

Economics G6222
Advanced Macroeconomic Analysis
Problem Set 1
Due February 8 in class

Computing the Equilibrium of a Small Open Economy

Consider an economy populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right],$$

where c_t denotes consumption in period t , and β and σ are parameters. The symbol E_t denotes the expectations operator conditional on information available in period t . Households are assumed to face the following sequential budget constraint:

$$c_t + d_t = y_t + \frac{d_{t+1}}{1+r}$$

where d_t is the level of debt assumed in period $t-1$ and due in period t , y_t is an exogenous and stochastic endowment, and r is a constant interest rate on external debt. Households are assumed to be subject to the following no-Ponzi-game constraint:

$$d_{t+1} \leq \bar{d},$$

where \bar{d} is a constant.

The only source of uncertainty in this economy is the endowment process, y_t . Assume that y_t follows a two state Markov process. Specifically, let the endowment take on values in the set $\{y_1, y_2\}$, with transition probability matrix π , such that $\pi_{ij} \equiv \text{prob}\{y_{t+1} = y_j | y_t = y_i\}$, for $i, j = 1, 2$.

Calibrate the model as follows: $\beta = 0.75$, $\sigma = 5$, $r = 0.2$, $\bar{d} = 5.6$, $y_1 = 1 - 0.066$, $y_2 = 1 + 0.066$, and $\pi_{ij} = 1/2$ for $i, j = 1, 2$ (i.e., the endowment is iid).

Approximate the policy function $d_{t+1} = D(y_t, d_t)$ by the method of value-function iteration. To this end, discretize the debt dimension of the state space by the vector $d = [d_1, d_2, \dots, d_{401}]$. Set $d_1 = 3$, $d_{401} = 5.6$, and $d_{i+1} - d_i = 0.0065$, for $i = 1, \dots, 400$.

Provide the answer to this problem set in the form of a 2×401 matrix of indices, denoted I_D , such that $I_D(i, j)$ denotes the position in the vector d of $D(y_i, d_j)$ for $i = 1, 2$ and $j = 1, \dots, 401$. So the entries of I_D belong to the set $\{1, 2, \dots, 401\}$.