Reputation and School Competition[†]

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Stratification is a distinctive feature of competitive education markets that can be explained by a preference for good peers. Learning externalities can lead students to care about the ability of their peers, resulting in across-school sorting by ability. This paper shows that a preference for good peers, and therefore stratification, can also emerge endogenously from reputational concerns that arise when graduates use their college of origin to signal their ability. Reputational concerns can also explain puzzling observed trends including the increase in student investment into admissions exam preparation, and the decline in study time at college. (JEL 121, 123, 126, J24)

A distinctive feature of competitive education markets is stratification (Hoxby 2009; Bound, Herschbein, and Long 2009). Theoretical models, notably Rothschild and White (1995); Epple and Romano (1998); and Epple, Romano, and Sieg (2006) explain stratification by introducing peer effects. They suppose that learning externalities lead students to care about the ability of other students at their school, resulting in equilibrium across-school sorting of students by ability.

The contribution of this paper is to show that a preference for good peers, and therefore stratification, can emerge endogenously from reputational concerns that arise when graduates use their college of origin to signal their ability. We also show that competition for a good reputation can explain some puzzling results in the empirical economics of education literature.

The importance of reputation effects in competitive markets was pointed out by Friedman (1962) and Akerlof (1970), who predict that firms with good reputations will expand and gain market share. This is indeed seen in many markets for consumer products, but educational markets often display a different pattern. Bound, Herschbein, and Long (2009) document that while the number of college applicants nearly doubled since the 1970s, elite colleges essentially did not grow but rather became increasingly selective. They suggest that in response, parents and students have invested ever more energy into the admissions process, engaging in possibly

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wasteful investments in what Ramey and Ramey (2010) label a "rug rat race." Why have the best suppliers restricted rather than expanded supply?

As Epple and Romano (1998) show, competition with peer effects indeed leads to the best schools being relatively small. But the evidence on whether students at more selective schools learn more is distinctly mixed, with careful analyses like Abdulkadiroğlu, Angrist, and Pathak (2014) suggesting little effect.¹ Why care so much about peers if they do not have a robust effect on skills?

Moreover, Babcock and Marks (2011) show that the past decades have seen a decline in the amount of time students spend studying. They observe this pattern at colleges throughout the quality spectrum. Why does effort at college seem to be falling while effort on getting into college is increasing?

We show that these facts can be explained by a simple model in which individuals of varying innate ability go to college and gain skill, which they then sell in a competitive labor market. Skill is determined by student innate ability, student *study effort* at school, and college value added. In addition, if schools are selective students can exert "*test prep*" effort to improve their admission chances. We assume, following Bound, Herschbein, and Long (2009), that test prep—in contrast to study effort—does not raise skill.

The model features no peer externalities; the only imperfection we introduce is that individual ability is not directly observable. As a result employers use whatever signals of skill are available, including the reputation of the college each individual graduated from. We define a college's reputation as the distribution of skill among its graduates. A college's reputation is thus not just a function of its value added but also of the ability of its students.

We show that in this setup individuals prefer colleges with good reputations; that is, they display an endogenous preference for better peers. This has two implications. First, since students can influence the market's perception of their ability by gaining admission to a more reputable college, increased stratification reduces study effort. This happens throughout the college reputation spectrum; for instance, if stratification arises from the co-existence of elite and non-selective schools, the mere existence of the former is informative on the ability of students in the latter. These results follow from the career concerns model of Holmström (1999). The basic idea is that in a competitive labor market wages reflect the market's best estimate of skill. If the market has imperfect information, then individuals have an incentive decreases with the precision of estimated skill.² Conversely, a desire for a selective college induces students to invest in test prep to game the admissions process. In short, a concern for reputation creates two distortions: too much test prep before admission, and too little study effort after admission.

¹ A number of studies use rigorous research designs to explore whether attending a more selective school raises test scores. No consistent pattern emerges; some papers like Cullen, Jacob, and Levitt (2006); Clark (2010); and Dobbie and Fryer (2011) find little or no impact; others like Jackson (2010) and Pop-Eleches and Urquiola (2013) find positive effects.

²This model has not been applied in the education context before. Gibbons and Murphy (1992) apply it to CEO compensation, and Alesina and Tabellini (2007) to the behavior of bureaucrats. Coate and Loury (1993) show how information regarding group ability (in their case created via affirmative action) can affect effort incentives.

Second, students' desire to pool with good peers provides schools with incentives to be selective and small. We call this the *anti-lemons* effect because it has the opposite impact of Akerlof's (1970) well-known lemons effect. Both phenomena are the result of asymmetric information. In the lemons model high quality sellers *exit* the market in order not to be pooled with low quality sellers. If one allowed reputations they might remain, grow, and dominate the market. In contrast, in education there is an incentive for firms to *enter* and acquire a reputation for quality by cream skimming the best students. In addition, schools with the best reputations choose to remain small.

Our agenda is as follows. Section I sets up a model that is sufficiently rich to capture reputation effects. Section II defines an allocation in this market. Section III presents two extreme allocations that illustrate key results. Section IV presents the anti-lemons effect. Section V discusses implications for competition-related policy, and Section VI concludes.

I. Basic Setup

Our model integrates a perfectly competitive college market with a competitive labor market in which wages equal the market's best estimate of worker productivity.³ The timing is divided into education and employment periods, with a sequence of choices as follows:

Period 0: Education:

- **0.1** Student innate ability is realized but not directly observed.
- **0.2** Students exert test prep effort to produce an admissions test score that colleges observe.
- **0.3** Each college sets a minimum admission standard in terms of this score.
- 0.4 Students choose a preferred school among those for which they qualify.
- **0.5** Individual skill is realized at college as a function of student innate ability, student study effort, and school value added.

Period 1: Employment:

- 1.1 Students produce an individual-specific signal of skill as they graduate.
- **1.2** Wages are set to expected skill conditional upon college reputation and the individual-specific measure of skill.

A. Ability and the Admissions Test

A large population of students is indexed by *i*; each individual has innate log ability denoted by α_i , where $\alpha_i \sim N(0, \sigma^2)$. It is convenient to work with the *precision* $\rho = \frac{1}{\sigma^2}$. As is common in labor economics, we will work with log variables and generally omit the term "log."

We follow Holmström (1999) in assuming that an individual does not directly observe her ability; rather she learns it during the school period. Students and colleges observe a signal of innate ability we call an *admissions test*; we assume employers do not observe this measure.⁴ Student *i*'s performance on this test is given by

(1)
$$\tau_i = \alpha_i + r_i + \epsilon_i^{\tau},$$

where $\epsilon_i^{\tau} \sim N(0, \sigma_{\tau}^2)$ and $\rho^{\tau} = \frac{1}{\sigma_{\tau}^2}$ is this measure's precision. Individual effort into producing the admission score—test prep—is r_i . The ex ante distribution of this score is given by $\tau_i \sim N(r_i, \sigma_{\tau 0}^2)$, where $\sigma_{\tau 0}^2 = \sigma^2 + \sigma_{\tau}^2$.

B. Skill

By attending college s, student i acquires skill

(2) $\theta_i = \alpha_i + e_i + v_s,$

where e_i is study effort at college, and v_s is college value added.⁵ We assume that in contrast to study effort, e_i , test prep, r_i , does not contribute to skill—it captures work at gaming the system. Finally, for simplicity we will suppose v_s is observed.

C. Colleges

We model perfect competition among schools in the standard manner—the number of available firms/schools exceeds demand. In particular, colleges are indexed by *s* and they are uniformly distributed over $S = S_L \cup S_H$, where $s \in S_H = (0, n_H)$ are schools with high value added, $v_s = v_H$, and $s \in S_L = (n_H, \overline{n} = n_H + n_L)$ have low value added, $v_s = v_L$, where $v_H > v_L$. The number of students is normalized to 1, and we assume that $n_H, n_L > 1$. Thus, both high and low value-added schools can potentially supply the whole market; the prevalence of high value-added schools will be an outcome of interest.

⁴ Although this assumption is stark, in general colleges might be better able to observe student innate ability than employers, or they may do so at lower cost. For example, in addition to collecting admissions test results, selective colleges may be able to obtain information on parental background, use alumni networks for interviewing, etc. ⁵Expression (2) is in logs; it can be derived from one in levels that interacts ability, effort, and value added.

We suppose that all colleges charge the same tuition, which is normalized to zero; effectively, colleges act as nonprofits.⁶ Their preferences can thus be reasonably defined as a function of enrollment, denoted n_s :

$$U_s = n_s \in [0,1].$$

In other words, colleges wish to serve more students. Note that (3) does not assume that colleges care about their reputation. A concern for reputation will be an endogenous feature of the equilibrium determined by student and school choices.

We assume colleges have a single strategy: to set an admissions standard τ_s , admitting any student with score $\tau \ge \tau_s$. A college wishing to be non-selective sets $\tau_s = -\infty$. If a school is oversubscribed, then it randomly selects students from the applicant pool up to its capacity constraint. We ensure perfect competition by supposing each school is capacity constrained at $n_s = 1$.

D. Student Preferences

Individual preferences are given by

(4)
$$u(r,e,w) = -d(r) - d(e) + \gamma w,$$

where w is the log wage in period 1 and γ is a discount factor. The cost of test prep is d(r), while the cost of study effort at college is d(e).⁷ Effort costs satisfy $d(\cdot) \ge 0$, $d'(\cdot) \ge 0$, and $d''(\cdot) > 0$. Notice that all students have the same cost of effort. Thus we shut down the main feature of the well-known Spence (1973) signaling model in which the cost of effort declines with ability; here, every individual goes to school and faces the same ex ante costs.

E. Signals of Skill and Wages

Skill is never directly observed; rather, the labor market infers it from two signals. The first is a noisy individual-specific measure that we call a *graduation test*:

(5)
$$t_i = \alpha_i + e_{s_i} + v_{s_i} + \epsilon_i^t,$$

where $\epsilon_i^t \sim N\left(0, \frac{1}{\rho^t}\right)$. Two aspects distinguish this measure from the admissions test, τ . First, it is a measure of skill rather than just of innate ability. Second, it is observed by all agents. This corresponds to a number of institutions in practice. For example, Colombia implements national (subject-specific) college graduation

⁶In MacLeod and Urquiola (2009) we show that for-profit, high productivity schools would charge higher tuition. US colleges engage in extensive price discrimination, and hence adding tuition greatly complicates the analysis while obfuscating the effect of reputation. Rothschild and White (1995) present a model of pricing without reputation effects; Epple, Romano, and Sieg (2006) develop a general equilibrium model of college provision, also without reputation effects.

⁷For simplicity we abstract from insurance motives, and so the wage rather than consumption enters utility. Note that without this assumption expression (4) could be derived from standard Cobb-Douglas preferences. This formulation can also be extended to allow for savings and intertemporal smoothing without changing our main conclusions.

tests that can give the market a sense of students' skills. There are also examples at other educational levels: Germany has highly publicized standardized high school graduation exams, and economics PhD students in the United States distribute "job market papers."

The second signal of skill is college reputation. There are many ways to model reputation (MacLeod 2007), but all share the feature that reputation is information that one obtains by knowing the *identity* of an agent. In our case, the market observes the identity of the college *s* individual *i* attended, denoted s_i . We will assume the reputation of a school *s* is the distribution of skill among its graduates. Below we consider two cases—no selection and perfect selection—for which mean skill is a sufficient statistic for school reputation. The online Appendix discusses a more complex case with a truncated distribution where the mean is no longer a sufficient statistic.

To summarize, individuals care about how their two signals, graduation test (t_i) and college identity (s_i) , affect their future earnings.⁸ We consider equilibria with the feature that ability is normally distributed at school s_i , in which case the Bayesian updating rule is linear in t_i . This, along with individual preferences defined by (4), implies that the marginal return to effort does not vary with ability. Hence, all students at the same school choose the same effort, e_{s_i} , and the log wage satisfies

(6)
$$w(s_i, t_i) = E\{\theta_i | s_i, t_i\} = E\{\alpha_i | s_i, t_i\} + e_{s_i} + v_s,$$

and can take on any real value.

II. Allocations

An *allocation* specifies student test prep and study effort, as well as the distribution of students across colleges. To capture perfect competition we follow Aumann (1966) in supposing that there is a continuum of agents. The actions of any single individual therefore have no effect upon any market level measure, only upon her wage. Similarly, the actions of any single school have no effect upon the market. Our setup allows further simplification: given that all students are ex ante identical, in equilibrium they will all select the same test prep level r.

The outcome of any mechanism is a match of students with admission test score τ to a set of schools that we denote by $F(s | \tau)$, the *school assignment function*. This is the cumulative probability distribution of schools to which a student with test score τ is allocated.⁹ We could also describe an allocation in terms of innate ability, α , but ability is never directly measured, while τ is observed. Further, regression discontinuity studies such as Abdulkadiroğlu, Angrist, and Pathak (2014) illustrate how one may empirically recover school-specific admissions standards, $\tau_s(\Gamma)$.¹⁰ In summary,

⁹Distribution functions are increasing and right continuous in *s*.

¹⁰For $n_s > 0$, $\tau_s(\Gamma) = \sup\{\tau | I(s,\tau') = 0, \forall \tau' < \tau\}$ where $I(s,\tau) = \max\{F_s(s^+|\tau), \lim_{s^0 \to s^-} (F(s|\tau) - F(s^0|\tau))\}$ is positive if there are students with ability τ assigned to school s. $F_s(s^+|\tau)$ denotes the derivative w.r.t. s from the right, which we assume to be well defined. If $n_s = 0$ or $I(s,\tau)$ is not well defined then the default is

⁸More generally individuals will consider and potentially manipulate other signals. For example, Bénabou and Tirole (2006) observe that people's reputation depends not only on their ability but also upon the extent to which they can be trusted. Hence, they allocate some of their time to pro-social activities.

we define an *allocation* by $\Gamma = \{r, e_s, F(s | \tau)\}_{\tau \in \Re, s \in S}$, and show that this provides all the information that is needed to compute agents' payoffs.

Given this definition, the rest of this section characterizes conditions under which an allocation is *efficient* and under which it is a *competitive equilibrium*. We begin by defining feasibility.

DEFINITION 1: An allocation $\Gamma = \{r, e_s, F(s | \tau)\}_{\tau \in \Re, s \in S}$ is feasible if $r, e_s \ge 0, s \in S$, and $F(s | \tau) = \operatorname{Prob}\{s_i \le s | \tau_i = \tau\}$ satisfies:

- (i) $F(s) = \int_{-\infty}^{\infty} F(s \mid \tau) f^N(\frac{\tau r}{\sigma_{\tau 0}}) \frac{1}{\sigma_{\tau 0}} d\tau$ is continuous and right differentiable in s.
- (ii) The derivative from the right exists and satisfies $F'(s^+) = n_s \leq 1$.

The first requirement, a technical one, ensures that the distribution is well defined; the second is the school capacity constraint, where $F'(s^+)$ is the density of students at school *s*.¹¹ In short, a feasible allocation specifies the distribution of ability at school *s*, and hence determines log wages, $w(s, t | \Gamma)$, via (6) under the assumption that students choose the effort defined by the allocation. The key point is that wages depend only on the school attended and the graduation test score.

A. Payoffs and the Efficient Allocation

All students are ex ante identical. Student *i* makes decisions at two points in time. First, she chooses test prep, r_i , before she knows her admissions score, τ_i . Second, after observing τ_i , she chooses a school, s_i , and sets her study effort, e_i . We assume she can only choose from the set of schools that will admit her.

We work backwards by analyzing school and effort choice conditional upon τ_i . First, define $t_i(s_i | \Gamma) = \alpha_i + e_{s_i} + v_{s_i} + \epsilon_i^t$ to be an individual's graduation test score if she attends school s_i under allocation Γ . After observing her admissions score τ_i , she chooses a school and study effort, correctly anticipating the consequence for future wages. Hence, her payoff as a function of all her choices— r_i, s_i, e_i —after observing her admission score, τ_i , is given by

(7)
$$u(r_i, s_i, e_i | \mathbf{\Gamma}, \tau_i) = -d(r_i) - d(e_i) + \gamma E \{ w(s_i, t_i(e_i | \mathbf{\Gamma}) + (e_i - e_{s_i})) | \mathbf{\Gamma}, \tau_i \}.$$

The market expects study effort e_{s_i} and cannot observe e_i . Thus if she chooses $e_i > e_{s_i}$, she raises her graduation test and hence her wage. The equilibrium effort at school s_i is determined by that point at which the optimal effort satisfies $e_i^* = e_{s_i}$.

 $[\]tau_s(\Gamma) = -\infty$, corresponding to school *s* being non-selective and the market holding pessimistic beliefs. This is discussed in more detail below. With perfect stratification this distribution will have a point mass for each test score $\tau \left(\forall \tau, \exists s \in S \text{ s.t. } \lim_{s^0 \to s^-} (F(s | \tau) - F(s^0 | \tau)) = 1 \right).$

¹¹ An example of an allocation would be $F(s | \tau) = \min\{s, 1\}$, which corresponds to F'(s) = 0 for $s \ge 1$ and $F'(s) = 1, s \in (0, 1)$. In this case, students are uniformly allocated across colleges $s \in (0, 1)$.

Second, define $\tau_i(\Gamma) = \alpha_i + r_i + \epsilon_i^{\tau}$, the admissions score of an individual who chooses test prep level r_i under allocation Γ . Assuming that upon entering school s_i she chooses e_{s_i} , her payoff at the time she chooses test prep is¹²

(8)
$$u(r_i | \Gamma) = \int_{\tau \in \Re} \int_{s \in S} u(r_i, s, e_s | \Gamma, \tau) dF(s | \tau_i(\Gamma) + (r_i - r)) f^N(\frac{\tau - r_i}{\sigma_{\tau 0}}) \frac{1}{\sigma_{\tau 0}} d\tau.$$

Because the mean innate ability is zero and the labor market is competitive, the aggregate effect of ability is integrated out. Hence the ex ante payoff at an allocation is

(9)
$$u(\Gamma) = -d(r) + \int_{s \in S} (\gamma(e_s + v_s) - d(e_s)) n_s \, ds.$$

With this we can state the efficient allocation—the one that maximizes utility of the representative individual in (9):

PROPOSITION 1: A feasible allocation $\Gamma^* = \{r^*, e_s^*, F^*(s \mid \tau)\}_{\tau \in \Re, s \in S}$, is efficient if and only if:

- (i) There is no effort into admissions test prep $(r^* = 0)$.
- (ii) The marginal cost of study effort is equal to the rate of time preference $(d'(e_s^*) = \gamma, \forall s)$.
- (iii) Students are allocated only to high value-added colleges: $n_s = 0$ for $s \in S_L$.

These conditions reflect that test prep has no economic value; the optimal study effort, e^* , should reflect the return from skill acquisition; and students should use only high value-added schools.

B. Equilibrium

We now define an equilibrium allocation at which individuals and schools cannot gain by altering their strategies. When $n_s > 0$ there exists a distribution of student ability at school *s*, and hence a student's payoff at this school is well defined. For an empty school ($n_s = 0$) there is no information on peer quality. Hence beliefs have to be specified as part of an equilibrium concept.

We suppose that students hold *pessimistic beliefs*. That is, when they observe a school advertising an admission criterion τ_s , they assume only individuals with test score τ_s would choose such a school. In the next section we show that under these beliefs, the expected payoff to choosing this school is well defined and denoted by $u^E(r, \tau_s, v_s | \Gamma)$. We can now define an equilibrium:

¹²Note that r in expression (8) refers to the test prep under allocation Γ .

DEFINITION 2: An allocation $\Gamma^* = \{r^*, e_s^*, F^*(s \mid \tau)\}$ is a perfectly competitive equilibrium if:

- (i) Students optimally chose test prep: $u(r^* | \Gamma^*) = \max_{r_i > 0} u(r_i | \Gamma^*)$.
- (ii) A student with admissions score τ optimally chooses:¹³
 - (a) A school from the set of feasible schools $s'(\tau \ge \tau_{s'}(\Gamma))$: $u(r^*, s, e_s^* | \Gamma^*, \tau) \ge u(r^*, s', e_{s'}^* | \Gamma^*, \tau)$.
 - (b) Study effort: $u(r^*, s, e_s^* | \Gamma^*, \tau) = \max_{e_i \ge 0} u(r^*, s, e_i | \Gamma^*, \tau).$
- (iii) For any student with score τ attending school s there does not exist $s' \in S$ with $n_{s'} < 1$ such that

$$u^{E}(r^{*}, \tau, v_{s'} | \Gamma^{*}) > u(r^{*}, s, e_{s}^{*} | \Gamma^{*}, \tau),$$

i.e., schools with excess capacity cannot alter admissions standards to attract more students.

Conditions (i) and (iib) ensure effort choices are optimal. The continuum assumption implies that a single student can always be added into a school, and hence (iia) ensures that students with the same test score get the same payoff at a competitive equilibrium. This implies that if condition (iii) is not satisfied then school s' can attract all students with test score τ , and fill the school.

III. Reputation, Wages, and Effort under Two Extreme Allocations

We now consider two extreme allocations. The first features a complete absence of choice—students are randomly assigned to any college. The second is one in which colleges are free to enter and set admission standards, and students are free to choose any college that will admit them.

A. College Reputation under the Non-Selective Allocation

The *non-selective* (*NS*) allocation is denoted by $\Gamma^{NS} = \left\{ r^{NS}, e_s^{NS}, F^{NS}(s|\tau) \right\}_{\tau \in \Re, s \in S}$. In this case, students are randomly assigned to colleges. Thus, for each test score τ , they are evenly distributed over all schools: $\frac{dF^{NS}(s|\tau)}{ds} = \frac{1}{\overline{n}}$ and $F^{NS}(s|\tau) = \frac{s}{\overline{n}}$ for all τ , *s*. Notice that $\tau_s(\Gamma^{NS}) = -\infty$ for all $s \in S$.

Although in general a college's reputation is the distribution of skill among its graduates, in this allocation it is given by mean skill. Specifically, random assignment implies that all colleges have the same mean innate ability: $\overline{\alpha}_s = 0, \forall s$. There

¹³Technically, this holds when $I(s, \tau) > 0$, as defined in footnote 10.

is variation in college value added (v_H or v_L) since all schools are used. College reputation therefore varies only with value added and is given by

(10)
$$R_s^{NS} = \overline{\alpha}_s^{NS} + e_s^{NS} + v_s,$$
$$= e_s^{NS} + v_s.$$

B. College Reputation under the Free Choice Allocation

The free choice (FC) allocation is denoted by $\Gamma^{FC} = \left\{ r^{FC}, e_s^{FC}, F^{FC}(s|\tau) \right\}_{\tau \in \Re, s \in S}$. Schools are free to set admissions standards and students can choose any school with space at which they meet the standard $\tau_s(\Gamma^{FC})$. Since in equilibrium all students choose the same test prep, r^{FC} , then if $\tau_i > \tau_j$, student *i* is more able than student *j*, where τ is a signal of innate ability with precision ρ_{τ} , i.e., $E\{\alpha_i | \tau_i\} = \frac{\rho_{\tau}}{\rho + \rho_{\tau}} (\tau_i - r^{FC})$ (DeGroot 1972, Section 9.5, Theorem 1). Given the excess supply of high value-added schools, one expects a free choice equilibrium to only feature such schools, and thus we set $n_s = 0 \ \forall s \in S_L$.

Below we shall show that this allocation is a perfectly competitive equilibrium that results in perfect segregation by admission score. Since high value-added schools are identical we can suppose, without loss of generality, that students with higher test scores attend schools with higher *s*. We can then assign students to schools in a one to one fashion to satisfy the schools' capacity constraint as follows. Begin with $G(\tau) = F^N(\frac{\tau - r^{FC}}{\sigma_{\tau 0}}) \in [0, 1]$, the cumulative distribution of τ for the whole population, where F^N is the standard normal distribution. Define τ_s by

$$au_s = G^{-1}\left(rac{s}{n_H}
ight), \quad s \in (0, n_H),$$

and let $F^{FC}(s|\tau) = \begin{cases} 1, & \tau \leq \tau_s \\ 0, & \tau > \tau_s \end{cases}$. This rule means that students with test score $\tau = \tau_s$ get allocated only to school *s*. This also implies that $F'(s) = n_s = \frac{1}{n_H} < 1$, so that students are evenly allocated over all high value-added schools.¹⁴ By construction, $\tau_s(\Gamma^{FC}) = \tau_s$.

The expected innate ability at school s is therefore normally distributed with mean

(11)
$$\overline{\alpha}_s^{FC} \equiv E\{\alpha \,|\, \tau = \tau_s\} = \frac{\rho^{\tau}}{\rho + \rho^{\tau}} (\tau_s - r^{FC}).$$

The reputation of school s under the free choice allocation is given by the statistic

(12)
$$R_s^{FC} = \overline{\alpha}_s^{FC} + e_s^{FC} + v_H$$

$$= \frac{\rho^{\tau}}{\rho + \rho^{\tau}} (\tau_s - r^{FC}) + e_s^{FC} + v_H.$$

¹⁴ As a check note that $F(s) = \int_{-\infty}^{\infty} F(s \mid \tau) f^N(\frac{\tau - r}{\sigma_{\tau 0}}) \frac{1}{\sigma_{\tau 0}} d\tau = \int_{-\infty}^{\tau_s} f^N(\frac{\tau - r}{\sigma_{\tau 0}}) \frac{1}{\sigma_{\tau 0}} d\tau = G(\tau_s) = \frac{s}{n_H}$; hence $F' = \frac{1}{n_H}$.

	Non-selective allocation	Free choice allocation
Wage	$w^{NS}(s_i, t_i) = \pi^{NS} R_{s_i}^{NS} + (1 - \pi^{NS}) t_i$	$w^{FC}(s_i, t_i) = \pi^{FC} R_{s_i}^{FC} + (1 - \pi^{FC}) t_i$
Weight on reputation	$\pi^{NS} = rac{ ho}{ ho+ ho'}$	$\pi^{FC} = rac{ ho + ho^ au}{ ho + ho^t + ho^ au}$

TABLE 1—WAGE DETERMINATION UNDER EACH ALLOCATION

At this allocation college reputation rises (without limit) as a function of admissions scores.

C. Wages

We can now describe the wages under these two allocations.

PROPOSITION 2: The wages of a student with graduation test score t_i under the non-selective and free choice allocations satisfy Table 1.

For each allocation, the second row in Table 1 displays the weight placed on reputation in wage determination, π . The weight placed on the graduation test score is $1 - \pi$. In both allocations reputation is given by the same statistic—the mean skill at a school—but the weight given that statistic depends upon the degree of stratification; stratification is greater at the free choice allocation, thus $\pi^{FC} > \pi^{NS}$. Intuitively, college membership conveys more information under free choice, and hence rational employers place greater weight on reputation.¹⁵

D. Equilibrium Effort

We find equilibrium effort under each allocation by backwards induction. First, student *i* with test prep r^* and admissions test score τ at school *s* chooses study effort e_i to maximize $u(r^*, s, e_i | \Gamma^*, \tau)$. Since students are risk neutral, optimal effort solves

(13)
$$d'(e_i) = \gamma \frac{\partial E(w \mid \tau_i, s, \Gamma)}{\partial e_i}.$$

From Proposition 2 this implies

(14)
$$d'(e^{NS}) = \gamma(1-\pi^{NS}) = \gamma \frac{\rho^t}{\rho^t + \rho},$$

(15)
$$d'(e^{FC}) = \gamma (1 - \pi^{FC}) = \gamma \frac{\rho^t}{\rho^t + \rho^\tau + \rho}$$

¹⁵This is consistent with empirical evidence. Specifically, in the United States there is more sorting by student ability across colleges than across high schools. Arcidiacono, Bayer, and Hizmo (2010) find that college identity is much more predictive of wages than high school identity.

Thus, the incentives for effort depend only upon the amount of stratification at the allocation, which in turn determines the sensitivity of the graduation signal to individual effort. In particular, for these two cases effort does not vary with school identity.

Notice that under pessimistic beliefs wages are given by w^{FC} because a school's admission standard is assumed to be equal to the mean test score at the school. Since this does not depend upon what is happening at other schools, the payoff from choosing school *s* is well defined and given by

$$u^{E}(r,\tau,v_{s} \mid \boldsymbol{\Gamma}) = -d(r) - d(e^{FC}) + \gamma \left(\frac{\rho^{\tau}}{\rho + \rho^{\tau}}(\tau - r) + e^{FC} + v_{s}\right),$$

where the future expected wage is computed at time of admission to the school.

Next we work out test prep. At the non-selective allocation, the payoff before taking the admissions test is

$$u(r_i \mid \Gamma^{NS}) = -d(r_i) - d(e^{NS}) + \gamma(e^{NS} + E\{v \mid \Gamma^{NS}\}),$$

where $E\{v | \Gamma\}$ is the mean value added at the allocation. Since test prep has no effect upon future wages it is optimal to set r = 0. In contrast, in the free choice allocation a better admissions score procures a better college reputation. Using Proposition 2 we get

$$u(r_{i} | \Gamma^{FC}) = -d(r_{i}) - d(e^{FC}) + \gamma E\{w | \Gamma^{FC}, r_{i}\}$$

= $-d(r_{i}) - d(e^{FC}) + \gamma \left(\pi^{FC} \frac{\rho^{\tau}}{\rho + \rho^{\tau}} (r_{i} - r^{FC}) + (e^{FC} + v_{H})\right).$

From this we get that the unique test prep effort solves

(16)
$$d'(r^{FC}) = \pi^{FC} \frac{\rho^{\tau}}{\rho^{\tau} + \rho} \gamma = \frac{\rho^{\tau}}{\rho^{t} + \rho^{\tau} + \rho} \gamma.$$

In words, the more precise the admissions test, the greater the effort into test prep. Summarizing:

PROPOSITION 3: Suppose the cost of effort is strictly convex, twice differentiable, and satisfies d(0) = d'(0) = 0; then the equilibrium test prep and study effort levels at the non-selective and free choice allocations satisfy:

- (i) Test prep effort: $r^{FC} > r^{NS} = 0$,
- (ii) Study effort: $0 < e^{FC} < e^{NS} < e^*$, where e^* is the efficient level of study effort (Proposition 1).

Proposition 3 shows that competition can distort effort, as previewed above. Specifically, it increases investment into test prep—the "rug rat race" emphasized by Ramey and Ramey (2010). In addition, the stratification it induces lowers study effort. Finally, expressions (14) and (15) show that in both allocations, study effort increases with the precision of the exit exam, ρ^t . This result is consistent with Bishop's (2006) point that emphasizing such tests can raise study time and earnings. It also implies that if the precision with which schools measure innate ability rises, then the distortion under the free choice allocation increases.

IV. The Anti-Lemons Effect

This section shows that the non-selective allocation is inherently unstable, while the free choice allocation is an equilibrium. Consider a non-selective allocation with observable value added. Since there is an excess supply of schools, some school *s* has additional capacity $n_s < 1$. Suppose this school has value added, v_s , and suppose it sets an admissions standard τ_s . Given pessimistic beliefs only individuals of type τ_s are expected to attend this school, which implies that wages are set by the free choice expression in Proposition 2. This in turn implies that a student with ability τ_s who opted for this school—instead of a non-selective school—would obtain the following change in utility (the difference between the two payoffs in Section IIID):

(17)
$$\Delta u = \left\{ \gamma e^{FC} - d(e^{FC}) - \left(\gamma e^{NS} - d(e^{NS}) \right) \right\} + \gamma \pi^{FC} \frac{\rho^{\tau}}{\rho + \rho^{\tau}} \tau_s.$$

The first term is negative due to lower effort incentives at the selective school. The second is the wage gain from attending a selective school. Since there is no upper bound to τ_s , it can be set sufficiently high such that a student with score τ_s prefers the selective school. Thus the free entry condition (3) of a perfectly competitive equilibrium is not satisfied by a non-selective allocation.

In contrast, under a free choice allocation all students choose the same study effort, e^{FC} , both at their current school and at any potential entrant. In addition, since there is an excess supply of high value-added schools the free entry condition ensures that low value-added schools have no students in equilibrium. Thus a student with test score τ_i would prefer a new entrant *s* if and only if it has high value added and $\tau_s > \tau_i$. This implies that the "Groucho Marx" condition is satisfied: a student prefers only schools outside the set of those that will actually admit him.¹⁶ Thus we have:

PROPOSITION 4: A non-selective allocation cannot be a competitive equilibrium. An allocation with perfect segregation can be a competitive equilibrium if and only if there are no low value-added schools. Hence, the free choice allocation, Γ^{FC} , is a perfectly competitive equilibrium.

¹⁶Erskine Johnson wrote: "In resigning from the Friars club Mr. Marx, incapable of passing up an opportunity for a gag-line, wrote: 'I don't want to belong to any club that will accept me as a member.'" ("Hollywood Notes," October 20, 1949, syndicated by Newspaper Enterprise Association.)

Proposition 4 shows that non-selective systems are inherently unstable. As soon as some school engages in selection, reputation no longer depends only on value added, and parental choice will lead to more stratification.

Focusing on two extreme allocations, Propositions 2–4 illustrate various elements required to think about competition in an education market. In the online Appendix (Proposition A.1) we also consider a school that serves a range of students, say with test scores $\tau \in [\tau^0, \tau^1]$. Given that study effort is chosen optimally, we use the envelope condition to show that students with test scores greater than τ^0 would prefer to raise the standard at such a school, i.e., to ask those students with scores τ^0 to leave. In other words, there is consumer demand for *more* selective schools.

We label these results the *anti-lemons effect*. In the lemons model of Akerlof (1970), asymmetric information leads to the exit of high quality sellers. If one were to allow reputations, then these sellers might remain, grow, and dominate the market. In contrast, in education the fact that a school's reputation is linked to the ability of its customers creates incentives for schools to enter and build their reputations on selection rather than value added. It also leads the schools with the best reputations to remain small.

Finally, the online Appendix also considers a scenario between the polar cases covered in Proposition 4: a mixed system with a combination of selective and non-selective schools. This can be an equilibrium as long as non-selective schools have a cost advantage. For example, in many jurisdictions public schools take any student and are subsidized by the government, while private schools select students but face an entry cost. Mixed scenarios are also relevant at the college level, as in many countries selective schools coexist with colleges that practice open admissions.

The online Appendix illustrates a competitive equilibrium with the following features. High ability students with scores above a cutoff $\overline{\tau}$ are matched to perfectly selective schools. Those with scores below $\overline{\tau}$ are in the non-selective sector. The existence of a selective sector leads the students in non-selective schools to exert *lower* study effort than they would in the absence of any stratification.

In short, to the extent that sorting in college markets makes school identity more predictive of innate ability, it leads to reduced effort throughout the reputation distribution, as found by Babcock and Marks (2011). For instance, Hoxby (2009) shows that a few decades ago low ability college students in the United States were much more likely to be pooled with high ability students. Since then, the integration of the college market (due to, for example, lower transport costs) led to sorting as the top schools became more selective and the bottom ones less so.¹⁷ MacLeod et al. (2015) show that in the past 15 years Colombia has experienced qualitatively similar trends: rising selectivity at top colleges and overall increasing stratification by ability—e.g., the R^2 on a regression of admissions test scores on a full set of college dummies has trended up during this period.

Finally, while Babcock and Marks (2011) analyze the evolution of effort over time, it is the case that in a cross-sectional sense effort should be higher at non-selective than at selective schools. The regression discontinuity designs that

¹⁷ Although Hoxby (2009, p. 108) does not provide a quantitative measure of sorting, the paper states, "[t]he re-sorting of students among colleges clearly caused high-aptitude students to experience peers who were them-selves increasingly of high aptitude. The reverse is true of students with low college aptitude."

have been used to analyze the effect of going to a more selective school or college would in principle lend themselves to testing this—one could compare effort among students just above or below an admissions standard τ_s , for example. On the other hand, such designs yield reduced form estimates, and differences in effort could conflate individual responses—labeled e_s in our framework—with others induced by the school. For example, if students at a higher ranked school work harder because they are assigned more homework, this might conceptually be viewed as part of school value added, v_s .

V. Managed Competition

Our results suggest that the structure of education markets can affect their performance. This point may be intuitively evident to policymakers. For example, charter schools in the United States are required to use admissions lotteries.¹⁸ In principle this shuts down stratification, making value added transparent because it is then the only source of variation in school reputation.¹⁹ Our model has clear implications on the effects of such a system:

PROPOSITION 5: Suppose that school value added is observable, and that schools are required to use admissions lotteries. At any competitive allocation with some high value-added schools ($n_s > 0$ for some s where $v_s = v_H$), there will be no low value-added schools ($n_s = 0$ if $v_s = v_L$). Furthermore there will be no test prep (r = 0), and study effort is $e^{NS} > e^{FC}$.

Proposition 5 illustrates that charter schools can have advantages. At the same time, some high value-added schools are needed to ensure that low value schools are driven from the market: if all schools with positive enrollment are of low value added, then high value-added schools cannot enter under pessimistic beliefs. In practical terms, if charter schools are in limited supply, then competition may still be imperfect, which in turn may help explain the mixed results concerning the effect of charters on performance. It also illustrates that entrants may invest in managing expectations regarding the likely student body.

VI. Conclusion

The markets for many goods have the feature that if a product is defective, the customer can return it for a refund or seek damages. In contrast, parents do not expect a refund if a school does not perform as expected, and suing for poor performance has been attempted with little success.²⁰ Given that students cannot directly

¹⁸For background on charter schools and their effects see Abdulkadiroğlu et al. (2011).

¹⁹While there is clear evidence that absolute achievement affects school choice (e.g., Black 1999 and Hastings and Weinstein 2008), the evidence that school value added does so is weak (e.g., Rothstein 2006; Mizala and Urquiola 2013; and Imberman and Lovenheim 2013).

²⁰ In the United States, Section 5 of the Uniform Commercial Code specifies in detail how parties are to behave when goods are defective (American Law Institute & National Conference on Uniform State Laws. 2009. Uniform Commercial Code: Official Text and Comments. Westlaw: St. Paul, MN). In contrast, Martinez (2011) discusses a New York City parent who sued a preschool for not providing adequate educational services. The judge dismissed the case.

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contract for quality, they must rely upon what they *expect* the quality of service to be: in other words the *reputation* of the school they consider.

This paper introduces a model of a perfectly competitive education market with reputation effects. We begin with the hypothesis that the labor market efficiently uses information on the college one attends to infer one's ability. The key assumptions are that students and employers care about and reward skill, and that ability is imperfectly observed.

This setup leads to two main results. First, reputational concerns can hinder as well as promote educational efficiency. While competition for a good reputation can reduce the demand at low value-added schools, it has unintended consequences on student effort: it increases unproductive test prep as individuals try to gain admission to selective schools, and it reduces study effort after admission. Second, a concern for reputation creates an incentive for colleges to build their reputations on the identity of their students: the anti-lemons effect. This leads competition to promote stratification (e.g., Hsieh and Urquiola 2006) and leads the top schools to remain small.

Our results suggest that policy could promote efficiency in school markets by restricting selection while allowing freedom of entry and student choice—a design along the lines of charter schools in the United States. This implication should be tempered by the potential gains that selective systems may produce. For example, there is social value in signals of ability if they allow for better matching of specific educational inputs to students, and for more productive matches in the labor market.²¹ Teasing out these countervailing effects is a challenging agenda for future research.

More generally, our model suggests there are gains to considering how signaling occurs via the identity of the school one attends, as opposed to simply the number of years of schooling one obtains, as in the Spence (1973) model. The model we have introduced is stark, yet it illustrates a complex relationship between competition and individual incentives, and hence may help explain why designing effective education markets is so difficult.²²

An additional path for future research arises because we have assumed that students are ex ante identical and face the same costs of effort. If the cost of test prep varies across students—e.g., if wealthier parents can more easily afford test prep then school admissions gets tipped in favor of given groups. This implies that admissions standards should be adjusted for group characteristics.

Finally, we have considered the case in which a college's reputation is measured by the ability of its graduates. More generally a school's reputation is multidimensional and, as the proliferation of rankings illustrates, the weight assigned to different aspects of performance can yield different rankings. Thus, there remains a great deal of work to understand the complex interplay between school reputation and competition in education markets.

²¹ This is analogous to the effects of the entry of middlemen, as studied by Biglaiser (1993) and Lizzeri (1999). ²² It is worth pointing out that the goal of Holmström (1999) is to illustrate that a manager's concern for a good reputation does not necessarily result in efficient incentives. Here we are extending this point to the case in which a firm's (school's) reputation has implications for its customers.

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