = \tilde{x}_{02} = 1$. The resulting best response for party X is then:

$$\begin{cases} \tilde{x}_{01} & \text{if } \tilde{x}_{01} < 1, \tilde{x}_{02} = 1 \\ 1 & \text{if } \tilde{x}_{01} = \tilde{x}_{02} = 1 \\ \tilde{x}_{02} & \text{if } \tilde{x}_{02} < 1, \tilde{x}_{01} = 1. \end{cases}$$

This function inherits continuity in $\overline{y}$ from the continuity of $\tilde{x}_{01}$ and $\tilde{x}_{02}$. An identical argument holds for party Y's best response.

Third, suppose $s \in [\min\{1/(n_1+n_2), 1/(n_2+n_3)\}, \max\{1/(n_1+n_2), 1/(n_2+n_3)\}]$. Without loss of generality, assume that $1/(n_1+n_2) < 1/(n_2+n_3)$. We again show that the pure strategy best responses satisfy the conditions of Brouwer's fixed point theorem. For party X's best responses the analysis is identical to the second case. For party Y, we show that the best response is single-valued and continuous.

Since $s \leq 1/(n_2+n_3)$, it is clear that $y_0 = 0$ in any best response. Noting that $y_1 = 0$ and

$y_2 = (1 - n_3 y_3)/n_2$ in any best response, we rewrite (3) as the party Y objective as follows:

$$E_3(\overline{x}, \overline{y}) = \left[\frac{x_2 - (1 - n_3 y_3)/n_2 + s x_0 + \alpha}{\theta} + \frac{1}{2}\right](x_3 - y_3 + s x_0 + \gamma_3) + y_3 + s y_0$$

This expression is obviously concave in y_3. Straightforward maximization yields the solution $y_3^* = \frac{1 + n_3 x_3 + (n_3 s - n_2 s) x_0 + n_3 \gamma_3 + n_2(\theta/2 - \alpha - x_2)}{2 n_3}$, which is obviously single-valued and continuous. ∎

Proof of Proposition 2. By Lemma 1(ii), the assumptions on s allow us to restrict attention to strategies where $x_0 = y_0 = 0$.

We now characterize the unique equilibrium platforms. The budget constraints and weak domination imply that $x_2 = (1 - n_1 x_1 - n_3 x_3)/n_2$ and $y_2 = (1 - n_1 y_1 - n_3 y_3)/n_2$. Substituting these into (3) for each group yields:

$$\begin{aligned}
E_1(\overline{x}, \overline{y}) &= \left[\frac{\alpha + (n_1 y_1 + n_3 y_3 - n_1 x_1 - n_3 x_3)/n_2}{\theta} + \frac{1}{2}\right](x_1 - y_1 + \gamma_1) + y_1 \\
E_3(\overline{x}, \overline{y}) &= \left[\frac{\alpha + (n_1 y_1 + n_3 y_3 - n_1 x_1 - n_3 x_3)/n_2}{\theta} + \frac{1}{2}\right](x_3 - y_3 + \gamma_3) + y_3.
\end{aligned}$$

Clearly, $\frac{\partial E_1}{\partial x_3}(\overline{x}, \overline{y}) < 0$ and $\frac{\partial E_3}{\partial y_1}(\overline{x}, \overline{y}) < 0$ for all $(\overline{x}, \overline{y})$, so $x_3^P = y_1^P = 0$. The expected utilities of group-1 and group-3 voters can then be written:

$$E_1(\overline{x}, \overline{y}) = \left[\frac{n_3 y_3 - n_1 x_1}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2}\right](x_1 + \gamma_1) \tag{31}$$

$$E_3(\overline{x}, \overline{y}) = \left[\frac{n_3 y_3 - n_1 x_1}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2}\right](-y_3 + \gamma_3) + y_3. \tag{32}$$

Expressions (31) and (32) are concave and univariate objectives in $x_1 \in [0, 1/n_1]$ and $y_3 \in [0, 1/n_3]$, respectively. Thus for any $\overline{x}$ (respectively, $\overline{y}$), there is a unique platform for party Y (respectively, X) that maximizes the utility of the pivotal voter in group 3 (respectively, 1). Each party's candidates must therefore choose the same platform in equilibrium.

Denoting the equilibrium transfer vectors $\overline{x}^P$ and $\overline{y}^P$, the first-order conditions on (31) and (32) produce:

$$x_1^P = \frac{n_3 y_3^P - n_1 \gamma_1 + n_2(\theta/2 + \alpha)}{2 n_1} \tag{33}$$

$$y_3^P = \frac{n_1 x_1^P + n_3 \gamma_3 + n_2(\theta/2 - \alpha)}{2 n_3}. \tag{34}$$

Solving these yields the stated unique equilibrium allocations. ■

Proof of Proposition 3. (i) Note that by Lemma 1(iii), the best responses by candidates in both parties under the stated condition are to offer only public goods when $s > 1/n_2$.

Consider the platform choices of party X candidates when $s \leq 1/n_2$. By Lemma 1(iv), for any optimal platform either $x_1^* = 0$ or $x_2^* = 0$. Observe that the left-hand side of expressions (8)-(9) in Lemma 2 are the numerators of the derivatives with respect to x_0 of the party X objectives $E_{11}(x_0, \overline{y})$ and $E_{12}(x_0, \overline{y})$ evaluated at $x_0 = 1$, where the objectives are restricted to $x_2 = 0$ and $x_1 = 0$, respectively. Since the denominators of the derivatives are strictly positive, the sign of each expression is sufficient for signing the derivative.

Evaluating (9) at $s = 1/n_2$, $\frac{dE_{12}}{dx_0}(1, \overline{y}) > 0$ if:

$$\frac{1 - y_0}{n_2} > y_2 - \alpha - \theta/2.$$

Since $\theta > \alpha \geq 0$ and the budget constraint implies $y_2 \leq (1 - y_0)/n_2$, this expression always holds. Thus by the concavity of $E_{12}(x_0, \overline{y})$, $x_0 = 1$ is the optimal strategy for party X candidates when $x_1 = 0$ and $s = 1/n_2$. By the continuity of $E_{12}(x_0, \overline{y})$ in s, there exists a nonempty set $\mathcal{S} \equiv [\underline{s}, 1/n_2)$ such that for all $s \in \mathcal{S}$, $\frac{dE_{12}}{dx_0}(1, \overline{y}) > 0$.

Now consider $E_{11}(x_0, \overline{y})$. Clearly, for all $s > 1/n_1$, the optimal strategy for party X candidates when $x_2 = 0$ is $x_0 = 1$. Since $n_1 > n_2$, the region $[1/n_1, 1/n_2] \cap \mathcal{S}$ is non-empty. Party X candidates will then choose $x_0 = 1$ regardless of whether their best response is to maximize $E_{11}(x_0, \overline{y})$ or $E_{12}(x_0, \overline{y})$ when $s \geq \min\{[1/n_1, 1/n_2] \cap \mathcal{S}\}$. Thus, for $n_1 > n_2$, party X candidates will offer only public goods for some s strictly less than $1/n_2$.

The analysis for party Y candidates is symmetric and therefore omitted. Combining the statements for both parties yields a threshold $\tilde{s}$.

(ii)-(iv) These expressions follow immediately from Lemma 1 and the derivations of (4) and (15). ■

Proof of Proposition 4. We first establish party X candidates' optimal platforms for $\gamma > \frac{(1-n_1 s)\theta}{2n_1 s}$. By Lemma 2, the solution for party X candidates is $x_1 = \max\{1/n_1, \tilde{x}_1\}$ if (8) holds, or:

$$(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \theta/2) + n_1s\gamma < 0.$$

The left-hand side of this expression is increasing in γ. Substituting in $\gamma = \frac{(1-n_1s)\theta}{2n_1s}$, this expression reduces to:

$$(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)y_2 < 0.$$

This expression cannot hold for any $s \in \left(\frac{1}{2n_1}, \frac{1}{n_1}\right)$, and thus there is no optimal platform where $x_1 > 0$ when $\gamma \geq \frac{(1-n_1s)\theta}{2n_1s}$. By Lemma 1(i), this implies that at any party X best response, the entire budget is used on x_0 and x_2, or equivalently, $n_2x_2 + x_0 = 1$.

To derive the value of x_2, we substitute $n_2x_2 + x_0 = 1$, $n_2y_2 + y_0 = 1$, and the assumed parameter restrictions on γ_1, γ_3, and α into the expressions for $\tilde{x}_2$ (6) and $\tilde{y}_2$ (17), which yields the following:

$$x_2 = y_2 + \frac{\gamma}{2n_2s} - \frac{\theta}{4(1-n_2s)} \tag{35}$$

$$y_2 = x_2 + \frac{\gamma}{2n_2s} - \frac{\theta}{4(1-n_2s)}. \tag{36}$$

There is clearly no generic interior solution for this system. Since $\frac{\gamma}{2n_2s} - \frac{\theta}{4(1-n_2s)} > (<)\ 0$ for $\gamma > (<)\ \frac{n_2s\theta}{2(1-n_2s)}$, the unique solution is $x_2 = y_2 = 1/n_2\ (= 0)$ for $\gamma > (<)\ \frac{n_2s\theta}{2(1-n_2s)}$.

Now consider the party X candidates' optimal platforms for $\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}$. Observe that $\frac{(1-n_1s)\theta}{2n_1s} < \frac{n_2s\theta}{2(1-n_2s)}$ for all $s > \frac{1}{n_1+n_2}$, which follows from the assumption that $s > \max\{\frac{1}{2n_1}, \frac{1}{2n_2}\}$. We first establish that $x_2 = 0$ for any best response. To see this, note first that the system (35)-(36) implies that for $\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}$, there can be no solution where $x_2 > 0$ and $y_2 > 0$; thus, $x_2 > 0$ requires $y_2 = 0$. By Lemma 2, party X chooses $x_2 > 0$ if and only if (9) holds, or:

$$(2n_2s^2 - s)(1 - y_0) - n_2s(y_2 - \theta/2) - (1 - n_2s)\gamma < 0. \tag{37}$$

The left-hand side of this expression is decreasing in γ. To show that (37) cannot hold for $\gamma \leq \frac{(1-n_1s)\theta}{2n_1s}$, it will be convenient to substitute in $y_2 = 0$ and $\gamma = \frac{n_2s\theta}{2(1-n_2s)}$. Then (37) can be satisfied for all $\gamma < \frac{n_2 s\theta}{2(1-n_2 s)}$ only if:

$$(2n_2 s^2 - s)(1 - y_0) < 0.$$

This expression cannot hold for any $s > \frac{1}{2n_2}$, and thus all best responses must satisfy $x_0 + n_1 x_1 = 1$. A symmetric analysis holds for party Y candidates.

To derive the value of x_1, observe that an interior solution must be given by $\tilde{x}_1$ and $\tilde{y}_1$, as defined by (4) and (15). The unique solution of this system is $x_1^* = y_3^* = \min\left\{\frac{1}{n_1}, \frac{(1-n_1 s)\theta}{2n_1 s} - \gamma\right\}$; these are clearly non-negative for all $\gamma \leq \frac{(1-n_1 s)\theta}{2n_1 s}$. ■

Proof of Proposition 5. By Lemma 1(ii), we require $s > \frac{1}{n_1+n_2}$ for $x_0 > 0$ in equilibrium. Since only x_1 or x_2 can be strictly positive at an optimal platform, it is sufficient to derive conditions under which the possible private good allocations for x_1 and x_2, $\tilde{x}_1$ and $\tilde{x}_2$, are not maximized. Observe that for $s > 1/n_i$, $x_i = 0$, and so we restrict attention to $s \leq 1/n_i$ for $i = 1, 2$. Using expression (4) and the fact that $y_2 \geq 0$, $\tilde{x}_1 < 1/n_1$ if:

$$\begin{aligned}\frac{n_1\gamma_1 + (2n_1 s - 1)(1 - y_0)}{2n_1(n_1 s - 1)} + \frac{\alpha + \theta/2 - y_2}{2n_1 s} &< \frac{1}{n_1}\\ sn_1\gamma_1 + s - s(2n_1 s - 1)y_0 + (\alpha + \theta/2)(n_1 s - 1) &> 0\end{aligned}$$

Noting that $(2n_1 s - 1)y_0$ is bounded from above by 1, this expression simplifies to:

$$s > \frac{\alpha + \theta/2}{n_1(\gamma_1 + \alpha + \theta/2)}.$$

Likewise, using expression (6) and the fact that $y_2 \leq 1/n_2$, $\tilde{x}_2 < 1/n_2$ if:

$$(n_2 s - 1)\gamma_1 - s(2n_2 s - 1)y_0 + s + n_2 s(\alpha + \theta/2 - 1/n_2) > 0$$

Noting that $(2n_2 s - 1)y_0$ is bounded from above by 1, this expression simplifies to:

$$s > \frac{\gamma_1}{n_2(\gamma_1 + \alpha + \theta/2) - 1}.$$

Combining the two expressions for s yields the result. ■

Proof of Proposition 6. Combining Lemmas 1, 2′, and 4, and expressions (4)-(18), we have the following choices for party X candidates. For $s \leq \min\{1/n_1, 1/n_2\}$, $x_2^* =$

$\min\{\max\{0, \tilde{x}_2\}, 1/n_2\}$ if $\gamma_1 \geq \overline{\gamma}_1$, and $x_1^* = \min\{\max\{0, \tilde{x}_1\}, 1/n_1\}$ if $\gamma_1 \leq \underline{\gamma}_1$. Likewise, we have the following choices for party Y candidates. For $s \leq \min\{1/n_3, 1/n_2\}$, $y_2^* = \min\{\max\{0, \tilde{y}_2\}, 1/n_2\}$ if $\gamma_3 \leq \underline{\gamma}_3$, and $y_3^* = \min\{\max\{0, \tilde{y}_3\}, 1/n_3\}$ if $\gamma_3 \geq \overline{\gamma}_3$.

For parts (i)-(iii), the interior platforms are derived from straightforward solutions of the linear systems implied by these equilibrium best response platforms. For part (iv), the system yields:

$$x_2 = y_2 + \frac{\gamma_1}{2n_2 s} - \frac{\theta + 2\alpha}{4(1 - n_2 s)} \tag{38}$$

$$y_2 = x_2 - \frac{\gamma_3}{2n_2 s} - \frac{\theta - 2\alpha}{4(1 - n_2 s)} \tag{39}$$

There is clearly no generic interior solution for this system. For γ_1 and $|\gamma_3|$ sufficiently large, the unique solution is the corner at $x_2 = 1/n_2$ and $y_2 = 1/n_2$. ■