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ABSTRACT

Friedman (1962) argued that a free market in which schools compete based upon their reputation would lead to an efficient supply of educational services. This paper explores this issue by building a tractable model in which rational individuals go to school and accumulate skill valued in a perfectly competitive labor market. To this it adds one ingredient: school reputation in the spirit of Holmstrom (1982). The first result is that if schools cannot select students based upon their ability, then a free market is indeed efficient and encourages entry by high productivity schools. However, if schools are allowed to select on ability, then competition leads to stratification by parental income, increased transmission of income inequality, and reduced student effort---in some cases lowering the accumulation of skill. The model accounts for several (sometimes puzzling) findings in the educational literature, and implies that national standardized testing can play a key role in enhancing learning.
Please accept my resignation. I don’t want to belong to any club that will accept me as a member. —Groucho Marx

By and large, I’m going to be picking from the law schools that basically are the hardest to get into. They admit the best and the brightest, and they may not teach very well, but you can’t make a sow’s ear out of a silk purse. If they come in the best and the brightest, they’re probably going to leave the best and the brightest, O.K.? —Antonin Scalia

1. INTRODUCTION

The ability of firms to acquire and maintain reputations for quality is a key ingredient for the efficient provision of complex goods and services in a market economy. Friedman (1962) further hypothesized that this ingredient is sufficient, namely, sellers’ concern for their reputation ensures that an unfettered market efficiently supplies such goods.¹ In this paper, we study the market for educational services and show that this hypothesis is true only under the appropriate conditions. Specifically, if schools cannot select students based upon their ability, then a free market is efficient and encourages entry by high productivity schools. However, if schools use an entrance exam to select students, then competition leads to stratification by parental income, increased transmission of income inequality, and reduced student effort—in some cases lowering the accumulation of skill.

These results follow from an “anti-lemons” effect that arises when firms can influence the quality of their good by positively selecting their buyers. Specifically, Akerlof (1970) showed that if the quality of goods is difficult to observe, then sellers with high quality goods exit the market, leaving behind only low quality “lemons” for sale. In contrast, the perceived quality of a school depends upon the quality of the buyers who purchase its services, resulting in a tendency for selective schools to drive non-selective ones from the market. Analogous phenomena are observed in other markets for service goods. For example, restaurants, social clubs, and law firms are perceived to be of high quality when they serve exclusive clients. What makes education unique is that the industry’s output (student achievement) depends upon both firm (school) productivity and buyer (student) effort.

This matters because Holmstrom (1999) has shown that the incentive effect of reputation depends upon the existence of uncertainty regarding ability.² When uncertainty is large, individuals have an incentive to work hard to show to the market that they are able. In contrast, if individual ability is

¹ See MacLeod (2007) for a review of the literature on reputation and quality assurance.
² This model is widely used in labor economics. See for example Gibbons and Murphy (1992), Farber and Gibbons (1996), and Altonji and Pierret (2001).
known, then effort cannot affect the market’s perception of one’s ability, reducing effort incentives. We apply this insight to school reputation. Schools with good reputation are attractive to students because admission to such a school signals high ability, thereby raising future income. In addition, however, admission to a selective school reduces uncertainty regarding ability, resulting in lower effort.

Finally, the model illustrates that a school’s reputation is a function of both the quality of its students and the school’s value added. This implies that parents may select a school with lower value added if this is counterbalanced by a sufficiently high quality student population. Hence, a concern for school reputation does not imply that parents will always choose schools with greater value added.

These anti-lemon effects reconcile two apparently contradictory empirical findings in the school choice literature. First, there is evidence that parents value school choice and prefer higher-achieving schools (Black (1999), Hsieh and Urquiola (2006), and Hastings and Weinstein (2008)). Second, there is no consistent evidence that introducing choice substantially improves learning, or that private schools have higher value added than public ones (McEwan et al. (2008) and Neal (2008)). In the reputation model, these are in fact the expected results when there is competition between selective schools. In contrast, if cream-skimming is limited, then choice can enhance performance, consistent with recent evidence regarding charter schools (Hoxby and Murarka (2008) and Abdulkadiroglu et al. (2009)). The model also predicts that European-style systems featuring national testing and competition are likely to perform relatively well, consistent with the evidence discussed in Neal (2008).

Our agenda is as follows. In the next section we introduce a model that supposes that individuals go to school and graduate with skills that depend additively on three factors: i) innate ability, ii) effort devoted to studying as opposed to non-academic activities like sports and student government, and iii) school value added. Upon graduation, each individual is employed at a wage that reflects the market’s best estimate of her skill. This estimate is based upon two signals: an individual-specific measure of skill in the form of a graduation test, and the reputation of the school she attended. A school’s reputation is simply the expected skill of its graduates.

On the supply side, schools produce two outputs: educational value added and amenities. Educational value added enhances skill, while amenities are consumption goods that raise students'
welfare but not their skills. Schools are assumed to be of low or high productivity. The latter produce value added at a lower marginal cost but are initially not sufficiently numerous to supply the whole market.

Next we study three scenarios. Section 3 considers a public school system in which the median voter chooses taxes and school characteristics. Public schools are assumed to be non-selective in that the distribution of innate ability is the same at all of them. This would result, for example, if schools were assigned students in a randomized fashion. This benchmark scenario implies that policies seeking to improve learning have to either increase the prevalence of high productivity schools, or raise students’ academic effort.

In Section 4 the base scenario is compared to one that features only for-profit schools, with the caveat that these must also be non-selective. For simplicity, students differ only with respect to income, which is assumed to be uncorrelated with ability. Compared to the public system, for-profit provision has two advantages. First, it allows schools to respond to consumer heterogeneity in terms of tastes for amenities; second, high productivity schools earn positive profits that provide incentives for further entry. In addition, subsidizing for-profit schools via vouchers is shown to be particularly beneficial to lower income students—not only does it increase their educational investment via redistribution, it also raises the likelihood that high productivity schools enter the market to serve them. In short, the second scenario shows that when there is no selectivity and school reputation therefore reflects only value added, the model captures Friedman’s (1962) intuition: private participation raises school productivity, and vouchers enhance the outcomes of low income individuals.

Finally, Section 5 introduces a third scenario with a system of non-selective public schools in which selective for-profit schools are given the opportunity to enter and choose students based on an admissions test that measures innate ability. If they enter, such schools’ reputation therefore varies with their productivity and their student composition. To highlight the effect of selectivity, this scenario makes three assumptions that would seem to foreclose for-profit entry: i) individuals differ only with respect to innate ability (thereby eliminating private schools’ ability to cater to heterogeneity in the demand for amenities), ii) all schools are equally productive, and iii) for-profit schools must operate unsubsidized.
For-profit entry turns out to be feasible, despite these assumptions, as long as private schools can cream skim the highest ability students from the public system. These individuals are willing to pay a premium for selective schools because employers are willing to pay higher salaries to these schools’ graduates. The resulting equilibrium is characterized by a strict hierarchy of schools, with the highest ability students going to the most selective for-profit schools, and the low ability ones remaining in the non-selective public sector.

Section 6 discusses the framework’s empirical and policy implications. There we note that the model abstracts from peer effects for several reasons. First, Epple and Romano (1998) have shown that peer effects lead to stratification in equilibrium. The present model also implies stratification, and hence provides an alternative hypothesis for this effect. It also makes additional predictions that can help disentangle peer effects from reputation effects. Second, peer composition can affect learning through different channels, such as lowering disruption in class (Lazear (2001)), or allowing material to be presented in an ability-appropriate manner (Duflo, Dupas, and Kremer (2008)). Rather than make a choice regarding the form that peer effects take, this paper focuses upon the implications of school reputation. Section 7 contains concluding comments.

2. Setup

This section sets out four basic elements of the model: i) individual utility and skill, ii) school characteristics, iii) the labor market and signals of skill, and iv) wages and student effort. The key market imperfection is that student innate ability and student effort are not directly observable, but can only be inferred from performance on tests that provide a noisy measure of individual skill.

2.1. Individual utility and skill. Consider a two period model in which individuals first go to school, where they exert costly effort. In the second period they work, and their wages reflect the skills acquired in the first period. Utility is given by:

\[
U_{is} = \log(c_{is}^0) + \delta_i \log(c_{is}^1) + \phi_i \log(z_{is}) + \Psi(e_{is}, a_i),
\]

where \(i\) indexes individuals, and \(s\) stands for the school they attend. \(c^0\) and \(c^1\) denote consumption in each period, and \(\delta\) is the discount rate. \(\phi\) stands for the taste for non-educational amenities, which are labeled \(z\) and are assumed to raise student welfare directly, but to not produce skills (manicured lawns or air conditioning might be examples).
The last term in (2.1) reflects that individuals must choose to allocate their effort between: i) academic effort, $e_{is}$, which refers to activities like doing homework and paying attention in class (more broadly, were one to consider parental actions, it could stand for time spent helping with homework, or expenditure on after-school tutoring), and ii) non-academic activities like sports, student government, watching television, or community service.\(^3\)

It is assumed that academic effort improves students' skill, while non-academic activities do not raise skill but provide a return, $\Psi(e_{is}, a_i) \geq 0$, that is increasing in the taste for these activities, $a_i$. Academic effort is costly and is rendered more so by increases in the taste for non-scholastic activities; specifically, $\frac{\partial \Psi}{\partial e_{is}} < 0$, $\frac{\partial^2 \Psi}{\partial e_{is} \partial e_{is}} < 0$, and $\frac{\partial^2 \Psi}{\partial e_{is} \partial a_i} < 0$. Finally, for most of the analysis heterogeneity in $a_i$ does not play a role, and accordingly we write $\Psi(e_{is})$ for simplicity.

It is assumed that individuals cannot save, and hence consumption is given by:

$$
\begin{align*}
    c^0_{is} &= Y_i - p_s, \\
    c^1_{is} &= W_{is},
\end{align*}
$$

where $Y$ is exogenous income (e.g., income students receive from their parents) and is divided between the expenditure students must incur to attend school, $p$, and other first period consumption, $c^0$. $W$ is the individual’s market wage in the second period.

An individual’s skill after attending school $s$ is denoted by $\theta_{is}$, and is determined by her innate ability, her academic effort, and her school’s value added. Specifically, skill is given by:

$$
\theta_{is} = \alpha_i + e_{is} + \beta_s,
$$

where $\alpha_i$ is innate ability and is independent of the school student $i$ attends; $e_{is}$ is academic effort, and $\beta_s$ is the value added school $s$ provides to all the students it enrolls. Innate ability, $\alpha_i$, is distributed normally with zero mean and precision $\rho^\alpha = \frac{1}{\sigma^2_\alpha}$, the reciprocal of the variance $\sigma^2_\alpha$. Precision and variance are used interchangeably below, depending upon which one results in the simpler formula.

Two assumptions implicit in this formulation of skill deserve discussion upfront. First, academic effort and school value added enter in a separable fashion. This assumption would seem hard

\(^3\) Although student effort has not been a focus of the literature, Bishop (2004) emphasizes its importance, and there is a growing empirical research on interventions to elicit effort (see Angrist, Lang, and Oreopoulos (2006), and Kremer, Miguel, and Thornton, forthcoming).
to test, as this might require exposing arguably comparable students to different levels of value added, and then studying differences in their measured effort. In fact, recent randomizations in developing countries, although not focused on the issue, achieve this to a large extent. Specifically, Banerjee, Cole, Duflo, and Linden (2007) and He, Linden, and MacLeod (2008) evaluate a series of interventions (e.g. targeted tutoring) and find them to significantly raise test scores. At the same time, these interventions are not associated with changes in attendance, a key measure of student effort in developing countries. Second, in contrast to seminal models in the literature (e.g., Epple and Romano, 1998) the specification of skill features no peer effects. We make this assumption for simplicity and return to a discussion of it later in the paper.

2.2. School characteristics. In providing instructional value added, $\beta_s$, and non-academic amenities, $z_s$, schools incur costs on a per-capita basis given by:

\[
C_s(\beta_s, z_s) = q_s C(\beta_s) + z_s
\]

where $C(\cdot)$ is a twice differentiable cost function satisfying $C', C'' > 0$, and $C(0) = 0$. School productivity, parametrized by $q_s > 0$, takes on two values corresponding to low and high productivity, $q_L$ and $q_H$ respectively, where $q_L > q_H$. Given that the marginal cost of providing value added is lower in the high productivity schools, these will generally supply greater value added.

The size of the student population is normalized to 1, and $n > 1$ is the number of schools. It is assumed that each school has one student. This implies that not all schools will be utilized, and hence there is real competition between schools. In addition, the initial fraction of schools that are of high productivity is fixed at $\lambda \in (0, 1)$, such that $\lambda n < 1$ and high productivity schools cannot serve the whole market. The question will be whether in equilibrium all high productivity schools enter the market, and whether these earn positive profits that provide an incentive for further high productivity entry. Per-student profit at school $s$ is:

\[
\Pi_s = p_s - q_s C(\beta_s) - z_s.
\]

\footnote{These assumptions are merely for simplicity. What is crucial is that the number of students is large, and that schools have limited capacity.}
Under perfect competition low productivity schools earn zero profit, while high productivity schools earn positive profits, the magnitude of which depends on market structure. Finally, let $I_s$ denote the resources schools devote to value added:

\begin{equation}
I_s = p_s - z_s - \Pi_s = q_s C(\beta_s),
\end{equation}

i.e., value added is tuition minus expenditures on amenities and profits, all in per-student terms.

### 2.3. The labor market and signals of skill.

Individuals are employed in a perfectly competitive labor market where they are paid a wage equal to the market’s best estimate of their skill. Following Jovanovic (1979) and Harris and Holmström (1982), we suppose information regarding worker ability is symmetric; workers and firms have the same beliefs regarding individual skill, and both efficiently use the available information to estimate it.

The market receives two signals of individual skill. First, it observes an individual-specific measure of learning called a graduation test. Its existence can be motivated by reference to the standardized high school graduation or university admissions exams in existence in countries including Germany, Malaysia, Romania, South Korea, and Turkey. Performance in these tests strongly influences college admissions and, eventually, job market success. Analogous motivation at a higher educational level comes from the “job market papers” that Economics Ph.D. students distribute as they enter the labor market. These provide an individual-specific signal of skill, and significantly influence students’ labor market outcomes.

Formally, the graduation test reflects an individual’s innate ability, her academic effort, her school’s value added, and an error term:

\begin{equation}
t_{is} = \alpha_i + \epsilon_{is} + \beta_s + \epsilon_{is}^t,
\end{equation}

where $\epsilon_{is}^t \sim N(0, \sigma_t^2)$, and hence the test has precision $\rho^t = \frac{1}{\sigma_t^2}$. Precision intuitively corresponds to test quality. When a test is uninformative, its precision is zero; greater precision implies a more accurate measure of skill. Precision is assumed to be finite, such that the graduation test never perfectly measures skill.

---

5 The expression for profit highlights one role for amenities. Specifically, in many jurisdictions schools operate under a zero profit constraint, and one way for schools to dissipate rents is through expenditures on amenities like nice grounds, field trips, and so forth.

6 In some cases like Germany, these exams are not national but jurisdiction-specific.
The second signal the market observes is the identity and therefore the reputation of the school each student attended.\footnote{We will assume that the identity of a student’s school of origin influences her compensation, as suggested by Hoekstra (2009) and Saavedra (2008), though Dale and Krueger (2002) find mixed evidence in this regard.} This is defined to be the expected skill of the school’s graduates:

\begin{equation}
R_s = E\{\theta_i|i \in s\} = E\{\alpha_i|i \in s\} + \hat{e}_s + \beta_s.
\end{equation}

Thus, a school’s reputation depends on its own value added, $\beta_s$, on the average innate ability of its students, $E\{\alpha_i|i \in s\}$, and on their average academic effort, $\hat{e}_s$. This effort is determined as an equilibrium outcome that depends upon the structure of the school system, as discussed below. Market participants are assumed to be able to anticipate the effect of the school system upon incentives, and hence they have correct expectations regarding the average effort, $\hat{e}_s$, and value added, $\beta_s$ at each school.

Finally, suppose that each school has a large number of students, and hence any given individual’s effort has a negligible impact upon her school’s reputation. For simplicity, this effect is set to zero, $\frac{\partial R_s}{\partial e_{is}} = 0$. In contrast, a student is able to raise her graduation test score; formally, $\frac{\partial t_{is}}{\partial e_{is}} = 1$.

2.4. **Wages and student effort.** Log wages in the second period, $w$, are equal to expected skill:

\begin{equation}
w_{is} = \log(W_{is}) = E\{\theta_{is}|t_{is}, R_s\},
\end{equation}

\begin{equation}E\{\alpha_i + e_{is} + \beta_{is}|t_{is}, R_s\}.
\end{equation}

Given that the cost of academic effort is separable from consumption, utility maximizing students who anticipate the effect of academic effort upon future wages choose effort to satisfy:

\begin{equation}-\Psi'(e_{is}) = \delta \frac{\partial E\{w_{is}|i \in s\}}{\partial e_{is}}.
\end{equation}

where $E\{w_{is}|i \in s\}$ is the expected wage when admitted to school $s$.

For later reference, it is useful to note the effort that would exist if skill were observable. In this case, the market would simply set wages equal to skill: $w_{is} = \theta_{is} = \alpha_i + e_{is} + \beta_s$, such that $\frac{\partial w}{\partial e} = 1$, and (2.7) implies that efficient effort, denoted by $e^*$, would satisfy:

\begin{equation}-\Psi'(e^*) = \delta \frac{\partial w_{is}}{\partial e_{is}},
\end{equation}

\begin{equation}= \delta.
\end{equation}
In this case students’ academic effort is independent of the school they attend. In general, since skill is imperfectly observed this level of effort will not be attainable. Rather, the incentive to study will vary with the structure of the market for educational services.

3. A SYSTEM OF NON-SELECTIVE PUBLIC SCHOOLS

The benchmark scenario is a public school system consisting of only non-selective schools. This means that the distribution of student innate ability is the same at every school, which would result, for instance, if students were randomly assigned to schools. It would also hold if students attended the school closest to them, and there were no spatial segregation in student ability. Such extreme non-selectivity is probably not observed in practice, but it provides an analytically useful benchmark. Further, this assumption is consistent with the lack of explicit ability-based admissions policies observed among public schools in many countries.

The lack of competition in the market for public schools is assumed to imply that expected school productivity is the mean productivity of all available schools: \( \bar{q} = \lambda q_H + (1 - \lambda)q_L \). For simplicity, suppose also that: i) all individuals have the same preferences and differ only by income (such that \( a_i = a, \delta_i = \delta, \) and \( \phi_i = \phi \) for all \( i \)), and ii) income, \( Y_i \), is independent of innate ability, \( \alpha_i \). These assumptions are not essential for the analysis of the public sector, but they simplify the comparison with for-profit schools below. Next we consider individual behavior in this setting.

3.1. Individual effort. Recall that skill is given by \( \theta_{is} = \alpha_i + e_{is} + \beta_s \), and that innate ability, \( \alpha_i \), is not observed. The market sets workers’ wages equal to their expected skill, and its beliefs are determined by two signals. First, the market observes the reputation of the schools students attend. Since in this scenario schools are not selective—all have an innate ability distribution \( \alpha_i \sim N(0, 1/\rho^\alpha) \)—school reputation is only a function of average student academic effort and school value added: \( R_s = E\{\theta_i | i \in s\} = \hat{e}_s + \beta_s \). Second, the market observes students’ graduation test, \( t = \alpha_i + e_{is} + \beta_s + e'_{is} = R_s + \alpha_i + e'_{is}, \) measured with precision \( \rho^t \).

In the perfectly competitive labor market considered, an individual’s wage is set equal to her expected skill conditional upon these signals. From Bayes’ rule one has that the wage is a weighted
average of the two signals:\(^8\)

\[
(3.1) \quad w_{is} = \pi^{(t)\alpha} t_{is} + \pi^{(a)t} R_s
\]

\[
(3.2) \quad = R_s + \pi^{(t)\alpha} (t_{is} - R_s)
\]

where \(\pi^{(t)\alpha} = \frac{\rho^t}{\rho^x + \rho^y} \in [0, 1]\) is the weight assigned to the graduation test, and \(\pi^{(a)t} = \frac{\rho^a}{\rho^x + \rho^y} \in [0, 1]\) is the weight attached to school reputation.\(^9\)

Expression (3.1) illustrates that an individual’s wage can be expressed as a convex combination of the two signals of skill, where the weight assigned to each signal depends upon its relative precision. Alternately, expression (3.2) shows that each individual’s wage is set equal to her school’s reputation plus an adjustment, \(\pi^{(t)\alpha} (t_{is} - R_s)\). This adjustment reflects the information contained in the graduation test score. If \(t_{is} > R_s\) then the student has performed better than the average student at school \(s\), and the market adjusts her wage upwards; if a student does poorly on the graduation test, the market adjusts its expectation downwards.

These expressions make clear that the benchmark scenario does not provide first best incentives for academic effort. The reason is that while a student’s effort can affect her test score, \(\left(\frac{\partial t_{is}}{\partial e_{is}} = 1\right)\), its effect on her school’s reputation is negligible \(\left(\frac{\partial R_s}{\partial e_{is}} = 0\right)\). A rational individual anticipates this, and the level of academic effort in a non-selective school system, \(e_{NS}\), therefore satisfies:

\[
(3.3) \quad -\Psi'(e_{NS}, a) = \delta \frac{\partial w_{is}}{\partial e_{is}} = \delta \frac{\partial w_{is}}{\partial t_{is}} \frac{\partial t_{is}}{\partial e_{is}} = \delta \pi^{(t)\alpha}.
\]

While academic effort is increasing in the precision of the graduation test, it is lower than the first best (given by \(-\Psi'(e^*, a) = \delta\)). The following proposition summarizes and expands these results.

**Proposition 1.** In a non-selective school system, students choose academic effort \(e_{NS}\) satisfying (3.4). This effort is lower than the first best, \(e^*\), given by (2.8). Academic effort is increasing in the precision of the graduation test, and decreasing in \(a\), the taste for alternative activities.

\(^8\) See DeGroot (1972), Theorem 1, Section 9.5.

\(^9\) This is a notation we will use henceforth, namely, the term \(\pi^{(e)y}z = \frac{\rho^e}{\rho^x + \rho^y + \rho^z}\) is the weight attached to signal \(x\) in a situation in which signals \(y\), and \(z\) are also present. Similarly, we will denote \(\pi^{(xyz)} = \frac{\rho^x + \rho^y + \rho^z}{\rho^x + \rho^y + \rho^z}\).
Proof. The first result follows from first order conditions,

\[-\Psi'(e^{NS}, a) = \delta \pi(t) < \delta = -\Psi'(e^{*}, a),\]

and from the fact that \(\Psi\) is concave in effort. The second follows from

\[
\frac{d(\Psi'(e^{NS}, a) + \delta \pi(t))}{da} = 0,
\]

from which we get

\[
\frac{de^{NS}}{da} = -\frac{\Psi_{ei}}{\Psi_{ee}} < 0.
\]

\[\square\]

Proposition 1 highlights that while ultimately a school system may be judged by the performance of its students, some of the factors that determine their outcomes are at least partially beyond schools' control. In particular, while schools can improve their value added, students' performance also depends upon how students allocate their time between academic and alternative activities.\(^{10}\)

This also suggests that the broader institutional settings in different jurisdictions can make a difference. If a neighborhood or country has in place institutions that reward non-academic endeavors like sports, student government, or gang activity, this will detract from academic effort. Second, if an educational system has poor measures of individual performance, then the market will set wages using other observable characteristics, such as the identity of the school or the district a student attended. In such cases, superior students from under-performing schools have no way to signal their skill, and will therefore rationally divert effort toward non-academic activities.

3.2. Public school characteristics. Consider the characteristics that schools in a non-selective public system would have if these were controlled by a median voter who selects the level of funding per student, \(p_s\), and the level of amenities, \(z_s\), to be provided by school \(s\). The focus is on these two characteristics as the primary control variables because they are directly observable by parents. Further, along with schools' equilibrium profit levels, these choices determine value added, \(\beta_s\), via the expression

\[
I_s = p_s - z_s - \Pi_s = q_s C(\beta_s).
\]

In choosing these parameters, voters know that school productivity is uncertain and has expected value \(\bar{q} = \lambda_s q_H + (1 - \lambda_s) q_L\), where \(\lambda_s\) is the probability that school \(s\) is of high productivity. Their formal problem is therefore:

\[
(3.5) \quad \max_{p_s, z_s} U(p_s, z_s, \Pi_s, e_s | \gamma_i, \lambda_s)
\]

\(^{10}\) This provides one explanation for why boarding schools are sometimes preferred by parents. These schools have better control over activities both within and out of the classroom. If appropriately designed, this environment may enhance performance relative to a day-school where outside activities are less strictly regulated.
where an individual’s utility, given characteristics $\gamma_i = \{Y_i, \phi, a, \delta\}$, is defined by:

\[
U(p_s, z_s, \Pi_s, e_s|\gamma_i, \lambda_s) = \log(Y_i - p_s) + \delta[\beta(p_s - z_s - \Pi_s, \lambda_s) + e_s] + \phi \log(z_s) + \Psi(e_s, a),
\]

and school value added is given by:

\[
\beta(I_s, \lambda_s) = \lambda_s \beta_H(I_s) + (1 - \lambda_s) \beta_L(I_s) = \lambda_s C^{-1} \left( \frac{I_s}{q_H} \right) + (1 - \lambda_s) C^{-1} \left( \frac{I_s}{q_L} \right).
\]

This problem can be simplified by noting two facts. First, in a public system like that in this benchmark scenario, voters will always set profits equal to zero. Second, the level of academic effort at school $s$, $e_s$, is chosen by students as a function of school selectivity and expected returns in the labor market. Given than in the present scenario schools are non-selective, individuals will choose effort $e^{NS}$ as defined in Proposition 1. These facts imply that problem (3.5) can be written as

\[
\max_{p_s, z_s} U(p_s, z_s, 0, e^{NS}|\gamma_i, \lambda_s).
\]

To work out the expenditure on value added and amenities, it is useful to define the marginal benefit from additional expenditure on value added:

\[
MB(I_s, \lambda_s) = \delta \frac{\partial \beta(I_s, \lambda_s)}{\partial I}.
\]

This represents the marginal future income gain from investing in educational value added today. It falls with increases in expenditure on value added ($\frac{\partial MB}{\partial I} < 0$), and rises with increases in the level of amenities, profits, and, importantly, productivity ($\frac{\partial MB}{\partial z} > 0$, $\frac{\partial MB}{\partial \Pi} > 0$, $\frac{\partial MB}{\partial \lambda} > 0$). With this definition, the solution to the optimal school choice problem is summarized as follows:

**Proposition 2.** In a non-selective public school system, student effort is $e^{NS}$, as defined by (3.4).

The per capita expenditure on value added, $I(Y, \phi, \lambda_s)$, as a function of income, $Y$, the taste for amenities, $\phi$, and school productivity, $\lambda$, is the unique solution to:

\[
1 = \frac{MB(I_s, \lambda_s)}{MB(I_s, \lambda_s)} = \frac{Y - I_s}{(1 + \phi)}.
\]

Moreover, the expenditure on value added is increasing in income ($\frac{\partial I}{\partial Y} > 0$) and school productivity, ($\frac{\partial I}{\partial \lambda} > 0$), but decreasing in the taste for amenities, ($\frac{\partial I}{\partial \phi} < 0$).\(\textsuperscript{11}\) The first order conditions imply

\(\textsuperscript{11}\) See Appendix A for the proof.
that the optimal first period consumption, \( c^0 \), amenities, \( z \), and tuition, \( p_s \) satisfy:

\[
(3.9) \quad c^0_s = G(Y, \phi, \lambda_s) = \frac{Y - I(Y, \phi, \lambda_s)}{(1 + \phi)},
\]

\[
(3.10) \quad z_s = Z(Y, \phi, \lambda_s) = \frac{\phi[Y - I(Y, \phi, \lambda_s)]}{(1 + \phi)},
\]

\[
(3.11) \quad p_s = P(Y, \phi, \lambda_s) = \frac{\phi Y + I(Y, \phi, \lambda_s)}{(1 + \phi)},
\]

where as stated \( I(Y, \phi, \lambda_s) \) denotes the equilibrium investment on value added. The other functions, \( G(\cdot) \), \( Z(\cdot) \), and \( P(\cdot) \) define the equilibrium level of consumption, amenities and tuition as a function of the exogenous parameters. Importantly, an increase in school productivity, \( \lambda_s \), results in greater school expenditure, \( p_s \), less consumption, \( c^0 \), and a lower level of amenities, \( z \).\(^{12}\)

In summary, the analysis of a non-selective public school system highlights two margins along which school systems' performance might be enhanced. The first is by increasing student academic effort, which could be achieved, for example, by raising the precision of an individual-specific measure of learning like the graduation test (or creating one, if it does not exist), or by reducing the benefits of non-academic activities. Second, school productivity could be increased. In particular, as we have set it up, in a purely public system not all high productivity schools are utilized, and without profits there are no rewards to further entry by high productivity institutions.

\(^{12}\) Additionally, an increase in the taste for amenities, \( \phi \), results in more amenities, \( z \), greater school expenditure, \( p \), and less consumption. An increase in income results in more consumption and more expenditure on amenities. To prove these, consider first the effect of productivity. If \( \lambda^0_s \) increases to \( \lambda^1_s \), from Proposition 2 we have:

\[
\frac{I_1 - I_0}{1 + \phi} = \frac{1}{MB(I(Y, \phi, \lambda^0_s)), \lambda^0_s) - \frac{1}{MB(I(Y, \phi, \lambda^1_s), \lambda^1_s)} > 0.
\]

From this we get:

\[
c^1_{s1} - c^{0}_{s0} = \frac{I_0 - I_1}{1 + \phi} < 0,
\]

\[
z_{s1} - z_{s0} = I_0 - I_1 + \frac{I_1 - I_0}{1 + \phi} = \frac{\phi}{1 + \phi} (I_0 - I_1) < 0,
\]

\[
p_{s1} - p_{s0} = \frac{I_1 - I_0}{1 + \phi} > 0.
\]

The effect of amenities is a straightforward substitution effect, while the effect of income follows from the fact that all goods are normal.
4. INTRODUCING COMPETITION VIA NON-SELECTIVE, FOR-PROFIT SCHOOLS

One widely discussed way of improving school system performance is by allowing competition by for-profit schools. Our framework illustrates that to be effective, such a reform needs to raise student academic effort and/or improve school productivity. To highlight the distinction between these margins, this section considers a fully private market in which for-profit schools are also required to be non-selective (selective schools are analyzed in the next section). The bottom line is that in this case our framework is consistent with Friedman’s (1962) intuition: competition raises average school productivity and improves learning. In short, a first result in this section reaffirms that school productivity is one of the factors that can drive competition in educational markets, with beneficial effects.

On the other hand, a purely private system also tends to exacerbate inequality because wealthier students purchase more education, and hence future income depends upon both students’ innate ability and the income of their parents. To address this issue, Friedman recommended the introduction of a voucher system that would ensure that all individuals purchase a minimum level of educational services. A second result in this section is that aside from achieving greater equality, a voucher system increases the incentive for high productivity schools to enter and serve students at the lower end of the income distribution.

To illustrate these points, this section continues to assume that students vary only with respect to income, \( Y \in (0, Y_{\text{max}}) \), which is assumed to have a continuous distribution, \( F(\cdot) \), with \( F(Y_{\text{max}}) = 1 \). Income is assumed to be independent of ability, such that even if there is sorting by income, the distribution of innate ability is the same at all schools. Given that schools are non-selective, combined with the separability of academic effort and skill, implies that the introduction of for-profit suppliers provides no additional information regarding an individual’s innate ability. Hence, the equilibrium effort will still be \( e^{NS} \) as given by Proposition 1.

4.1. Unsubsidized for-profit schools. Consider a market consisting of \( n \) unsubsidized for-profit schools, with \( n_H < 1 \) high productivity schools, and hence \( n - n_H \) low productivity schools. The fact that high productivity institutions are in short supply implies they earn positive profits \( \Pi(n_H) \geq 0 \), while low productivity schools’ are (normalized to) zero. When \( \Pi(n_H) > 0 \) there is an incentive for high productivity schools to enter the market; the level of profits provides a measure of its intensity.
Since value added and amenities are normal goods, higher income individuals desire more of these, and schools will therefore be segregated by income. Further, since high productivity schools can provide more value added, higher income students will outbid lower income ones for these. This implies that the scarce supply of high productivity schools will serve high income individuals first, namely all individuals with income \( Y \geq \bar{Y} \), where:

\[
(4.1) \quad n_H = 1 - F(\bar{Y}).
\]

The low productivity schools serve students with income \( Y < \bar{Y} \). For this to be an equilibrium, it must be the case that the individual with income \( \bar{Y}(n_H) \), who is on the margin between a high and a low productivity school, is indifferent between the two. The utility of this student at a low productivity school (from expression 3.5) is given by:

\[
U_L(n_H) = \max_{p,z} U(p, z, 0, e^{NS}\bar{Y}(n_H), 0).
\]

Her utility at a high productivity school is:

\[
(4.2) \quad U_H(n_H, \Pi) = \max_{p,z} U(p, z, \Pi, e^{NS}\bar{Y}(n_H), 1).
\]

If profits were zero in both cases, utility would clearly be higher at the high productivity school \( U_H(n_H, 0) > U_L(n_H) \). Since utility is monotonically decreasing with profit, there exists a unique profit function, \( \Pi(n_H) \geq 0 \), such that:

\[
(4.3) \quad U_L(n_H) = U_H(n_H, \Pi(n_H)).
\]

Furthermore, as \( n_H \) increases, \( \Pi(n_H) \) falls until \( \Pi(1) = 0 \), and hence high productivity schools’ profits decrease with the entry of more high productivity schools, reaching zero when they cover the market. The properties of this equilibrium are summarized in Proposition 3.

**Proposition 3.** Suppose individuals vary only with respect to income, and that income is uncorrelated with ability. Then, there exists an equilibrium for privately supplied schooling where:

1. All students with income \( \bar{Y}(n_H) \) or greater attend the high productivity schools, where \( \bar{Y}(n_H) \) is determined such that \( n_H = 1 - F(\bar{Y}(n_H)) \). The equilibrium profit of high productivity schools, \( \Pi(n_H) \), satisfies (4.3) and is strictly decreasing with \( n_H \), with \( \Pi(1) = 0 \) (the top
segment in Figure 4.1 illustrates that profits are monotonically decreasing with the number of high productivity schools).

(2) An individual with income $Y$ chooses the school that charges price $p_Y$, offers amenities $z_Y$, and supplies value added $\beta_Y$ satisfying:

$$z_Y = Z(Y - \pi_Y, \phi, \lambda_Y),$$

$$p_Y = P(Y - \pi_Y, \phi, \lambda_Y)$$

$$\beta_Y = \beta(I(Y - \pi_Y, \phi, \lambda_Y), \lambda_Y)$$

where $\lambda_Y = 1$ and $\pi_Y = \Pi(n_H)$ if $Y \geq \check{Y}(n_H)$, and both equal to zero if not.

In contrast to the fully public system, therefore, the private system entails the efficient use of all available high productivity schools. While the rents in this system take away resources from amenities and value added, these would be dissipated with the entry of high productivity schools.

In addition, as in Epple and Romano (1998), the market is stratified by income, with higher income individuals consuming more amenities and value added. In the case of amenities, which are pure consumption goods, this may not be a major concern. However, in the case of value added, stratification by income implies that if two individuals have the same innate ability, then the one with wealthier parents will consume more value added and have higher future income. Thus, a fully
private system tends to reinforce the inter-generational transmission of inequality. Friedman (1962) recognized this problem, and recommended that all students be offered vouchers that could be used only for the purchase of educational services.

4.2. Vouchers. To explore the consequences of this, consider a system in which each student is given a voucher of value $V$ that can only be used to buy school services from a private provider. The system raises revenues via a constant marginal tax, $v$, distributing them equally such that the voucher per student is (recall the student population is normalized to 1):

$$V = \int_0^{Y_{\text{max}}} v \times y \times f(y)dy = vY,$$

where $Y = E\{Y\}$ is mean income.\(^{13}\) In practice voucher systems sometimes require that schools run only on the voucher subsidy, and sometimes they allow them to charge supplementary tuition. We consider a case in which tuition payments are allowed; it will be clear how our results apply to the case with no add-ons.

In comparison to a fully private system, a voucher scheme thus has two effects. First, it constrains some lower income individuals to consume more education than they would otherwise; second, it redistributes income toward the less wealthy. Formally, the notional income of an individual as a function of her exogenous income $Y$ and the voucher amount is:

$$Y^v(Y, V) = V + \left(1 - \frac{V}{Y}\right)Y = Y + V\left(1 - \frac{Y}{Y}\right).$$

To work out the effect of vouchers, consider first the case where there are only low productivity schools. Figure 4.2 illustrates the relationship between income, notional income and tuition in this case. The pre-voucher scheme income, $Y$, appears as the 45 degree line, with $Y^v(Y, V)$ having an intercept at $V$, the value of the voucher, and a flatter slope due to the redistribution of income toward individuals with $Y < \bar{Y}$. Let $P(Y^v(Y, V), \phi, 0)$ be the willingness to pay under vouchers, which Figure 4.2 illustrates is likewise flatter than $P(Y, \phi, 0)$, due to redistribution. Denote $Y^{VL}$ the pre-redistribution income at which an individual would voluntarily pay a tuition equal to the

\(^{13}\) One could also consider voucher amounts that vary by student income rather than being uniform, both are observed in practice. For a treatment of ability-contingent vouchers, see Epple and Romano (2002).
voucher at a low productivity school:

\begin{equation}
P(Y^v(Y^{VL}, V), \phi, 0) = V,
\end{equation}

i.e., an individual with income $Y^{VL}$ would be willing to pay tuition equal to the voucher even without voucher funding (though net of the redistribution). Hence, even though low income individuals would choose more education than they would without redistribution ($P(Y^v(Y, V), \phi, 0) > P(Y, \phi, 0)$ for incomes below the mean), at the lowest income levels given by $Y < Y^{VL}$, individuals would still prefer to consume less education than $V$. Thus, to address inter-generational inequality, vouchers necessarily constrain the choices of the lowest income individuals.

The separability assumption also implies that individuals with incomes $Y < Y^{VL}$ consume the same level of amenities and value added, given by:

\begin{align*}
z^{VL} &= Z(Y^v(Y^{VL}, V), \phi, 0), \\
\beta^{VL} &= \beta(I(Y^v(Y^{VL}, V), \phi, 0), 0).
\end{align*}

In short, at low incomes parents consume more education under vouchers, partially due to redistribution (the difference between $P(Y, \phi, 0)$ and $P(Y^v(Y, V), \phi, 0)$ in Figure 4.2), and partially due to the minimum expenditure constraint. For incomes above $Y^{VL}$, individuals are willing to pay
additional tuition for higher levels of amenities and value added. For incomes above the mean, the level of school expenditure is lower than would be observed in a purely private system.

4.3. **Entry.** Vouchers thus address one of the perceived shortcomings of a fully private system: low educational consumption by low income students, an effect that is well understood. In addition, the voucher system more effectively encourages entry by high productivity schools. To see this, consider the effect of increasing the number of high productivity schools, \( n_H \). Our previous analysis showed that in a purely private system, these schools earn a rent in equilibrium, and that this rent is generated in part via higher tuition. Figure 4.3 plots the prices schools charge as a function of income, and returns to considering both low and high productivity schools. The top, solid segments refer to a fully private system in which high productivity schools serve individuals with incomes greater than or equal to \( \tilde{Y} = \tilde{Y}(n_H) \) (Section 4.1), and charge a higher price than these students would pay at low productivity institutions. As income falls tuition approaches zero, and hence the premium that high productivity schools can command falls as well. The effect on their profits is illustrated in Figure 4.1, which plots profits against the number of high productivity schools in operation, \( n_H \) (the top, solid segment of the figure refers to the fully private system).

Moving on to the voucher system, the lower dotted lines in Figure 4.3 show that even when there are only low productivity schools, individuals with incomes greater than \( Y^{VL} \) choose to pay tuition
above the voucher—they choose the same level of educational expenditure as in the purely private system save for the redistribution effect. This also implies, under the hypothesis that the voucher is not set too high, that some parents are willing to pay tuition greater than the voucher even to attend a low productivity school.

Suppose that high productivity schools can enter, and consider a student on the margin between a low and a high productivity school. At a low productivity school, her utility would be:

\[ U^L_V(n_H) = \max_{p \geq V, z} U(p, z, 0, e^{NS}|Y^v(\tilde{Y}(n_H), V), 0). \]

At a high productivity school her utility would be:

\[ U^H_V(n_H, \Pi) = \max_{p \geq V, z} U(p, z, \Pi, e^{NS}|Y^v(\tilde{Y}(n_H), V), 1). \]

The equilibrium profit makes the marginal student indifferent between the two choices:

\[ U^V_H(n_H, \Pi^V(n_H)) = U^V_L(n_H). \]

When the number of high productivity schools is small, these will serve only the highest income individuals. This implies that like a purely private system, a voucher system will be characterized by stratification, with high income parents being served by the high productivity schools.

However, with vouchers the relationship between profit and the number of high productivity schools is different from that observed in a purely private scheme. To see this, note that with vouchers high productivity entrants will eventually serve a student who would pay tuition equal to the voucher at a low productivity school; this happens when their number reaches \( n^V_L \) and the associated marginal student has income \( Y^V_L = \tilde{Y}(n^V_L) \). At this point, the marginal student is indifferent between the two schools, but the productivity advantage of the high productivity school implies that it will charge tuition at least slightly above the voucher, \( P(Y^v(Y^V_L), \phi, 1) > V \), and that it will earn strictly positive profits.

Figure 4.4 illustrates the intuition behind this. Point A is the level of value added and amenities that would be chosen at a low productivity school. A high productivity school earns profits that make the student indifferent between point A and the combination at a high productivity school, point C (where the x-intercept illustrates that this school makes positive profits).\(^{14}\)

\(^{14}\) Notice that at a high productivity school operating under a zero profit constraint, value added would be higher while the level of amenities would be lower, as illustrated by point B.
Further entry will continue to drive down tuition until it reaches the voucher level, which happens at income $Y^{VL'}$ in Figure 4.3. At this point the associated number of high productivity schools is $n_H^{VL'}$ (that is $Y^{VL'} = \hat{Y}(n_H^{VL'})$). After this, tuition cannot fall any further, and hence the profits of high productivity schools do not decline more until all students are served. This point is illustrated in the lower, dotted segment of Figure 4.1. In short, while profits in the purely private system decline strictly monotonically, in the voucher system they are constant for a range. The implication is that a voucher scheme provides enhanced incentives for high productivity schools to enter and serve low income individuals. The intuition is that as long as students' outside option is a low productivity school, and as long as they are forced to spend at least the voucher on tuition, the high productivity schools will be strictly preferred and able to earn a positive rent (Figure 4.4).

To summarize, the results in this section confirm and bolster Friedman’s (1962) intuition that a for-profit system combined with voucher subsidies would enhance skill accumulation. However, the assumption that schools are non-selective turns out to be crucial to this result, as explored in the next section.

5. Selective for-profit schools and the Anti-Lemons Effect

This section relaxes the assumption that for-profit schools must be non-selective, and shows that this substantially qualifies the conclusions regarding the positive effects of competition (discussed
Specifically, consider a non-selective public system in which school characteristics are chosen by a median voter, and suppose that private schools can enter and select their student population based upon an entrance exam that measures innate ability. Would these schools find it profitable to enter such an environment?

To focus on some of the key aspects selectivity raises, this section makes a series of assumptions that would seem to essentially foreclose such entry. First, suppose that all students have the same characteristics \( \gamma = \{ Y, \phi, a, \delta \} \), and hence differ only with respect to innate ability.\(^{15}\) This eliminates for-profit schools’ ability to cater to heterogeneity in tastes for amenities. Second, assume all schools are of the same productivity, so that for-profit entrants cannot offer consumers an advantage in this dimension. Third, suppose that for-profit schools must operate unsubsidized, such that individuals using them have to pay tuition in addition to the taxes that give them access to public schools.

Despite all this, private entry is profitable, which reflects two aspects. First, when a school is selective, its reputation derives not just from its value added, but also from the composition of its student body; all else equal, schools that are able to select students of high innate ability will enjoy good reputations. Second, employers will rationally offer higher salaries to graduates from more selective schools, and hence higher ability students will be willing to pay for-profit schools’ tuition as long as these are sufficiently selective. In general, equilibrium will therefore be characterized by the coexistence of a stratified for-profit selective sector that contains the highest innate ability students, and a non-selective public sector containing those whose ability was too low to secure admission into a selective school.

Finally, this section illustrates that while such sorting is not inefficient per se (recall we assume no peer effects), it does have two negative consequences. First, it lowers academic effort in both sectors. This reflects that selectivity allows the school system to transmit a clearer signal of students’ skill in the form of school reputation. This lowers students’ incentive to exert academic effort to manipulate the other signal, the graduation test score. Second, as a result, lower innate ability students receive lower incomes than they would if the school system were entirely non-selective.

5.1. **Selectivity and private entry.** Suppose for-profit schools can select students using an *admissions test* that measures innate ability with error:

\[
\tau_i = \alpha_i + \epsilon_i^T
\]

\(^{15}\) We also assume that before they have been tested, all students have the same belief regarding this attribute.
where the error term is normally distributed with precision $\rho^\tau$; $\epsilon_i^\tau \sim N \left( 0, \sigma_i^2 \right)$ and $\sigma_i^2 = \frac{1}{\rho^\tau}$. As with the graduation test, suppose that the precision of the admission test is bounded above. Additionally, assume that the market is sufficiently thick so that competition among selective schools leads each one to specialize in admitting students with a test score exactly equal to $\tau_s$.

Suppose also that students’ performance on the admissions test is observed only by schools, and is not available to employers. Although this assumption is stark, the motivation is that schools might have an advantage in certifying innate ability. For example, while an outside agency might try to administer admissions exams and disseminate their results, it might find it hard to replicate the admissions process at selective schools.

The key impact of introducing selective admissions is that schools’ reputations then partially depend on their student composition. To see this, recall that when all schools are non-selective, the market expects that students, regardless of their school of origin, have expected innate ability equal to the mean, $E\{\alpha_i\} = 0$. When school $s$ admits only students with an entrance score of $\tau_s$, however, the expected innate ability among its graduates is:

\begin{equation}
E\{\alpha_i|s\} = \pi^{(\alpha)\tau} E\{\alpha_i\} + \pi^{(\tau)\alpha} \tau_s = \pi^{(\tau)\alpha} \tau_s
\end{equation}

where $\pi^{(\tau)\alpha} = \frac{\rho^\alpha}{\rho^\alpha + \rho^\tau}$ is an increasing function of the precision of the admissions test.\(^\text{16}\)

When a school is selective, its reputation (always given by the market’s correct expectation of its graduates’ skill) is therefore:

\begin{equation}
R_s = E\{\alpha_i|i \in s\} + \hat{e}_s + \beta_s = \pi^{(\tau)\alpha} \tau_s + \hat{e}_s + \beta_s.
\end{equation}

In short, in the educational industry the perceived quality of a firm can vary with the quality of its buyers. Education is not the only sector with this feature; private clubs, New York apartment coops, and consulting firms are all partially judged by the characteristics of their clients.

One key consequence of this is that knowledge that a student attended a selective school lowers the variance of her estimated skill. Specifically, given the assumption of a linear learning model ($\theta_{is} = \alpha_i + e_{is} + \beta_s$), the posterior distribution of skill among the graduates from school $s$ is

\(^{\text{16}}\) As defined above, $\pi^{(\alpha)\tau} = \frac{\rho^\alpha}{\rho^\alpha + \rho^\tau}$. 

normally distributed and given by: \( \theta \sim N(R_s, 1/(\rho^\alpha + \rho^\tau)) \) (DeGroot, 1972). The implications are summarized in the following proposition:

**Proposition 4.** Upon admission to a school \( s \) that takes only students with admission test score \( \tau_s \), a student has expected log wage:

\[
(5.4) \quad w_s = \pi(\tau) \alpha \tau_s + \hat{e}_s + \beta_s.
\]

Conditional on obtaining a graduation test score, \( t_i \), this individual’s wage is:

\[
(5.5) \quad w_{is}(R_s,t_{is}) = \pi(t) \tau_i + \pi(\tau) t_{is} R_s.
\]

The first result (5.4) reflects that upon acceptance to a selective school, a student’s expected log wage is equal to the sum of the average effort of the students at the school, the school’s value added, and a term that is strictly increasing in school selectivity. In particular, note that \( \frac{\partial w_s}{\partial \tau_s} = \pi(\tau) \alpha > 0 \), i.e., future wages increase without limit as a function of selectivity.\(^{17}\) This implies that if a for-profit school is sufficiently selective, some parents will be willing to pay its tuition even if the school otherwise has no productivity advantage, and even if they can use a non-selective public school at no cost. The implication is that it will be feasible for a very selective school to enter the market.\(^{18}\)

This implies that if it is profitable for a school with selectivity \( \tau_s \) to enter, then it is profitable for all schools with higher selectivity to enter.

Consider a related question: if a selective school entered the market “at the top,” say by admitting only students of the highest ability, would it seek to grow by accepting students of lower ability? Expression (5.4) implies that expected wages rise with school selectivity, and hence once a student has been admitted to a selective school it is not in her interest that the school admit students of lower ability, although she would always welcome higher ability peers.

Putting these facts together yields that if for-profit selective entry is allowed, the market will be characterized by some cutoff point, \( \tau \), such that there are selective schools for all levels of admissions test performance greater than or equal to \( \tau \), which we explicitly derive below.\(^{19}\) The assumption of

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17 Increases in the precision of the entrance exam also raise the returns to enrolling in a more selective school.

18 For another illustration, let \( U(e^*|\tau_s, \rho^\tau) \mid \tau_s, \rho^\tau) \) be the equilibrium utility of a student accepted at a school \( s \) with selectivity \( \tau_s \) and admission test precision \( \rho^\tau \). From (5.4) and the envelope theorem, \( \frac{\partial U}{\partial \tau_s} = \delta \pi(\tau) \alpha > 0 \).

19 Note that the fact that income grows without bound as \( \tau_s \) rises implies that \( \tau < \infty \). Further, the assumption here is that all students take the same admissions test with with the same precision. If schools could set their own precision, they would set it as high as possible, since the envelope theorem implies that a small change in the precision has
thick markets implies that each school will specialize in students with admissions scores exactly equal to \( \tau_s \), since students with higher scores choose more selective schools, while incumbents experience a loss in utility if students with low innate ability are admitted. Thus, we may conclude:

**Proposition 5.** With free entry of private selective schools with a common admission exam and the same productivity, the equilibrium is characterized by an admissions test score \( \bar{\tau} \) and precision \( \rho^\tau \) such that all students with a score \( \tau_i \geq \bar{\tau} \) attend a selective school, while the rest attend non-selective public schools.

5.2. Selectivity and student effort. Now consider the consequences of selectivity on effort. The second result in Proposition 4 illustrates the effect of the graduation test upon wages in this context:

\[
\frac{\partial w_s}{\partial t_i} = \pi^{(t)\alpha} = \frac{\rho^t}{\rho^t + \rho^s + \rho^\alpha} > 0.
\]

As before, an increase in the precision of the graduation test, \( \rho^t \), increases the sensitivity of wages to students’ performance on this test. In contrast, an increase in the precision of the admissions test, \( \rho^\tau \), reduces the sensitivity of wages to performance on the graduation test. This in turn reduces the incentives for effort experienced by students in selective schools. More specifically:

**Proposition 6.** A student attending a selective school chooses academic effort to satisfy:

\[
-\Psi'(e_s, a) = \delta \pi^{(t)\tau} < \delta \pi^{(t)\alpha},
\]

where \( \pi^{(t)\alpha} = \frac{\rho^t}{\rho^s + \rho^\tau + \rho^\alpha} < \frac{\rho^t}{\rho^s + \rho^\alpha} = \pi^{(t)\alpha} \). Given the concavity of \( \Psi \), it follows that an increase in the precision of the admissions test, or a decrease in the precision of the graduation test, results in lower academic effort.\(^{20}\)

\(^{20}\) Holmstrom (1999) was the first to make the point that a better reputation may reduce performance incentives. Gibbons and Murphy (1992) find evidence consistent with this for CEO compensation. Namely, more senior CEOs with better reputations should be less concerned about their careers, and hence firms should rationally provide them with more performance pay; this is consistent with the data that Gibbons and Murphy analyze.
Again, the intuition reflects that selectivity allows the school system to transmit a clearer signal of students’ skill in the form of school reputation. This lowers students’ incentive to exert academic effort to manipulate the other signal (the only one they can affect): the graduation test score.

To summarize, thus far this section illustrates that if schools can select students, then free entry and competition can entail negative effects (Section 4 illustrated they can have positive effects as well). In the extreme, for-profit entry is feasible even if private schools are not more productive, and this entry will result in lower academic effort among students attending selective schools. This is the essence of the anti-lemons effect—entry by selective schools that derive their reputation for high quality from selectivity. In the next two subsections, we work out the equilibrium of the market with selection. A necessary preliminary to pinning down $\bar{\tau}$ is determining the effort of students who do not gain entry into a selective school.

5.3. Academic effort in non-selective schools when they co-exist with selective ones. Proposition 6 shows that students at selective schools will exert lower academic effort than they would in a non-selective school system. In this subsection, we characterize the consequences that entry by selective for-profit schools has on the academic effort of students “left behind” in the non-selective public sector. This requires some technical detail because these students, by failing to secure admission into a selective school, are revealed to come from an adversely selected pool of individuals. The bottom line is that these students will also display lower effort than they would in a non-selective system.

To see this, let $\bar{\tau}$ denote the cutoff score such that all students with admission test performance below this level remain in the non-selective sector. Let $\bar{s}$ denote any school in the non-selective sector, and let $\hat{e}_\bar{s}$ be the equilibrium academic effort level observed among them. The expected log wage of individuals who cannot make it into a selective school is therefore:

$$w_{i\bar{s}}(t_{i\bar{s}}) = E\{\alpha_i|\tau_{\bar{s}} < \bar{\tau}, t_{i\bar{s}}\} + \hat{e}_\bar{s} + \beta_{\bar{s}}$$

where in a manner analogous to that seen above, the first term on the right hand side reflects that the market will use the sector of origin to estimate students’ innate ability.
This expression can be explicitly computed because it is the expected value of a normally distributed variable given that another normally distributed variable with which it is correlated is truncated. The details of this computation are in Appendix B. There we show the following.

**Proposition 7.** The expected log wage of an individual entering a non-selective school is:

$$w(\tau) = -\frac{\sigma^2}{\sigma_x} \Gamma\left(-\frac{\tau}{\sigma_x}\right) + \hat{e}_s + \beta_s,$$

where $\Gamma(x) = \frac{f(x)}{1-F(x)}$ is the hazard function and $f$ and $F$ are density and cumulative distribution functions for the normal distribution.

This illustrates that an expansion of the selective sector (a decrease in $\tau$) lowers the expected wage of students in the non-selective sector. This implies that “cream-skimming” can hurt those left in the non-selective public sector, even without peer effects.

One can bound the hazard function, from which it is possible to show that for $\bar{\tau} \leq 0$:

$$\pi (\tau)^{\alpha} \hat{e}_s + \beta_s \geq w(\tau) \geq \pi (\tau)^{\alpha} \hat{e}_s + \beta_s.$$

As $\bar{\tau}$ falls, $w(\bar{\tau})$ approaches a linear function of $\bar{\tau}$, with slope $\pi (\bar{\tau})^{\alpha} < 1$. As before, to determine the effect on the incentives for effort, one needs to compute the contribution of the graduation test to wages. It is not possible to obtain a simple closed form solution, so consider an approximation (also detailed in the appendix). Specifically, the marginal impact of academic effort on wages for students in the non-selective schools is:

$$\Psi'(\hat{e}_s(\tau), a) = \delta \frac{\partial w(\tau)}{\partial e} \simeq \delta \pi (t)^{\alpha} \left(1 - \pi (t)^{\alpha} \Gamma'(\frac{-\pi (t)^{\alpha} \sigma^2}{\sigma_x} \Gamma\left(-\frac{\tau}{\sigma_x}\right) + \bar{\tau})\right).$$

The hazard rate grows without limit, and hence as $\bar{\tau}$ falls, the expected log wage falls without limit. The marginal hazard rate satisfies $\Gamma' \in [0, 1]$, and therefore we have:

$$\pi (t)^{\alpha} \geq \frac{\partial w(\tau)}{\partial e} \geq \pi (t)^{\alpha} \tau.$$

This result has two key implications. First, as long as a selective sector exists, the incentives for academic effort in the non-selective sector will never exceed those that would prevail in the complete absence of selection. Second, when the selective sector is small ($\bar{\tau}$ is high), the marginal incentive for academic effort in the non-selective sector is higher than in selective schools. As the selective
sector grows, however, it falls to that observed among selective schools. Again, given the existence of a selective sector, increasing its size reduces performance incentives in the non-selective sector, adversely impacting the most disadvantaged students.

5.4. Equilibrium. We now compute the equilibrium admission cutoff, \( \bar{\tau} \), where students with scores above this level attend selective private schools, while the rest remain in the non-selective public sector. Let \( p^m \) be the cost of a public school in the latter, where the utility of a student who uses it is given by:

\[
U^{Pub}(\tau) = \arg\max_{z, \beta} \log(Y - p^m) + \delta [\beta + \hat{e}_s(\tau) - \frac{\sigma^2_{\alpha}}{\sigma_\tau} \Gamma \left( -\frac{\tau}{\sigma_\tau} \right)] + \phi \log(z) + \Psi(\hat{e}_s(\tau), a),
\]

where, \( \hat{e}_s(\tau) \) solves (5.8). The total cost of operating the public schools is given by \( p^m(\bar{\tau}) = (z^m + \bar{q}C(\beta^m))F(\bar{\tau}), \) which is the per-student amount (\( \bar{q} \) is the average quality of these schools) multiplied by \( F(\bar{\tau}), \) the fraction of the population that attends public school. Suppose that \( \bar{\tau} > 0 \) so that the median voter actually attends public school.

Let \( \bar{z}_s, \bar{\beta}_s, \) and \( \bar{p}^m_s \) denote the solution to this problem, and observe that their optimal values are independent of \( \bar{\tau}. \) Note also that only effort and the final term in (5.9) vary with \( \bar{\tau}, \) and hence the utility of individuals who enter the non-selective public sector can be rewritten as:

\[
U^{Pub}(\bar{\tau}) = \bar{u}^{Pub} + E(\bar{\tau}) - \delta \sigma^2_{\alpha} \Gamma \left( -\frac{\bar{\tau}}{\sigma_\tau} \right),
\]

where \( E(\bar{\tau}) = \delta e_s(\bar{\tau}) + \Psi(e_s(\bar{\tau}), a), \) and \( \bar{u}^{Pub} \) collects the remaining constant terms.\(^{21}\) Notice that the payoff in the non-selective sector is strictly increasing with \( \tau, \) and converges to the utility students would experience if there were only non-selective public schools (Section 3).

At an equilibrium, it must be the case that an individual with admission exam score \( \bar{\tau} \) is indifferent between the public and private sector. Because a student attending a private school must nonetheless pay the taxes that entitle him to use the public sector, the utility from attending a private school with entrance requirement \( \bar{\tau} \) is:

\[
U^{Prv}(\bar{\tau}) = \bar{u}^{Prv} + E^{Prv} + \delta \pi(\tau)^{\alpha} \bar{\tau},
\]

where:

\[
\bar{u}^{Prv} = \arg\max_{z, \beta} \log(Y - z - \bar{q}C(\beta) - p^m(\bar{\tau})) + \phi \log(z) + \delta \beta,
\]

\(^{21}\) More precisely, \( \bar{u}^{Pub} = \log(Y - p^m_s) + \phi \log(z_s) + \delta \beta_s. \)
\[ E^{Prv} = \delta e_s + \Psi(e_s, a), \]

and \( e_s \) is the unique solution to (5.7). Notice that \( \tilde{u}^{Prv} + E^{Prv} < \tilde{u}^{Pub} + E(\tau) \), such that without selection, the public school would always be preferred due to the higher cost of private school. However, as stated the payoff to the private school increases without bound as \( \tau \) increases, and hence if a school is sufficiently selective it will always be preferred to a public school.

5.5. **Vouchers.** Finally, consider the impact of giving all students vouchers equal to \( p_s^{m} \), and of allowing them to choose any school they wish. With this, the cost of a selective school would be the same as that of a non-selective school. All students with high test scores strictly prefer selective to non-selective schools if they come at the same price. By construction, the average ability of an individual in the non-selective sector is less than \( \bar{\tau} \), and now there is no additional cost associated with the selective sector. One therefore has an extreme form of the *anti-lemons* effect: the non-selective schools are driven from the market, leaving only selective schools. Given that this reduces the incentives for effort, this outcome is strictly worse than a pure, non-selective public system. Therefore, in the absence of heterogeneity in school productivity, the anti-lemons effect leads to a highly stratified school system with strictly worse outcome in terms of student performance.

Finally, note that introducing income heterogeneity would significantly complicate the analysis while adding little insight. If the market were sufficiently thick then in the absence of vouchers high income individuals would leave the public system, regardless of ability. For this group, there would be stratification by both ability and income. The introduction of vouchers would simply allow the stratification by ability to extend to individuals with lower incomes.

6. **Discussion**

The reputation model explores that educational systems not only produce skills but also serve as settings for the transmission of information on individual ability. As this section discusses, taking this into account has numerous policy implications and helps to account for puzzles in the literature.

6.1. **The impact of competition.** There is clear evidence that parents value school choice and schools with higher achievement. For instance, in 1981 Chile essentially implemented Friedman’s (1962) voucher proposal, and the private sector’s market share subsequently increased by about 45 percentage points (McEwan, Urquiola, and Vegas (2008)). In the U.S., Black (1999) and Figlio and Lucas (2004) find that parents are willing to pay more for residences tied to schools with higher
test scores, and Hastings and Weinstein (2008) present evidence that parents’ school choices react to information on achievement.

In the reputation model, however, a parental preference for higher performing schools does not necessarily imply that competition will improve outcomes—Section 5, for example, illustrates that extensive private entry can be associated with no gains in average school productivity. Such a disappointing outcome is consistent with stylized evidence surrounding the first example cited, Chile’s introduction of vouchers. Specifically, at the start of the 1980’s Chile’s school system in many ways resembled the scenario of Section 5: it had a large, generally non-selective public sector, and a smaller selective private one; on average private schools enjoyed much better reputations than their public counterparts. In 1981 the government introduced an unrestricted voucher scheme by which any student could in principle attend any subsidized school, public or private, religious or secular. Importantly, private voucher schools were allowed to implement a wide range of admissions policies.

The reputation model predicts that these measures would result in substantial entry, with the private sector cream-skimming the best students from the public sector, and private schools themselves becoming stratified. The evidence is consistent with this. First, mainly for-profit private schools presently account for most enrollments, up from about a ten percent share at the time of the reform. Second, Hsieh and Urquiola (2006) suggest that this growth was associated with the “middle class” largely following upper income households into the private sector, with the lowest income students remaining in public schools. Third, at present one observes clear hierarchies of schools by income.22

There is no consistent evidence that this reform had a substantial net effect on test scores, despite the large reallocation of students to the private sector. While some studies find positive effects (for example Gallego (2006)), Hsieh and Urquiola (2006) find private entry had little if any impact on average test scores or years of schooling. Consistent with this, Chile’s international testing performance has not displayed major changes since the 1970s, and disappointment with the evolution of learning outcomes is widespread.23

22 For instance, Mizala, Romaguera, and Urquiola (2007) suggest that a full set of school dummies accounts for about 80 percent of the variation in student income in Chile.
23 McEwan, Urquiola, and Vegas (2008) revisit the disappointing impact of this reform on testing results, and in a comment on the piece, Gallego (2008) points out that a lack of improved learning is indeed one of the “stylized facts” of Chile’s experience with vouchers.
Further, the reputation model predicts that the growth of the private sector would have very different consequences on those transferring into it and on those left behind. On the one hand, students attending selective private voucher schools might experience an increase in wages, as Bravo, Mukhopadhyay, and Todd (2008) suggest in fact happened. On the other hand, the growth of the selective sector would generally lower the welfare of individuals remaining in the public sector. Consistent with this, over the past few years Chilean public high school students, despite having access to schools that have improved substantially at least in terms of amenities, have taken to at times violently demanding changes in the laws that govern the school sector. In short, the reputation model suggests that the Chilean voucher system was perhaps structured in a way that led schools to compete on selectivity (or amenities) rather than productivity in the generation of skill.

For some contrast, consider the case of Sweden. In the early 1990s, Sweden allowed independent schools to begin receiving per-student subsidies equal to about 80 percent of those given to public schools. Independent schools can have explicit religious affiliations, and can be operated for-profit. In relevant dimensions, therefore, the Swedish system is quite similar to Chile’s. However, Swedish private schools must be operated on a “first-come, first-served” basis, and cannot select students based on ability, income, or ethnicity. Although in practice the allocation of students to schools is probably never random, the reputation model suggests that this design would produce less stratification and greater effects on learning. The literature is broadly consistent with Swedish private schools on average not being that different from public schools in terms of socioeconomic composition (Sandstrom and Bergstrom (2005)), in stark contrast with the outcome in Chile. In terms of impacts on learning, the evidence is mixed (see Sandstrom and Bergstrom (2005) and Bohlmark and Lindahl (2008)).

Additionally, Sweden has experienced less private entry than Chile—at present the private enrollment rate is about 15 percent in Sweden relative to about 55 percent in Chile. This could reflect a variety of factors, including that Sweden’s reform is more recent. Nonetheless, it is also consistent with the possibility that without selection, private entrants have to build competitive strategies that rely on catering to heterogeneity in parental tastes for value added or amenities, as opposed to strategies than emphasize cream skimming.

For a final case, note that competition in the U.S. also takes forms that the reputation model predicts would have heterogeneous effects on learning. As Hoxby (2000) points out, “Tiebout”
choice between independent districts is probably the most important form of school choice in the U.S. One might expect this mechanism to be associated with stratification, since admission into a “good” district depends exclusively on households’ ability to buy or rent a house within its confines. In terms of stratification, Clotfelter (1999) and Urquiola (2005) suggest that increased district availability leads to sorting; there is mixed evidence on its impact on performance (see Hoxby (2000) and Rothstein (2006)).

In contrast, consider competition induced by charter schools, which generally have significantly less latitude in selecting students than private schools. The reputation model would suggest these would have a more positive impact, as Hoxby and Murarka (2008) and Abdulkadiroglu, Angrist, Cohodes, Dynarski, Fullerton, Kane, and Phatak (2009) recently find using randomized designs.

The bottom line is that competition may lead to enhanced learning, but the specifics of market design may matter. If so, then it is not surprising, for example, that the literature on whether private schools are more effective has produced mixed findings. More broadly, the private advantage itself may be endogenous, and more likely to reflect greater value added in settings in which market design gives private schools greater incentives to differentiate along dimensions related to value added.

6.2. Resource policies. The reputation model is also consistent with disappointment surrounding another major set of educational policies: those focused on providing schools with more inputs. This reflects that while such initiatives can enable schools to provide greater value added, they do not fundamentally affect the incentives faced by students or schools.

In fact, the claim that resource-oriented policies can disappoint is almost non-controversial. Specifically, while recent research shows that certain inputs raise learning, the broader picture is one in which most countries have seen their educational expenditures climb substantially over recent decades, often with scant testing gains to show for it. This has led Pritchett (2003) to argue that there has been a generalized decline in school productivity across the OECD; while Hoxby (2002) suggests that U.S. school productivity has declined by 50 percent since the 1970s.

6.3. Testing policies. The reputation model highlights three determinants of individual educational performance: innate ability, effort, and school value added. The interventions discussed thus far in this section—competition and resources—are generally aimed at improving value added, and

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24 To illustrate, Angrist, Bettinger, Bloom, King, and Kremer (2002) present experimental evidence of a private school advantage in Colombia, while the evidence in the U.S. has been more mixed (e.g. Howell and Peterson (2002), and Krueger and Zhu (2004)).
have only indirect effects on the other two. Of these, innate ability is often considered to be outside
the domain of education (but is affected by public health policies, e.g., Currie (2009)). The third
determinant, student effort, has not been the object of much analysis, although there are exceptions;
for example, Bishop (2004) has long highlighted its importance. The point here is that competition,
if accompanied with stratification, can reduce the incentives for effort.

The model makes this point starkly because, for simplicity, it assumes that upon entry to a
selective school there is no further selection. In practice, this is not the case for at least some
students. For example, the best students from selective high schools cannot all go to the best
universities, and they compete with each other to obtain entry to these institutions. Similarly,
students at top graduate programs are sometimes ranked. One might therefore expect the incentive
effects of selection to be mitigated for such groups, a prediction that is consistent with casual
observation that motivating the best students is rarely a challenge.

The reputation model also suggests that national testing can be useful in motivating the rest—
the claim is that educational systems that provide individual-specific measures of learning tied to
outcomes parents care about will tend to develop school systems geared toward higher achievement.
Consistent with this, Woessmann (2007) suggests that countries with standardized graduation or
college admissions exams perform better than expected in international tests. In anecdotal evidence,
few observers disagree that such high stakes national examinations result in high levels of student
and parental effort.\(^\text{25}\) For instance, Romania and South Korea display extensive private tutoring
industries that parents use to supplement their children’s learning at school.\(^\text{26}\)

At the opposite extreme, consider cases in which the labor market observes no independent signal
of individual attainment, a scenario that is particularly relevant in the U.S. In terms of the model, in
such a setting the assumption of perfect selection means that all individuals who attend a selective
school \(s\) will have admissions scores, \(\tau_i\), equal to \(\tau_s\), and wages \(w_{is} = \pi(\tau)^a \tau_s + \epsilon_s + \beta_s\). This implies
that the wages of individuals who attend selective schools will vary with their admissions score:

\(^{25}\) The New York Times (2008) reports that some South Korean children react to failure in college-entry exams by
enrolling in “boot camp”-type institutions that heavily restrict all activities but preparing for the next annual round
of examinations. Analogous effort is also observed in the U.S., but in more isolated settings. For instance, admissions
into the top public high schools in New York City are test-based, and as a result children planning to apply for them
often devote much energy to preparation.

\(^{26}\) These observations further suggest that while No Child Left Behind might have moved the U.S. in the right direction
by increasing the availability and use of educational performance data, the initiative’s impact might be enhanced if
it made greater use of individual-specific information and incentives. For example, at present its testing results have
few consequences on individuals as opposed to schools. Similarly, the main national testing effort in the U.S., the
NAEP (National Assessment of Educational Progress), does not even generate scores at the student level.
\[ \frac{\partial w_i}{\partial \tau_i} = \pi(\tau)\alpha > 0. \] In contrast, students in the non-selective school have a starting wage \( w_{i\bar{s}} = w(\bar{\tau}) \), and hence for them \( \frac{\partial w_{i\bar{s}}}{\partial \tau_i} = 0 \).

In short, if the market observes no signal analogous to a graduation test score, then the correlation of workers’ innate ability and their initial wages will be very different depending on the sector they attended—if they went to a selective school, the correlation will be positive; if they did not, it will be zero. Over time, employers will learn about individuals’ true skill (e.g. Farber and Gibbons (1996) and Lange (2007)), and hence the wages of individuals from non-selective schools will also come to reflect \( \tau_i \) later in their careers, but this will not be the case initially.

Arcidiacono, Bayer, and Hizmo (2008) present evidence that is quite consistent with these predictions. They analyze individuals’ scores in the AFQT, which is basically an aptitude test used by the armed forces, the results of which are generally not available to employers; hence, in the context of our model, this is a reasonable proxy for \( \tau_i \). Given that most high school graduates who enter the labor force upon graduation attended lower quality public schools, one can suppose that these students come from a non-selective sector. In contrast, college graduates can be considered to originate in a selective sector. This reflects that college admissions in the U.S. are very competitive and at least partially rely on academic ability (e.g., they consider SAT scores). Further, it is easy for employers to observe graduates’ colleges of origin. To summarize, the implication is that the starting wages of college graduates should be significantly correlated with AFQT scores; wages for high school graduates who entered the labor market immediately should initially not be correlated with AFQT scores, but become so over time. This is exactly what Arcidiacono, Bayer, and Hizmo (2008) find.\(^{27}\)

This differential effect implies that the return to skill for high school graduates is delayed relative to college graduates. If individuals with lower cognitive skills indeed prefer more immediate rewards, then ideally high school graduates should face higher rather than lower immediate rewards for performance.\(^{28}\) Yet, the U.S. educational system may be doing largely the opposite: high schoolers

\(^{27}\) More specifically, Arcidiacono, Bayer, and Hizmo (2008) emphasize that analyses of employer learning should perhaps not pool data from all education levels, since employers learn slowly about the ability of high school graduates, while the ability of college graduates is “directly revealed.” Our model suggests that this direct revelation may largely reflect that colleges are on average more selective than high schools (particularly than those high schools whose students typically enter the labor market immediately upon graduation). If this is the case, our model further suggests that actually controlling for the precise college of origin would reduce the observed correlation between AFQT and wages. Whether or not this effect would go to zero (as in the extreme our model would predict) is an empirical question.

of higher innate ability are rewarded with entry into elite colleges and then high paying jobs; in contrast, the more skilled individuals in the group of high school graduates (who do not go on to college) face delayed rewards.

Finally, the model highlights the fact that raising test scores, say by strengthening national testing, entails some trade-offs. If students spend more time preparing for a national test, then they necessarily spend less time at other activities such as debating clubs and sports. This may explain why many observers in countries that perform well on international tests express concern that children spend too much time studying, while the U.S., despite its poor performance in tests like PISA, produces high school graduates that go on to be part of one of the most successful higher educational systems.

6.4. Pro-social behavior. In the reputation model, academic effort is assumed to describe activities that enhance an individual’s labor market outcome. The model explicitly allows for variation in the return to non-academic activities via the preference term $\Psi(e, a)$. There is a recent economics literature that explicitly considers some of the factors that affect the parameter $a$.

Benabou and Tirole (2006) observe that people’s reputation depends not only on their ability, but also upon the extent to which they can be trusted. Hence, if it is important for individuals to signal such characteristics then they will allocate some of their time to pro-social activities. We do not explicitly model this effect here, but their point implies that schools have an incentive to also acquire a reputation for producing pro-social individuals. The anecdotal evidence is consistent with this in that private schools often have programs promoting pro-social behavior. At Andover, an elite private school in the U.S., the admissions process explicitly considers whether applicants are “nice.”

Akerlof and Kranton (2002) make the point that individuals create distinctive group identities within a school. These groups in turn can affect the extent to which individuals invest into study versus activities rewarded by the group (such as delinquent behavior). Fryer (2005) extends these ideas to explore the economic implications for identity with a particular racial group. If “acting white” is associated with study, then individuals who believe there is a higher return from group membership would reduce study time, and hence overall labor marker performance.

This literature illustrates that schools have the potential to shape the perceived trade-off between study and other activities. In the model, these would be reflected in the $\beta$ term. The main result
still applies—namely if schools are non-selective then it is easier for the market to measure the extent to which they are able to create an environment that produces high value. On the flip side, if pro-social behavior is valued by the market, then schools can also increase their reputations by selecting students that exhibit more pro-social behavior (Andover’s solution), which in turn will make it more difficult for other schools to teach such skills.

6.5. **Peer effects.** Models that emphasize peer effects, such as Manski (1992), Epple and Romano (1998), and Ferreyra (2007), also predict stratification in equilibrium. From the perspective of parents, peer effects and reputation may be viewed as two good reasons for choosing a selective school—at such a school children benefit from high quality peers and can expect good future job opportunities because of the school’s reputation. In practice, therefore, peer effect and reputation concerns may reinforce each other, making it hard to distinguish between them.

There is nonetheless evidence suggesting that the reputation model should be considered seriously. First, it is consistent with the observation that parents care a great deal about peer composition, yet the literature often fails to find clear evidence of peer effects (e.g. Oreopoulos (2003), and Katz, Kling, and Liebman (2006)). Second, if peer effects have some of the commonly considered functional forms, then one might expect countries that display significant stratification to have high average levels of learning, or at least very good test scores “at the top.” Specifically, if students benefit from not interacting with low-ability peers (e.g., Epple and Romano (1998)), then one would expect good outcomes for the students at the best schools. On the other hand, if peer-related gains come mainly from homogeneity (e.g. Duflo, Dupas, and Kremer (2008)), then one might expect countries with extensive sorting to perform well across the distribution.

To our knowledge, there is no standardized measure of educational stratification that would allow for cross-country comparisons in this spirit. Nevertheless, and on a speculative note, one might expect countries with extensive school choice, like the U.S. or Chile, to be relatively stratified. The evidence is that both of these countries do not perform well for their income levels. For instance, Murfin (2007) considers PISA 2003 performance among 20 high income countries including the U.S. In terms of average 8th grade Math performance, the U.S. places last. When the comparison considers only students at the 95th percentile in each of these 20 countries, the U.S. places second to last. The latter result is particularly surprising given that upper income households in the U.S. have access to substantial school choice.
6.6. **The impact of information.** Policy makers increasingly appreciate that in order to measure school quality one must take selection into account. As a result, several jurisdictions (e.g., New York City) are carrying out efforts to publicize information that approximates school value added. These efforts complement much more widespread initiatives that simply disseminate information on schools’ absolute testing performance.

What is possibly less well appreciated is that public and private objectives may be different; namely, while policy makers might wish households used information to choose schools with higher value added ($\beta$), parents may be more concerned with reputation ($R_s = \pi^{(r)} + \alpha \tau_s + \hat{\epsilon_s} + \beta_s$), of which value added is only one component. If so, then parents may prefer a given school to another that has higher value added but is less selective. Consistent with this, recent research suggests that while parents react to information on absolute achievement (e.g. Hastings and Weinstein (2008)) they are less sensitive to data that might approximate value added (e.g. Rothstein (2006) and Mizala and Urquiola (2008)).

We do not wish to suggest that one should not attempt to measure or disseminate data on absolute achievement or value added, but that the impact of such initiatives on overall school productivity might be less clear or pronounced than policy makers hope. In the extreme, these policies might supply another example of the list described by Steve Kerr (1975) in his classic article *On the folly of rewarding A while hoping for B.*

As an alternative to improving schools via information provision, policy makers might use data on value added to introduce remedial interventions or financial penalties for poorly performing schools (as New York City is currently doing). Alternatively, they might insist that schools be non-selective, in which case a school’s reputation would provide a “clean” signal of performance. The latter scheme would face multiple obstacles, among them that it is difficult to truly allow parental choice while constraining schools to be non-selective.

6.7. **A summary of empirical implications.** This section closes by summarizing the empirical implications it covered. Specifically, if the reputation model holds for a school market:

- Parents will have a clear preference for schools with higher absolute achievement—this will not necessarily translate into a preference for schools with greater value added.
- If schools can select students based upon ability then:
– School choice will result in stratification, with the highest ability/income children going to the most desirable and productive schools.
– School choice will result in lower student effort, and in lower incomes for students who do not gain admission to selective schools. (Note that if peer effects exist, then changes in the distribution of students will have additional effects on the level and distribution of achievement.)

• If schools cannot select on ability, the introduction of school choice will unambiguously raise school performance and student outcomes.
• All else equal, educational attainment will be higher in school systems that make use of individual-specific measures of learning observed by the labor market.
• Selective school systems will reveal individual ability more effectively than non-selective systems. In terms of labor market outcomes, this implies that innate ability will be immediately correlated with wages even when the market observes no individual-specific measures of learning. In contrast, in non-selective systems without individual-specific measures of learning, this correlation will only emerge over time.

7. Conclusion

In this paper, we study the characteristics of a competitive market for education in which schools are able to acquire a reputation for quality, as measured by the achievements of their graduates. When schools are able to select students based upon innate ability there is an “anti-lemons” effect: namely entry by relatively small schools that serve students within a specific ability range. This leads to stratification, where the most able students attend the schools with the best reputations and subsequently earn the highest incomes, while the least able remain in the worst schools. In general, this is not an efficient solution for two reasons. First, all students face weaker incentives for academic effort than they would in a non-selective setting. Second, reputation effects dilute schools’ incentive to enhance productivity, since a low value added school can always enhance its reputation by being more selective. In contrast, if schools are non-selective, competition leads to an efficient outcome.

Contrary to Friedman’s (1962) claim, these results illustrate that reputation effects are not sufficient to ensure that free markets ensure the efficient provision of complex goods. Analogous phenomena have been observed in other markets. For example, Dranove, Kessler, McClellan, and
Satterthwaite (2003) show that health report cards can result in cream skimming of patients by physicians, leading to under provision of services to the most needy individuals. Similarly, the recent financial meltdown made clear that reputation effects are not sufficient to ensure that financial firms behave in a prudent fashion. Thus, as Posner (2009) argues in his discussion of the financial crisis, some form of regulation or market design must supplement reputation effects to ensure the success of the market system.

Our results also illustrate the challenges one faces when attempting to enhance school performance. For instance, it is well appreciated that schools’ average test scores are not a good measure of their value added. As a consequence, jurisdictions such as New York City are publicizing estimates of value added. However, our model predicts that parents care not only about schools’ value added, but also about schools’ student composition. Hence, their reactions to these informational initiatives may be weaker than policymakers hope.

The reputation model also has implications regarding the political economy of school reform. For example, once stratification takes hold, parents with children at schools with good reputations will rationally resist efforts to make schools less selective. This may make it difficult to enhance school performance by reducing selectivity. However, the model also predicts that the introduction of more rigorous national testing provides an alternate way to enhance performance. This prediction is consistent with Bishop (1997), who has long advocated the importance of enhancing individual incentives.

Finally, our model may help explain why it is so difficult to enhance school performance in urban areas where competition for admission into selective schools leaves many students behind in schools with adversely selected populations. The model predicts that these students should expect a lower return from academic study, and hence will rationally allocate their time to non-academic activities such as sports, part time jobs, crime, and parenthood. Understanding the link between school selectivity and academic performance among those in the lowest ranked schools is an important topic for future research.
Appendix A. Optimal Investment into Value Added

Proposition 8. In a optimal non-selective public school system the expenditure on value added is increasing in income \((\frac{\partial}{\partial Y} > 0)\) and school productivity, \((\frac{\partial}{\partial \lambda} > 0)\), but decreasing in the taste for amenities, \((\frac{\partial}{\partial \phi} < 0)\).

Proof. It is straightforward to show this result graphically. In Figure A.1 the x-axis is investment in value added, \(I\). The equilibrium level of instruction is the intersection of the curves \(\frac{1}{MB(I, \lambda)}\) and \(\frac{Y-I}{(1+\phi)}\). The latter is downward sloping and intersects the x-axis at \(Y = I\). If productivity increases from \(\lambda_s\) to \(\lambda_s'\), this raises the marginal benefit of instruction and shifts the \(\frac{1}{MB}\) curve to shift right, such that spending on value added rises from \(I_0\) to \(I_1\). If the taste for amenities increases from \(\phi\) to \(\phi'\), it causes the curve \(\frac{Y-I}{(1+\phi)}\) to shift down, and expenditure on value added to go from \(I_0\) to \(I_2\). A similar argument shows that an increase in income leads to an increase in spending on value added. \(\square\)

![Figure A.1. Optimal Investment into Value Added](image-url)
APPENDIX B. DERIVATION OF THE SELECTION EFFECT

The effect of selection upon expected ability can be computed using results on conditional expectations of normal random variates with truncation. From Birnbaum (1950) we have:

\[ E\{X|Z \geq z\} = \mu \Gamma(z), \]

where \(X\) and \(Z\) are standard normal random variables with zero mean and unit variance; \(\mu = E\{XZ\}\), \(\Gamma(z) = \frac{f(z)}{1 - F(z)}\) is the inverse mills ratio or hazard rate, and \(f, F\) are the p.d.f. and c.d.f. for the standard normal distribution, respectively. If \(X\) and \(Z\) have normal distributions with variances \(\sigma_X^2\) and \(\sigma_Z^2\), respectively, then:

\[ (B.1) \]

\[ E\{X|Z \leq z\} = E\{X\} - \frac{\text{cov}\{X,Z\}}{\sigma_Z} \Gamma\left(\frac{E\{Z\} - z}{\sigma_Z}\right). \]

The expected future wage of a person who is admitted to the non-selective sector is given by:

\[ w_s(\tau) = E\{\alpha|\tau \leq \bar{\tau}\} + \hat{e}_s + \beta_s. \]

We have that \(E\{\alpha\} = E\{\tau\} = 0\) and \(\text{cov}\{\alpha, \tau\} = \sigma^2_\alpha\). Applying (B.1) implies:

**Proposition.** The expected innate ability and wage, \(w(\tau)\), of a student entering a non-selective school where \(\tau \leq \bar{\tau}\) is:

\[ E\{\alpha|\tau \leq \bar{\tau}\} = -\frac{\sigma^2_\alpha}{\sigma_\tau} \Gamma\left(-\frac{\bar{\tau}}{\sigma_\tau}\right) \]

\[ w(\tau) = -\frac{\sigma^2_\alpha}{\sigma_\tau} \Gamma\left(-\frac{\bar{\tau}}{\sigma_\tau}\right) + e_s + \beta_s, \]

where \(e_s\) and \(\beta_s\) are the equilibrium effort and value added in this sector.

Moreover, we also have from Birnbaum (1950) that \(x + \frac{1}{2} \geq \Gamma(x) \geq x \) for \(x \geq 0\), and \(\lim_{x \to -\infty} \Gamma(x) = 0\). Thus we have:

**Proposition.** For \(\bar{\tau} \leq 0:\)

\[ \pi(\tau)\alpha + e_s + \beta_s \geq w(\bar{\tau}) \geq \pi(\tau)\alpha + \frac{1}{\pi(\tau)\alpha} + e_s + \beta_s. \]

Notice that as \(\bar{\tau}\) falls, then so does the expected wage. Hence, as the selective sector increases in size, the expected income of individuals in the non-selective sector falls.

The equilibrium effort in the non-selective sector depends upon how effort is rewarded when its students enter the labor market. An individual’s choice does not affect the average effort in the sector, only her test score, \(t_{\bar{s}}\), upon leaving school. By construction we know that \(\frac{\partial t_{\bar{s}}}{\partial e_s} = 1\), so the next step is to work out the effect of test scores on future wages. Let \(w(t, \bar{\tau})\) be the expected wage of an individual from the non-selective sector who enters the labor market with a test score of \(t\).
Under the assumption that all individuals have the same taste for non-academic activities, \( a \), the level of effort, \( \hat{e}_s(\bar{\tau}) \), in this sector is the solution to:

\[-\Psi'(\hat{e}_s(\bar{\tau}), a) = \delta \frac{\partial w(t, \bar{\tau})}{\partial e} \equiv \delta E \left\{ \frac{\partial w(t, \bar{\tau})}{\partial t} \right\}.\]

We begin by finding \( w(t, \bar{\tau}) \). It is the solution to:

\[ w(t, \bar{\tau}) = E \{ \alpha | t, \tau \leq \bar{\tau} \} + \hat{e}_s(\bar{\tau}) + \beta. \]

Given that the last two terms do not depend upon \( t \) we need only work out the expected ability:

\[ E \{ \alpha | t, \tau \leq \bar{\tau} \} = E \{ E \{ \alpha | t, \tau \} | t, \tau \leq \bar{\tau} \} = \pi(t)\alpha(t - \beta_s - \hat{e}_s) + \pi(\tau)\alpha(t, \tau \leq \bar{\tau}) + \pi(\tau)\alpha(t, \tau \leq \bar{\tau}) \]

To compute the final expectation notice that we can view \( \tau \) as a random variable conditional upon the test score \( t \), such that:

\[ E \{ \tau | t \} = E\{ \alpha | t, s\} + E\{ e^\tau | t \} = \pi(t)\alpha(t - \beta_s - \hat{e}_s), \]

where \( \pi(t)\alpha \) is the optimal weight when only the test score is known. Note also that the variance of \( \tau \) is equal to the conditional variance given \( t \) plus the variance of the error term:

\[ \text{var} \{ \tau | t \} = \text{var} \{ \alpha | t \} + \text{var} \{ e^\tau \} = \frac{1}{\rho^\alpha + \rho^t} + \sigma^2_{\tau} \equiv \sigma^2_{\tau|t}. \]

We can now use formula (B.1) with \( X = Z = \tau \), to compute:

\[ E\{ \tau | t, \tau < \bar{\tau} \} = \pi(t)\alpha_t - \sigma_{\tau|t} \Gamma \left( \frac{\pi(t)\alpha_t - \tau}{\sigma_{\tau|t}} \right). \]

Using this result we have that the expected skill of an individual who obtains a test score \( t \) and who went to school in a non-selective sector with cutoff score \( \bar{\tau} \) is:
(B.2) \( w(t, \tau) = E\{\theta_s|t, \tau < \overline{\tau}\} = \pi^{(t)\alpha} t - \pi^{(\tau)\alpha} \sigma_{\tau|t} \Gamma \left( \frac{\pi^{(t)\alpha}(t - \beta s - \bar{e}) - \overline{\tau}}{\sigma_{\tau|t}} \right) + \left(1 - \pi^{(t)\alpha}\right)(\beta s + e_s) \)

We can compute a simple formula that captures the main effects on effort incentives by supposing that \( \Gamma' \) is approximately linear locally, and observing that \( E\{t - e_s - \beta_s|\tau \leq \overline{\tau}\} = E\{\alpha|\tau \leq \overline{\tau}\} = -\frac{\sigma^2}{\sigma_{\tau}} \Gamma \left( -\frac{\overline{\tau}}{\sigma_{\tau}} \right) \). From this we get.

**Proposition 9.** The marginal incentive for effort that students face in such schools is approximately:

\[
\frac{\partial w(\tau)}{\partial e} = E\left\{ \frac{\partial w(t, \tau)}{\partial t} \right\} \simeq \pi^{(t)\alpha} - \pi^{(\tau)\alpha} \pi^{(t)\alpha} \Gamma' \left( -\frac{\pi^{(t)\alpha} \sigma^2_{\tau}}{\sigma_{\tau|t}} \right) \left( -\frac{\overline{\tau}}{\sigma_{\tau}} - \overline{\tau} \right). 
\]
References


