

# An Organizational Theory of State Capacity\*

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January 7, 2023

## Abstract

A burgeoning literature recognizes that the efficacy of the state is crucial for economic growth and citizen welfare. However, much of that literature abstracts away from the institutional details underlying state capacity. We develop a theory that provides a working definition of state capacity—the ability to handle administrative problems of varying complexity, such as tax collection—and how it is provided and maintained. We conceive of the state as a knowledge hierarchy, or an information-processing institution that passes problems up a set of organizational layers until a layer with the required expertise solves it. Knowledge hierarchies are costly to establish and operate, and politicians differ in policy preferences and public goods valuations. We embed this structure in a simple political economy framework, where politicians may idle parts of the state depending on electoral prospects, thus reducing output. In conjunction with high partisanship, this gives the state designer incentives to distort the state away from efficient levels of capacity and specialization.

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\*Preliminary. We thank Charles Angelucci, Dana Foarta, Bob Gibbons and seminar audiences at Carlos III Madrid, CUNEF, Princeton, Florida State, Harvard, UCSD, Yale, the Berkeley Formal Theory and Comparative Politics, NBER Orgs, Nottingham Political Economy, and Utah Orgs and PE Conferences for helpful comments.

# 1 Introduction

Intuitive notions of state capacity have been shown empirically to be important for determining both political and economic outcomes.<sup>1</sup> Rigorous theoretical underpinnings—including a definition of state capacity which can encompass intuitive notions—have lagged behind empirics. In particular, current state-of-the-art theory equates state capacity with realized tax revenue, does not contemplate changes in state capacity over time, and attributes the dynamics of state capacity to macro-level shocks to preferences—such as wars and depressions—rather than political processes.

We propose a broader definition of state capacity—the problem-solving capacity of the state—and, with tools from organizational economics, use it to address some of the lacuna described above. In our model, problems are solved by a bureaucracy (based on the knowledge hierarchy model of Garicano, 2000), and the maintenance of that bureaucracy over time, and hence its structure, is the result of political processes. This allows us to distinguish *state capacity*—the extent of problems that a state could solve—from *output* which may be limited by political factors and imperfect state maintenance. These patterns of maintenance lead to distortions in state design; for example, if the state designer fears what the political opposition may do upon taking power, she may create inefficient structures that either insulate the state from political interference, or are excessively expensive for the opposition to operate.

Our model sheds light on the capacity of contemporary states—from roughly 1945 on, conflict has been less frequent, and consumed a smaller portion of state revenue than in prior periods. Moreover, even in the late modern period the building of state capacity was not a one-way process—it rose more or less continuously in the Britain, while alternatively building and ebbing in France (Brewer 1988). Further, it ties together notions of state capacity with its fundamental organizational structures—the bureaucracy. Finally, by introducing a more general notion of state capacity, it allows for the possibility of conceptualizing states as not only tax authorities, but rather as problem solvers that must implement a census, prosecute crimes, or grapple with the problems created by climate change or a global pandemic.

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<sup>1</sup>See Johnson and Koyama (2017) for a review.

Three sets of facts motivate the main components of our work. First, shortfalls between capacity and output are both common and deliberate. In many developing countries, tax authorities struggle to reach potential revenues (Schreiber 2019), and politicians have acted over time to diminish revenue collecting capacity. In the U.S., the Internal Revenue Service lost about 25% of its enforcement staff and 15% of its real budget between 2010 and 2017. Unsurprisingly, the audit rate for individual taxpayers plummeted from 0.9% to 0.5%. These reductions did not affect taxpayers evenly. High-income filers saw their audit rates fall in part due to the under-resourcing of auditing and investigative units, while very low-income filers who could claim the Earned Income Tax Credit were audited at higher rates. Thus, the Republican-controlled Congress managed to re-orient the IRS’s enforcement resources to avoid one type of tax avoidance in favor of another (ProPublica 2018, 2020, US CBO 2020). This is inconsistent with the extant literature that views state capacity as an investment in extractive capabilities that are never voluntarily relinquished (Besley and Persson 2009, Acemoglu, García-Jimeno, and Robinson 2015).

Second, accounts of state capacity focus on more than the ability to raise revenue, and recognize that organizations—in the form of functioning, non-corrupt judicial and executive institutions—are crucial for its realization. For example, the development literature has recently begun to focus on the role of personnel in helping the state achieve its goals (Finan, Olken, and Pande 2017). Historically, the development of a powerful, centralized Prussian state was accompanied by extensive institutional reforms that could bring the human capital of enlightenment-educated bureaucrats to handle increasingly complex problems (Johnson 1975). In the contemporary world, highly developed states are generally associated with large bureaucracies. As an example, the share of European Union national workforces employed in public service provision (possibly through private sector providers) in 2016 ranged from 20% in Romania to 40% in Denmark and Sweden (Thijs, Hammerschmid, and Palaric 2017).

Third, even similar macro shocks to preferences do not necessarily lead to similar state capacity outcomes. For example, Brewer (1988) traces the evolution of British and French state capacity over the 18th century. During this time, both nations fought numerous wars—often against each other—yet this led to a drastic increase in state capacity and

output for the British state, and the eventual bankrupting of the French state and ensuing revolution. Why? Brewer traces the difference to two important factors. First, the British were primarily a naval power, and the French primarily a land power. As navies required persistent funding for maintenance of expensive capital (ships), state output was needed even during peaceful periods. Alternatively, armies could be raised and de-mobilized relatively rapidly. This factor, coupled with a second—a relative lack of internal rivalries within Britain—resulted in a professional and centralized tax bureaucracy which could experiment with a wide array of taxation instruments. The French, instead, relied on less efficient practices, such as tax farming, that would both reward local aristocrats and could be revoked in between wars. Although the shocks to preferences were similar, their translation into state capacity and output were affected by fine details that limited, or allowed, policy persistence.

To incorporate these observations, this paper examines state capacity, and the evolution of state output, when politicians control some structural and operational parameters of state institutions. We consider a society composed of two groups that are tied to a party that represents them in government. Importantly, groups and their associated parties or politicians are internally homogenous, but have heterogeneous preferences over state output, and have the institutional tools to hobble the state.

As a theoretical foundation, we turn to *knowledge hierarchies*, which offer a simple way to model expertise and production in complex organizations (Garicano 2000, Garicano and Rossi-Hansberg 2006). In the basic formulation, an organization such as a firm faces problems with a difficulty level drawn from a known distribution on  $\mathbb{R}$ . The organization consists of a series of layers, each of which is endowed with the ability to solve a subset of possible problems. All problems “enter” through the first layer, which solves those within its knowledge set and sends the remainder on to the next layer at a cost. Each successive layer behaves in a similar fashion, until no layers are left. The central trade-off in designing knowledge hierarchies is between personnel and operational costs. Since the workers in each layer must be capable of handling that layer’s most complex task, more layers increase specialization and decrease personnel costs. But more layers also increase the average number of times that problems are passed up, increasing operational costs.

Our game begins with a founding period, which represents an initial shock that enables

the creation of a new state function. In it, one group establishes the knowledge hierarchy. This is the basic structure of the state, which determines both the extent of its possible problem-solving ability, or its capacity, and its personnel costs. In every subsequent period, each group attains political power with exogenous probability. While in power, the incumbent party receives a signal about the likelihood of maintaining political power, and then decides which layers of the knowledge hierarchy to activate or idle for the following period. Activated layers can solve problems but impose operational costs. While the model is general, we can analyze closed forms using a special case in which activating and idling layers is costless. Personnel and operational costs are shared by both groups through a tax that falls evenly on the members of both parties. Importantly, idle layers still impose personnel costs. This reflects the idea that governmental leaders can temporarily subvert state offices more easily than they can fire civil servants or neutralize public sector unions.

In each period, the problem-solving done by the set of active layers translates directly into the realized production of a public good, or output. Parties differ in their valuations of the public good, and also value the public good more highly when they are in power than when they are not. We refer to the latter difference, resulting from differences in policy preferences between the incumbent and opposition groups, as *partisanship* (Kasara and Suryanarayan 2020). Thus, when deciding whether to diminish or enhance state capacity the group in power faces a trade off. Enhancing state capacity will be beneficial if the incumbent is returned to power, or if the incumbent’s partisanship is low. Frequent transitions of power, or large policy differences between the parties, will lead the incumbent to reduce output below the state’s capacity.

Our first results consider a politics-free setting, where the state designer does not need to worry about idling. The resulting state has intuitive properties. As in Besley and Persson (2009), capacity increases with the prospects of retaining power. Because of training costs, the knowledge hierarchy that solves the easiest problems and possibly excludes the most difficult ones. Low operational costs add layers and increase the state’s specialization. Capacity increases when partisanship is low, and a specialized, high-capacity, high-output state emerges when there is high consensus over the value of public goods. Structurally, the primary feature of the knowledge hierarchy is that capacity and depth are related: higher

capacity states are also more specialized states.

How does state maintenance work? We next take a given state hierarchy and derive strategies for activating and idling layers. In equilibrium, each party activates more layers as its electoral prospects improve. Because of their higher operational costs, the layers associated with the most difficult problems are idled first, and thus are least likely to be active over time. High operational costs, partisanship, and differences in public goods valuations all amplify the consequences of elections and reduce the chances that a given layer will stay active. There is no general relationship between political competitiveness and a layer's average output. State maintenance is highest in politically competitive societies when there is general agreement over policy, and lowest in politically competitive societies in the presence of disagreement.

Our main results show that the political process can distort the design of the state in several important ways. These distortions break the link between capacity and specialization. When the opposition's public goods valuation is so low that it rarely activates any layer, the founding group creates a small but specialized state and simply acquiesces to the loss of state functions when the opposition gains control. If the opposition becomes willing to activate the lowest layers, then a designer with low partisanship insulates the state by reducing depth and reallocating capacity downwards to the lowest layers. This under-specialized design exploits the opposition's willingness to allow the most basic governmental functions, creating a "bottom heavy" state. By contrast, a highly partisan founding party shifts capacity toward the top layers and increases specialization instead. In this case, high operational costs deter the opposition from performing too many state functions. This produces a smaller but "top heavy" state that obstructs the opposition from accomplishing too much.

Partisanship also plays a role in the relationship between capacity and the probability of holding power. Holding power is especially important to the state-builder when partisanship is high, and thus capacity is increasing in the probability they maintain power. This relationship becomes weaker, and may even reverse, as partisanship declines. This relationship becomes weaker, and may even reverse, as partisanship declines.

This paper complements an extensive literature on state capacity in economics and historical political economy (Tilly 1990, Besley and Persson 2011, Johnson and Koyama 2017, Berwick and Christia 2018). Much of this work focuses on the development of states across broad swaths of time, and on war as a driver of capacity. Moreover, the economic literature has focused on the raising of revenue as a measure of state capacity. Both war and taxes are clearly of first-order importance. To this literature we add the ability to examine the ebbs and flows of state capacity over relatively short time periods in response to political factors. Moreover, we explicitly tie state capacity to the instrument of its achievement, the bureaucracy.

The more limited theoretical literature on state and bureaucratic capacity has tended to focus on the state as the product of investment in extractive ability. For example, in Besley and Persson (2009), politicians invest in legal and taxation capabilities in anticipation of future needs, while in Acemoglu, García-Jimeno, and Robinson (2015), central and local politicians jointly invest in extractive capabilities in an environment with local spillovers. Gennaioli and Voth (2015) treat centralization as a main determinant of extraction, and model a ruler’s decision over whether to overcome local resistance to centralization. Another approach considers capacity as a parameter in agency problems involving the bureaucracy (e.g., Huber and McCarty 2004, Ting 2011, Foarta 2021). To our knowledge, the existing theoretical work does not explore the implications of internal organizational structure, or the existence of political actors expressly interested in diminishing state capacity.

Our conception of state capacity as problem-solving ability helps to unify a range of related policies or institutional features. In addition to revenue extraction, studies have emphasized legal protections, fiscal centralization, personnel levels, personnel quality, and general administrative resources (Derthick 1990, Besley and Persson 2009, Brown, Earle, and Gehlbach 2009, Grindle 2012, Dal Bó, Finan, and Rossi 2013, Fukuyama 2013, Dincecco and Katz 2014, Bolton, Potter, and Thrower 2016, Garfias 2018). The focus on organizational structure also allows the possibility of integrating the insights of studies that address the institutional details of tax collection (Almunia and Lopez-Rodriguez 2018, Bachas, Jaef, and Jensen 2019, Cullen, Turner, and Washington 2021).

Our paper is also complementary to the large literature on the bureaucracy in political

science, and to a more limited extent, economics. In the organizational economics literature, knowledge hierarchies explicitly sidestep the agency problems that are endemic to firms (Garicano 2000), and have received frequent attention (e.g., Moe 1989, Gailmard and Patty 2012, Gibbons and Roberts 2014). While this naturally limits the range of organizational problems that can be examined, the framework has the virtues of endogenizing internal organizational structure and allowing for a tractable characterization of output. In fact, its central tension between specialization and generalization is reflected in historical decisions about the design of government personnel systems (e.g., Silberman 1993). Our paper also makes a limited contribution to the organizational economics literature as well: aside from Garicano and Rossi-Hansberg (2012), ours is the only one we are aware of to examine knowledge hierarchies across time. While their model is built on a competitive equilibrium framework, ours is built on one with exogenous political frictions.

We organize the paper as follows. The next section introduces the basic model. Section 3 establishes a base case in which a state designer chooses a knowledge hierarchy in the absence of political interference. Section 4 takes the state structure as given and works through how parties activate and idle layers of the knowledge hierarchy. Section 5 uses these results to derive the design of the knowledge hierarchy. Next, Section 6 examines comparative statics on state capacity. Section 7 concludes with a discussion of how our model can be used to understand some of the examples provided above.

## **2 Model: Problems and Knowledge Hierarchies**

We develop a simple model of the establishment and evolution of state capacity over an infinite horizon. State capacity is useful for addressing a range of policy problems depending on how it is deployed by the government. A particularly salient class of policy problems is the collection of tax revenue, which we use throughout as a running example. Much of the model is a fairly straight-forward instantiation of the knowledge hierarchy model of Garicano (2000). Our major departure is the ability of governing parties to sabotage the state by idling specific layers of the knowledge hierarchy.



**Parties:** The players are two infinitely-lived parties  $k \in \{1, 2\}$  that represent different groups in society, each consisting of a share  $r_k$  of the (unit measure of) population. Both parties discount the payoff in future periods by a factor  $\delta \in (0, 1)$ . Throughout we will use the terms party and group interchangeably. This terminology does not necessarily imply that the society is a democracy: groups may lose or gain power through different means. In each period  $t$ , party  $k$  is in power with probability  $r_k$ .

In each period Nature reveals a realized probability  $\rho_{k,t}$  that  $k$  will be in power the next period. This probability is drawn independently according to distribution  $F_k(\cdot)$  in each period. The expected probability of being in power is  $\mathbb{E}[\rho_{k,t}] = r_k$ , and the complementary probability that  $k$ 's opposition is elected is  $1 - \rho_{k,t}$ . For some results and numerical applications, we adopt the following simple assumption about the distribution of probabilities of holding power.

**Assumption 1** [Uniformly Distributed  $\rho$ .]  *$F_k(\cdot)$  is uniform with support  $[r_k - \varepsilon, r_k + \varepsilon]$ , where  $\varepsilon \in (0, \min\{r_k, 1 - r_k\})$ .*

**Knowledge Hierarchies:** To perform tasks such as revenue collection in each period, the state has to solve a continuum of revenue collection problems. Each problem has a difficulty level  $z \in Z \equiv [0, \bar{Z}]$ , with lower values of  $z$  representing simpler problems. Problems are distributed according to  $G(z)$  with corresponding density  $g(z)$  on  $Z$ . Note that  $G(z)$  is un-normalized, that is,  $\int dG(z)$  is generally not equal to one. Throughout we assume  $G(z) \sim \bar{Z} \times U[0, \bar{Z}]$ , so that each unit of problem space contains a unit measure of problems. However, we maintain the  $G(\cdot)$  notation to clarify the role of the distribution of problems.

Problem solving is performed by an organization, called a knowledge hierarchy, that divides its workers into an integer number  $J \geq 0$  of (possibly disjoint) layers. Each layer  $j$  is associated with a set  $Z_j = [z_j, \bar{z}_j] \subseteq Z$  of problems that it can solve. Each solved problem results in an equal amount of tax revenue for the incumbent politician to spend. The measure of workers required to address layer  $j$ 's problems is  $G(\bar{z}_j) - G(z_j)$ . Workers within a layer cannot discriminate between problems. Let  $\mathcal{Z} = \cup_i Z_i$  represent the union of knowledge sets, which is equivalently the set of problems solvable by the knowledge

hierarchy. We will refer to  $J$  as the knowledge hierarchy's *depth*, and to  $Z_j$  as layer  $j$ 's *knowledge set*. Knowledge sets are ordered in increasing sequence according to  $z_j$ .

A knowledge hierarchy solves problems by passing them up through the layers until it reaches a layer that can address it. Each layer may be *active* or *idle*, where the former can solve problems in its knowledge set and the latter cannot. The knowledge hierarchy automatically rejects any problem that cannot be addressed by one of its active layers.

To give a concrete example, suppose that all layers are active in a given period and a new problem is drawn with complexity  $z$ . If  $z \notin \mathcal{Z}$ , then the problem is automatically dropped from consideration. If  $z \in Z_1$ , then the problem is solved by layer 1. Otherwise, layer 1 passes the problem on to layer 2. Layer 2 addresses the problem if it is in  $Z_2$ , or else passes it on to layer 3, and so on until a problem layer  $J$  is reached. Observe that if  $z_{j+1} < \bar{z}_j$ , then the bureaucrats in level  $j + 1$  will never process any problems with  $z < \bar{z}_j$ . Thus, we assume without loss of generality that  $z_{j+1} \geq \bar{z}_j$ .

**Costs:** The state structure imposes two kinds of costs, which are born equally by all members of a (unit measure) population. First, a problem of difficulty  $z$  must be handled by a bureaucrat with personnel cost  $cz$  each period, where  $c > 0$ . As bureaucrats within a layer cannot discriminate among received tasks, they must be able to solve any problem in their knowledge set  $Z_j$ , as well as all easier problems. The total personnel cost of layer  $j$  over all of time is thus

$$\sum_{t=1}^{\infty} \delta^t c \bar{z}_j \int_{z_j}^{\bar{z}_j} dG(z) = \frac{\delta}{1 - \delta} c \bar{z}_j (G(\bar{z}_j) - G(z_j)) \quad (1)$$

Specialization (in the sense of having multiple layers) therefore economizes on total personnel costs, as fewer workers need to be trained and paid to solve the most difficult problems. To reflect the difficulty of firing government personnel (whether due to unions, political opposition, civil service rules, or bureaucratic inertia), this cost is fixed when the state is established and incurred for each layer in every period, regardless of whether the layer is idle.<sup>2</sup>

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<sup>2</sup>This is qualitatively equivalent to  $c$  representing a one-time personnel cost. If personnel costs are not incurred when a layer is idled this will change the incentives somewhat for idling and activating layers, although a suitably modified version of Proposition 2 will still hold. Other propositions will be qualitatively

Second, processing each problem imposes operational costs, which may represent a reduced form for internal agency problems. We implement this in a simple way by assuming a cost  $h > 0$  whenever a layer receives a problem to process, regardless whether it is actually able to solve it.<sup>3</sup> There is no cost for a layer to “send” a problem on to the next layer. Thus, any problem solved by layer  $j$  ultimately incurs a search cost of  $jh$ . The design of an optimal knowledge hierarchy balances operational and personnel costs: increasing layers saves on personnel costs, but creates higher operational costs, whereas decreasing layers saves on operational costs, but at the expense of higher personnel costs.

**Benefits:** The expected utility of a member of party  $k$  from solving problems in layer  $j$  in period  $t$  is:

$$\int_{z_j}^{\bar{z}_j} w_{k,t}^l dG(z) = w_{k,t}^l (G(\bar{z}_j) - G(z_j)). \quad (2)$$

This benefit depends on the parameter  $w_{k,t}^l$ , which is the marginal value of government spending for party  $k$  in period  $t$  when party  $l$  is in power. Its possible values are:

$$w_{1,t}^l = \begin{cases} 1 & \text{if } l = 1 \text{ is in power} \\ 1 - \pi_1 & \text{if } l = 2 \text{ is in power} \end{cases} \quad w_{2,t}^l = \begin{cases} q(1 - \pi_2) & \text{if } l = 1 \text{ is in power} \\ q & \text{if } l = 2 \text{ is in power} \end{cases} \quad (3)$$

The parameter  $q > 0$  represents the fact that different groups may differ in their valuation of public goods or government policy. As it is possible for  $q > 1$ , either party could value policy more highly. Thus, it is without loss of generality to consider the case where Party 1 constructs the knowledge hierarchy. Note that if  $jh > w_{k,t}^l$ , Party  $k$  receives negative marginal value from layer  $j$ .

The parameters  $\pi_k \in [0, 1]$  represent *partisanship*, or differences in policy preferences between the two groups that we allow to be asymmetric (for example, Mann and Ornstein 2012, Grossman and Hopkins 2016). High values of  $\pi_1$  and  $\pi_2$  correspond to an environment in which parties can target resources to their supporters, for example when groups are highly aligned with geographic or cultural cleavages. Low values of  $\pi_1$  and  $\pi_2$  correspond to unaffected.

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<sup>3</sup>Idle layers continue to impose personnel costs and operational costs for passing problems upwards, but do not incur operating costs for problems it would have solved if active, since those problems are dropped. Such costs might arise from the activities by a “skeleton staff” that can pass problems along but cannot solve them.

a situation where both parties favor public goods that are valuable to both groups. Finally, if one value is low, and the other is high, then one group favors universalistic programs, while the other prefers to target its own supporters. We say that the groups are *polarized* if some  $\pi_k$  is high.

**State Maintenance, Capacity, and Output:** A central feature of the model is the endogeneity of the state’s problem-solving capacity over time. The structure of the state—the depth  $J$  and the boundaries of layers  $z_j$  and  $\bar{z}_j$  for all  $j \leq J$ —is established at the beginning of the game. However, upon observing its probability of staying in power in the next period, each incumbent has some control over which layers will function in the following period.

From the perspective of the state designer, activation and idling are important for their effects on the long-run average probability of each layer remaining active. We define the ex ante average probability that Party  $k$  activates a given layer  $j$  by as  $\sigma_k^j = (1 - \delta) \sum_{t=1}^{\infty} \delta^t \mathbb{E}[a_{k,t}^j]$ , where  $a_{k,t}^j = 1$  (0) represents the choice by Party  $k$  to activate (idle) layer  $j$  in period  $t$ , conditional upon holding power. We assume that  $\sigma_k^j$  has the following properties:

- (i)  $\sigma_k^j$  is decreasing in  $j$  and independent of all  $z_j, \bar{z}_j$
- (ii)  $\frac{\partial \sigma_2^j}{\partial q} > \frac{\partial \sigma_2^{j'}}{\partial q}$  for  $j > j'$  if  $\sigma_2^j \in (0, 1)$ ; and  $\frac{\partial \sigma_2^j}{\partial q} \geq 0$ , with strict inequality for  $\sigma_2^j \in (0, 1)$
- (iii)  $\frac{\partial \sigma_k^j}{\partial r_k} \geq 0$ ,  $\frac{\partial \sigma_k^j}{\partial r_{-k}} \leq 0$ ,  $\frac{\partial \sigma_k^j}{\partial \pi_k} \leq 0$ , and  $\frac{\partial \sigma_k^j}{\partial \pi_{-k}} \geq 0$

Property (i) simply states that lower layers are less likely to be active, and that a layer’s activation rate is unaffected by the problems it solves. The activation rate can instead be driven by the benefits and costs of problems solved there. By (ii), Party 2’s activation rates are increasing in  $q$  at interior values, with higher layers showing more responsiveness than lower layers. And by (iii), each party’s likelihood of activating any layer weakly increases with its probability of holding power improves and lack of polarization, and weakly decreases with the other party’s. Various microfoundations are consistent with these properties. For the numerical examples and some results we adopt Assumption 2, which essentially states that parties can choose any layers to be active in the following layer without constraint.

**Assumption 2** [Costless Activation and Idling.] *Both parties can activate or idle any layer at zero cost.*

As we show in Section 4, this produces equilibrium strategies that are consistent with properties (i)-(iii), and also allows for closed-form solutions of party strategies.

We refer to the process of idling layers as *sabotage*, and the general process of activating and idling layers as *state maintenance*. We define *state capacity* as the total measure of problems that the state could solve, if all layers were open. In any period, *output* is the measure of all problems that the active layers of the state actually solve. Output is always weakly less than total state capacity, and we refer to the difference as the *output gap*.

**Timing:** The party power in period  $t = 0$  can create the structure of the state  $\mathcal{Z}$  that will operate for the remainder of the game. The strategy space for Party 1 in period  $t = 0$  is the powerset of all compact intervals of  $Z$ . The timing of the rest of that stage, and the ones that follow is:

1. The party in power raises revenue with its inherited state, and allocates that revenue towards their policy aim(s).
2.  $\rho_{k,t}$  is realized.
3. The party in power determines which layers of the state will be idled or active in the next period.
4. A power transition may occur, with the party in power determined according to probability  $\rho_{k,t}$ .

The party  $k$  stationary strategy for activation and idling in each period is  $\mathbf{a}_k : [0, 1] \times \{0, 1\}^J \rightarrow \{0, 1\}^J$ , mapping realizations of the probability of holding power and the set of current active layers to the set of (future) layer configurations. In the special case of Assumption 2, the set of currently active layers is irrelevant and no state variable is necessary. In a game where the set of active layers in the subsequent period depends on active layers in the current period, strategies a Markov Perfect equilibrium will depend on the configuration of active layers.

### 3 Baseline Model: No Sabotage

We solve for the structure of the state, as a function of the parameters  $\pi_k$ ,  $q$ ,  $r_k$ ,  $\varepsilon$ ,  $\delta$ ,  $c$ , and  $h$  in three steps. First, in this section, we solve for the optimal state hierarchy without the possibility of sabotage: every layer is activated in every period, regardless of who is in charge (i.e.,  $\sigma_k^j = 1$  for both  $k$  and all  $j$ ). Solving this model will provide a useful baseline. The next section examines the state maintenance decisions of both parties, and establishes conditions under which state maintenance will be high or low—that is when average output will be close to, or far from, capacity. Finally, in Section 5, we solve for the optimal knowledge hierarchy in the presence of sabotage, and examine the ways in which the possibility of political sabotage causes very different patterns of state design.

The absence of sabotage means that output will be constant across periods, making it much easier to solve for the optimal state structure. That solution is further aided by the fact that, as a special case of the model with sabotage, the knowledge hierarchy will have no gaps (that is,  $\bar{z}_j = z_{j+1}$  for  $j = 1, \dots, J-1$ ), and  $z_1 = 0$ , as established in Lemma 1.

The expected benefits and costs of a particular knowledge hierarchy in the baseline case are quite simple. In particular, the expected per-period benefit of a given state structure to party  $k$  is given by

$$B_k(z) \equiv G(z) \sum_{l=1}^2 r_l w_k^l,$$

which varies only with  $z$ , as Lemma 1, plus the assumption of no sabotage, implies that all problems in  $[0, z]$  will be solved in every period. While the benefit to members of group  $k$  will depend on who is in power in a given period, the distribution of party power, and thus the expected benefit of state capacity, is fixed.

Further, we can write the cost of a state with capacity  $z$  as:

$$C(z) \equiv \min_{J; \bar{z}_1, \dots, \bar{z}_J} \sum_{j=1}^J (jh + c\bar{z}_j)(G(\bar{z}_j) - G(\bar{z}_{j-1})).$$

As this special case of our fuller model, the fact that  $C(z)$  is increasing and convex in  $z$  is implied by the proof of Lemma 2 and Proposition 5. As, in our setup, Party 1 always designs the state (w.l.o.g), that the state design will be determined by setting  $B'_1(z) = C'(z)$ . This

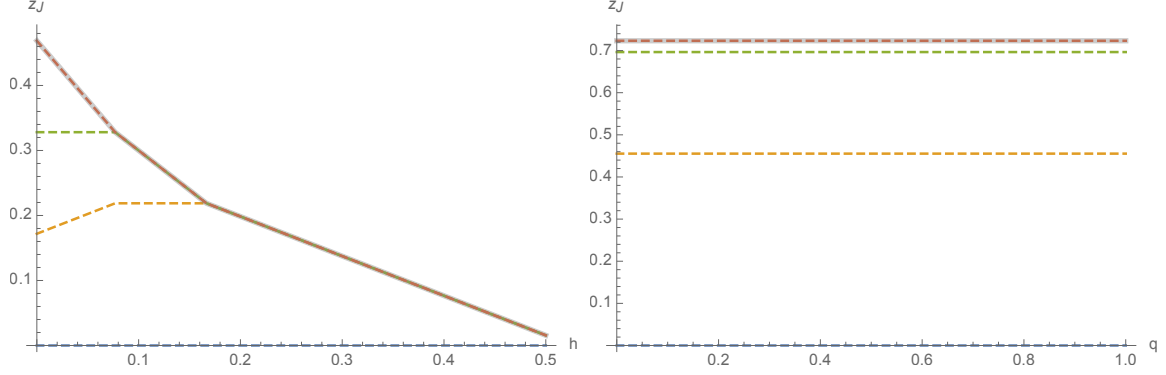


Figure 1: Baseline Knowledge Hierarchies and Output as a Function of  $q$ . Parameters are  $r_1 = 0.5$  for both panels, and  $\pi_1 = 0.05$ ,  $c = 0.8$  on the left, and  $\pi_1 = 0.1$ ,  $c = 0.7$ ,  $h = 0.15$  on the right. Dashed lines indicate layer boundaries, and gray solid lines indicate average output over time.

makes it straight-forward to solve for comparative statics of state capacity.

**Proposition 1** *Under the baseline model setting in which  $\sigma_k^j = 1$  for all  $j \in J$  and  $k \in \{1, 2\}$  state capacity is:*

- (i) *Monotonically increasing in  $r_1$ ,*
- (ii) *Monotonically decreasing in  $\pi_1$ ,  $c$ , and  $h$ ,*
- (iii) *Unaffected by  $q$ ,  $\pi_2$ ,  $\varepsilon$ , and  $\delta$ .*

*Further, state complexity is weakly increasing when state capacity is increasing, and weakly decreasing when state capacity is decreasing.*

The proof of this result is straightforward: any parameter that causes the benefit of state capacity to Group 1  $B_1(z)$  to decrease, or the cost  $C(z)$  to increase, causes state capacity to decrease, and vice-versa. For example, an increase in  $h$  causes  $C(z)$  to increase for all values of  $z$ , and consequently reduces state size and complexity, as shown in the left-hand panel of Figure 1. As  $q$  and  $\pi_2$  only affect the benefits of state capacity for Party 2, they do not affect a state constructed by Party 1 at all, and thus state structure is constant in these parameters, as illustrated in the left-hand panel of Figure 1. In the full model, when Party 2 can engage in political sabotage by idling some layers when it is in power, parameters only affecting Party 2 will affect state design.

The structure of Proposition 1 can also be used to speculate about how a social planner might construct and maintain a state, without fully specifying its objective. In particular, it would not make sense for a social planner to construct capacity they did not intend to use in some period, thus, the social planner’s problem will have some resemblance to our baseline model in that there would be no idling. Moreover, when the cost of state capacity increased ( $c$  or  $h$  increased) the social planner would decrease the size of the state. Finally, and in contrast to Proposition 1, it is likely that a social planner would take both parties’ values of public goods into account, so state capacity would be increasing in  $q$ , and decreasing in  $\pi_2$  (similar to  $\pi_1$ ). Thus, Proposition 1 can serve as baseline comparative statics for both a “no sabotage” case, and for what would happen with state capacity under some sort of social planner. However, in the presence of sabotage, state design becomes much more complex. Thus we turn to the state maintenance strategies of both parties when idling is possible, before turning out attention to a full characterization of state design.

## 4 State Maintenance

As our opening discussion illustrates, existing state structures are subject to political manipulation, which may lead to values of  $\sigma_k^j$  less than one. To see how political manipulation achieves this, we characterize stationary strategies for activating and idling the layers of a given knowledge hierarchy  $\mathcal{Z}$ . To maintain tractability, this section uses Assumptions 1 and 2.

### 4.1 Activating and Idling

Because idling and activating are costless, the current set of layers in either state is irrelevant for future payoffs. As such, we derive a strategy that only depends on the characteristics of the knowledge hierarchy (number of layers, and their boundaries), and the probability that party  $k$  is in power in the following period(s)  $\rho_{k,t}$ .

The period  $t + 1$  expected value of an active  $j$ th layer  $Z_j = [z_j, \bar{z}_j]$  to an incumbent party  $k$  is the measure of problems solved by that layer ( $G(\bar{z}_j) - G(z_j)$ ), times the value to individuals in that group— $w_k^l$ , which will depend on who is in power in the next period—of



public goods paid for with the revenue generated from solving that measure of problems:

$$(\rho_{k,t}w_k^k + (1 - \rho_{k,t})w_k^{-k} - jh)(G(\bar{z}_j) - G(z_j)).$$

As personnel costs ( $c$ ) are effectively sunk, the marginal value of problems solved in a given layer depend only on communication costs, which increase with a layer's depth  $j$ . As such, only the highest layers of any knowledge hierarchy will be idled.

Define  $p_k^j$  as the realized value for party  $k$  of  $\rho_{k,t}$  that makes it indifferent between keeping either the lowest  $j - 1$  or lowest  $j$  layers active. Denote by  $\bar{v}_k$  and  $\underline{v}_k$  the ex ante average payoff party  $k$  expects conditional upon winning and losing, respectively. Finally let  $\bar{v}_k^j$  and  $\underline{v}_k^j$  denote the Party  $k$  continuation value conditional upon winning and losing, respectively, when the party in power in the current period inherits  $j$  active layers.

This produces a system of  $2(2J - 1)$  linear equations. The expected value for each  $j$  and  $k$  can be written as:

$$\begin{aligned}\bar{v}_k^j &= \sum_{i=1}^j (w_k^k - ih)(G(\bar{z}_i) - G(z_i)) + \delta(r_k \bar{v}_k + (1 - r_k) \underline{v}_k) \\ \underline{v}_k^j &= \sum_{i=1}^j (w_k^{-k} - ih)(G(\bar{z}_i) - G(z_i)) + \delta(r_k \bar{v}_k + (1 - r_k) \underline{v}_k).\end{aligned}\tag{4}$$

These expressions omit personnel costs ( $c$ ), as those are incurred whether a party wins or losses, and hence do not affect idling and activating. Party  $k$ 's expected payoff for a depth- $j$  knowledge hierarchy and realized probability of retaining power  $\rho_{k,t}$  is then  $\rho_{k,t} \bar{v}_k^j + (1 - \rho_{k,t}) \underline{v}_k^j$ .

Next, the ex ante expected values are:

$$\begin{aligned}\bar{v}_k &= \sum_{j=1}^{J-1} \left( F_k(p_k^{j+1}) - F_k(p_k^j) \right) \bar{v}_k^j \\ \underline{v}_k &= \sum_{j=1}^{J-1} \left( F_{-k}(p_{-k}^{j+1}) - F_{-k}(p_{-k}^j) \right) \underline{v}_k^j.\end{aligned}\tag{5}$$

Under the assumption that realized election probabilities are distributed uniformly on  $[0, 1]$ , the parenthesized expressions in (5) become  $p_k^{j+1} - p_k^j$  and  $p_{-k}^{j+1} - p_{-k}^j$ , respectively.

For any  $j$  between 1 and  $J$ ,  $p_k^j$ —the probability of returning to office that makes  $k$  indifferent between keeping  $j - 1$  and  $j$  layers active—must satisfy:

$$p_k^j \bar{v}_k^{j-1} + (1 - p_k^j) v_k^{j-1} = p_k^j \bar{v}_k^j + (1 - p_k^j) v_k^j, \quad (6)$$

Note that as  $p_k^j$  is a stationary indifference condition, it is independent of the distribution of election probabilities. As such, the expected probability—before  $\rho_k$  is realized—Party  $k$  activates layer  $j$  is  $\sigma_k^j \equiv 1 - F_k(p_k^j)$ . Solving for  $p_k^j$  produces the following interior probability cutoff:

$$p_k^j = \frac{w_k^{-k} - jh}{w_k^{-k} - w_k^k} = \begin{cases} -\frac{(1 - \pi_1) - jh}{\pi_1} & \text{for Party } k = 1, \\ -\frac{q(1 - \pi_2) - jh}{q\pi_2} & \text{for Party } k = 2, \end{cases} \quad (7)$$

where (7) follows from substituting in the value functions (4) into (6).

These thresholds, and the associated probabilities of activation, vary in equilibrium only in response to  $j$ —with higher  $j$ 's requiring a higher threshold, and thus having a lower probability of activation. They are fixed across periods due to the absence of activation or idling costs, and are also independent of the distribution of problems  $G(z)$ .

Expression (7) conveys several intuitions about activation and idling. Since the denominator of (7) is always negative, Party  $k$  always activates layer  $j$  if  $w_k^{-k} > jh$ . If, on the other hand,  $w_k^{-k} < jh$ , then the cutoff will be increasing, and probability of activation decreasing, in partisanship  $\pi_k$ .<sup>4</sup> Intuitively, partisanship makes maintaining the state less attractive as the opposition provides relatively little policy benefit relative to the cost of its operation. When  $jh > 1$ , Party 1 will never activate layer  $j$  along the equilibrium path. As Party 1 will not design a state to include a layer it never uses, we can use this fact to establish a loose, but useful bound on  $J$ :  $J < (1 - \pi_1)/h$  (and hence  $jh < 1 - \pi_1$  for all  $j < J$ ).

A major driver in the difference between the thresholds used by the parties comes from the asymmetry of valuation of public goods  $q$ . The thresholds of Party 2 are decreasing—and corresponding probabilities of activation  $\sigma_2^j$  increasing—in  $q$ . When partisanship is symmetric ( $\pi_1 = \pi_2$ ), Party 2 is less inclined to maintain the state when it values public goods less than Party 1 ( $q < 1$ ), but more inclined to do so when  $q > 1$ .

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<sup>4</sup>This is true if  $1 - jh > 0$ , which is implied by  $J < 1/h$ .

Figure 2 illustrates a party's expected value for different values of  $\rho_{k,t}$ . The function is piecewise linear and convex, which reflects the fact that the stakes of party control are greater when more layers are active. Thus, more layers are activated as the party's prospects for holding power improve.

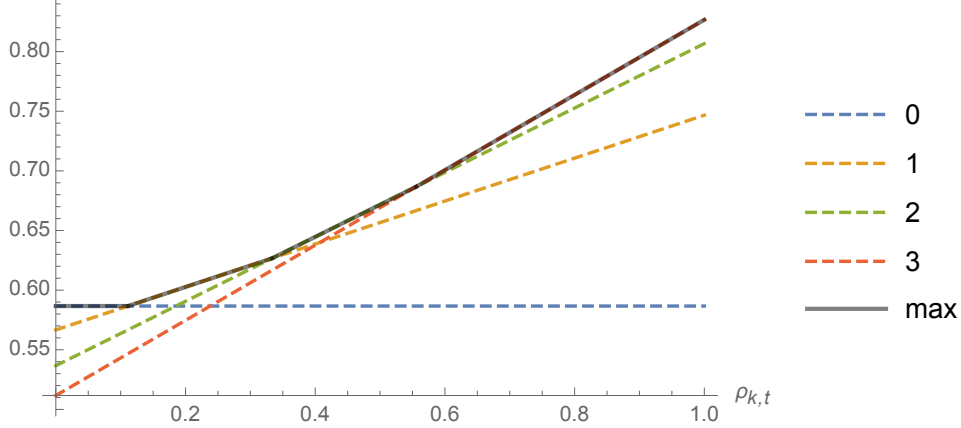


Figure 2: Value Functions by Depth,  $J = 3$ . Each line plots continuation value for 0, 1, 2, 3 layers, given equilibrium future choices. Parameter values are  $h = 0.2$ ,  $r_k = 0.5$  ( $\rho_{k,t} \sim U[0, 1]$ ),  $q = 1.0$ ,  $\pi_1 = \pi_2 = 0.1$ , and the knowledge hierarchy layers are  $(0, 0.2, 0.3, 0.35)$ . The  $p_k^j$  cutoffs for choosing 1, 2, and 3 layers are  $1/9$ ,  $3/9$ , and  $5/9$ , respectively.

## 4.2 Comparative Statics on Maintenance

We now examine how average activation probabilities change with respect to various parameters. These probabilities will affect the period 0 value of constructing a layer of given depth and width—in addition to some parameters affecting this decision directly—as discussed in the next two sections.

Along the equilibrium path, layer  $j$ 's expected output is its probability of activation, or:

$$\sum_k r_k \sigma_k^j = 1 - r_1 F_1(p_1^j) - r_2 F_2(p_2^j). \quad (8)$$

That is, the average probability that a layer is active is simply the probability that Group 1 has a realized probability of maintaining power high enough to warrant (re-)activation, multiplied by the probability that the group is in power in the first place, plus the probability

that Group 2 has a realized probability of maintaining power high enough to (re-)activate that layer, times the probability that they are in power.

Our first result uses this expression to establish some basic comparative statics on the probability of activating some layer  $j$  for a given state structure.

**Proposition 2** [Activating and Idling.] *For a state represented by knowledge hierarchy  $\mathcal{Z}$ , the probability that layer  $j$  is active is:*

- (i) *Weakly decreasing in  $h$  and  $\pi_k$ , and weakly increasing in  $q$ ,*
- (ii) *0 for an interval of  $r_1$  only if  $q < \frac{jh\pi_1}{\pi_1 + \pi_2((1-jh) - \pi_1(1-2\varepsilon))}$ . This interval is internal to  $(0,1)$  only if  $jh < q(1 + \varepsilon\pi_2)$ .*
- (iii) *1 for an interval of  $r_1$  only if  $q > \frac{jh\pi_1}{\pi_1 + \pi_2((1-jh) - \pi_1(1+2\varepsilon))}$ . This interval is internal to  $(0,1)$  only if  $jh > \max[1 - \pi_1(1 + \varepsilon), q(1 - \pi_2(1 + \varepsilon))]$ .*

A number of parameters have straight-forward and intuitive effects on state maintenance, as described in part (i) of the proposition. With high partisanship ( $\pi_k$ ), and low public goods valuations ( $q$ ), the prospect of an opposition-run state can be unappealing an incumbent parties. An incumbent that will likely lose power will thus be inclined to idle the highest layers of the state. High operational costs ( $h$ ) reduce the value of active layers further and therefore have an effect similar to increasing partisanship.

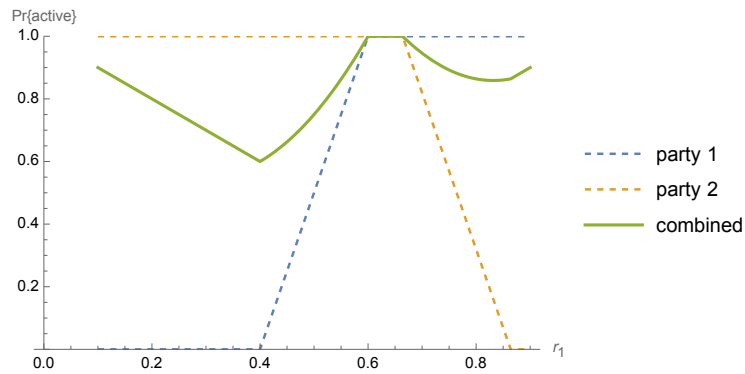
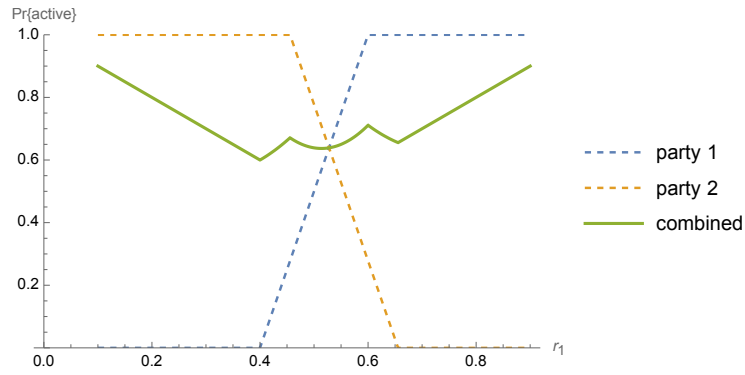
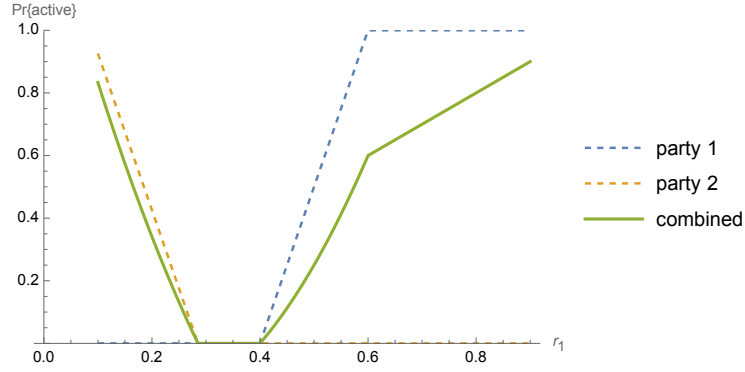
On the other hand, state maintenance is always non-monotonic in the probability that one party or the other ( $r_k$ ) remains in power, although moderate values of those parameters may either minimize or maximize realized output.<sup>5</sup> In particular, parts (ii) and (iii) show that moderate values of  $r_k$  can be associated with a layer being always idled ( $\sigma_k^j = 0$ ) or always activated ( $\sigma_k^j = 1$ ). These occur when  $r_k + \varepsilon \leq p_k^j$  or  $r_k - \varepsilon \geq p_k^j$ , respectively. Parts (ii) and (iii) then follow from finding conditions under which the ranges of  $r_k$  overlap, but do not contain the entire  $(0, 1)$  interval.

To understand why moderate values of  $r_k$  can be associated with a layer being either always idled or always activated, we focus on the public-goods valuation ( $q$ ) of Party 2, and illustrate its workings in Figure 3. When Party 2 does not value public goods highly ( $q$  low)

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<sup>5</sup>Note that the probability of a transition of power is given by  $2r_1(1 - r_1)$ , which is maximized at  $r_1 = 1/2$ , and minimized at  $r_1 = 0$  or  $1$ .

Figure 3: Maintenance may be highest or lowest in societies with a high-probability of power transitions, depending on  $q$ .



Notes: All three panels use  $j = 1$ ,  $h = 0.5$ ,  $\varepsilon = 0.1$ ,  $\pi_1 = 1$ , and  $\pi_2 = 0.9$ . In Panel A  $q = 0.6$ , in Panel B  $q = 1$ , and in Panel C  $q = 1.6$ .

then it will only activate layers when it is relatively sure of re-election. At the same time, if  $\pi_1$  is high, Party 1 will only activate the layer if it is relatively sure to stay in power. This implies that a given level of the knowledge hierarchy may cease function when  $r_1$  is moderate, as shown in Panel A. In other words, a dominant party or group is needed to maintain a highly-functional state. On the other hand, when Party 2 values public goods at a relatively equal level to Party 1, it will become willing to keep all layers active, even when its chances of staying in power are modest, making moderate values of  $r_1$  best for state maintenance, as shown in Panel C. In between these two extremes, moderate probabilities of maintaining power lead to moderate levels of state maintenance, as shown in Panel B.

Election probabilities also interact with the other parameters. High communication costs ( $h$ ) and high partisanship ( $\pi_k$ ) make parties less willing to keep layers activated unless they are relatively sure of holding onto power, creating scenarios where a dominant party or group is needed for state maintenance, as in Panel A. On the other hand, if communication costs are low, and/or the public goods valuations are high, then maintenance will be highest when both parties have a moderate chance of being in power, as in Panel C.

The election-induced incentives to maintain the state have clear implications for realized output and hence state design. The following remark shows that Party 1's partisanship can drive a wedge between capacity and output in any state that it would create, as it would dislike layers that Party 2 would activate. The bound follows simply from manipulating  $r_1\sigma_1^j(1-jh) + r_2\sigma_2^j(1-\pi_1-jh)$ , or Party 1's ex ante expected value of problems solved in layer  $J$ , and thus we state the result without proof.

**Remark 1** [Output and Capacity.] *If  $\pi_1 > 1 - jh$ , then for any layer  $j$  designed by Party 1:*

$$\sigma_2^j < -\frac{r_1\sigma_1^j(1-jh)}{r_2(1-\pi_1-jh)}.$$

Party 1 takes these relationships into account when designing the state. In particular, when its partisanship is high, it may shrink layers that Party 2 will activate and expand those that it will idle. Likewise, lower partisanship will give it an incentive to increase the size of layers that Party 2 is likely to activate. The subsequent sections examine the implications of these incentives for the capacity and specialization of the state.

## 5 State Design

We proceed in two steps to solve for the optimal knowledge hierarchy / knowledge sets  $\mathcal{Z}$ , given the opening and closing strategies described above. First, Section 5.1 establishes some simple results useful for both characterizing the optimal hierarchy, and for providing an intuition for its structure. In particular, the optimal knowledge hierarchy will have no “gaps” (that is  $\bar{z}_j = z_{j+1}$ ), that the lowest knowledge set will contain 0 (that is,  $z_1 = 0$ ), and that knowledge sets will be decreasing in length. Thus, to build the optimal knowledge hierarchy one can anchor it at zero, and then stack layers on top of it until an optimal next layer would have negative length (that is,  $\bar{z}_{J+1} < \bar{z}_J$ ). At that point,  $J$ , the last layer that the state designer wishes to assign positive length, is the top layer of the optimal hierarchy. Second, Section 5.2 uses these results to explicitly define the optimal structure.

### 5.1 Basic Results

The initial design of the knowledge hierarchy depends Party 1’s anticipation of which layers will stay active over time. As established above, parties determine which layers to activate or idle for the following period based on their realized probability of holding onto power. Only the lowest layers are activated, and the reelection probability cutoff  $p_k^j$  for activating layer  $j$  is given by (7).

We next establish the optimal state structure from the perspective of the Party 1 in the initial period ( $t = 0$ ). Our first result greatly simplifies the analysis by pinning down the possible types of solutions. In an optimal knowledge hierarchy, knowledge sets are “stacked,” or arranged sequentially with no overlaps and no gaps in between. These stacks are “anchored” at 0 to emphasize the easiest (low  $z$ ) problems. (The proof of this and all other results can be found in the Appendix.)

**Lemma 1** [Stacking and Anchoring Knowledge Sets.] *In an optimal depth- $J$  knowledge hierarchy,  $\bar{z}_j = z_{j+1}$  for  $j = 1, \dots, J - 1$  and  $z_1 = 0$ .*

Lemma 1 simplifies the derivation of optimal knowledge hierarchies by allowing us to restrict attention to stacked and anchored knowledge sets. Accordingly, we adopt the no-

tation that each  $Z_j$  takes the form  $[\bar{z}_{j-1}, \bar{z}_j]$  for each layer  $j$ . These knowledge sets form a partition of  $[0, \bar{z}_J]$ , where  $\bar{z}_J$  is the capacity of the state.

The party in power in period  $t = 0$  thus has the following maximization problem:

$$\begin{aligned} \max_{J, \bar{z}_0, \dots, \bar{z}_J} U_1(\mathcal{Z}) &= \sum_{t=1}^{\infty} \delta^t \sum_{j=1}^J \left[ \sum_{k=1}^2 r_k \sigma_k^j(w_1^k - jh) - c\bar{z}_j \right] (G(\bar{z}_j) - G(\bar{z}_{j-1})), \quad (9) \\ \text{s.t. : } &0 \leq \bar{z}_0 \leq \bar{z}_1 \leq \dots \leq \bar{z}_J \leq \bar{Z}. \end{aligned}$$

Note that  $\bar{Z}$  is assume to be large enough that the last constraint never binds.

It will be useful to focus initially on the construction of optimal knowledge hierarchies for a given  $J$ . While the objective (9) is concave in each  $z_j$ , the constraints generally ensure that it is not possible to have a solution where all  $z_j \in (0, 1)$ . The next result shows that anchoring at  $z_0 = 0$  has the fortunate consequence making the objective function globally concave for all remaining  $z_j$  terms.

**Lemma 2** Concavity of Depth- $J$  Knowledge Hierarchies. *Fixing  $J$  and  $z_0 = 0$ , the party's objective (9) is concave over knowledge sets satisfying  $\bar{z}_0 \leq \bar{z}_1 \leq \dots \leq \bar{z}_J$ .*

Due to the concavity of  $U_1(\mathcal{Z})$ , we can solve for the optimal knowledge hierarchy, up to the constraints in (9) by using the first order conditions. For layers  $j = 1, \dots, J-1$ , and making use of the fact that  $G(z) \sim \bar{Z} \times U[0, \bar{Z}]$  these can be written as:

$$\begin{aligned} \bar{z}_j &= \frac{\bar{z}_{j-1} + \bar{z}_{j+1}}{2} + \frac{1}{2c} \sum_{k=1}^2 r_k \left[ \sigma_k^j(w_1^k - jh) - \sigma_k^{j+1}(w_1^k - (j+1)h) \right] \\ &\vdots \\ \bar{z}_J &= \frac{\bar{z}_{J-1}}{2} + \frac{1}{2c} \sum_{k=1}^2 r_k \sigma_k^J(w_1^k - Jh). \end{aligned} \quad (10)$$

The quantity  $(w_1^k - jh)$  is simply Party 1's marginal value of an active layer  $j$  when party  $k$  is in power. This must be sufficiently positive, in expectation, for Party 1 to create a  $j$ th layer.

The expressions in (10) convey a simple intuition for the effect of costs and election probabilities. As the probability  $\sigma_k^j$  of activating layers does not depend on personnel costs



(c), increasing these costs will shift the layer  $j$  bound  $\bar{z}_j$  downwards toward the midpoint between  $\bar{z}_{j-1}$  and  $\bar{z}_{j+1}$ . Increasing communication costs ( $h$ ) and partisanship ( $\pi_k$ ) have the opposite effect, as these increase the threat of idling the  $j + 1$ -th layer and thus give the institution designer an incentive to increase the size of the  $j$ th layer.

Solving the system (10) in terms of  $\bar{z}_J$  produces a unique interior layer boundary for each knowledge set:

$$\bar{z}_j = \frac{j}{J}\bar{z}_J + \frac{1}{cJ} \sum_{k=1}^2 r_k \left( J \sum_{i=1}^j \sigma_k^i(w_1^k - ih) - j \sum_{i=1}^J \sigma_k^i(w_1^k - ih) \right). \quad (11)$$

We can further solve for  $\bar{z}_J$ , producing the following expression for capacity:

$$\bar{z}_J = \frac{1}{c(J+1)} \sum_{k=1}^2 r_k \sum_{i=1}^J \sigma_k^i(w_1^k - ih). \quad (12)$$

Thus far, we have not addressed the possibility of corner solutions, where  $z_j = z_{j+1}$ . Optimal knowledge hierarchies with degenerate layers are possible, in part because the state designer may want to prevent (through high operational costs) her opponent from activating layers when she is out of power. Such designs are both relatively rare and cumbersome to characterize, so the subsequent analysis focuses on interior solutions to the state designer's problem.

## 5.2 Full Characterization

To complete the derivation of the optimal knowledge hierarchy, we determine the optimal depth  $J^*$ . The main challenge of this exercise is that  $J^*$  can change with the parameters of the model. We thus introduce a general property of optimal layer arrangements that is useful for developing an intuition for the maximum possible depth.

Trivially, Party 1 strictly prefers  $J$  over  $J - 1$  layers when it is willing to allocate a positive measure of problems to layer  $J$ . As Party 1 could feasibly add a degenerate  $J$ th layer to an optimal depth- $J - 1$  knowledge hierarchy without incurring additional personnel or operational costs, adding any nondegenerate layer must be strictly beneficial. To see when the  $J$ th layer has positive size, we solve  $\bar{z}_J - \bar{z}_{J-1} \geq 0$  in terms of  $z_J$  using expressions (11)

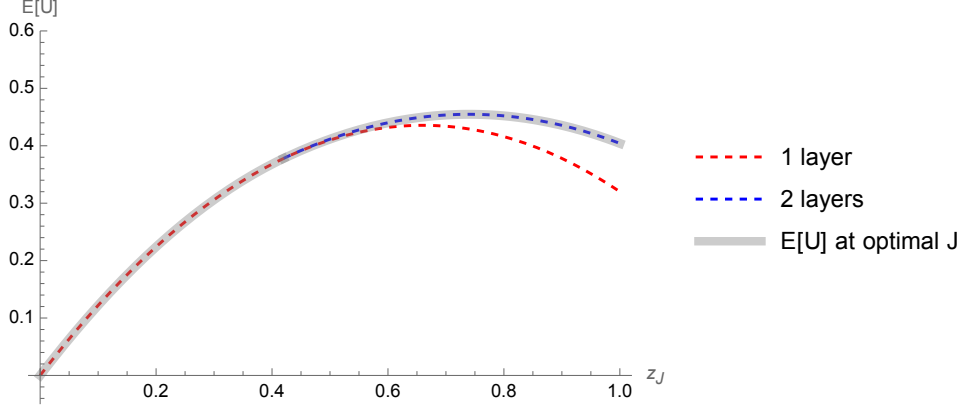


Figure 4: Layers, Capacity, and Expected Utility. Here,  $h = 0.2$ ,  $c = 1.0$ ,  $\delta = 0.75$ ,  $\pi_1 = 0.5$ ,  $\pi_2 = 0.75$ ,  $q = 0.25$ ,  $r_1 = 0.5$ , and election probabilities are distributed according to  $U[0, 1]$ . The gray curve is the expected utility of a knowledge hierarchy as a function of capacity when  $J$  and all internal layers are chosen optimally. Two layers become optimal at  $z_J = 0.42$ ; the optimal knowledge hierarchy is  $J = 2$  and  $\bar{z}_2 = 0.74$ .

and (12). This produces the minimum capacity for a knowledge hierarchy that sustains  $J$  layers:

$$\underline{z}_J = \frac{1}{c} \sum_{k=1}^2 r_k \left( \sum_{i=1}^{J-1} \sigma_k^i(w_1^k - ih) - (J-1)\sigma_k^J(w_1^k - Jh) \right). \quad (13)$$

For a state of depth  $J$ , a capacity of at least  $\underline{z}_J$  is obviously necessary for an interior solution to the state designer's problem.

It is straightforward to show that higher-depth states require larger capacity, and so  $\underline{z}_J$  is increasing in  $J$ . Once capacity exceeds  $\underline{z}_J$ , the optimal depth becomes at least  $J$ . Thus the optimal depth is  $J$  when  $\bar{z}_J$  lies in the interval  $[\underline{z}_J, \underline{z}_{J+1})$ . The weakly monotonic relationship between the optimal depth and problem-solving ability effectively reduces the politician's problem to maximizing over capacity. This fixes the maximum number of non-empty layers for an optimal stacked and anchored knowledge hierarchies, with internal boundaries uniquely characterized by equation (11).

To put this another way, for any capacity level  $\bar{z}_J$ , there is an optimal number and arrangement of the layers that uniquely minimize costs, given the concavity of the cost function. This arrangement maximizes Party 1's utility, as its benefits are fixed for a given  $\bar{z}_J$ . This is illustrated in Figure 4, where one layer minimizes costs for  $\bar{z}_J$  less than 0.42

and two layers minimize costs above 0.42. The overall expected utility function, consisting of the curve for  $J = 1$  for  $\bar{z}_J < 0.42$  and the curve for  $J = 2$  for  $\bar{z}_J > 0.42$ , is smooth and concave.

The final result in our characterization provides a unique solution for the depth and capacity of the knowledge hierarchy designed by Party 1.

**Proposition 3** [Optimal Capacity and Depth.] *In the unique optimal knowledge hierarchy for the Party 1 politician,*

$$\bar{z}_{J^*} = \frac{1}{c(J^* + 1)} \sum_{k=1}^2 r_k \sum_{i=1}^{J^*} \sigma_k^i(w_1^k - ih), \quad (14)$$

in which  $J^*$  is the depth such that  $\bar{z}_{J^*} \in [\underline{z}_{J^*}, \underline{z}_{J^*+1})$ , or equivalently the value of  $J$  such that:

$$\sum_{k=1}^2 r_k \sigma_k^J(w_1^k - Jh) > \frac{\sum_{k=1}^2 r_k \sum_{i=1}^J \sigma_k^i(w_1^k - ih)}{J + 1} > \sum_{k=1}^2 r_k \sigma_k^{J+1}(w_1^k - (J + 1)h).$$

The proof of Proposition 3 shows that, as Figure 4 suggests, Party 1's objective is concave even when  $J$  is chosen optimally. Thus there is a unique  $J$  and corresponding  $\bar{z}_J$  that satisfies its first-order condition. For given values of  $\sigma_k^i$ , the optimal depth is straightforward to calculate.

## 6 State Structure

This section describes the principal features of the state as a function of its political environment. We first characterize the main determinants of state capacity, and then show how the depth or specialization of the state responds to the designer's anticipation of opposition strategies.

### 6.1 Capacity, Costs, and Preferences

To begin, Figure 5 illustrates how state structures vary with some parameters of the model. This and all subsequent illustrations follow Assumptions 1 and 2 of uniform election probabilities and costless activation and idling. It plots Party 1's optimal knowledge sets, as

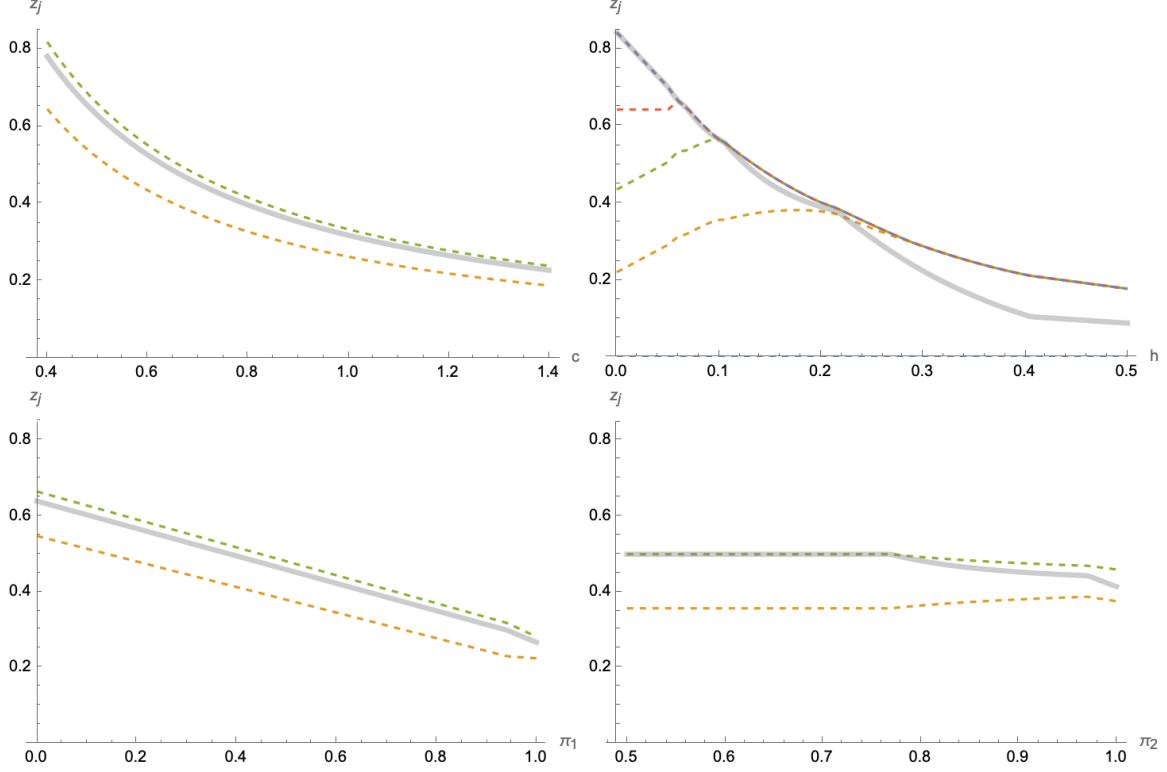


Figure 5: Optimal Knowledge Hierarchies and Output. Panels plot layers of optimal knowledge hierarchy established by Party 1, as a function of  $c$ ,  $h$ , and  $\pi_k$ . Default parameters are  $c = 0.7$ ,  $h = 0.15$ ,  $\pi_1 = 0.5$ ,  $\pi_2 = 0.75$ ,  $q = 0.5$ ,  $\delta = 0.5$ ,  $r_k = 0.5$ , and  $\epsilon = 0.25$ . Dashed lines indicate layer boundaries, and gray solid lines indicate average output over time.

well as average output after accounting for idling induced by realized election probabilities. An immediate observation in each example is that layers are occasionally idled, and thus output often falls short of capacity.

The figure shows that costs and partisanship have intuitive effects on the state's potential and realized problem-solving ability. In particular, higher costs and partisanship generally reduce capacity. The following comparative statics on  $\bar{z}_{J^*}$  follow directly from expression (14), and so we state them without proof.

**Remark 2** [Partisanship, Personnel Costs, and Capacity.] *At the optimal depth  $J^*$ ,  $\bar{z}_{J^*}$  is decreasing in  $\pi_k$ ,  $c$ , and  $h$ .*

Next, Figure 6 shows that capacity (as well as specialization) may be increasing or

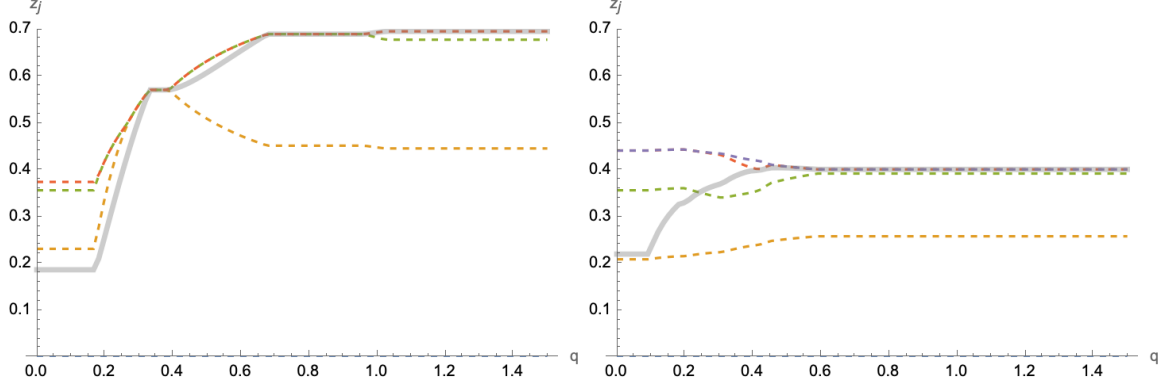


Figure 6: Optimal Knowledge Hierarchies and Output as a Function of  $q$ . Parameters are  $c = 0.7$ ,  $r_k = 0.5$ ,  $\pi_2 = 0.75$ ,  $\delta = 0.5$ , and  $\epsilon = 0.25$ . On the left,  $\pi_1 = 0.1$  and  $h = 0.15$ , and on the right,  $\pi_1 = 0.9$  and  $h = 0.0875$ . Dashed lines indicate layer boundaries, and gray solid lines indicate average output over time.

decreasing in  $q$ . In particular, it strongly suggests that the effect of  $q$  on capacity depends on Party 1's partisanship ( $\pi_1$ ). When  $\pi_1$  is low, Party 1 loses little from being out of power. An opposition with a high public goods valuation will tend to activate layers and thereby induce the creation of a larger state. When  $\pi_1$  is high, Party 1 does not benefit from the opposition's operation of the state, and compensates in part by reducing training costs. Thus, higher values of  $q$  can reduce capacity.

Proposition 4 formalizes the relationship between  $q$ ,  $\pi_1$ , and state capacity by showing that  $\bar{z}_{J^*}$  (given by expression (14)) decreases in  $q$  under high partisanship, and increases otherwise. The result makes use of the fact that  $J$  is integer-valued, and thus the optimal depth of the state is locally constant with respect to  $q$ .

**Proposition 4** [Opposition Characteristics and Capacity.] *At the optimal depth  $J^*$ , there exists  $\tilde{\pi}_1 < 1$  such that  $\bar{z}_{J^*}$  is weakly decreasing (increasing) in  $q$  if  $\pi_1 > (<) \tilde{\pi}_1$ .*

## 6.2 Capacity and Transitions of Power

One sensible conjecture is that the ability to hold power will increase Party 1's return from state-building (e.g., Besley and Persson 2009). This logic implies that state capacity should increase in  $r_1$ . Our model produces this effect as well, but the possibility of idling sometimes changes this finding.

The prospects for holding power matter for the state designer when the stakes of political control are high. The role of  $r_1$  therefore interacts with Party 1 partisanship. A highly partisan Party 1 invests in greater capacity as  $r_1$  increases. It also creates greater capacity if Party 2 is unlikely to activate layers whose operation harms Party 1 through high operational costs. A less partisan Party 1 benefits from the state even when out of power, if Party 2 is willing to activate layers. This reduces the importance of holding power and thus also its effect on capacity-building.

The next proposition establishes conditions that isolate each of these relationships. The first part concerns when capacity increases with  $r_1$ . A sufficient condition is for Party 2 never to activate any layer, due either to high partisanship or low public goods valuation. The second part concerns the responsiveness of capacity to  $r_1$ . A subtlety of this result is the fact that  $r_1$  enters Party 1's objective both directly through its probability of holding power and indirectly through each layer's long run probability of activation. For reduced partisanship to attenuate the relationship between  $r_1$  and capacity, it must also not encourage Party 1 to activate layers more often, which would amplify the effect of  $r_1$ . The attenuation is therefore most pronounced when Party 1 is already willing to activate the top layer with certainty.

**Proposition 5** [Transitions of Power and Capacity.]

- (i) If  $\sigma_2^1 = 0$  or  $\pi_1 > 1 - h$ , then at the optimal depth  $\bar{z}_{J^*}$  is weakly increasing in  $r_1$ .
- (ii) If  $\sigma_1^{J^*} = 1$ , then  $\frac{\partial \bar{z}_{J^*}}{\partial r_1}$  is weakly increasing in  $\pi_1$ .

Figure 7 illustrates how the forces in Proposition 7 combine to affect state capacity as Party 1's partisanship changes. Reducing  $\pi_1$  both increases capacity and flattens the effect of  $r_1$ . Interestingly, in this example the relationship between capacity and  $r_1$  not only weakens but reverses as partisanship decreases. To understand why this reversal can occur, it is again important to take Party 2's idling incentives into account. Under the parameters assumed in the figure, Party 2 activates layer 1 with probability  $2.37 - 2r_1$  and layer 2 with probability  $1.57 - 2r_1$ . Thus as Party 1 increasingly values public goods when Party 2 is in power, it shifts more problem-solving capacity to the first layer, which is less likely to be idled. A low-capacity state — driven by the a shrinkage of the second layer—compensates

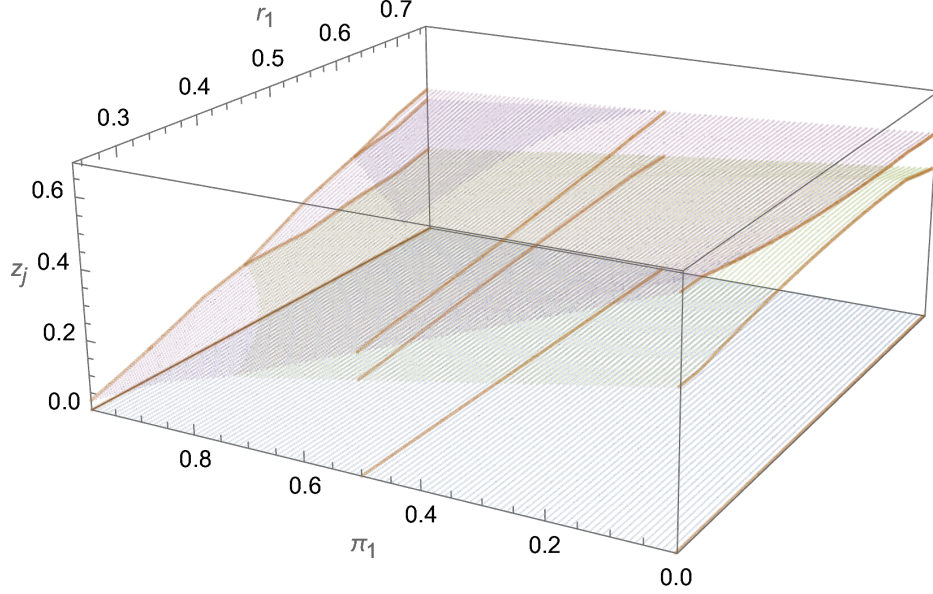


Figure 7: Optimal Knowledge Hierarchies as a Function of  $r_1$  and  $\pi_1$ . Figure plots layers of optimal knowledge hierarchy established by party 1 at different values of  $\pi_1$ . Parameters are  $c = 0.8$ ,  $h = 0.15$ ,  $\pi_2 = 0.75$ ,  $q = 0.5$ ,  $\delta = 0.5$ , and  $\epsilon = 0.25$ .

for the high personnel costs incurred by a very large first layer.

### 6.3 Specialization

Figures 5 and 6 additionally suggest that the depth or specialization of the state does not bear a clear relationship with capacity. For example, Figure 5 clearly shows the inverse relationship between capacity and specialization also present in our baseline model. In particular, the cost parameters have different implications for specialization. Higher personnel costs ( $c$ ) reduce capacity but leave depth unchanged. By contrast, higher operational/communication costs ( $h$ ) reduce the gains from specialization, thereby reducing both depth and capacity.

The most interesting feature of Figure 6 is the non-monotonic relationship between capacity and the underlying number and arrangement of layers. Specialization can both increase and decrease with the state's potential problem-solving ability. These patterns are due to politics, as the prospect of idling creates a complex set of tradeoffs for Party 1's design.

As an example, consider the left panel of Figure 6, where Party 1’s partisanship is low. When Party 2’s public good valuation  $q$  is very low, they receive little benefit from the state whether they are in or out of power—so they will always idle the entire state when they get the chance. In a world where only Party 1 participates in governance, the state designer chooses an optimally specialized arrangement of layers for a low level of capacity. As  $q$  increases Party 2 is sometimes willing to keep lower layers open, and the state designer capitalizes by expanding the state while reducing depth. By bundling together tasks at lower layers, the state designer “insulates” capacity from politics. This ensures greater, although less efficient, state output. As  $q$  continues to increase, Party 2 becomes willing to keep more layers open, and so the state designer increases both the size and complexity of the state. Finally, for  $q > 1$ , both groups are willing to keep the same number of layers open, and so state capacity and output stabilize at a high level.

This pattern is not unique. As the right panel of Figure 6 shows, the evolution of capacity and depth depend on partisanship. There, Party 1’s partisanship is high and an initial increase in  $q$  creates the opposite effect: capacity shrinks while depth increases. This structure reduces the harm of Party 2 control by concentrating tasks in higher layers that Party 2 is unlikely to activate. As  $q$  increases further and Party 2 activates even higher layers, the state designer collapses the upper layers and settles on a knowledge hierarchy with both reduced depth and capacity. In both cases, output gaps disappear entirely at high values of  $q$ , but at very different levels of capacity.

Proposition 6 captures these opposing arcs by addressing Party 1’s incentive to increase or decrease depth at different levels of partisanship. The result examines the behavior of overall capacity  $\bar{z}_{J^*}$ , in relation to the minimum capacity levels that sustain  $J^*$  and  $J^* + 1$  layers (respectively  $\underline{z}_{J^*}$  and  $\underline{z}_{J^*+1}$ , as given by expressions (12) and (13)). The following definitions are useful for describing the main behaviors of the knowledge hierarchy with respect to changes in  $q$ .

**Definition 1** (i) If  $\bar{z}_{J^*} - \underline{z}_{J^*}$  is increasing and  $\underline{z}_{J^*+1} - \bar{z}_{J^*}$  is decreasing in  $q$ , then the knowledge hierarchy is expanding.

(ii) If  $\bar{z}_{J^*} - \underline{z}_{J^*}$  is decreasing and  $\underline{z}_{J^*+1} - \bar{z}_{J^*}$  is increasing in  $q$ , then the knowledge hierarchy is contracting.



(iii) If  $\bar{z}_{J^*} - \underline{z}_{J^*}$  and  $\underline{z}_{J^*+1} - \bar{z}_{J^*}$  are constant in  $q$ , then the knowledge hierarchy is constant.

An expanding knowledge hierarchy moves “closer” to admitting a  $J^* + 1$ th layer. Likewise, a contracting knowledge hierarchy moves closer to losing its  $J^*$ th layer. Importantly, expansion and contractions do not necessarily imply anything about capacity levels: as Figure 6 illustrates, an expanding knowledge hierarchy may lose capacity as  $q$  increases, and a contracting one may gain capacity.

**Proposition 6** [Structure and Polarization.] *There exist  $\hat{\pi}_1$  and  $\hat{\pi}$ , where  $\hat{\pi}_1 < \hat{\pi} < \tilde{\pi}_1 < 1$ , such that if  $\pi_1 < \hat{\pi}_1$ , the state is:*

$$\left\{ \begin{array}{ll} \text{constant} & \text{if } \sigma_2^i = 0 \text{ for all } i \\ \text{contracting, with capacity increasing in } q & \text{if } \sigma_2^{J^*} = 0 \text{ and } \sigma_2^i > 0 \text{ for some } i \\ \text{expanding, with capacity increasing in } q & \text{if } \sigma_2^{J^*} < 1 \text{ and } \sigma_2^i > 0 \text{ for all } i \\ \text{constant} & \text{if } \sigma_2^i = 1 \text{ for all } i. \end{array} \right.$$

And if  $\pi_1 > \hat{\pi}_1$  the state is:

$$\left\{ \begin{array}{ll} \text{constant} & \text{if } \sigma_2^i = 0 \text{ for all } i \\ \text{expanding, with capacity decreasing in } q & \text{if } \sigma_2^{J^*} = 0 \text{ and } \sigma_2^i > 0 \text{ for some } i \\ \text{contracting} & \text{if } \sigma_2^{J^*} < 1 \text{ and } \sigma_2^i > 0 \text{ for all } i \\ \text{constant} & \text{if } \sigma_2^i = 1 \text{ for all } i. \end{array} \right.$$

Proposition 6 shows that Party 1’s partisanship ( $\pi_1$ ) and Party 2’s willingness to activate layers the knowledge hierarchy ( $\sigma_2^i$ ), and in particular the top layer, play central roles in determining internal structure. When Party 2 never activates the top layer — whether due to high partisanship, operational costs, or low public goods valuation — specialization and capacity are inversely related. If Party 1 is not very partisan it uses low specialization to take advantage of Party 2’s willingness to activate the lowest layers. Thus, a large but “bottom heavy” state that insulates basic tasks from politics emerges. The inefficiently large lower layer(s) contract the upper layers and may lead to their elimination. A more partisan Party 1 shifts tasks to upper layers that Party 2 is unlikely to activate. Rather

than insulating tasks from politics, a “top heavy” state makes tasks inaccessible to the opposition. The increased personnel costs of large upper layers are offset by both increased depth and reduced capacity.

A Party 2 that is willing activate the top layer with positive probability largely reverses the preceding patterns. When  $\pi_1$  is low, political consensus across parties is high, and the state designer creates extra layers solely to realize greater operational efficiency. Thus, greater capacity complements greater specialization. When  $\pi_1$  is high, the inevitability of Party 2 using the state to its full potential eliminates Party 1’s benefit from a top heavy state, thereby resulting in contraction. The effect on capacity is ambiguous, but as the right panel Figure 6 illustrates, reduced capacity may accompany this contraction.

## 7 Discussion

Despite a recent surge in scholarly interest in the topic, theoretical accounts of state capacity have taken little notice of several key aspects of governmental functions and organizations. In addition to revenue collection, modern governance requires the skilled performance of a wide range of activities, each of which presents distributions of sub-problems of varying levels of difficulty. Modern states are also typically subject to regular political processes. Politicians with heterogeneous preferences may interfere with its functioning, and recent decades have seen the emergence of numerous political movements that are expressly interested in reducing the power of the state. Everyday state maintenance is therefore important even after the critical moment of state foundation.

Our theory of state capacity posits the bureaucracy as the primary link between politicians and the solution of social problems. Knowledge hierarchies provide a flexible and tractable way to model the throughput of such complex organizations. Existing applications focus on the structure of firms, but its natural trade-offs between specialization and coordination pervades public sector organizations as well. By placing knowledge hierarchies in a simple dynamics political economy setting, our model provides a novel application for the framework.

The ability of politicians to idle parts of the state allows the model to distinguish

meaningfully between output and capacity. Incumbent politicians idle layers of the state according to their policy preferences and chances of retaining power, and political competitiveness does not necessarily minimize the risk of idling. These idling strategies in turn produce distortions in the capacity and specialization dimensions of state design. Most notably, high capacity and specialization coincide only when social groups are not polarized and agree on the value of policy. When the founding party or group faces an opposition that cares less about the policy in question, this relationship is broken: higher capacity and less specialization, or lower capacity and greater specialization can result, depending on the founder’s partisanship. Partisanship also affects the relationship between capacity and electoral prospects. An electorally advantaged founding group maximizes capacity under high partisanship, but under low partisanship it is an electorally disadvantaged group that may do so.

Our framework presents numerous directions for additional work. In addition to providing results on state structure, the model should provide observable implications on realized output. There is also room for exploring robustness with respect to policy or organizational technologies. For example, groups may have different valuations over specific problems, as opposed to over policy production in general. As the IRS example in the introduction illustrates, liberal and conservative groups might prefer to idle different parts of the state because their activities affect core constituents. Alternatively, some organizations may feature top-to-bottom propagation of problems, rather than bottom-to-top. It is finally worth considering in general how the technology of knowledge hierarchies can fit into other political or institutional contexts.

## 8 Appendix

**Proof of Proposition 2.** The ex ante probability that layer  $j$  is active in any given period is  $\sum_k r_k \sigma_k^j$ . Ignoring corner solutions, for parties 1 and 2, the conditional probability  $\sigma_k^j \equiv 1 - F_k(p_k^j)$  of keeping a layer active is:

$$\begin{aligned}\hat{\sigma}_1^j &= \frac{1}{2\varepsilon} \left[ r_1 + \varepsilon - \frac{jh - w_1^2}{2\varepsilon(w_1^1 - w_1^2)} \right] = \frac{(r_1 + \varepsilon)\pi_1 - (jh - (1 - \pi_1))}{2\varepsilon\pi_1} \\ \hat{\sigma}_2^j &= \frac{1}{2\varepsilon} \left[ r_2 + \varepsilon - \frac{jh - w_2^2}{2\varepsilon(w_2^2 - w_2^1)} \right] = \frac{(1 - r_1 + \varepsilon)q\pi_2 - (jh - q(1 - \pi_2))}{2\varepsilon q\pi_2}.\end{aligned}$$

Noting that  $r_1 = 1 - r_2$ , each  $r_k \hat{\sigma}_k^j$  is convex in  $r_1$ . Moreover, it is apparent that for  $r_1 \in [0, 1]$ ,  $\hat{\sigma}_1^j > 0$  only if  $r_1$  is sufficiently high, and  $\hat{\sigma}_2^j > 0$  only if  $r_1$  is sufficiently low. Let  $\underline{r}_k^j$  denote the value of  $r_1$  such that  $\sigma_k^j = 0$ , and  $\bar{r}_k^j$  denote the value of  $r_1$  such that  $\sigma_k^j = 1$ , if such values exist. Then we have the following:

$$\sigma_1^j = \begin{cases} 0 & \text{if } r_1 \leq \underline{r}_1^j \\ \hat{\sigma}_1^j & \text{if } r_1 \in (\underline{r}_1^j, \bar{r}_1^j) \\ 1 & \text{if } r_1 \geq \bar{r}_1^j \end{cases} \quad \sigma_2^j = \begin{cases} 1 & \text{if } r_1 \leq \bar{r}_2^j \\ \hat{\sigma}_2^j & \text{if } r_1 \in (\underline{r}_2^j, \bar{r}_2^j) \\ 0 & \text{if } r_1 \geq \underline{r}_2^j. \end{cases} \quad (15)$$

We now establish (i). First,  $\hat{\sigma}_1^j$  and  $\hat{\sigma}_2^j$  are both weakly decreasing in  $h$ , thus,  $\sum_k r_k \sigma_k^j$  is also weakly decreasing in  $h$ . Second,  $\hat{\sigma}_2^j$  is weakly increasing in  $q$ , and  $\hat{\sigma}_1^j$  is constant in  $q$ , thus,  $\sum_k r_k \sigma_k^j$  is weakly increasing in  $q$ . Third, taking derivatives with respect to  $\pi_k$  produces:

$$\frac{d\hat{\sigma}_1^j}{d\pi_1} = -\frac{1 - jh}{2\varepsilon\pi_1^2} \quad \frac{d\hat{\sigma}_2^j}{dw_2} = -\frac{q - jh}{2\varepsilon q\pi_2^2}.$$

This implies that the  $\hat{\sigma}_k^j$  are monotonic in  $\pi_k$  on  $[0, 1]$ . Further, if  $\frac{d\hat{\sigma}_1^j}{d\pi_1} > 0$ , then  $jh > 1$ . As  $\hat{\sigma}_1^j$  is maximized when  $r_1 + \varepsilon = 1$ ,  $\frac{d\hat{\sigma}_1^j}{d\pi_1} > 0 \Rightarrow \hat{\sigma}_1^j < 0$ , and thus  $\sigma_1^j = 0$  (a similar argument works for  $\sigma_2^j$ ). Thus,  $\hat{\sigma}_k^j$  must be weakly decreasing in  $\pi_k$ , which implies that  $\sum_k r_k \sigma_k^j$  is also weakly decreasing in  $\pi_k$ .

(ii)  $\sum_k r_k \sigma_k^j = 0$  iff the interval  $[\underline{r}_2^j, \bar{r}_1^j]$  is non-empty. Solving for the value of  $r_1$  such that  $\hat{\sigma}_1^j = 0$  produces  $\underline{r}_1^j = 1 - \frac{1-jh}{\pi_1} - \varepsilon$ . Similarly, solving for the value of  $r_1$  such that  $\hat{\sigma}_2^j = 0$  produces  $\underline{r}_2^j = \frac{q-jh}{q\pi_2} + \varepsilon$ . As  $\underline{r}_2^j$  is increasing in  $q$ , the interval is nonempty if

$q < \frac{jh\pi_1}{\pi_1 + \pi_2((1-jh) - \pi_1(1-2\varepsilon))}$ . For this interval to be internal both  $\underline{r}_1^j < 1$  and  $\underline{r}_2^j > 0$ . The fact that the first holds is implied by the layer's existence. The second holds if  $jh < q(1 + \varepsilon\pi_2)$ .

(iii)  $\sum_k r_k \sigma_k^j = 1$  iff the interval  $[\bar{r}_1^j, \bar{r}_2^j]$  is non-empty. Solving for the value of  $r_1$  such that  $\hat{\sigma}_1^j = 1$  produces  $\bar{r}_1^j = 1 - \frac{1-jh}{\pi_1} + \varepsilon$ . Similarly, solving for the value of  $r_1$  such that  $\hat{\sigma}_2^j = 1$  produces  $\bar{r}_2^j = \frac{q-jh}{q\pi_2} - \varepsilon$ . Since  $\bar{r}_2^j$  is increasing in  $q$ , the interval is nonempty if  $q > \frac{jh\pi_1}{\pi_1 + \pi_2((1-jh) - \pi_1(1+2\varepsilon))}$ . For this interval to be internal both  $\bar{r}_1 > 0$  and  $\bar{r}_2 < 1$ . The first will hold if  $jh > 1 - \pi_1(1 + \varepsilon)$ , and the second if  $jh > q(1 - \pi_2(1 + \varepsilon))$ , so both will hold when  $jh > \max[1 - \pi_1(1 + \varepsilon), q(1 - \pi_2(1 + \varepsilon))]$ . ■

**Proof of Lemma 1.** We prove the result by evaluating the ex-ante expected payoff to a member of Party 1 from the knowledge hierarchy.

Note that if  $[z_j, \bar{z}_j] \cap [z_{j'}, \bar{z}_{j'}] \neq \emptyset$ , for some layer  $j$  and  $j'$ —that is, two layers overlap—then the party could do strictly better by eliminating all workers in the intersection in either layer  $j$  or  $j'$ . Thus an optimal knowledge hierarchy must have non-overlapping layers.

Recall that Party 1 designs the knowledge hierarchy in period 0. Consider two successive layers with a gap between them, that is,  $Z_j = [\bar{z}_{j-1} + \eta, \bar{z}_j]$ , for some  $\eta \in (0, \bar{z}_j - \bar{z}_{j-1})$ . The ex-ante expected value of a given layer  $j$  is:

$$\left[ \sum_{t=1}^{\infty} \delta^t \sum_{k=1}^2 r_k \sigma_k^j (w_1^k - jh) - c\bar{z}_j \right] (G(\bar{z}_j) - G(\bar{z}_{j-1} + \eta)) > 0$$

which is positive for any layer that is created in equilibrium. Denoting the term in square braces by  $\phi_j$ , we have  $\phi_j > 0$  in equilibrium as  $\bar{z}_j > \bar{z}_{j-1} + \eta$ . The derivative of the above expression with respect to  $\eta$  is then  $-g(\bar{z}_{j-1} + \eta)\phi_j$ , which is negative as  $g(\cdot)$  has full support. Thus, Party 1 will want to set  $\eta = 0$ , leading to no gaps. The fact that  $z_1 = 0$  follows from the same argument applied to  $z_1 = \eta$ . ■

**Proof of Lemma 2.** There are two cases. In the first,  $z_0 = 0$  and  $\bar{z}_J < 1$ . The Hessian of (9) with respect to  $\bar{z}_1, \dots, \bar{z}_J$  is  $\frac{\delta}{1-\delta}$  times the matrix

$$\begin{array}{cccccc} \bar{z}_1 & \bar{z}_2 & \bar{z}_3 & \dots & \bar{z}_{J-1} & \bar{z}_J \end{array}$$

$$\begin{array}{c|cccccc}
\bar{z}_1 & -2c & c & 0 & 0 & 0 & 0 \\
\bar{z}_2 & c & -2c & c & 0 & 0 & 0 \\
\bar{z}_3 & 0 & c & -2c & \dots & 0 & 0 \\
\vdots & 0 & 0 & \dots & \dots & \dots & 0 \\
\bar{z}_{J-1} & 0 & 0 & 0 & \dots & -2c & c \\
\bar{z}_J & 0 & 0 & 0 & 0 & c & -2c
\end{array}$$

Thus, the Hessian is negative definite for any  $c > 0$ . ■

**Proof of Proposition 3.** Let  $U_1^*(z^s)$  be the Party 1 politician's utility over such knowledge hierarchies at the optimal depth for a given capacity  $z^s$ . Our first objective is to show that  $U_1^*(z^s)$  is concave and differentiable. We construct  $U_1^*(z^s)$  piecewise by determining the expected value of an optimal knowledge hierarchy at the optimal depth  $J^*$  for each possible capacity level  $z^s$ . As argued in the text, the optimal depth is the maximum number of layers such that each has non-negative length.

Let  $U_1^*(\bar{z}_J, J)$  denote the politician's objective for given values of  $\bar{z}_J$  and  $J$ , where all layer boundaries below  $J$  are arranged optimally. By Lemma 1,  $\bar{z}_J$  is also the capacity of the knowledge hierarchy. We can then rewrite the objective (9) in terms of  $\bar{z}_j$  as follows:

$$U_1^*(\bar{z}_J, J) = \frac{\delta}{1-\delta} \sum_{j=1}^J \left[ \sum_{k=1}^2 r_k \sigma_k^j(w_1^k - jh) - c\bar{z}_j \right] (\bar{z}_j - \bar{z}_{j-1}).$$

$U_1^*(\bar{z}_J, J)$  is weakly increasing in  $J$ , since a politician can do as well as a depth- $J$  knowledge hierarchy with a depth- $J+1$  knowledge hierarchy with a degenerate  $J+1$ th knowledge set of length 0.

To complete the expression of  $U_1^*(\bar{z}_J, J)$  we write each  $\bar{z}_j$  in terms of  $z_0$  and  $\bar{z}_J$ . To reiterate from (11), each  $\bar{z}_j$  can be written as follows:

$$\bar{z}_j = \frac{j}{J} \bar{z}_J + \frac{1}{cJ} \sum_{k=1}^2 r_k \left( J \sum_{i=1}^j \sigma_k^i(w_1^k - ih) - j \sum_{i=1}^J \sigma_k^i(w_1^k - ih) \right).$$

It will be convenient to denote the last part of the preceding expression as follows:

$$\kappa_{j,J} = \frac{1}{cJ} \sum_{k=1}^2 r_k \left( J \sum_{i=1}^j \sigma_k^i(w_1^k - ih) - j \sum_{i=1}^J \sigma_k^i(w_1^k - ih) \right) ..$$

Observe that  $\kappa_{j,J}$  is independent of all  $\bar{z}_j$ , and  $\kappa_{0,J} = \kappa_{J,J} = 0$ .

As noted in the text, expression (13) gives  $\underline{z}_J$ , which is the minimum capacity necessary to sustain  $J$  layers of non-negative length. As defined,  $U_1^*(\underline{z}_J, J) = U_1^*(\underline{z}_J, J-1)$ . We can therefore construct  $U_1^*(z^s)$  as a continuous function as follows:

$$U_1^*(z^s) = U_1^*(\bar{z}_J, J) \quad \text{for } z^s \in [\underline{z}_J, \underline{z}_{J+1}) .$$

We now show that  $U_1^*(z^s)$  is differentiable and concave using first- and second-order conditions. Omitting terms that are independent of  $\bar{z}_j$  and using the fact that  $z_0 = 0$ , the politician's objective for a given  $J$  simplifies to:

$$U_1^*(\bar{z}_J, J) = \frac{\delta}{1-\delta} \sum_{j=1}^J \left[ \sum_{k=1}^2 \left( r_k \sigma_k^j(w_1^k - jh) - c \left( \frac{j\bar{z}_J}{J} + \kappa_{j,J} \right) \right) \left( \frac{\bar{z}_J}{J} + \kappa_{j,J} - \kappa_{j-1,J} \right) \right] .$$

Differentiating with respect to  $\bar{z}_J$  produces the following expression, which we designate by parts:

$$\frac{dU_1^*}{d\bar{z}_J} = \sum_{j=1}^J \left[ \underbrace{\left( \sum_{k=1}^2 \frac{r_k \sigma_k^j(w_1^k - jh)}{J} \right)}_A - \underbrace{\frac{2cj\bar{z}_J}{J^2}}_B - \underbrace{\frac{c}{J} ((j+1)\kappa_{j,J} - j\kappa_{j-1,J})}_C \right] , \quad (16)$$

in which part 'C' equals zero:

$$\sum_{j=1}^J \frac{c}{J} ((j+1)\kappa_{j,J} - j\kappa_{j-1,J}) = \frac{c}{J} ((J+1)\kappa_{J,J} + \kappa_{0,J}) = 0. \quad (17)$$

We now show that:

$$\frac{dU_1^*(\underline{z}_J, J-1)}{d\bar{z}_J} = \frac{dU_1^*(\underline{z}_J, J)}{d\bar{z}_J} .$$

To show this, we compare differences between the values of each part at  $J$  and  $J-1$  layers.

For part ‘A’ of (16) we have for  $J - 1$  and  $J$ , respectively:

$$\begin{aligned} \sum_{j=1}^{J-1} \sum_{k=1}^2 \frac{r_k \sigma_k^j(w_1^k - jh)}{(J-1)} &= \sum_{j=1}^{J-1} \sum_{k=1}^2 \frac{r_k \sigma_k^j(w_1^k - jh)J}{J(J-1)} \\ \sum_{j=1}^J \sum_{k=1}^2 \frac{r_k \sigma_k^j(w_1^k - jh)}{J} &= \sum_{j=1}^J \sum_{k=1}^2 \frac{r_k \sigma_k^j(w_1^k - jh)J}{J(J-1)} - \sum_{j=1}^J \sum_{k=1}^2 \frac{r_k \sigma_k^j(w_1^k - jh)}{J(J-1)}. \end{aligned}$$

Taking the difference between the right-hand side terms produces:

$$\sum_{k=1}^2 \left( \sum_{j=1}^{J-1} \frac{r_k \sigma_k^j(w_1^k - jh)}{J(J-1)} - \frac{r_k \sigma_k^J(w_1^k - Jh)(J-1)}{J} \right).$$

Next, for part ‘B’ of (16), we substitute in values of  $\underline{z}_J$  (13) for  $J - 1$  and  $J$ , respectively:

$$\begin{aligned} \sum_{j=1}^{J-1} \frac{2cj}{(J-1)^2} \sum_{k=1}^2 \frac{r_k}{c} \left( \sum_{i=1}^{J-1} \sigma_k^i(w_1^k - ih) - (J-1)\sigma_k^J(w_1^k - Jh) \right) &= \\ \frac{J}{J-1} \sum_{k=1}^2 r_k \left( \sum_{i=1}^{J-1} \sigma_k^i(w_1^k - ih) - (J-1)\sigma_k^J(w_1^k - Jh) \right) &= \\ \sum_{j=1}^J \frac{2cj}{J^2} \sum_{k=1}^2 \frac{r_k}{c} \left( \sum_{i=1}^{J-1} \sigma_k^i(w_1^k - ih) - (J-1)\sigma_k^J(w_1^k - Jh) \right) &= \\ \frac{(J+1)}{J} \sum_{k=1}^2 r_k \left( \sum_{i=1}^{J-1} \sigma_k^i(w_1^k - ih) - (J-1)\sigma_k^J(w_1^k - Jh) \right). & \end{aligned}$$

Taking the difference between these terms produces:

$$\begin{aligned} \frac{1}{J(J-1)} \sum_{i=1}^{J-1} \sum_{k=1}^2 r_k \sigma_k^i(w_1^k - ih) - (J-1)r_k \sigma_k^J(w_1^k - Jh) &= \\ \sum_{k=1}^2 \left( \sum_{i=1}^{J-1} \frac{r_k \sigma_k^i(w_1^k - ih)}{J(J-1)} - \frac{(J-1)r_k \sigma_k^J(w_1^k - Jh)}{J} \right). & \end{aligned}$$

Thus, the costs and benefits in ‘A’ and ‘B’ cancel. Thus  $U_1^*(\bar{z}_J, J)$  and  $U_1^*(\bar{z}_J, J-1)$  are tangent at  $\underline{z}_J$ , and so  $U_1^*(z^s)$  is differentiable at  $z^s = \underline{z}_J$ .

We next show that each  $U_1^*(\bar{z}_J, J)$  is concave in  $\bar{z}_J$ . The second derivative of  $U_1^*(\bar{z}_J, J)$  with respect to  $\bar{z}_J$  is  $-2(J+1)c/J$ , which is clearly negative. Thus  $U_1^*(z^s)$  inherits concavity from the constituent  $U_1^*(\bar{z}_J, J)$  functions.



What remains is characterizing the optimal capacity and depth. For any given  $J$ , the unique interior solution is derived by straightforward differentiation to produce expressions (11) and (12). Since  $U_1^*(z^s)$  is concave, at an interior solution the maximizer  $\bar{z}_{J^*}$  must coincide with the unique value of  $J$  such that  $\bar{z}_{J^*} \in [\underline{z}_J, \underline{z}_{J+1})$ . Otherwise, the optimal state is at a corner. If  $\frac{dU_1^*(0)}{dz^s} < 0$  then  $\bar{z}_{J^*} = 0$  and no state is optimal.

To characterize  $J^*$  and obtain the expression in the statement of the result, observe that the optimal position of the last layer boundary,  $\bar{z}_J$  (as given by (12)) must satisfy (i)  $\bar{z}_J - \underline{z}_J \geq 0$ ; and (ii)  $\underline{z}_{J+1} - \bar{z}_J \geq 0$ . Substituting, conditions (i) and (ii) can be rewritten respectively as:

$$\sum_{k=1}^2 \frac{r_k J \left[ (J+1)(w_1^k - Jh)\sigma_k^J - \sum_{i=1}^J (w_1^k - ih)\sigma_k^i \right]}{(J+1)c} \geq 0 \quad (18)$$

$$\sum_{k=1}^2 \frac{r_k J \left[ \sum_{i=1}^J (w_1^k - ih)\sigma_k^i - (J+1)(w_1^k - (J+1)h)\sigma_k^{J+1} \right]}{(J+1)c} \geq 0. \quad (19)$$

Each of these expressions respectively hold iff:

$$\sum_{k=1}^2 r_k (J+1) \sigma_k^J (w_1^k - Jh) \geq \sum_{k=1}^2 \sum_{i=1}^J r_k \sigma_k^i (w_1^k - ih) \quad (20)$$

$$\sum_{k=1}^2 r_k (J+1) \sigma_k^{J+1} (w_1^k - (J+1)h) \leq \sum_{k=1}^2 \sum_{i=1}^J r_k \sigma_k^i (w_1^k - ih) \quad (21)$$

Since both  $\sigma_k^i$  and  $w_1^k - ih$  are non-increasing in  $i$ , it is clear that each increment in  $J$  adds at most  $\sum_k r_k \sigma_k^J (w_1^k - Jh)$  and  $\sum_k r_k \sigma_k^{J+1} (w_1^k - (J+1)h)$  to the left-hand sides of (20) and (21), respectively. It adds exactly  $\sum_k r_k \sigma_k^J (w_1^k - Jh)$  to their right-hand sides. Thus (18) holds only for  $J$  sufficiently small, and (19) holds only for  $J$  sufficiently large.  $J^*$  is the unique value of  $J$  satisfying both. ■

**Proof of Proposition 4.** Differentiating the expression for  $\bar{z}_{J^*}$  (14) with respect to  $q$  produces:

$$\frac{\partial \bar{z}_{J^*}}{\partial q} = \frac{r_2}{c(J^* + 1)} \sum_{i=1}^{J^*} (1 - \pi_1 - ih) \frac{\partial \sigma_2^i}{\partial q}. \quad (22)$$

For a fixed  $J^*$ , this derivative will be positive iff  $\sum_{i=1}^{J^*} (1 - \pi_1 - ih) \frac{\partial \sigma_2^i}{\partial q} > 0$ .

Observe that  $\frac{\partial \sigma_2^i}{\partial q} > 0$  at an interior solution,  $\frac{\partial \sigma_2^i}{\partial q} = 0$  at a corner solution, and  $\frac{\partial \sigma_2^i}{\partial q}$  is independent of  $\pi_1$ . Thus for any  $J^*$ ,  $\frac{\partial \bar{z}_{J^*}}{\partial q} > (=)(<) 0$  if:

$$\pi_1 < (=)(>) \tilde{\pi}_1 \equiv 1 - \frac{h \sum_{i=1}^{J^*} i \frac{\partial \sigma_2^i}{\partial q}}{\sum_{i=1}^{J^*} \frac{\partial \sigma_2^i}{\partial q}}. \quad (23)$$

The value of  $\tilde{\pi}_1$  is clearly contained within  $(1 - Jh, 1)$ . Note finally that in the special case where  $\sigma_2^1, \dots, \sigma_2(j') \in (0, 1)$ , this threshold simplifies to  $1 - h(2j' + 1)/3$ . ■

**Proof of Proposition 5.** Differentiating the expression for  $\bar{z}_{J^*}$  (14) with respect to  $r_1$  produces:

$$\frac{\partial \bar{z}_{J^*}}{\partial r_1} = \frac{1}{c(J^*+1)} \left[ \sum_{i=1}^{J^*} (1-ih) \left( r_1 \frac{\partial \sigma_1^i}{\partial r_1} + \sigma_1^i \right) + \sum_{i=1}^{J^*} (1-\pi_1-ih) \left( (1-r_1) \frac{\partial \sigma_2^i}{\partial r_1} - \sigma_2^i \right) \right].$$

(i) Observe that by assumption,  $\frac{\partial \sigma_1^i}{\partial r_1} \geq 0$  and  $\frac{\partial \sigma_2^i}{\partial r_1} \leq 0$  at interior  $\sigma_1^i$ . Furthermore, in equilibrium  $1 - J^*h > 0$ , and therefore  $(1-ih)(r_1 \frac{\partial \sigma_1^i}{\partial r_1} + \sigma_1^i) > 0$  for all  $i$ .

$\frac{\partial \bar{z}_{J^*}}{\partial r_1} > 0$  then holds if:

$$(1-\pi_1-ih) \left( (1-r_1) \frac{\partial \sigma_2^i}{\partial r_1} - \sigma_2^i \right) \geq 0 \text{ for all } i. \quad (24)$$

Two conditions are sufficient to ensure (24). First, the condition obviously holds if  $\sigma_2^i = 0$ . Because  $\sigma_k^i$  is weakly decreasing in  $i$ ,  $\sigma_2^1 = 0$  is sufficient. Second, observe that  $(1-r_1) \frac{\partial \sigma_2^i}{\partial r_1} - \sigma_2^i \leq 0$  for all  $i$ . Thus if  $1 - \pi_1 < h$  then (24) also holds.

(ii) To show when  $\frac{\partial \bar{z}_{J^*}}{\partial r_1}$  is increasing in  $\pi_1$ , observe that  $\frac{\partial \sigma_1^i}{\partial \pi_1} = \frac{\partial \sigma_2^i}{\partial \pi_1} = 0$  at a corner values. We take the partial derivative with respect to  $\pi_1$ :

$$\frac{\partial^2 \bar{z}_{J^*}}{\partial r_1 \partial \pi_1} = \frac{1}{c(J^*+1)} \left[ \sum_{i=1}^{J^*} (1-ih) \frac{\partial \sigma_1^i}{\partial \pi_1} - \sum_{i=1}^{J^*} \left( (1-r_1) \frac{\partial \sigma_2^i}{\partial \pi_1} - \sigma_2^i \right) \right]. \quad (25)$$

If  $\sigma_1^i = 1$  for all  $i$ , then  $\frac{\partial \sigma_1^i}{\partial \pi_1} = 0$  and the bracketed term in (25) is non-negative. Since  $\sigma_k^i$  is weakly decreasing in  $i$ , this condition reduces to  $\sigma_1^{J^*} = 1$ . ■

**Proof of Proposition 6.** We prove the result by examining the behavior of  $\bar{z}_J$  in relation

to  $\underline{z}_J$  and  $\underline{z}_{J+1}$ , the minimum thresholds for  $J$  and  $J+1$  layers.

First, define  $z_J^- \equiv \bar{z}_J - \underline{z}_J$  for a given  $J$ . By the argument in the proof of Proposition 3, an optimal non-empty  $J$ -layer knowledge hierarchy is feasible if  $z_J^- > 0$ . Using expressions (12) and (13),  $z_J^-$  evaluates to:

$$z_J^- = \frac{J}{c(J+1)} \left( J \sum_{k=1}^2 r_k \sigma_k^J (w_1^k - Jh) - \sum_{k=1}^2 r_k \sum_{i=1}^{J-1} \sigma_k^i (w_1^k - ih) \right). \quad (26)$$

A necessary condition for depth to decrease in  $q$  is  $\frac{\partial z_J^-}{\partial q} < 0$ . Differentiating  $z_J^-$  with respect to  $q$  produces:

$$\frac{\partial z_J^-}{\partial q} = \frac{Jr_2}{c(J+1)} \left( J(1 - \pi_1 - Jh) \frac{\partial \sigma_2^J}{\partial q} - \sum_{i=1}^{J-1} (1 - \pi_1 - ih) \frac{\partial \sigma_2^i}{\partial q} \right). \quad (27)$$

Next, define  $z_J^+ \equiv \underline{z}_{J+1} - \bar{z}_J$ , again using (12) and (13).

$$z_J^+ = \frac{J}{c(J+1)} \left( \sum_{k=1}^2 r_k \sum_{i=1}^J (w_1^k - ih) \sigma_k^i - (J+1) \sum_{k=1}^2 r_k (w_1^k - (J+1)h) \sigma_k^{J+1} \right). \quad (28)$$

A necessary condition for depth to increase in  $q$  is  $\frac{\partial z_J^+}{\partial q} < 0$ . Differentiating  $z_J^+$  with respect to  $q$  produces:

$$\frac{\partial z_J^+}{\partial q} = \frac{Jr_2}{c(J+1)} \left( \sum_{i=1}^J (1 - \pi_1 - ih) \frac{\partial \sigma_2^i}{\partial q} - (J+1)(1 - \pi_1 - (J+1)h) \frac{\partial \sigma_2^{J+1}}{\partial q} \right). \quad (29)$$

In what follows we make use of the assumptions that  $\frac{\partial \sigma_2^i}{\partial q} > 0$  for interior  $\sigma_2^i$ ;  $\frac{\partial \sigma_2^i}{\partial q} = 0$  for corner  $\sigma_2^i$ ,  $\frac{\partial \sigma_2^i}{\partial q}$  is non-decreasing in  $i$ ; and  $\sigma_2^i$  is decreasing in  $i$ . Note that the special case of costless activation and reactivation and uniformly distributed re-election probabilities, interior values of  $\sigma_2^i$  are given by  $\frac{1}{2} + \frac{r_2 q \pi_2 - jh + q(1 - \pi_2)}{2\varepsilon q \pi_2}$  and satisfy these assumptions. There are four cases, depending on the values of  $\sigma_2^i$ , or equivalently increasing values of  $q$ .

(i) If  $\sigma_2^i = 0$  for all  $i$  (for which  $\sigma_2^1 = 0$  is sufficient), then obviously  $\frac{\partial z_J^-}{\partial q} = \frac{\partial z_J^+}{\partial q} = 0$ . Thus there can be no changes in  $J^*$  until  $q$  increases enough such that  $\frac{\partial \sigma_2^1}{\partial q} > 0$ .

(ii) Suppose  $\sigma_2^j = 0$  but  $\sigma_2^i > 0$  for some  $i$ . Then by manipulating (27),  $\frac{\partial z_J^-}{\partial q} > 0$  iff:

$$\pi_1 > \underline{\pi}_1^- \equiv 1 - \frac{h \sum_{i=1}^{J-1} i \frac{\partial \sigma_2^i}{\partial q}}{\sum_{i=1}^{J-1} \frac{\partial \sigma_2^i}{\partial q}}.$$

Likewise, manipulating (29),  $\frac{\partial z_J^+}{\partial q} > 0$  iff:

$$\pi_1 < \underline{\pi}_1^+ \equiv 1 - \frac{h \sum_{i=1}^J i \frac{\partial \sigma_2^i}{\partial q}}{\sum_{i=1}^J \frac{\partial \sigma_2^i}{\partial q}}.$$

Since  $\sigma_2^j = 0$ , we then have  $\pi_1^+ = \pi_1^- = \tilde{\pi}_1$ , where  $\tilde{\pi}_1$  (23) is the threshold value of  $\pi_1$  for capacity growth in the proof of Proposition 4. We conclude that for  $\pi_1 > \tilde{\pi}_1$ ,  $z_+$  is decreasing,  $z_-$  is increasing, and  $z_{J^*}$  is decreasing in  $q$ . And for  $\pi_1 < \tilde{\pi}_1$ ,  $z_+$  is increasing,  $z_-$  is decreasing, and  $z_{J^*}$  is increasing in  $q$ .

(iii) Suppose  $\sigma_2^j \in (0, 1)$  (which implies  $\sigma_2^i > 0$  for  $i < J$ ). Manipulating (27),  $\frac{\partial z_J^-}{\partial q} > 0$  iff:

$$\pi_1 < \bar{\pi}_1^- \equiv 1 - \frac{h \left[ J^2 \frac{\partial \sigma_2^j}{\partial q} - \sum_{i=1}^{J-1} i \frac{\partial \sigma_2^i}{\partial q} \right]}{J \frac{\partial \sigma_2^j}{\partial q} - \sum_{i=1}^{J-1} \frac{\partial \sigma_2^i}{\partial q}}.$$

To verify that the denominator in the preceding expression is positive, note that  $J \frac{\partial \sigma_2^j}{\partial q} > \sum_{i=1}^{J-1} \frac{\partial \sigma_2^i}{\partial q}$  follows from the assumption that  $\frac{\partial \sigma_2^i}{\partial q}$  is non-decreasing in  $i$ . By a similar argument, the numerator is positive as well.

Likewise, manipulating (29),  $\frac{\partial z_J^+}{\partial q} > 0$  iff:

$$\pi_1 > \bar{\pi}_1^+ \equiv 1 - \frac{h \left[ \sum_{i=1}^J i \frac{\partial \sigma_2^i}{\partial q} - (J+1)^2 \frac{\partial \sigma_2^j}{\partial q} \right]}{\sum_{i=1}^J \frac{\partial \sigma_2^i}{\partial q} - (J+1) \frac{\partial \sigma_2^j}{\partial q}}.$$

Thus for  $\pi_1 < \hat{\pi}_1 \equiv \min\{\bar{\pi}_1^-, \bar{\pi}_1^+\}$ ,  $z_+$  is decreasing and  $z_-$  is increasing in  $q$ . Furthermore, it is straightforward to verify that  $\bar{\pi}_1^- < 1 - Jh$  and  $\bar{\pi}_1^+ < 1 - (J+1)h$ . Since by expression (23)  $\tilde{\pi}_1 > 1 - Jh$ ,  $\pi_1 < \hat{\pi}_1$  implies that  $\pi_1 < \tilde{\pi}_1$ , and so by Proposition 4,  $z_{J^*}$  is increasing in  $q$ .

Similarly, for  $\pi_1 > \hat{\pi}_1 \equiv \max\{\bar{\pi}_1^-, \bar{\pi}_1^+\}$ ,  $z_+$  is increasing and  $z_-$  is decreasing in  $q$ .

(iv) When  $\sigma_2^i = 1$  for all  $i$  (for which  $\sigma_2^j = 1$  is sufficient),  $\frac{\partial z_J^-}{\partial q} = \frac{\partial z_J^+}{\partial q} = 0$ . Thus there can be no changes in  $J^*$  for any  $q$  satisfying this condition.

Combining the results produces the reported comparative statics on  $q$  for  $\pi_1 < \hat{\pi}_1$  and  $\pi_1 > \hat{\pi}_1$ . ■

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