

Online Appendix for “The Effects of Permanent Monetary Shocks on Exchange Rates and Uncovered Interest Rate Differentials”

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This appendix presents a detailed derivation of the model presented in Section 2 of Schmitt-Grohé and Uribe (2021).

1 The Model

The small open economy is populated by a large number of identical households with preferences over consumption, C_t , and hours worked, N_t . Lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \quad (\text{A.1})$$

where $\beta \in (0, 1)$ denotes the subjective discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and E_t denotes the expectations operator conditional on information available at time t . The household’s sequential budget constraint is

$$P_t C_t + (1 + i_{t-1}) D_{t-1} + \mathcal{E}_t (1 + i_{t-1}^*) D_{t-1}^* \leq W_t N_t + \Pi_t + D_t + \mathcal{E}_t D_t^* - \mathcal{E}_t \psi(D_t^*), \quad (\text{A.2})$$

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where P_t denotes the nominal price of the consumption good in period t , W_t denotes the nominal wage rate in period t , $\mathcal{E}_t$ denotes the domestic currency price of one unit of foreign currency in period t , Π_t denotes nominal profit income of the household from ownership of firms in period t , D_t denotes domestic currency debt at the end of period t , D_t^* denotes foreign currency debt at the end of period t , i_t is the nominal interest rate on domestic currency debt held from period t to period $t + 1$, i_t^* is the nominal interest rate on foreign currency debt held from period t to period $t + 1$, and $\psi(D_t^*)$ represent portfolio adjustment costs in period t . The household takes the processes for P_t , i_t , i_t^* , $\mathcal{E}_t$, W_t , and Π_t and the initial conditions $(1 + i_{-1})D_{-1}$ and $(1 + i_{-1}^*)D_{-1}^*$ as given.

The household's problem consists in maximizing (A.1) subject to (A.2) and some borrowing limit that prevents it from engaging in Ponzi schemes. Let λ_t/P_t denote the Lagrange multiplier on the period- t budget constraint. The first-order conditions associated with the household's utility maximization problem are then given by (A.2) holding with equality and

$$C_t^{-\sigma} = \lambda_t, \quad (\text{A.3})$$

$$C_t^\sigma N_t^\varphi = W_t/P_t, \quad (\text{A.4})$$

$$1 = \beta(1 + i_t)E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}, \quad (\text{A.5})$$

and

$$1 = \beta \frac{(1 + i_t^*)}{1 - \psi'(D_t^*)} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}. \quad (\text{A.6})$$

Consumption, C_t is assumed to be CES aggregate of domestic goods, $C_{H,t}$ and foreign (imported) goods, $C_{F,t}$,

$$C_t = \left[(1 - \nu)^{\frac{1}{\eta}} C_{H,t}^{1 - \frac{1}{\eta}} + \nu^{\frac{1}{\eta}} C_{F,t}^{1 - \frac{1}{\eta}} \right]^{\frac{1}{1 - \frac{1}{\eta}}}, \quad (\text{A.7})$$

where $\eta > 0$ and $\nu \in (0, 1)$. Given a desired quantity of consumption C_t , households will

distribute their purchases of domestic and foreign goods so as to minimize expenditures $P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$, where $P_{H,t}$ is the domestic currency price of the domestic good and $P_{F,t}$ is the domestic currency price of the foreign good. The cost minimization gives rise to the following demand functions

$$C_{H,t} = (1 - \nu) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad (\text{A.8})$$

and

$$C_{F,t} = \nu \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \quad (\text{A.9})$$

where P_t satisfies

$$P_t = [(1 - \nu)P_{H,t}^{1-\eta} + \nu P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$$

and

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}.$$

Domestic goods, $Y_{H,t}$, are a composite of a continuum of domestically produced intermediate goods, denoted $Y_{H,t}(i)$ for $i \in [0, 1]$. Specifically,

$$Y_{H,t} = \left[\int_0^1 Y_{H,t}(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\frac{1}{\epsilon}}},$$

where $\epsilon \geq 1$.

The demands for the individual varieties $Y_{H,t}(i)$ can again be obtained from a cost minimization problem and are given by

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} Y_{H,t}, \quad (\text{A.10})$$

where $P_{H,t}(i)$ denotes the domestic currency price of intermediate good of variety i . The

price of the domestic composite good, $P_{H,t}$, then satisfies

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \quad (\text{A.11})$$

and the value of the domestic composite good equals the value of the intermediate inputs, that is,

$$P_{H,t} Y_{H,t} = \int_0^1 P_{H,t}(i) Y_{H,t}(i) di.$$

Each variety of the domestic intermediate input is produced by a monopolistically competitive firm via the production function $N_t(i)$, where $N_t(i)$ is employment in firm i . The demand for intermediate input i is given by equation (A.10). Firms must satisfy demand at the posted price $P_{H,t}(i)$, that is,

$$N_t(i) \geq \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} Y_{H,t}. \quad (\text{A.12})$$

Nominal prices are sticky à la Calvo-Yun. Each period only a fraction $(1 - \theta) \in (0, 1]$ of randomly chosen firms can reoptimize prices. Let $\tilde{P}_{H,t}$ denote the price a firm that can reoptimize in period t chooses. When a firm cannot reoptimize its price in period t , it is assumed to adjust its price based on lagged inflation. Specifically, we assume that if in period t a firm cannot reoptimize its price, its period t price is given by

$$P_{H,t}(i) = P_{H,t-1}(i) \Pi_{H,t-1}, \quad (\text{A.13})$$

where $\Pi_{H,t-1} \equiv P_{H,t-1}/P_{H,t-2}$ denotes the rate of inflation of the price of the domestic good in period $t - 1$.

Suppose a firm can reoptimize its price in period t . In choosing the optimal price, $\tilde{P}_{H,t}$, the firm will take into account not only what its choice of price implies for current profits but also for future profits in those states of the world in which it cannot reoptimize the price and instead adjusts it according to the indexation scheme given in (A.13). Suppose a firm

last reoptimized its price in period t . Then in period $t + j$ for $j \geq 1$, its price is equal to

$$P_{H,t+j}(i) = \tilde{P}_{H,t} \Psi_{t,t+j}; \quad \text{where } \Psi_{t,t+j} \equiv \prod_{k=0}^{j-1} \Pi_{H,t+k}.$$

In addition define $\Psi_{t,t} \equiv 1$.

The probability of being able to reoptimize prices is assumed to be independent of any fundamental of either the firm or the economy as a whole. Thus, the probability that a firm in period $t + j$ cannot reoptimize and has last reoptimized in period t is θ^j . Firm i chooses $\tilde{P}_{H,t}$ so as to maximize the expected present value of profits in those nodes of the state space in which it is not given the opportunity to reoptimize its price. Formally, $\tilde{P}_{H,t}$ is chosen so as to maximize

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{\tilde{P}_{H,t} \Psi_{t,t+j}}{P_{H,t+j}} \right)^{-\epsilon} Y_{H,t+j} \left(\tilde{P}_{H,t} \Psi_{t,t+j} - W_{t+j} \right),$$

where $Q_{t,t+j}$ is the period- t domestic currency price of one unit of domestic currency in a particular state of the world in period $t + j$ divided by the probability of occurrence of that state. The value of $Q_{t,t}$ is equal to one. In equilibrium,

$$Q_{t,t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{P_t}{P_{t+j}}.$$

The first-order condition with respect to $\tilde{P}_{H,t}$ is

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{\tilde{P}_{H,t} \Psi_{t,t+j}}{P_{H,t+j}} \right)^{-\epsilon} Y_{H,t+j} \left[\frac{\epsilon - 1}{\epsilon} \tilde{P}_{H,t} \Psi_{t,t+j} - W_{t+j} \right] = 0. \quad (\text{A.14})$$

The term in the square parenthesis is equal to marginal revenue minus marginal costs. Notice that the optimality condition is independent of firm-specific characteristics. This implies that any firm that gets to change prices in period t will charge the same price regardless of its price history.

For the characterization of equilibrium, it is convenient to rewrite (A.14) in a recursive fashion. To this end, let X_t^1 and X_t^2 , respectively, be given by

$$X_t^1 \equiv E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{\tilde{P}_{H,t} \Psi_{t,t+j}}{P_{H,t+j}} \right)^{-\epsilon} Y_{H,t+j} \frac{\epsilon-1}{\epsilon} \tilde{P}_{H,t} \Psi_{t,t+j}$$

and

$$X_t^2 \equiv E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left(\frac{\tilde{P}_{H,t} \Psi_{t,t+j}}{P_{H,t+j}} \right)^{-\epsilon} Y_{H,t+j} W_{t+j}.$$

We can then write (A.14) as

$$X_t^1 = X_t^2. \quad (\text{A.15})$$

Next writing X_t^1 and X_t^2 recursively yields

$$X_t^1 = \frac{\epsilon-1}{\epsilon} \tilde{P}_{H,t} \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} + \theta E_t Q_{t,t+1} \left(\frac{\tilde{P}_{H,t} \Pi_{H,t}}{\tilde{P}_{H,t+1}} \right)^{1-\epsilon} X_{t+1}^1 \quad (\text{A.16})$$

and

$$X_t^2 = W_t \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} + \theta E_t Q_{t,t+1} \left(\frac{\tilde{P}_{H,t} \Pi_{H,t}}{\tilde{P}_{H,t+1}} \right)^{-\epsilon} X_{t+1}^2. \quad (\text{A.17})$$

From the definition of the price of the domestic composite good, equation (A.11), one can obtain the evolution of inflation in domestically produced goods prices

$$\Pi_{H,t}^{1-\epsilon} = \theta \Pi_{H,t-1}^{1-\epsilon} + (1-\theta) \left(\frac{\tilde{P}_{H,t} \Pi_{H,t}}{\tilde{P}_{H,t}} \right)^{1-\epsilon}. \quad (\text{A.18})$$

The foreign demand for domestic goods, denoted $C_{H,t}^*$, is assumed to be given by

$$C_{H,t}^* = \nu \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^*, \quad (\text{A.19})$$

where $P_{H,t}^*$ denotes the foreign currency price of the domestically produced good, P_t^* denotes the foreign price of consumption (the foreign CPI index), and C_t^* denotes the level of foreign aggregate demand. Both P_t^* and C_t^* are exogenous variables from the point of view of the

small open economy.

Equilibrium in the market for the domestically produced composite good requires

$$Y_{H,t} = C_{H,t} + C_{H,t}^*. \quad (\text{A.20})$$

Let

$$N_t \equiv \int_0^1 N_t(i) di \quad (\text{A.21})$$

and integrate equation (A.12) holding with equality to obtain

$$N_t = b_t Y_{H,t}, \quad (\text{A.22})$$

where

$$b_t \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} di.$$

The variable b_t measures price dispersion and can be interpreted as capturing inefficiencies caused by nominal price rigidity. We can express b_t recursively as

$$b_t = \theta b_{t-1} \left(\frac{\Pi_{H,t-1}}{\Pi_{H,t}} \right)^{-\epsilon} + (1 - \theta) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon}. \quad (\text{A.23})$$

In equilibrium aggregate profits of domestic intermediate goods producing firms are given by

$$\Pi_t \equiv \int_0^1 \Pi_t(i) di = \int_0^1 [P_{H,t}(i) Y_{H,t}(i) - W_t N_t(i)] = P_{H,t} Y_{H,t} - W_t N_t,$$

where the last equality follows from (A.10), (A.11), and (A.21).

We assume that domestic currency bonds, D_t , can only be held by domestic agents and that they are in zero net supply ($D_t = 0$). In equilibrium, the sequential budget constraint of households, (A.2), then takes the form

$$P_t C_t + \mathcal{E}_t(1 + i_{t-1}^*) D_{t-1}^* = P_{H,t} Y_{H,t} + \mathcal{E}_t D_t^* - \mathcal{E}_t \psi(D_t^*). \quad (\text{A.24})$$

The law of one price is assumed to hold for both domestic and foreign goods. This implies that

$$P_{H,t} = \mathcal{E}_t P_{H,t}^* \quad (\text{A.25})$$

and

$$P_{F,t} = \mathcal{E}_t P_{F,t}^*. \quad (\text{A.26})$$

Note that $P_{H,t}^*$ is endogenously determined in the model whereas $P_{F,t}^*$ is exogenously given.

The nominal interest rate is set by the monetary authority according to the following Taylor-type interest rate feedback rule

$$(1 + i_t) = \beta^{-1} \Pi_{H,t}^{\alpha_\pi} \left(\frac{Y_{H,t}}{Y_H} \right)^{\alpha_y} e^{z_{m,t} + (1-\alpha_\pi) X_t^m}, \quad (\text{A.27})$$

where α_π denotes the Taylor coefficient on domestic inflation and α_y denotes the Taylor coefficient on deviations of domestic output from its nonstochastic steady-state value, Y_H . The variable $z_{m,t}$ denotes the temporary monetary disturbance and the variable X_t^m denotes the permanent one. The latter shock is the novel element of the present analysis.

An equilibrium then is a set of stochastic processes for $C_t, C_{H,t}, C_{H,t}^*, C_{F,t}, N_t, \lambda_t, b_t, Y_{H,t}, W_t, P_{H,t}, P_{H,t}^*, P_{F,t}, \tilde{P}_{H,t}, X_t^1, X_t^2, D_t^*, P_t, \Pi_{H,t}, \mathcal{E}_t, i_t$, satisfying (A.3)-(A.9), (A.15)-(A.20), (A.22)-(A.27), and

$$\Pi_{H,t} = (P_{H,t}/P_t)(P_{t-1}/P_{H,t-1})(P_t/P_{t-1}), \quad (\text{A.28})$$

given $Q_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}}$, exogenous processes $\{z_t^m, X_t^m, C_t^*, P_{F,t}^*, P_t^*, i_t^*\}$ and initial conditions $D_{-1}^*, i_{-1}^*, P_{H,-1}, \Pi_{H,-1}$, and b_{-1} . The transitory and permanent monetary disturbances are assumed to evolve over time as

$$z_{t+1}^m = \rho_{zm} z_t^m + \epsilon_{t+1}^z$$

and

$$(X_{t+1}^m - X_t^m) = \rho_{Xm}(X_t^m - X_{t-1}^m) + \epsilon_{t+1}^x,$$

where $\rho_{zm}, \rho_{Xm} \in [0, 1)$ and ϵ_t^i for $i = z, x$ are i.i.d. mean zero innovations with unit standard deviation.

2 Complete Set of Equilibrium Conditions

$$C_t^{-\sigma} = \lambda_t \tag{A.3}$$

$$C_t^\sigma N_t^\varphi = W_t/P_t \tag{A.4}$$

$$1 = \beta(1 + i_t)E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \tag{A.5}$$

$$1 = \beta \frac{(1 + i_t^*)}{1 - \psi'(D_t^*)} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \tag{A.6}$$

$$C_t = \left[(1 - v)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + v^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}} \tag{A.7}$$

$$C_{H,t} = (1 - \nu) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \tag{A.8}$$

$$C_{F,t} = \nu \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \tag{A.9}$$

$$X_t^1 = X_t^2 \tag{A.15}$$

$$X_t^1 = \frac{\epsilon - 1}{\epsilon} \tilde{P}_{H,t} \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} + \theta E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left(\frac{\tilde{P}_{H,t} \Pi_{H,t}}{\tilde{P}_{H,t+1}} \right)^{1-\epsilon} X_{t+1}^1 \tag{A.16}$$

$$X_t^2 = W_t \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} Y_{H,t} + \theta E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left(\frac{\tilde{P}_{H,t} \Pi_{H,t}}{\tilde{P}_{H,t+1}} \right)^{-\epsilon} X_{t+1}^2 \tag{A.17}$$

$$\Pi_{H,t}^{1-\epsilon} = \theta \Pi_{H,t-1}^{1-\epsilon} + (1 - \theta) \left(\frac{\tilde{P}_{H,t} \Pi_{H,t}}{P_{H,t}} \right)^{1-\epsilon} \tag{A.18}$$

$$C_{H,t}^* = \nu \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \quad (\text{A.19})$$

$$Y_{H,t} = C_{H,t} + C_{H,t}^* \quad (\text{A.20})$$

$$N_t = b_t Y_{H,t} \quad (\text{A.22})$$

$$b_t = \theta b_{t-1} \left(\frac{\Pi_{H,t-1}}{\Pi_{H,t}} \right)^{-\epsilon} + (1 - \theta) \left(\frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} \quad (\text{A.23})$$

$$P_t C_t + \mathcal{E}_t (1 + i_{t-1}^*) D_{t-1}^* = P_{H,t} Y_{H,t} + \mathcal{E}_t D_t^* - \mathcal{E}_t \psi(D_t^*) \quad (\text{A.24})$$

$$P_{H,t} = \mathcal{E}_t P_{H,t}^* \quad (\text{A.25})$$

$$P_{F,t} = \mathcal{E}_t P_{F,t}^* \quad (\text{A.26})$$

$$(1 + i_t) = \beta^{-1} \Pi_{H,t}^{\alpha_\pi} \left(\frac{Y_{H,t}}{Y_H} \right)^{\alpha_y} e^{z_{m,t} + (1 - \alpha_\pi) X_t^m} \quad (\text{A.27})$$

$$\Pi_{H,t} = (P_{H,t}/P_t)(P_{t-1}/P_{H,t-1})(P_t/P_{t-1}) \quad (\text{A.28})$$

3 Equilibrium Conditions in Stationary Variables

Let

$$e_t = \frac{\mathcal{E}_t P_t^*}{P_t}$$

$$x_t^m \equiv e^{X_t^m},$$

$$w_t = W_t/P_t$$

$$p_{H,t} = P_{H,t}/P_t$$

$$p_{F,t} = P_{F,t}/P_t$$

$$\tilde{p}_{H,t} = \tilde{P}_{H,t}/P_{H,t}$$

$$x_t^1 = X_t^1/P_{H,t}$$

$$x_t^2 = X_t^2/P_t$$

$$\tilde{\pi}_t = P_t/P_{t-1}/x_t^m$$

$$\tilde{\pi}_{H,t} = P_{H,t}/P_{H,t-1}/x_t^m$$

$$\check{i}_t = \frac{1 + i_t}{x_t^m}$$

and use (A.3) to eliminate λ_t . We assume that because the domestic country is small relative to the rest of the world $P_{F,t}^* = P_t^*$. We also assume that P_t^* , C_t^* , and i_t^* are stationary. Then the equilibrium conditions in terms of stationary variables are:

$$C_t^\sigma N_t^\varphi = w_t \quad (\text{A.29})$$

$$1 = \beta \check{i}_t E_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{1}{\tilde{\pi}_{t+1}} \frac{x_t^m}{x_{t+1}^m} \quad (\text{A.30})$$

$$1 = \beta \frac{(1 + i_t^*)}{1 - \psi'(D_t^*)} E_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t^*}{P_{t+1}^*} \frac{e_{t+1}}{e_t} \quad (\text{A.31})$$

$$C_t = \left[(1 - v)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + v^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right]^{\frac{1}{1-\frac{1}{\eta}}} \quad (\text{A.32})$$

$$C_{H,t} = (1 - \nu) p_{H,t}^{-\eta} C_t \quad (\text{A.33})$$

$$C_{F,t} = \nu p_{F,t}^{-\eta} C_t \quad (\text{A.34})$$

$$p_{H,t} x_t^1 = x_t^2 \quad (\text{A.35})$$

$$x_t^1 = \frac{\epsilon - 1}{\epsilon} \tilde{p}_{H,t}^{1-\epsilon} Y_{H,t} + \theta \beta E_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\tilde{\pi}_{H,t+1}}{\tilde{\pi}_{t+1}} \left(\frac{\tilde{p}_{H,t} \tilde{\pi}_{H,t} x_t^m}{\tilde{p}_{H,t+1} \tilde{\pi}_{H,t+1} x_{t+1}^m} \right)^{1-\epsilon} x_{t+1}^1 \quad (\text{A.36})$$

$$x_t^2 = w_t \tilde{p}_{H,t}^{-\epsilon} Y_{H,t} + \theta \beta E_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\frac{\tilde{p}_{H,t} \tilde{\pi}_{H,t} x_t^m}{\tilde{p}_{H,t+1} \tilde{\pi}_{H,t+1} x_{t+1}^m} \right)^{-\epsilon} x_{t+1}^2 \quad (\text{A.37})$$

$$(\tilde{\pi}_{H,t})^{1-\epsilon} = \theta (\tilde{\pi}_{H,t-1} x_{t-1}^m / x_t^m)^{1-\epsilon} + (1 - \theta) (\tilde{p}_{H,t} \tilde{\pi}_{H,t})^{1-\epsilon} \quad (\text{A.38})$$

$$C_{H,t}^* = \nu \left(\frac{p_{H,t}}{e_t} \right)^{-\eta} C_t^* \quad (\text{A.39})$$

$$Y_{H,t} = C_{H,t} + C_{H,t}^* \quad (\text{A.40})$$

$$N_t = b_t Y_{H,t} \quad (\text{A.41})$$

$$b_t = \theta b_{t-1} \left(\frac{\check{\pi}_{H,t-1} x_{t-1}^m}{\check{\pi}_{H,t} x_t^m} \right)^{-\epsilon} + (1 - \theta) \check{p}_{H,t}^{-\epsilon} \quad (\text{A.42})$$

$$C_t + e_t(1 + i_{t-1}^*) D_{t-1}^* / P_t^* = p_{H,t} Y_{H,t} + e_t D_t^* / P_t^* - e_t \psi(D_t^*) / P_t^* \quad (\text{A.43})$$

$$p_{F,t} = e_t \quad (\text{A.44})$$

$$\check{\mathbf{y}}_t = \beta^{-1} \check{\pi}_{H,t}^{\alpha_\pi} \left(\frac{Y_{H,t}}{Y_H} \right)^{\alpha_y} e^{z_{m,t}} \quad (\text{A.45})$$

$$\check{\pi}_{H,t} = \frac{p_{H,t}}{p_{H,t-1}} \check{\pi}_t \quad (\text{A.46})$$